Chaotic Dynamics - CSCI 5446

Problem Set 3

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Problem 0

The Figure 1 represents the an interesting part of "Newton's method on $x^3 - 1 = 0$" plot from the Julia site.

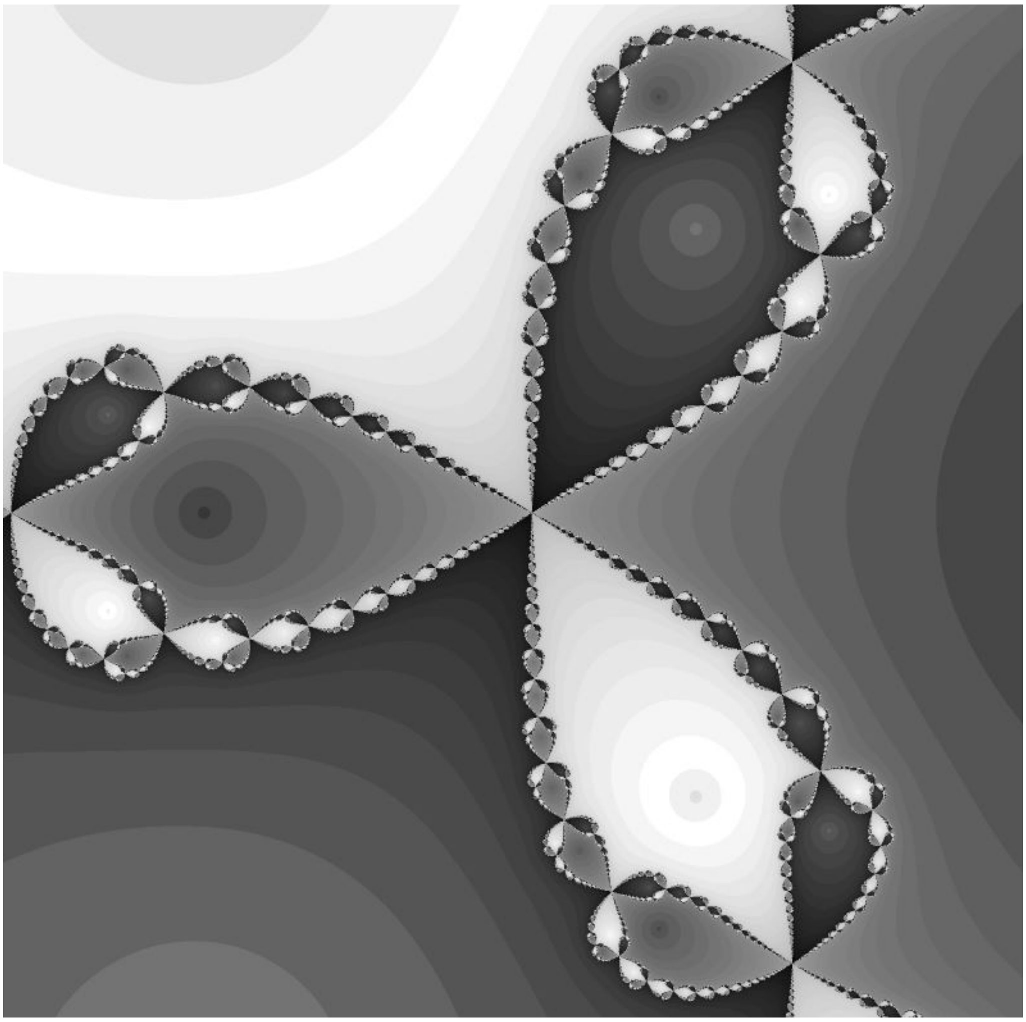

Figure 1: Julia plot

Problem 1

Capacity Dimension of the middle sixth removed Cantor Set.

K	$N(\epsilon)$	ϵ
0	1	1
1	5	1/6
2	25	1/36
.	.	.
K	5^K	$(1/6)^K$

$$d_c = \lim_{\epsilon \to 0} \frac{\log(N(\epsilon))}{\log(\frac{1}{\epsilon})}$$

$$d_c = \lim_{k \to \infty} \frac{\log(5^k)}{\log(6^k)}$$

$$d_c = \frac{\log(5)}{\log(6)} = 0.898$$

Problem 2

(a) Transform the following third order ODE into three first-order ODEs

$$2x''' - 3\tan(\frac{x''}{2}) + 16\log(x') - x = 0$$

Let the helper variables be,

$$x' = y$$
$$y' = z$$

First Order ODE's

$$x' = y$$
$$y' = z$$
$$z' = \frac{1}{2}[3\tan(\frac{z}{2}) - 16\log(y) + x]$$

(b) Transform the following set of first-order ODEs into a single higher-order ODE,

$$x' = y$$
$$y' = z$$
$$z' = yz + \log(y)$$

Higher Order ODE is

$$x''' - x'x'' - \log(x') = 0$$

(c) Both the Systems in (a) and (b) are nonlinear.

