Chaotic Dynamics - CSCI 5446

Problem Set 7

Santhanakrishnan Ramani

April 29, 2016

Problem 1

The Jacobian $D_{\vec{x}}\vec{F}$ for the Lorenz system is

$$D_{\vec{x}}\vec{F} = \begin{vmatrix} -a & a & 0 \\ r-z & -1 & -x \\ y & x & -b \end{vmatrix}$$

Problem 2

The associated variational system $\dot{\delta} = D_{\vec{x}}\vec{F}\delta$

$$\dot{\delta} = \begin{vmatrix} -a & a & 0 \\ r-z & -1 & -x \\ y & x & -b \end{vmatrix} \begin{vmatrix} \delta_{xx} & \delta_{yx} & \delta_{zx} \\ \delta_{xy} & \delta_{yy} & \delta_{zy} \\ \delta_{xz} & \delta_{yz} & \delta_{zz} \end{vmatrix} = \begin{vmatrix} -a.\delta_{xx} + a.\delta_{xy} & -a.\delta_{yx} + a.\delta_{yy} & -a.\delta_{zx} + a.\delta_{zy} \\ (r-z).\delta_{xx} - \delta_{xy} - x.\delta_{xz} & (r-z).\delta_{yx} - \delta_{yy} - x.\delta_{yz} & (r-z).\delta_{zx} - \delta_{zy} - x.\delta_{zz} \\ y.\delta_{xx} + x.\delta_{xy} - b.\delta_{xz} & y.\delta_{yx} + x.\delta_{yy} - b.\delta_{yz} & y.\delta_{zx} + x.\delta_{zy} - b.\delta_{zz} \end{vmatrix}$$

Problem 3

Integrating the Lorenz variational equation using RK4 with a = 16, r = 45, b = 4, and a timestep of .001 for 100 steps using the given initial conditions we get,

(a) For, $[x\, y\, z\, \delta_{xx}\, \delta_{xy}\, \delta_{xz}\, \delta_{yx}\, \delta_{yy}\, \delta_{yz}\, \delta_{zx}\, \delta_{zy}\, \delta_{zz}] = [0\ 1\ 2\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$, component of evolved matrix δ,

$$\delta = \begin{vmatrix} 2.4039 & 1.9189 & -0.0297 \\ 5.1751 & 4.2072 & -0.0893 \\ 0.4808 & 0.3741 & 0.6651 \end{vmatrix}$$

and the column sums of δ are 8.0598, 6.5002, 0.5461 respectively.

(b) For, $[x\, y\, z\, \delta_{xx}\, \delta_{xy}\, \delta_{xz}\, \delta_{yx}\, \delta_{yy}\, \delta_{yz}\, \delta_{zx}\, \delta_{zy}\, \delta_{zz}] = [10\ -5\ 2\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$, component of evolved matrix δ,

$$\delta = \begin{vmatrix} 2.1310 & 1.6392 & -0.4858 \\ 3.8600 & 3.0083 & -1.1106 \\ 3.1219 & 2.5325 & -0.0181 \end{vmatrix}$$

and the column sums of δ are 9.1130, 7.1800, -1.6145 respectively.

(c) For, $[x\, y\, z\, \delta_{xx}\, \delta_{xy}\, \delta_{xz}\, \delta_{yx}\, \delta_{yy}\, \delta_{yz}\, \delta_{zx}\, \delta_{zy}\, \delta_{zz}] = [0\ -1\ 2\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$, component of evolved matrix δ,

$$\delta = \begin{vmatrix} 2.4039 & 1.9189 & 0.0297 \\ 5.1751 & 4.2072 & 0.0893 \\ -0.4808 & -0.3741 & 0.6651 \end{vmatrix}$$

and the column sums of δ are 7.0983, 5.7519, 0.7841 respectively.

(d) Looking at the evolved matrix δ and their respective columns sums for the above three initial conditions, I found the variations grew faster in all the directions x, y, z for the initial condition given in (b). The z variation shrinked for initial conditions given in (b), whereas grew and remained less than one for the other two initial conditions. Keeping the x & z of initial condition constant and mirroring the y value, had a similar effect on x & y variation, but opposite effect on z variation.