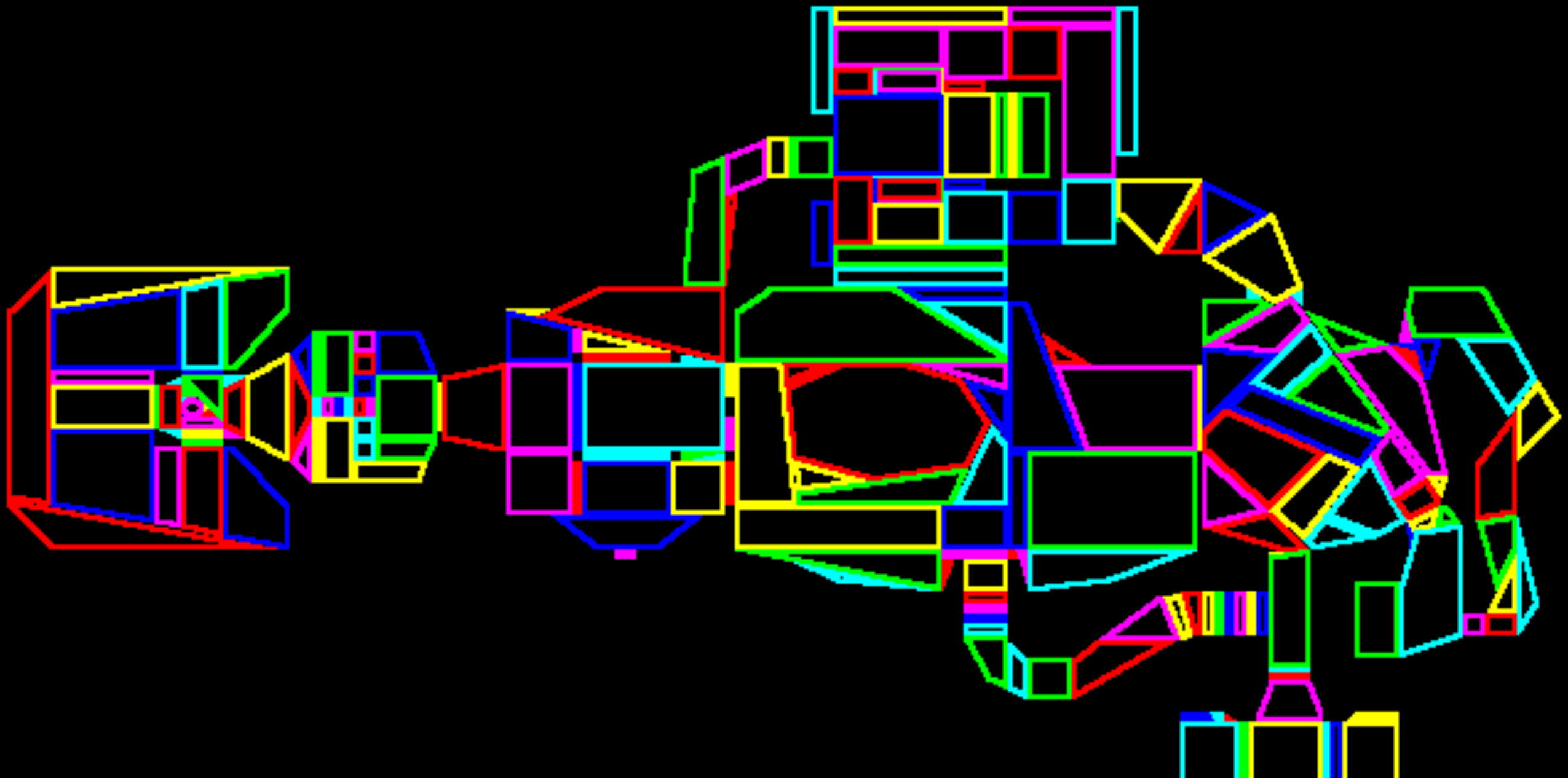

CSC418/2504 Computer Graphics
Some Slides/Images adapted from Marschner and Shirley

Today: Boundary Volume Hiearachies



Announcements

Assignment 1 grades Monday at the latest

Assignment 3 is due on Friday

Assignment 4 is out now

A4 requires OpenGL – *we officially support this on CDF only.*

TA Office Hours: Wednesday 1:30pm – 2:30pm, BA5256

TA Email Address: *csc418tas@cs.toronto.edu*

Any Questions ?

Today: Boundary Volume Hierachy

Common Geometric Queries on Graphics

Bounding Volumes

- Spheres
- Boxes

Object-Partitioning Hierarchies

- Sphere Trees
- AABB Trees

Space-Partitioning Hierarchies

- Uniform Spatial Subdivision
- Axis-Aligned Spatial Subdivision

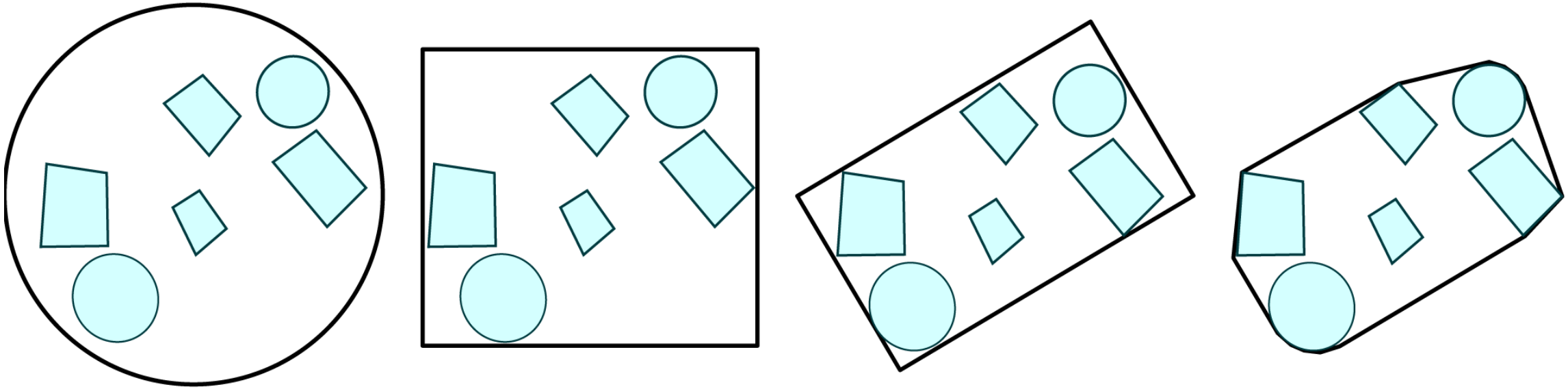
BD-Tree | James and Pai

Bounding Volumes (BVs)

"Simple" geometry that fully encloses a **collection** of other geometry

Sphere

Object Oriented Bounding Box
(OOBB)



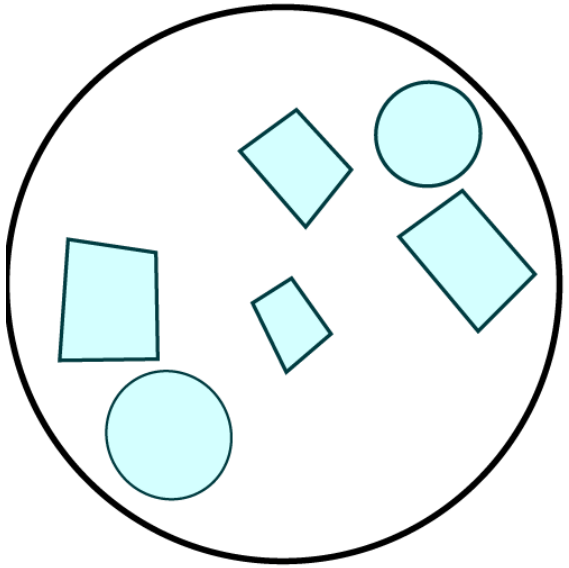
Axis-Aligned Bounding Box
(AABB)

