Chaotic Dynamics - CSCI 5446

Problem Set 8

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March 10, 2016

Problem 1

The figure 1 below represents the state space plot where the value of ω was constructed using divided differences first-order forward on data1. Since, the drive is off and it is a damped pendulum the plot should be a clean spiral. But the one in the figure below, is almost spiral but not clean, because of the numerical error and sampling rate used for reconstruction of ω using θ.

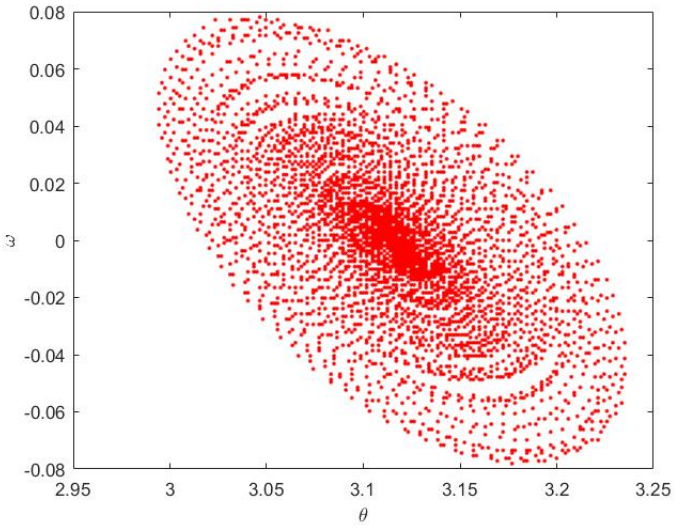


Figure 1: state space plot

Problem 2

(a) The figure 2 represents the reconstructed state plot on data2 using $\tau = 0.15$ and m=7. The plot below looks likes a chaotic attractor.

