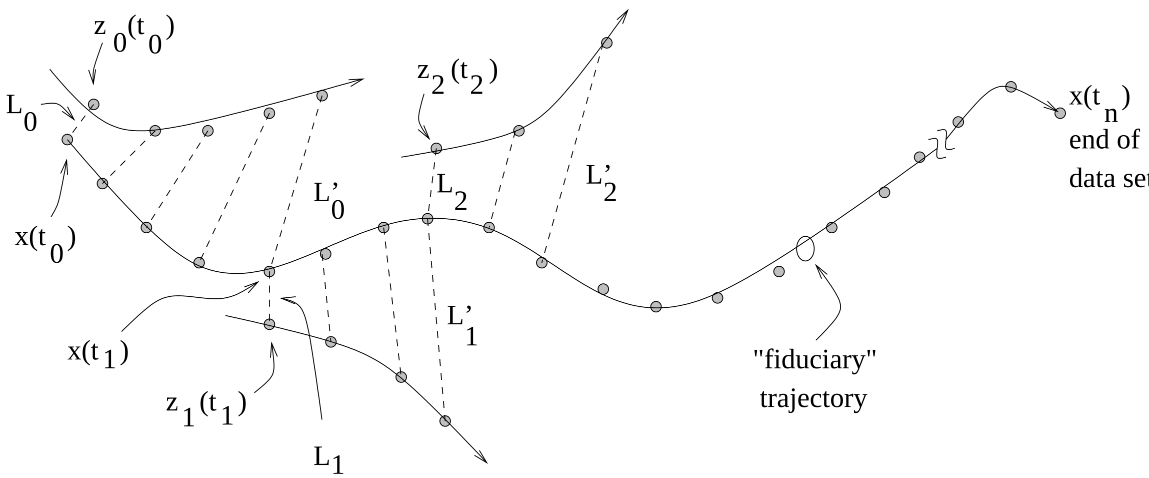
University of Colorado
Department of Computer Science

Chaotic Dynamics – CSCI 4446/5446

Wolf's algorithm for computing Lyapunov exponents from data:



Algorithm:

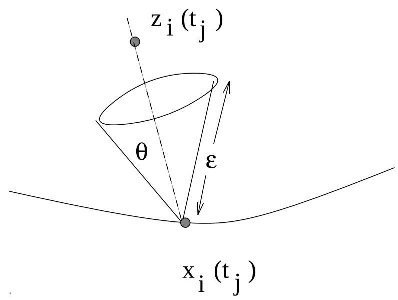
1. Embed the data set.
2. Pick a point $x(t_0)$ somewhere in the middle of the trajectory.
3. Find that point's nearest neighbor. Call that point $z_0(t_0)$.
4. Compute $||z_0(t_0) - x(t_0)|| = L_0$.
5. Follow the "difference trajectory" — the dashed line — forwards in time, computing $||z_0(t_i) - x(t_i)|| = L_0(i)$ and incrementing i, until $L_0(i) > \epsilon$. Call that value L'_0 and that time t_1.
6. Find $z_1(t_1)$, the "nearest neighbor" of $x(t_1)$, and loop to step 4. Repeat the procedure to the end of the fiduciary trajectory ($t = t_n$), keeping track of the L_i and L'_i.

Use this formula to compute λ_1, the biggest (positive) Lyapunov exponent:

$$\lambda_1 \approx \frac{1}{N\Delta t} \sum_{1}^{M-1} log_2 \frac{L'_i}{L_i}$$

...where M is the number of times you went through the loop above, and N is the number of timesteps in the fiduciary trajectory. $N\Delta t = t_n - t_0$.

Schematic of the directional "nearest neighbor" computation:



Start with $\theta = \pi/9$ and increase if necessary.

References:

- Please see the "Lyapunov exponents" section of the "Reading assignments for PS8-10 (Nonlinear Time Series Analysis)" handout.

  Here are some other references, if you'd like more detail:

- P. Bryant *et al.*, "Lyapunov exponents from observed time series," *Physical Review Letters*, **65**:1523-6 (1990).

- J.-P. Eckmann *et al.*, "Liapunov exponents from time series," *Phys. Rev. A* **34**:4971 (1986).

- M. Sano, "Measurement of the Lyapunov Exponents from a Chaotic Series," *Physical Review Letters* **55**:1082-1085 (1983).

- A. Wolf, "Quantifying chaos with Lyapunov exponents," in *Chaos*, Princeton University Press, 1986.