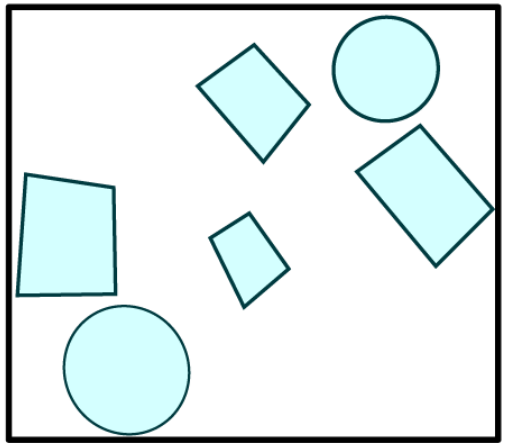
Convex Hull

Bounding Volumes (BVs)

"Simple" geometry that fully encloses a **collection** of other geometry

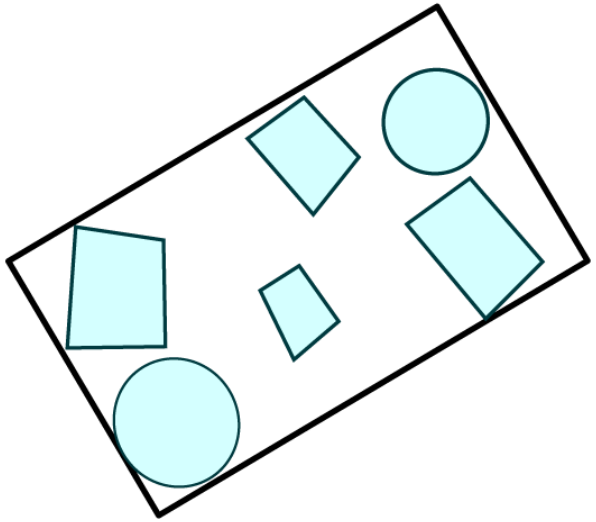
Sphere (A lot)

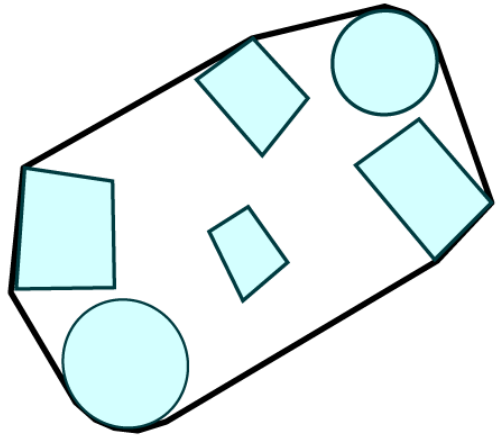




Axis-Aligned Bounding Box
(A lot)

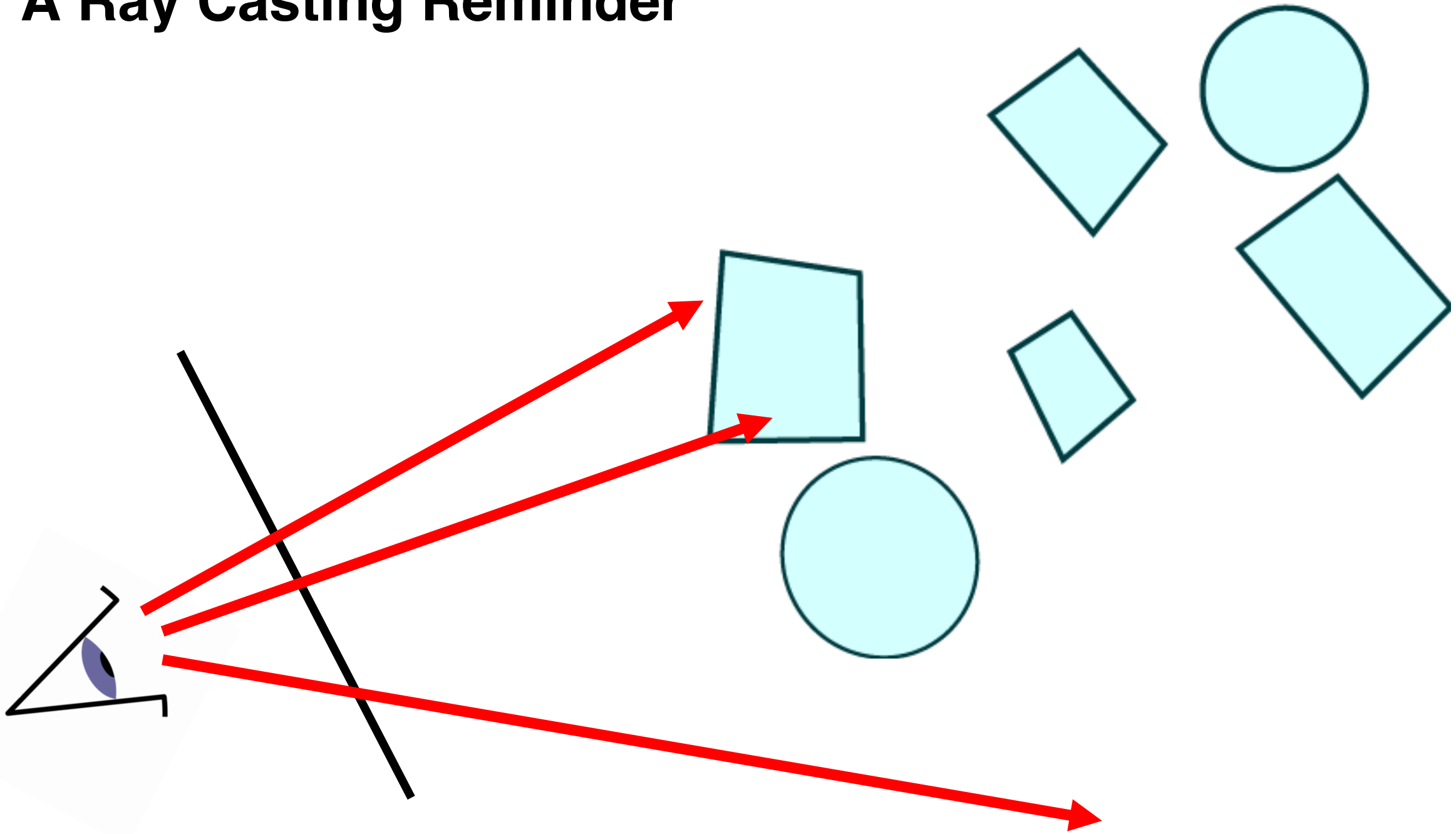
Object Oriented Bounding Box
(OOBB) (A little)