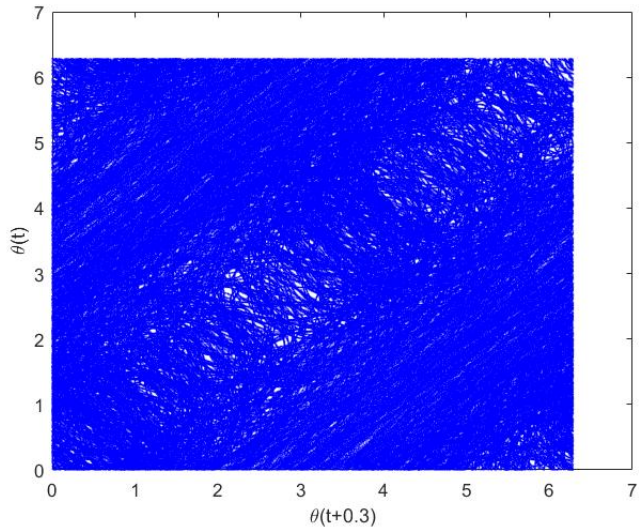
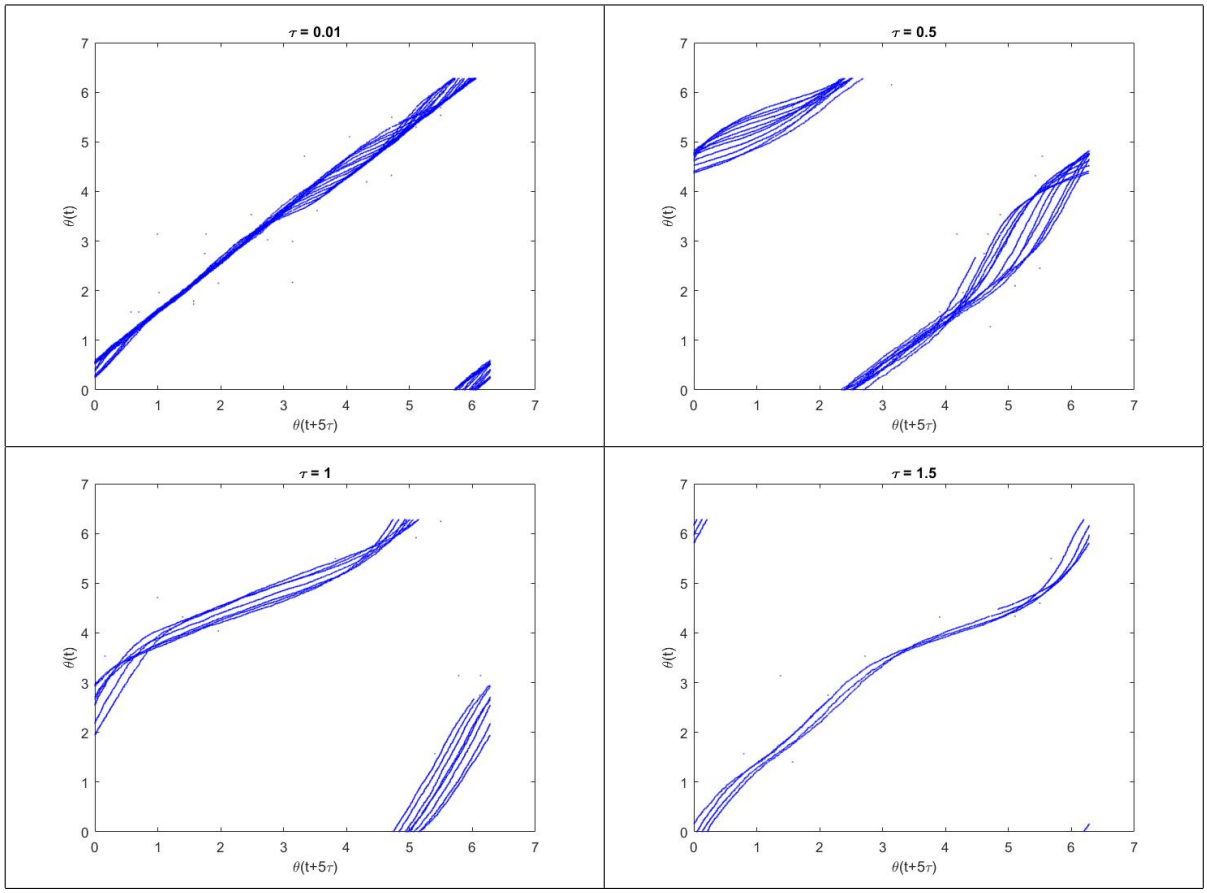


Figure 2: reconstructed state space plot

(b) The below figure represents the reconstructed state plot on data3 using m=7 and raising τ from 0.01 to 1.5 secs. The plots below looks like a periodic attractor switching between two regions. We could see from the below plots becoming sparser as the τ value is increased, and two periods region size grows and shrinks.



Problem 3

(a) According to Taken's theorem $m > 2d$. To characterize a driven pendulum the dimension required is 3, whereas for a undriven pendulum the dimension required is 2, so by taken's theorem $m > 6$ for driven and $m > 4$ for undriven pendulum, in order to achieve successful embedding.

(b) When m=2 the condition necessary to reconstruct a chaotic dynamical system from a sequence of observation of the state of a dynamical system is not sufficient according to Taken's theorem, as we will need atleast m = 7. Having m=25 is of no use as the dynamics of the reconstructed vector becomes deterministic way before, and the value of m preferred is atmost 2d+1 or sometimes even less.

(c) When $\tau = 10^{-16}$ the teeth of the comb just to reconstruct will be very close and some of the dynamics behavior will be missed as $\theta(t) = \theta(t + i\tau)$ will be more or less the same, and we will be seeing kind of a straight line. Whereas when $\tau = 10^6$ the teeth distance would be very high and we will just seeing some random points in the graph that too only if the sample size is greater than 10^6.

