University of Colorado
Department of Computer Science

Chaotic Dynamics – CSCI 4446/5446
Spring 2016

Problem Set 3

Issued:	26 January 2016
Due:	2 February 2016

Reading: *Strogatz, chapter 11; Kantz & Schreiber, section 6.1; Section 1 of the ODE Notes*; Baker&Gollub, pp111-120.

Bibliography:

- M. Barnsley, *Fractals Everywhere*, Academic Press, 1988. This is on library reserve for this course.
- J. Crutchfield *et al.*, "Chaos," *Scientific American*, 12/86
- P. Cvitanovic, *Universality in Chaos*, Adam Hilger, 1984. This is a great reprint collection; it's also on library reserve for this course.
- C. Grebogi, E. Ott, and J. A. Yorke, "Chaos, Strange Attractors and Fractal Basin Boundaries in Nonlinear Dynamics," *Science* **238**:632-638 (1987).
- R. Jensen, "Classical Chaos," *American Scientist*, 3/87. This is the paper that got me interested in this field.
- B. Mandelbrot, *The Fractal Geometry of Nature*, Freeman, 1983. Heavy going, but seminal.
- H.-O. Peitgen and P. H. Richter, *The Beauty of Fractals*, Springer-Verlag, 1986. Truly beautiful images. I have a copy in my office if you want to take a look.

Problems:

0. Go to the `googlelabs.com` Julia site that's listed on the course webpage and zoom into the "Newton's method on $x^3 - 1 = 0$". (This site doesn't work with all browsers, by the way, or all internet connections. If that's the case for you, please email me.) Find some really interesting area and turn in a plot of it.

1. Calculate (paper and pencil, not code) the capacity dimension for a middle-sixth-removed Cantor set.

2. (a) Transform the following third-order ODE into three first-order ODEs:

$$2\frac{d^3}{dt^3}x(t) - 3\tan(\frac{1}{2}\frac{d^2}{dt^2}x(t)) + 16\log(\frac{d}{dt}x(t)) - x(t) = 0$$

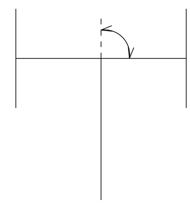
(b) Transform the following set of first-order ODEs into a single higher-order ODE:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= yz + \log y \end{aligned}$$

(c) Are the systems in parts (a) and (b) of this problem linear or nonlinear? Why?

3. This problem concerns a self-similar "fractal tree." Everyone should do all four parts of this problem; CSCI 5446 students have an extra task in two of these parts.

(a) Write a program that begins with a vertical line segment (the trunk), then iterates the following operation: draw two line segments at right angles from the top of the previous segment, each 0.6 times the length of that segment. This figure shows the trunk plus two iterations:



Iterate until you can no longer discern differences between successive segments. Turn in a plot of this object.

> **Optional for CSCI 4446 people:** calculate the capacity dimension of the fractal composed of the leaves of this tree (not the trunk and branches, just the *leaves*) in the limit of large iteration numbers.

(b) What happens to the tree if each segment is exactly *half* as long as the previous one, instead of 0.6 as long? What if the ratio is larger than $\sqrt{2}$? What about if it is smaller than 0.5? (There's no need to turn in plots; just describe what you see and hypothesize about the implications.)

> **Optional for CSCI 4446 people:** compute the capacity dimensions for these cases.

(c) Now adapt your program to accept different segment length ratios for the left and right branches, as well as different angles between the left and right branches and the trunk (see the figure above for a definition of how the right-branch angle is measured). Try 0.7 and 0.65 for the lengths and 60 and 40 degrees for the angles. Turn in a plot of this object. Hint: if you want to rotate a vector through an angle, you premultiply it by the matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(d) Experiment with some other interesting angles and ratios and turn in a couple of representative plots. If you wish, you may also modify your algorithm to *change* those values (randomly or following some pattern) as the depth increases.