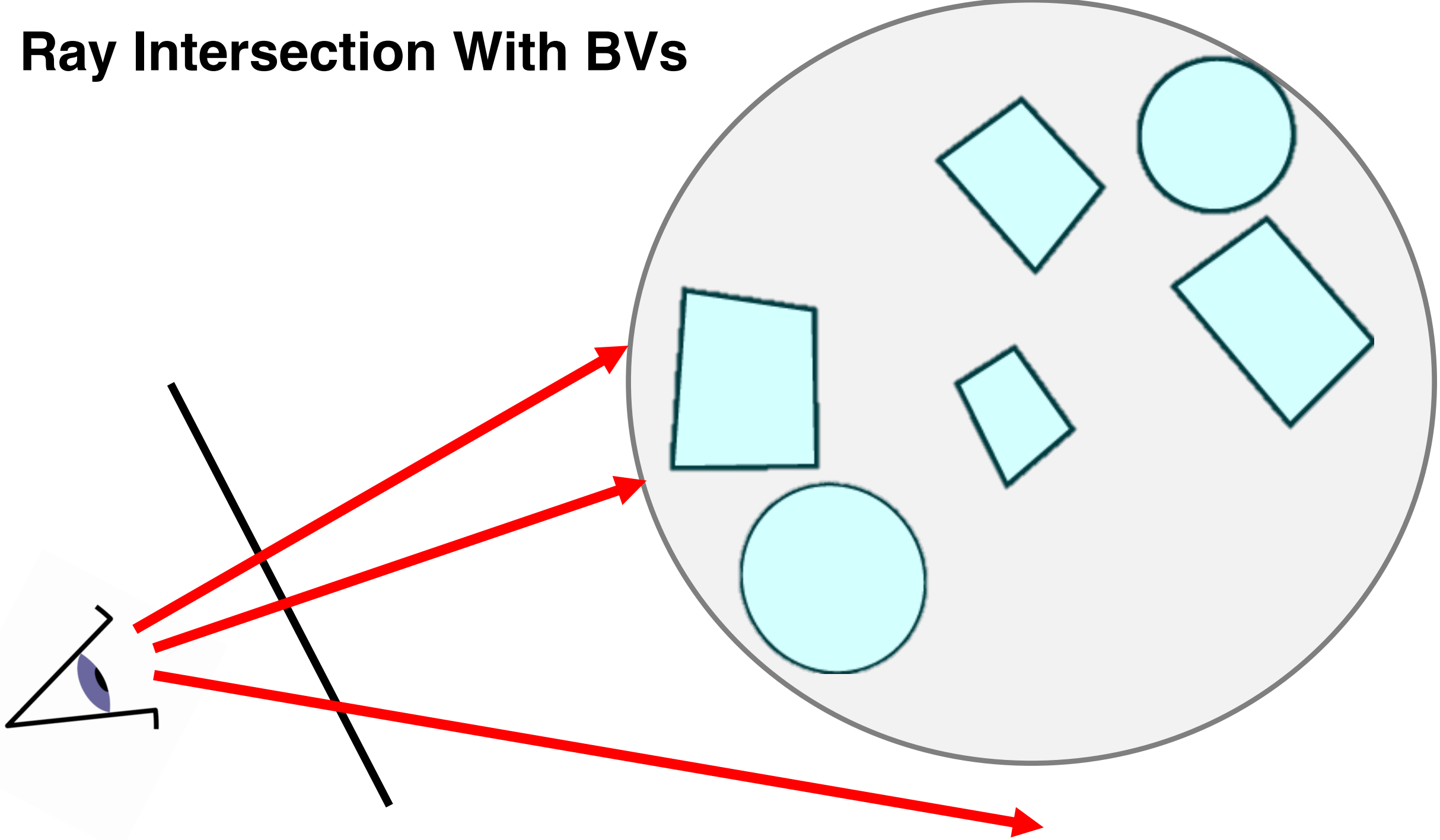


Convex Hull
(Nope)

