Chaotic Dynamics - CSCI 5446

Problem Set 1

Santhanakrishnan Ramani

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Problem 3

The following figures are some interesting plots obtained by plotting x_n vs n, x_{n+1} vs x_n, x_{n+2} vs x_n, for the logistic function,

$$x_{n+1} = Rx_n(1 - xn)$$

The Figure 1 show us that for the value of R = 2.8 and initial condition $x_0 = 0.2$, the system converges to a stable fixed point after some initial transient, in language of dynamical systems this is commonly referred as Fixed Point Attractor.

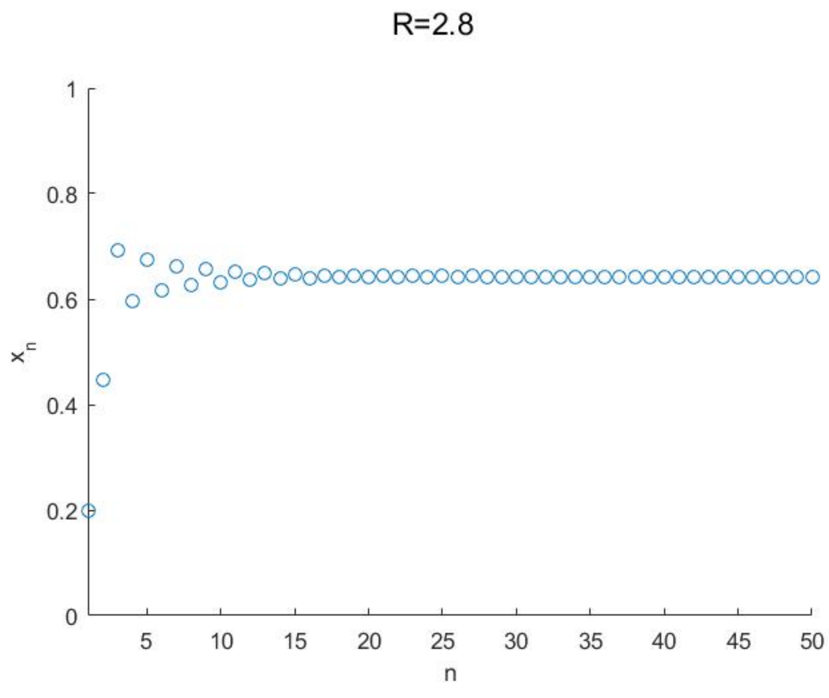

Figure 1: Fixed Point

The Figure 2 show us that for the value of R = 3.2 and initial condition $x_0 = 0.2$, the system oscillates in two period cycle after some initial transient, the period-2 cycle is clearly evident from Figure 3, in language of dynamical systems this is commonly referred as Limit Cycle Attractor.