Reason

When the System was expressed in terms of first order ODEs,
(a) has *log* term in state variable z',
(b) has product term yz in state variable z'
which makes the whole system non linear.

Problem 3

(a) The Figure 2 represents the self-similar "Fractal Tree" that was obtained by running the code for 13 iterations.

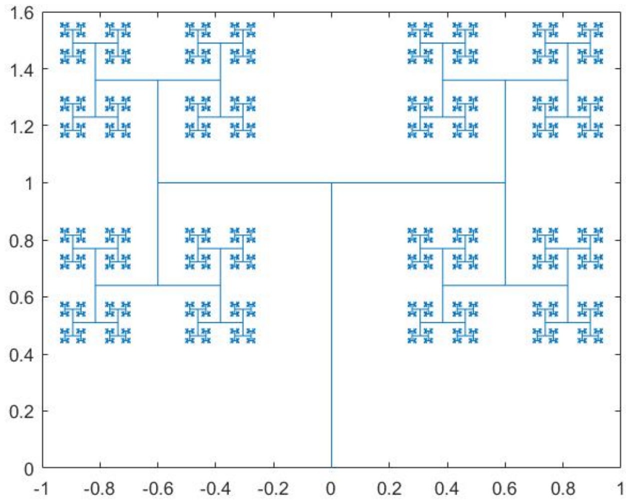

Figure 2: Fractal Tree

Capacity dimension of the fractal composed of the leaves of this tree.

K	$N(\epsilon)$	ϵ
0	1	1
1	2	3/5
2	4	9/25
.	.	.
K	2^K	$(3/5)^K$

$$d_c = \lim_{\epsilon \to 0} \frac{\log(N(\epsilon))}{\log(\frac{1}{\epsilon})}$$

$$d_c = \lim_{k\to\infty} \frac{\log(2^k)}{\log((5/3)^k)} \; d_c = \frac{\log(2)}{\log(5/3)} = 1.357$$

(b) When the length of line segment is half as the previous line segment, then the tree looks to be thin having lesser number of branches. (Ratio = 0.5)

$$d_c = \frac{\log(2^k)}{\log(2^k)} = 1$$

When the ratio is greater than $\sqrt{2}$, the tree looks like a grid increasing in size and gap for each iteration and volume of the tree is huge. (Ratio = 1.5)

Dimension can't be found as $\epsilon \to \infty$ rather than $\epsilon \to 0$

When the ratio is less than 0.5, the tree is very sparse with lesser number of branches and leaves and very small volume. (Ratio = 0.40)

$$d_c = \frac{\log(2^k)}{\log((5/2)^k)} = 0.756$$

(c) The Figure 3 represents the self-similar ”Fractal Tree” that was obtained by running the code for 10 iterations with 60 and 40 degrees for the angles and 0.7 and 0.65 for the lengths of respective sides.

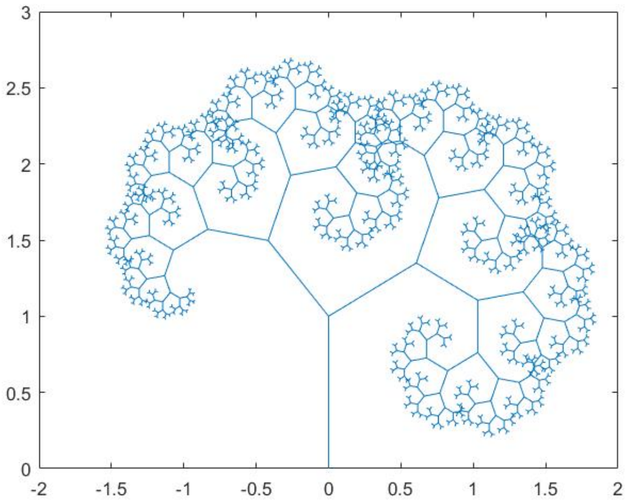

Figure 3: Fractal Tree

(d) Some interesting plots for different angles and length.