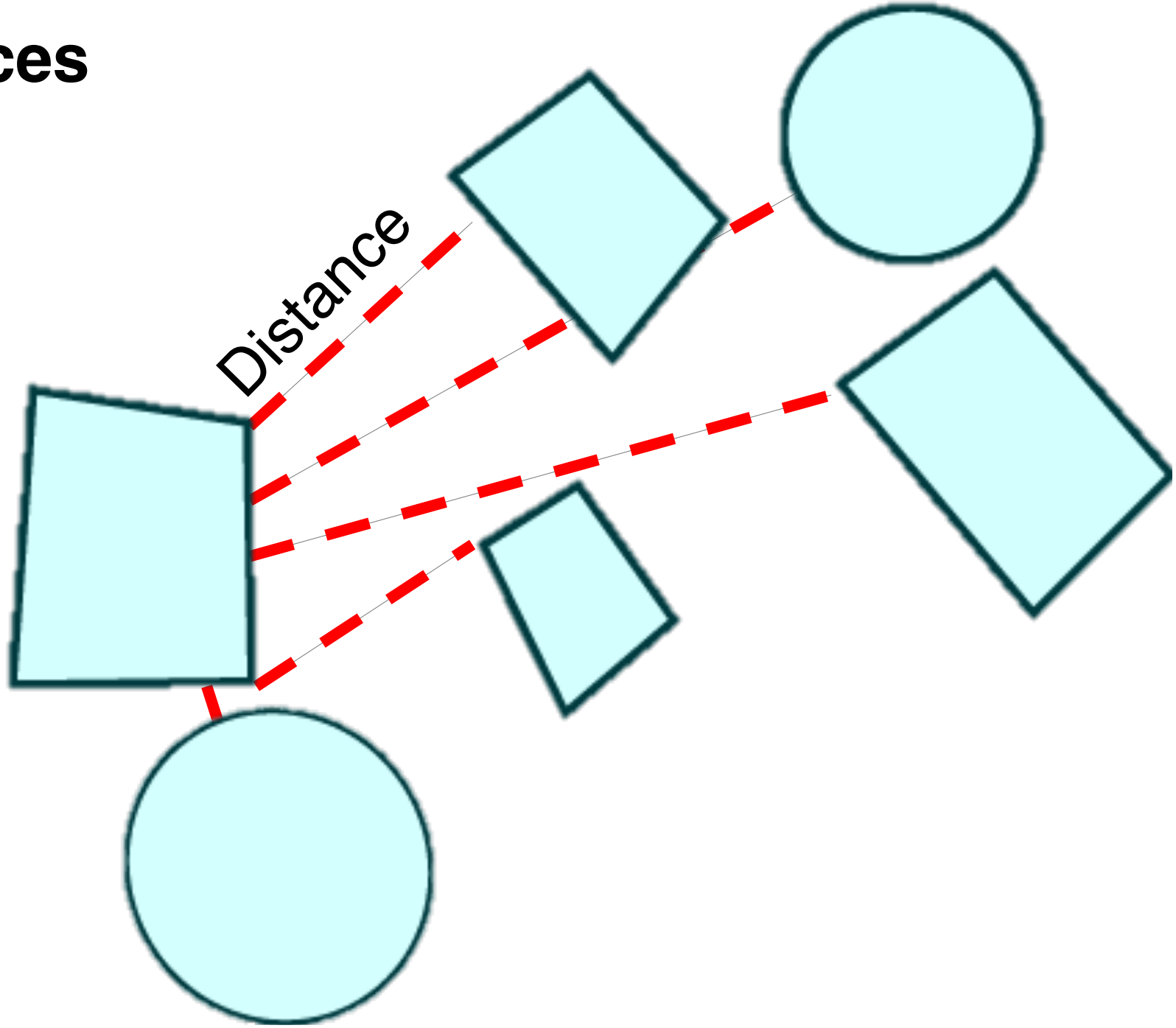
A Ray Casting Reminder



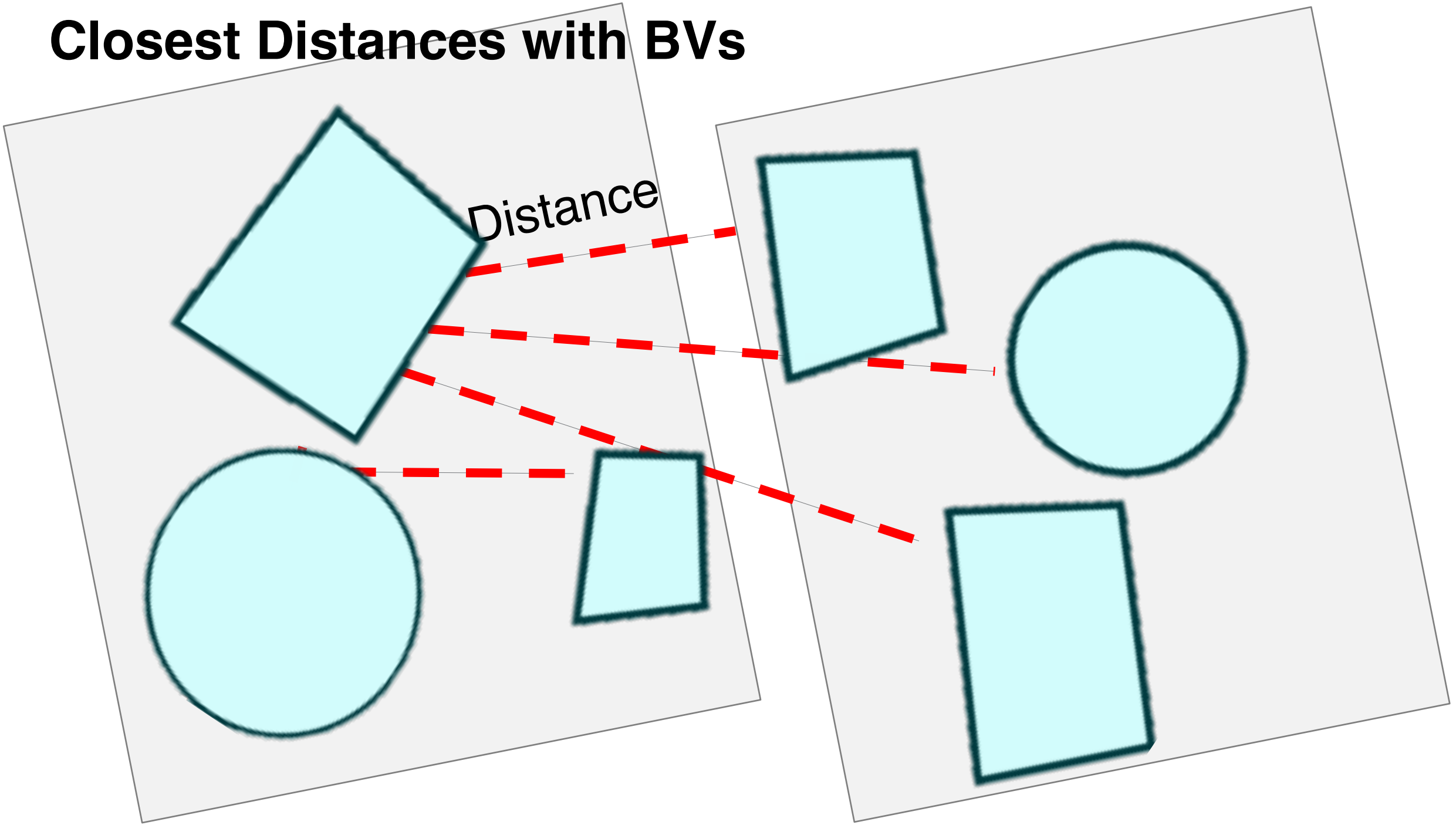
Ray Intersection With BVs



Closest Distances



Closest Distances with BVs



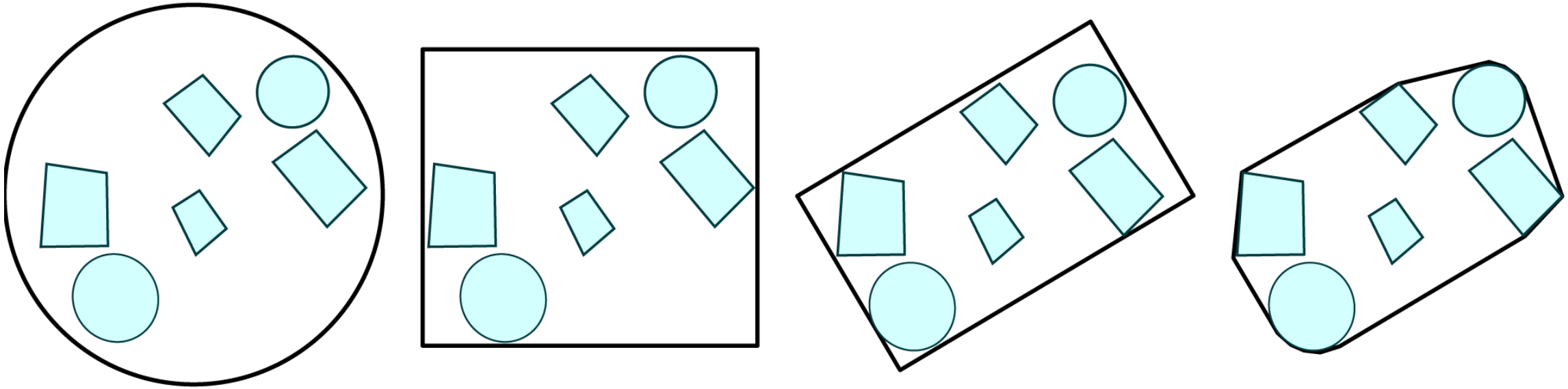
Bounding Volumes (BVs)

"Simple" geometry that fully encloses a **collection** of other geometry

Should fit geometry tightly

Sphere

Object Oriented Bounding Box (OOBB)

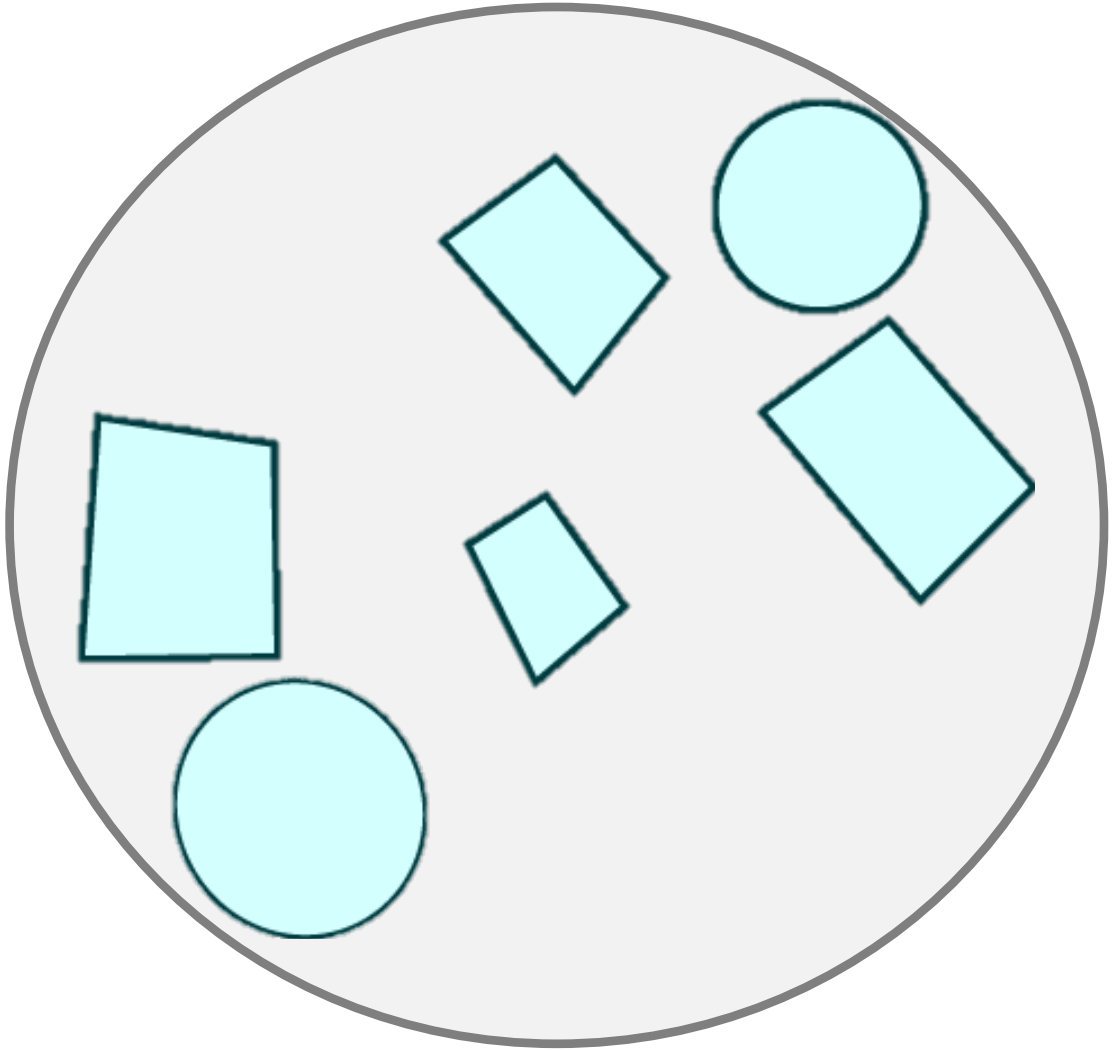


Axis-Aligned Bounding Box (AABB)