Problem 4

Using the TISEAN package to estimate τ parameter, I first ran the mutual tool with its default values. The default "max time delay" parameter was 20, and wasn't able to clearly find the first minimum in this curve.

Then I played a little with "max time delay" parameter , and increased it's value till 750, to be sure that I have got the first minimum of the curve. I found the min at the sample interval 301 which equals 0.602 secs as the sample are taken every 0.002 secs. Below is the plot of τ vs mutual information. The mutual call I made to produce the data,

mutual -D 1000 data2.first250sec -o out.dat

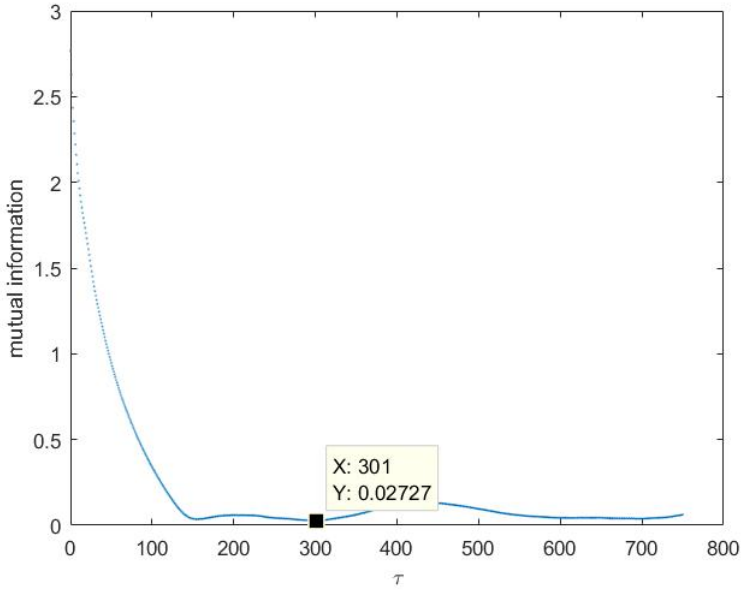


Figure 3: τ vs mutual information

Problem 5

In order to find a good value for embedding dimension m, I used TISEAN package false_nearest tool. I found that good value of m = 8, as it is first value of m whose ratio of false_neighbours went below 10%. Below is the plot of m vs ratio of false_neighbours. The call I made to produce the data.

false_nearest -M 1,10 -d 75 data2.first250sec -o out.dat

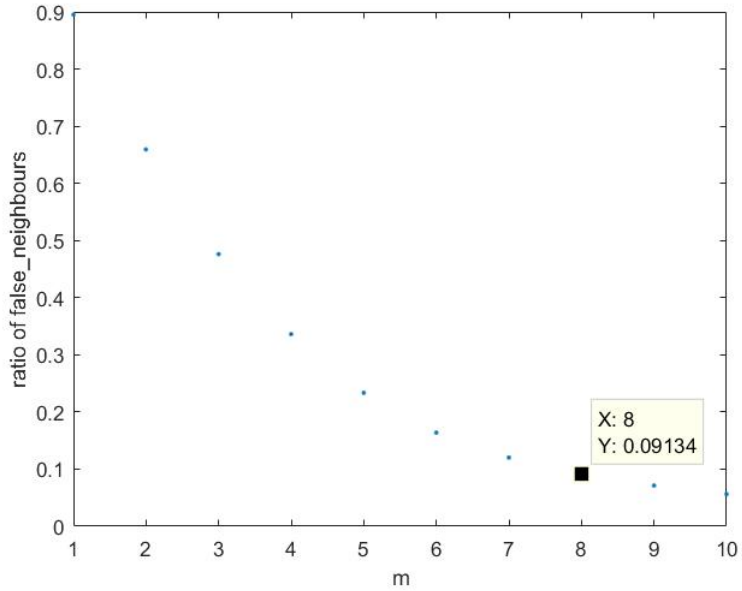


Figure 4: m vs ratio of false_neighours