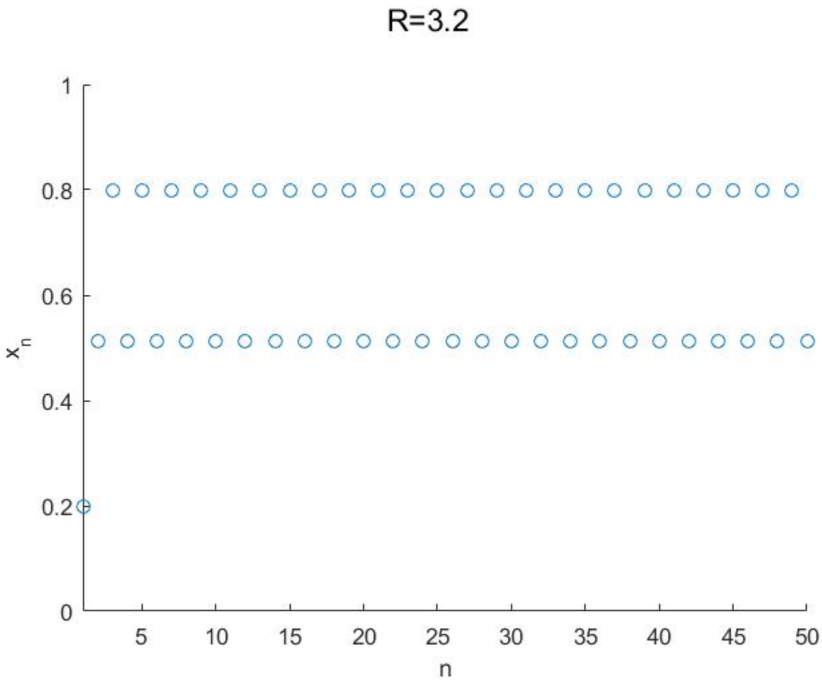

Figure 2: 2-Period Cycle

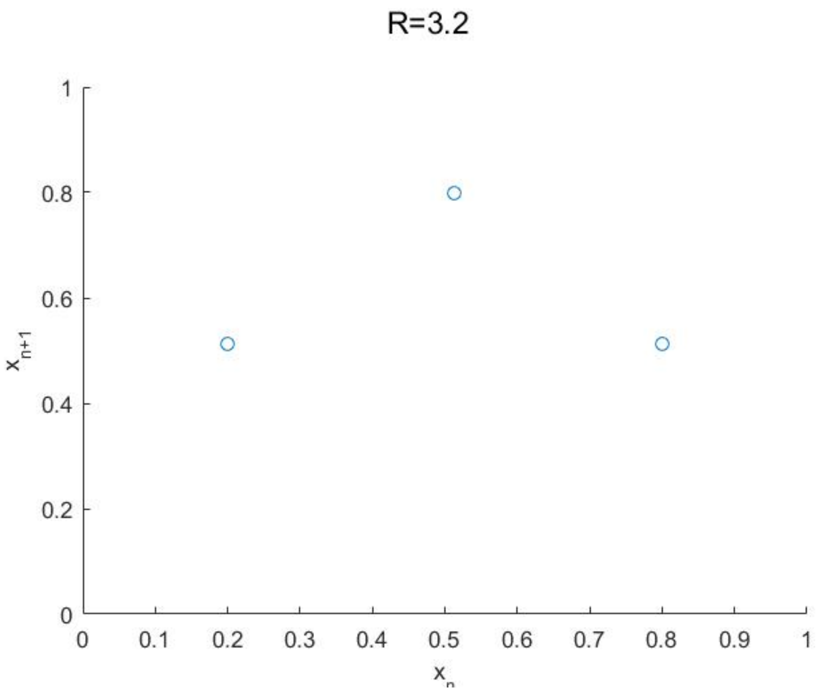

Figure 3: 2-Period Cycle

The Figure 4 and 5 show us that for the value of R = 3.97 the system exhibits Chaotic Behavior, and the behavior is highly dependent on the initial condition which is evident from the below two figures.