Convex Hull

Building a Bounding Sphere

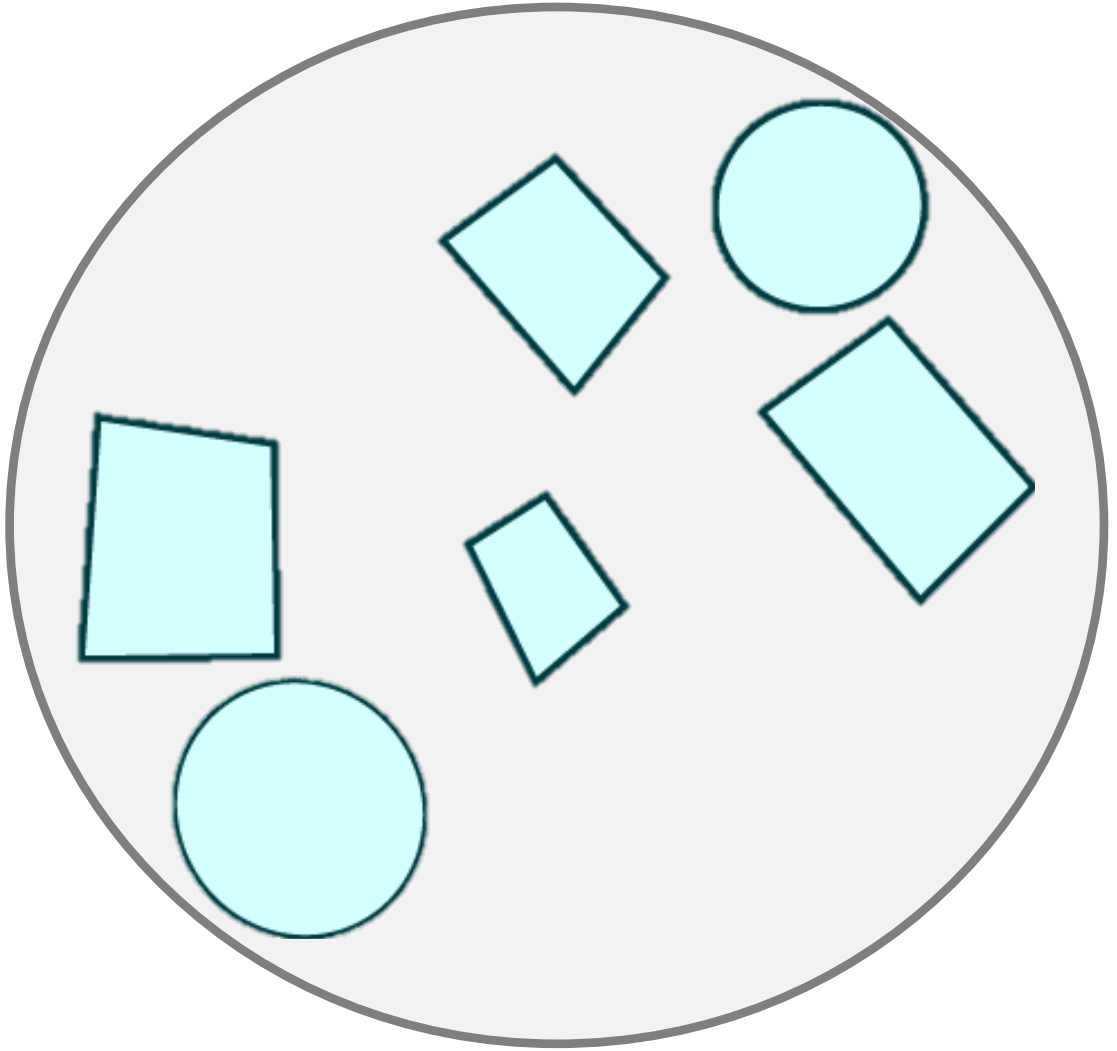
Parameters of a Sphere ?



Building a Bounding Sphere

Parameters of a Sphere:

