Chaotic Dynamics - CSCI 5446

Problem Set 4

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Problem 2

(a) The Figure 1 represents the state-space trajectory emanating from the point $[\theta, \omega] = [3, 0.1]$ with $m = 0.1kg\,, l = 0.1m\,, \beta = 0\,, \alpha = 0\,, A = 0\,\&\, h = 0.005$.

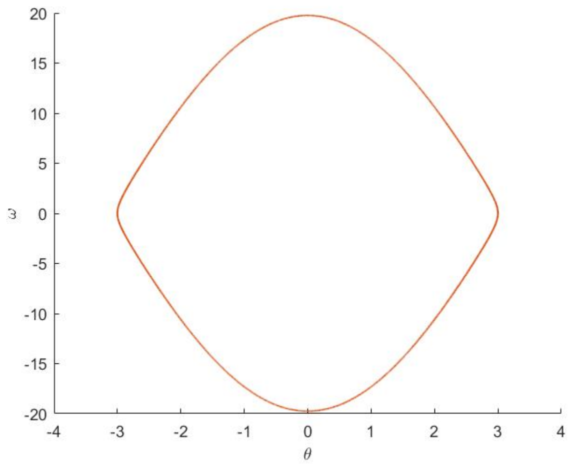

Figure 1: state-space trajectory

Yes, the initial condition is near the Equilibrium point $[\pi, 0]$, and it is unstable.

(b) The Figure 2 represents the state-space trajectory emanating from the point $[\theta, \omega] = [0.01, 0]$ with $m = 0.1kg\,, l = 0.1m\,, \beta = 0\,, \alpha = 0\,, A = 0\,\&\, h = 0.005$.

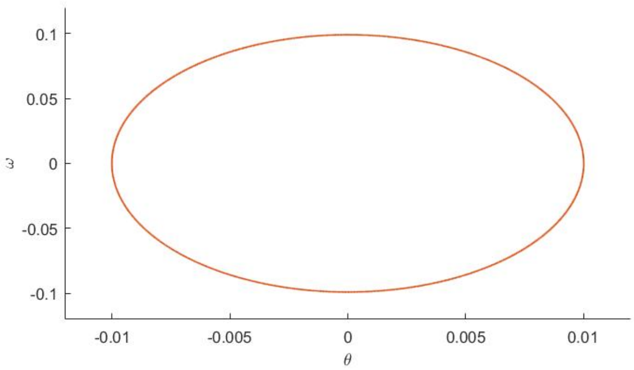

Figure 2: state-space trajectory

Yes, the trajectory look more like a perfect ellipse than the trajectory of part (a), the reason being the angular velocity is zero, and for smaller angular displacement $\sin(\theta) \approx \theta$, so the rate of change of angular velocity will also be uniform.

Problem 3

The Figure 3 represents the state-space portrait of the system using the coefficient values in Problem 2.

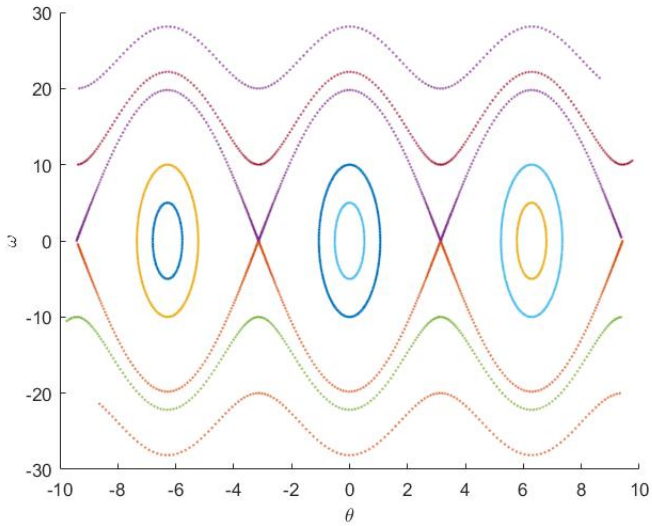

Figure 3: state-space portrait

Problem 4

The Figure 4 represents the state-space portrait of the system in Problem 3 , excpect the value of $\beta = 0.25$. I was able to see the effect of damping clearly affecting the dynamics of the pendulum as it comes to stop by converging to a nearby fixed point after certain rotations which wasn't the case in the previous problem.