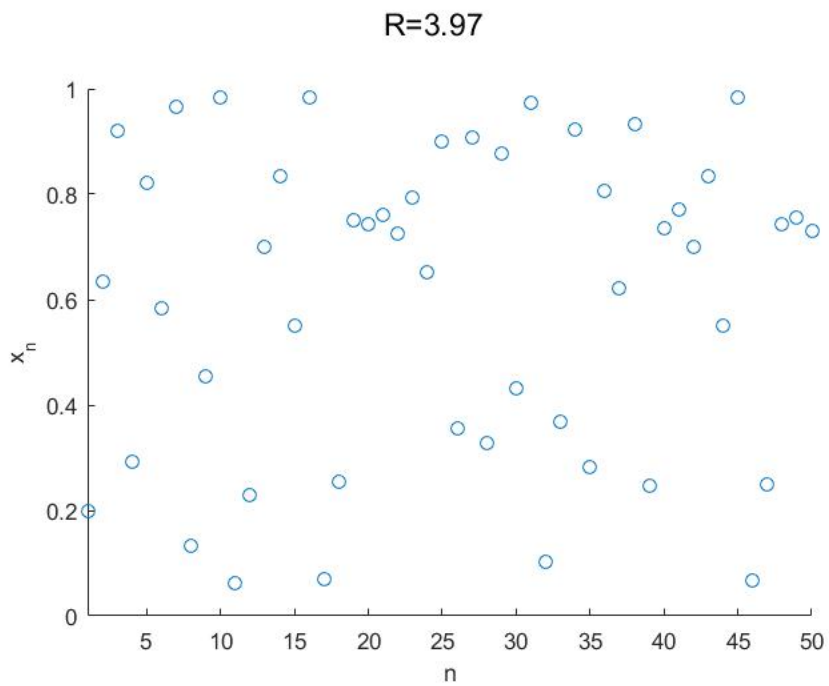

Figure 4: Chaotic Behavior $x_0 = 0.2$

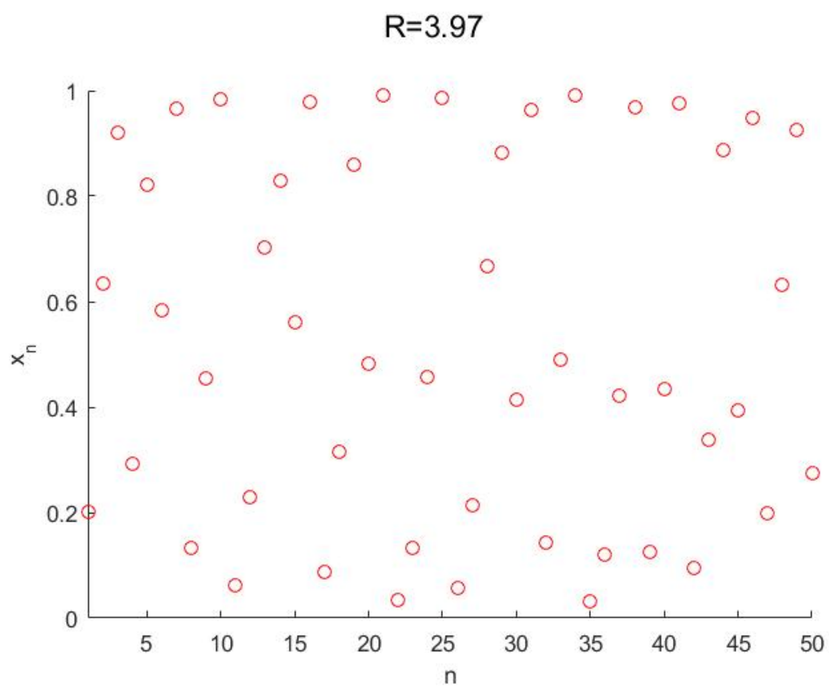

Figure 5: Chaotic Behavior $x_0 = 0.200001$

The Figure 6 show us the system behavior for the value of R = 4 and $x_0 = 0.2$, it is clearly evident from the x_n vs n graph the behavior is chaotic. The x_{n+1} vs x_n and x_{n+2} vs x_n plots shows the stretching and folding structure of the logistic map.

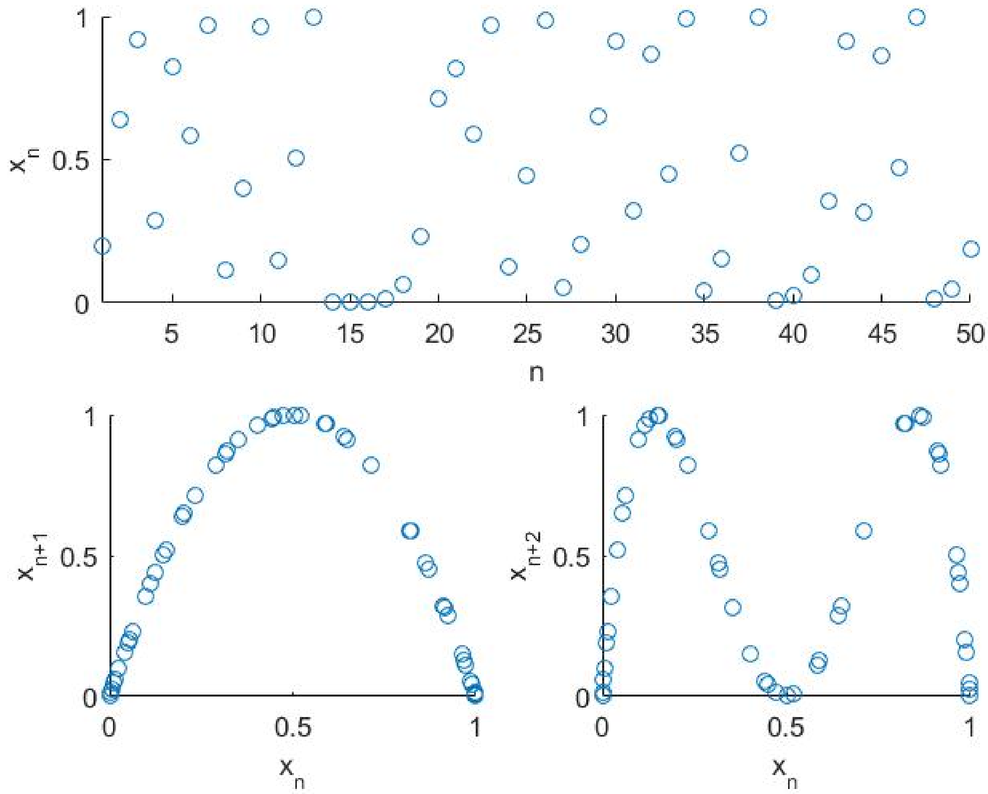

Figure 6: R = 4

When R > 4?