1. Center = ?
2. Radius = ?

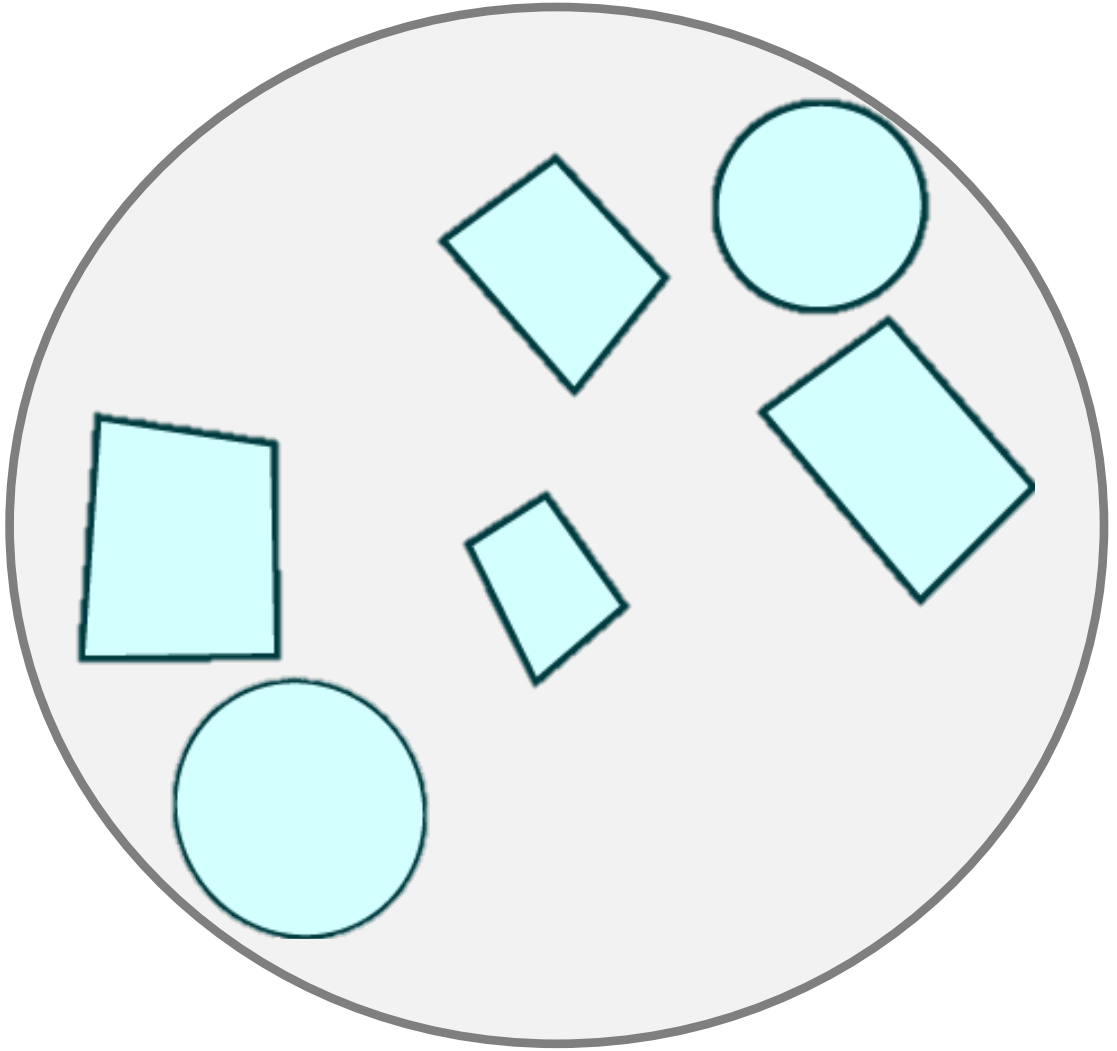


Building a Bounding Sphere

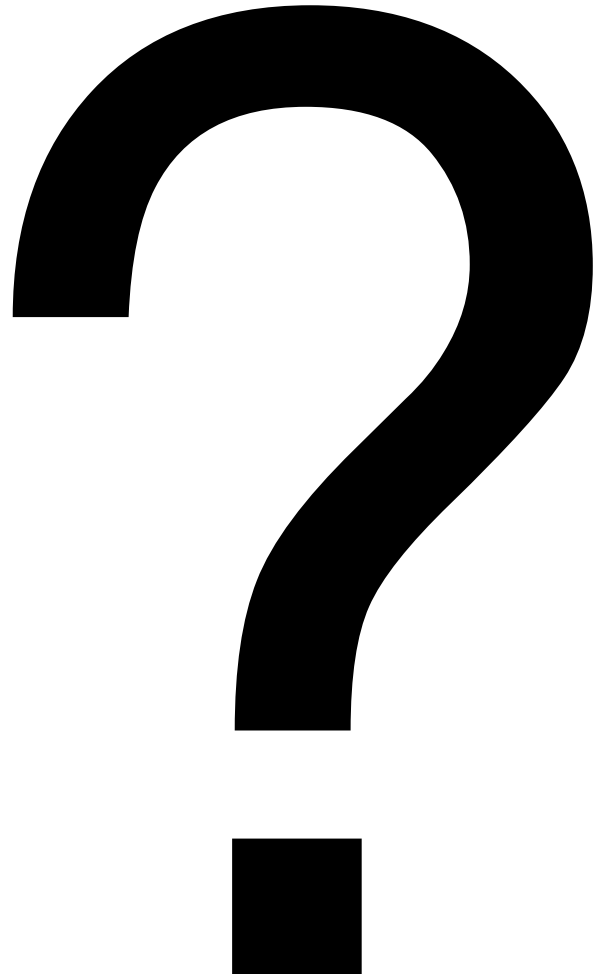
Parameters of a Sphere:

1. Center = $\mathbf{c} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{v}^i$
2. Radius = $r = \max\left(\mathbf{v}^i - \mathbf{c}\right)$

$$\mathbf{v}^i \in \text{Vertices}$$



Ray-Sphere Intersection



Ray-Sphere Intersection

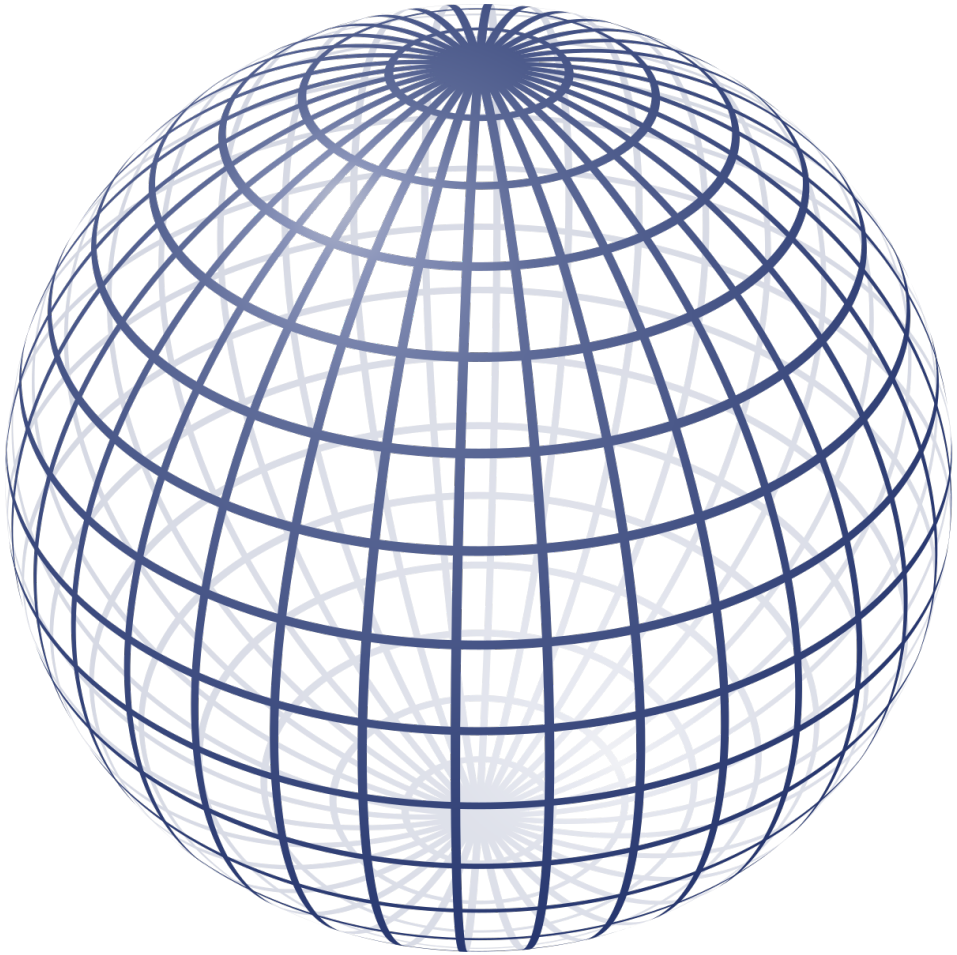
$$\mathbf{p}(t)^T \mathbf{p}(t) - r^2 = 0$$

$$a \cdot t^2 + b \cdot t + c = 0$$

$$a = (\mathbf{s} - \mathbf{e})^T (\mathbf{s} - \mathbf{e})$$

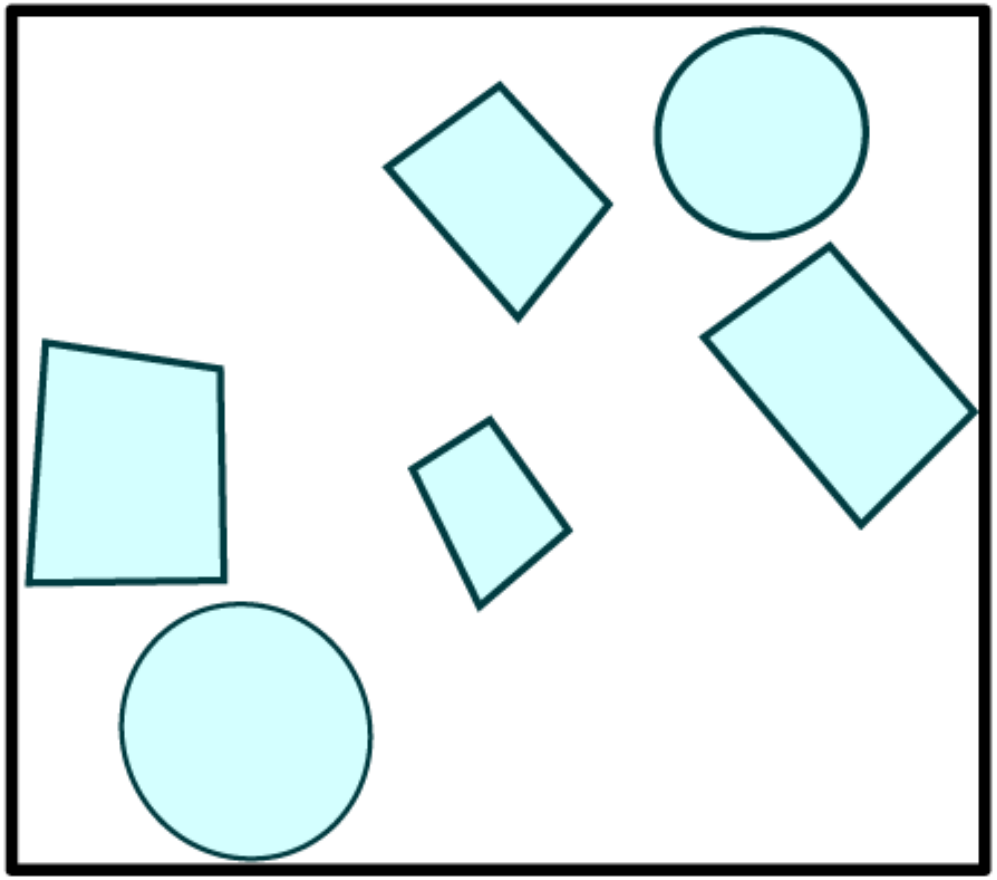
$$b = 2\mathbf{e}^T (\mathbf{s} - \mathbf{e})$$

$$c = \mathbf{e}^T \mathbf{e} - r^2$$



Building and Axis-Aligned Bounding Box (AABB)

?