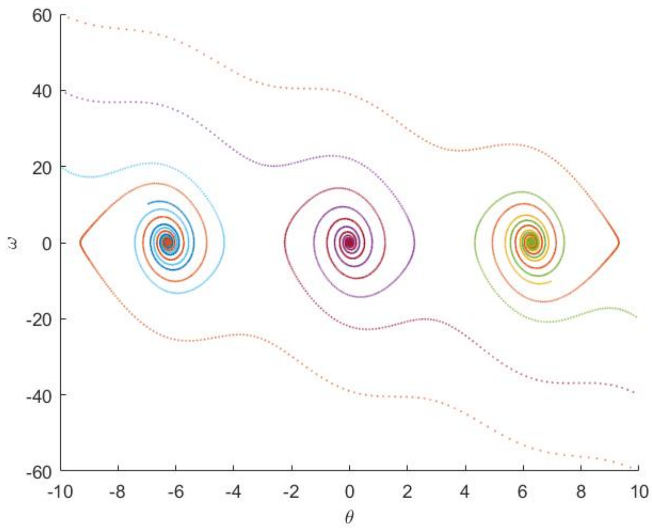

Figure 4: state-space portrait

For higher values of β the convergence was instant, and for lower values of β the converge was seen after few rotations.

Problem 5

The Figure 5 represents the state-space portrait of the system in Problem 4, but plots $\theta \; mod \; 2\pi$. I was able to see the values starting near the unstable fixed point $[(2n+1)\pi, 0]$ converges to a nearby fixed point $[(2n)\pi, 0]$

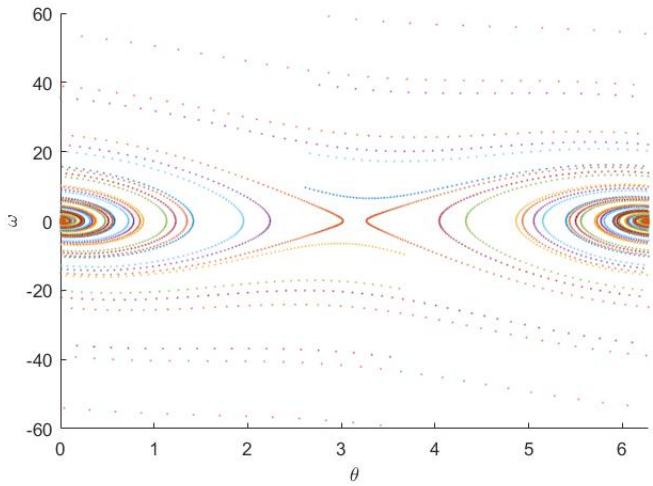

Figure 5: state-space portrait

Problem 6

The Figure 6 represents a chaotic orbit when $A = 1.1$, $\alpha = 7.4246$, $\beta = 0.25$

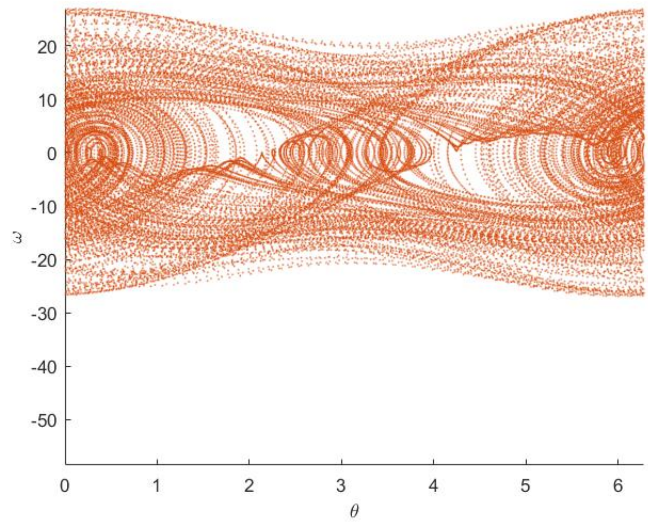

Figure 6: state-space portrait

As you vary the value of A by increasing it every time by some value you see a pattern of Periodic Orbit and Chaotic Orbit alternating, and each time a Periodic orbit reappears a bifurcation has taken place and there is no symmetry between the periods.

Problem 7

On increasing the value of timeStep by using the values in Problem 2, I found out that it starts to spiral inwards and converge to a fixed stable point initially, and once the value of timeStep is greater than 0.3 it starts to diverge.