In this case, the function value goes out of the interval [0,1] and the system starts to diverge and reaches infinity for almost all initial values.

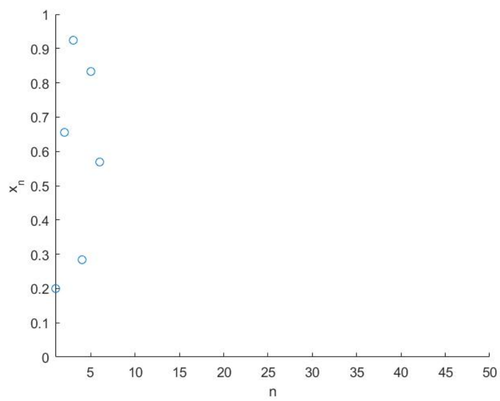

Figure 7: R > 4

When R = 2.5?

In this case the system behavior/trajectory is identical after some initial transient for any initial condition. This is evident from Figure 8, all systems with the different initial condition converges to the same stable fixed point. In Dynamical Systems terminology this is called the **basin of attraction**.

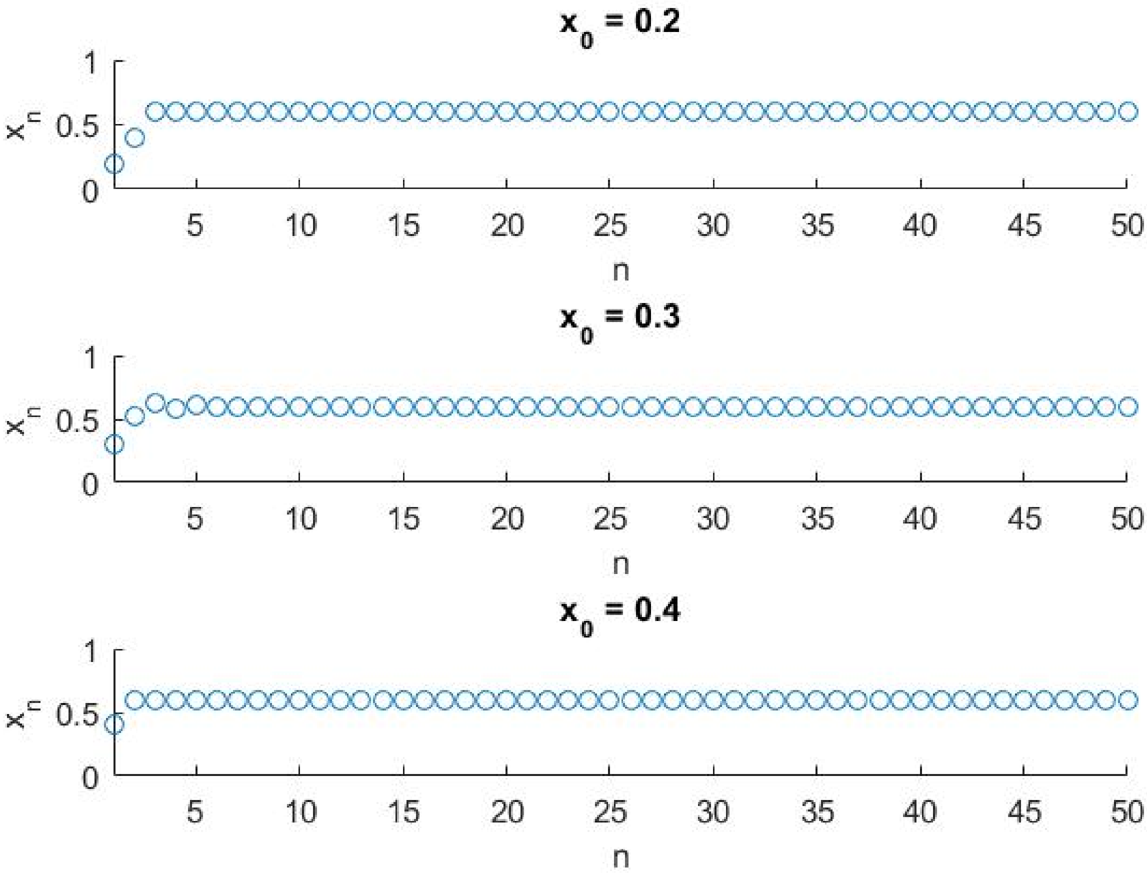

Figure 8: R = 2.5