Building and Axis-Aligned Bounding Box (AABB)

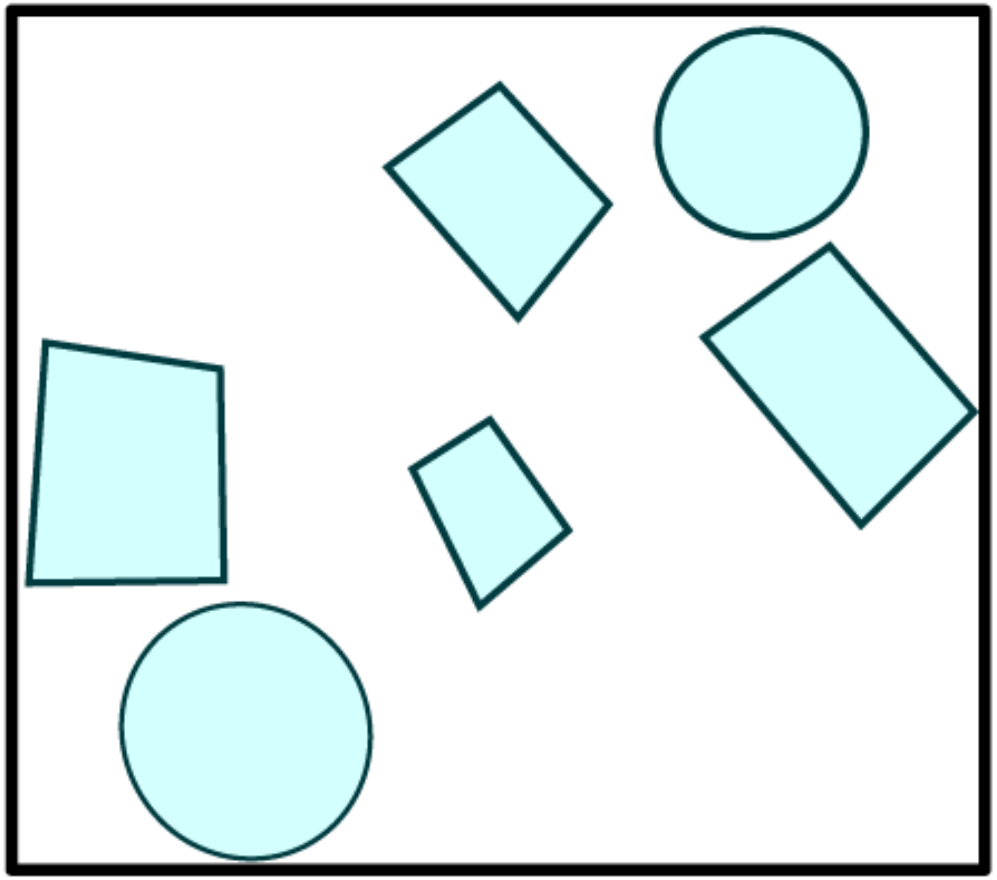
$$x_{min} = \min\left(v_x^i\right)$$

$$x_{max} = \max\left(v_x^i\right)$$

$$y_{min} = \min\left(v_y^i\right)$$

$$y_{max} = \max\left(v_y^i\right)$$

$$\mathbf{v}^i \in \text{Vertices}$$

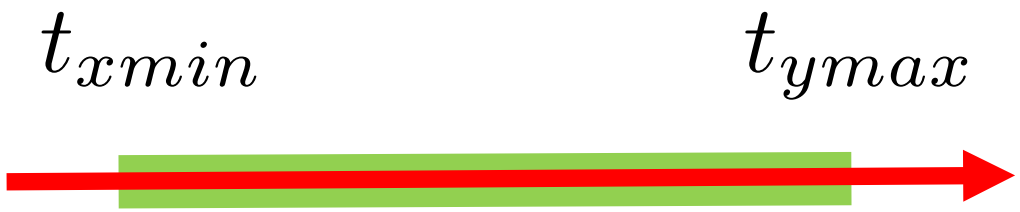


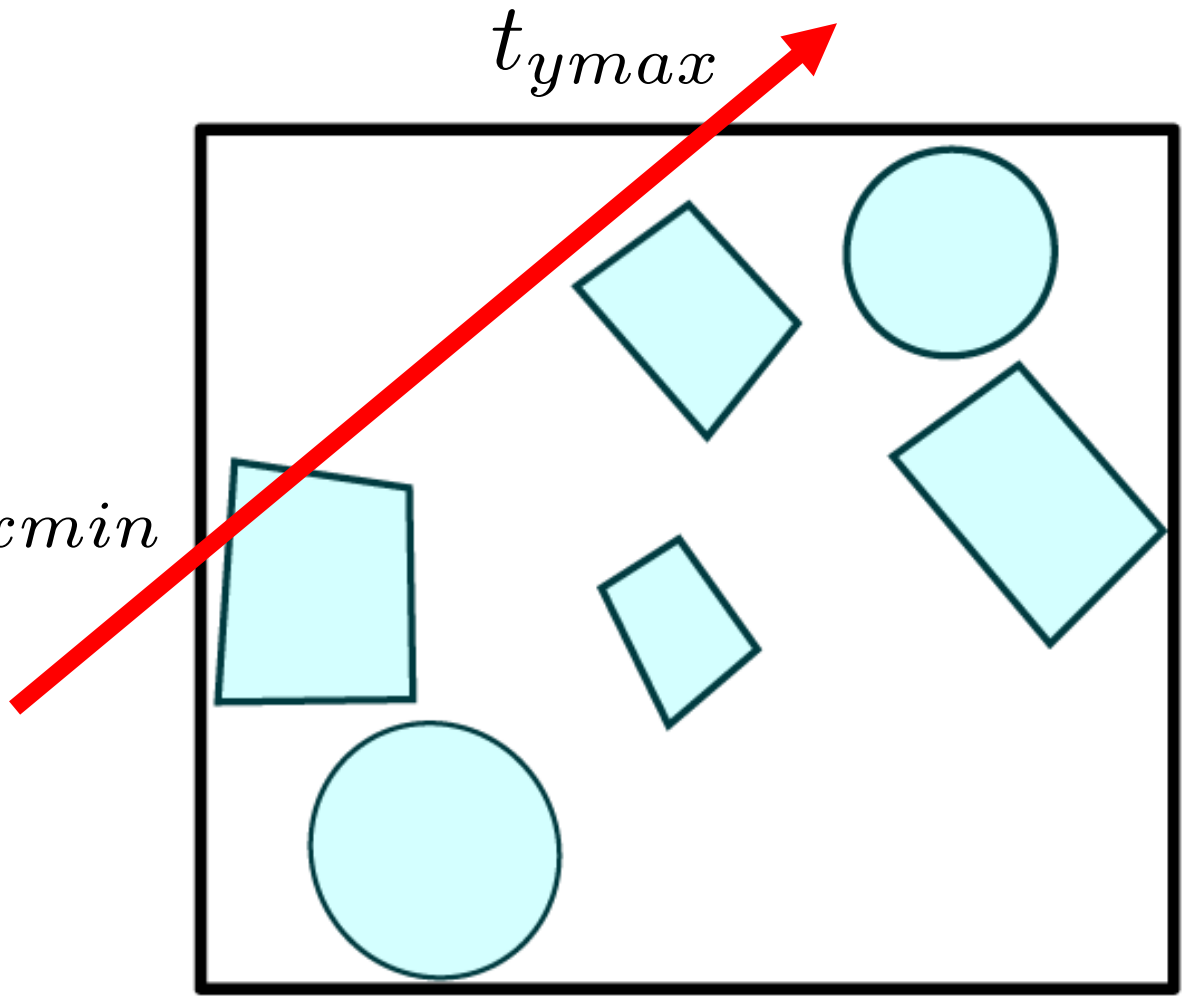
Ray-AABB Intersection

$$
\begin{aligned}
t_{\text{xmin}} &= (x_{\text{min}} - x_e)/x_d \\
t_{\text{xmax}} &= (x_{\text{max}} - x_e)/x_d \\
t_{\text{ymin}} &= (y_{\text{min}} - y_e)/y_d \\
t_{\text{ymax}} &= (y_{\text{max}} - y_e)/y_d
\end{aligned}
$$

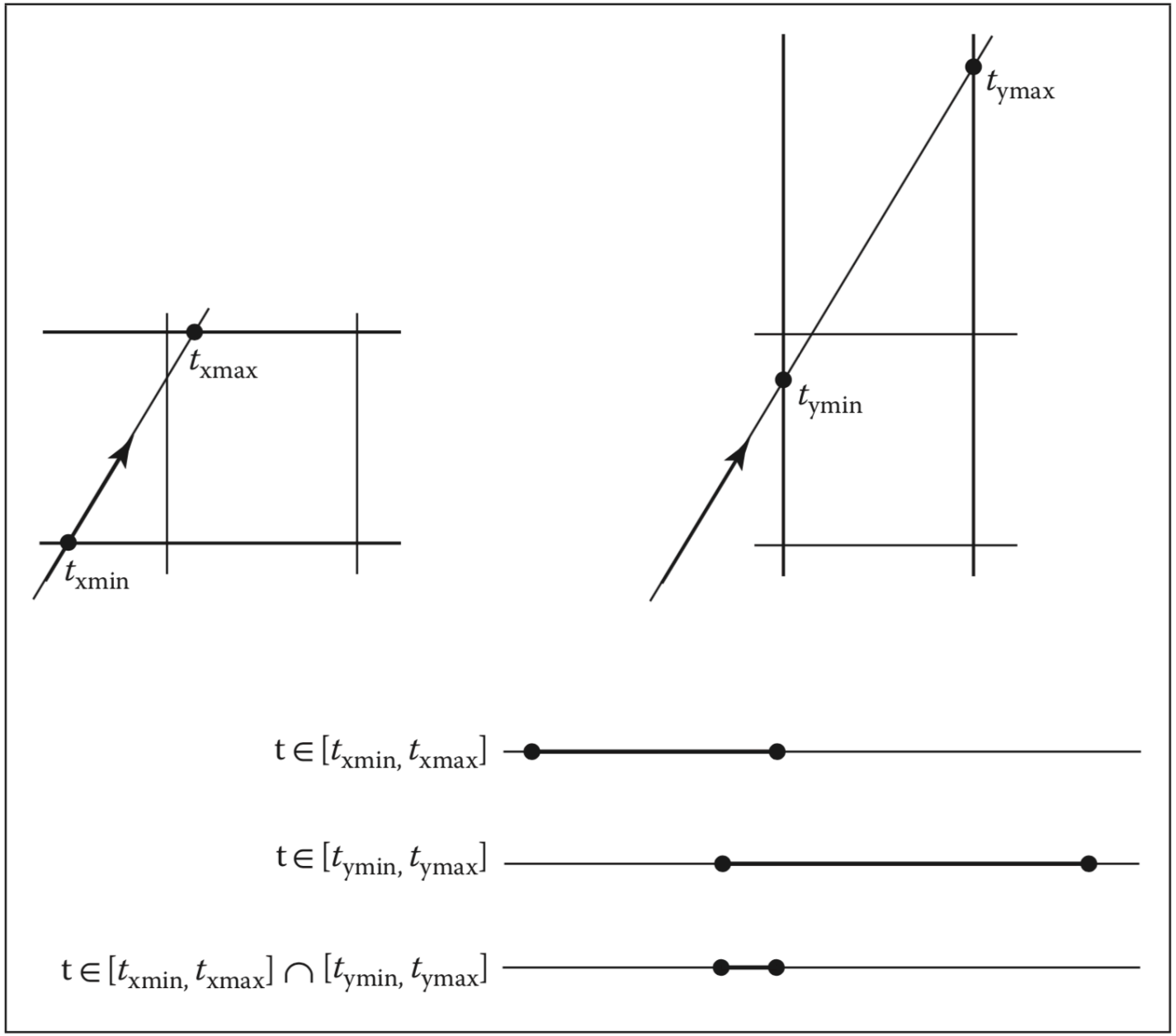
Ray-AABB Intersection

$t_{xmin} < t_{ymax}$

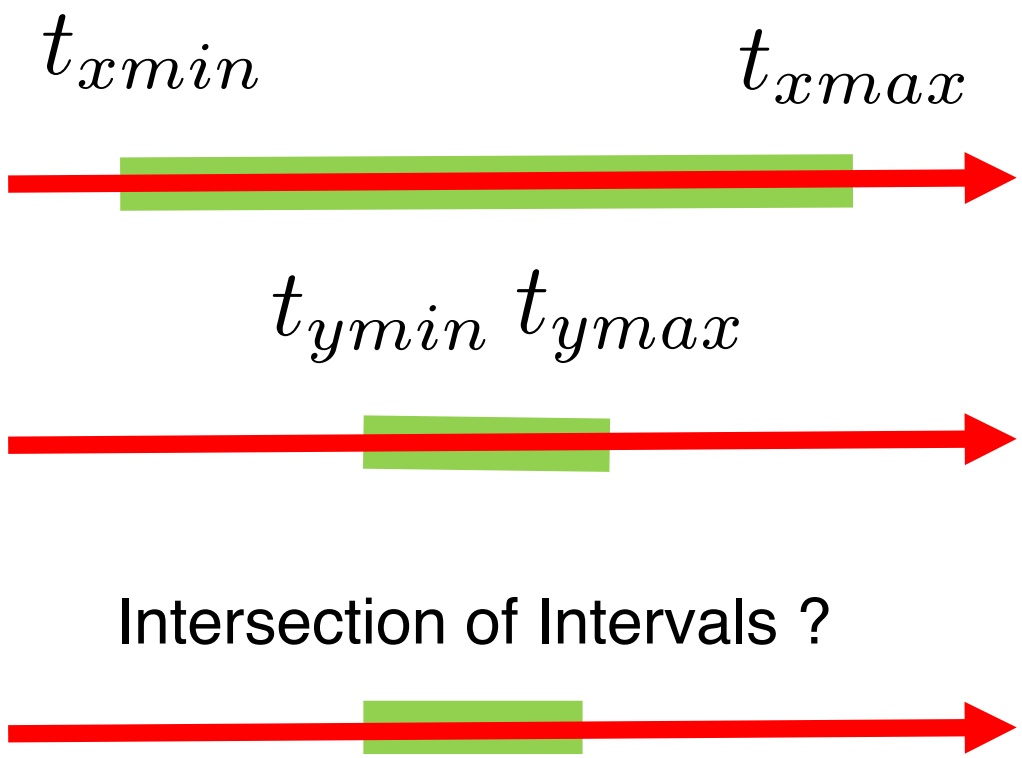




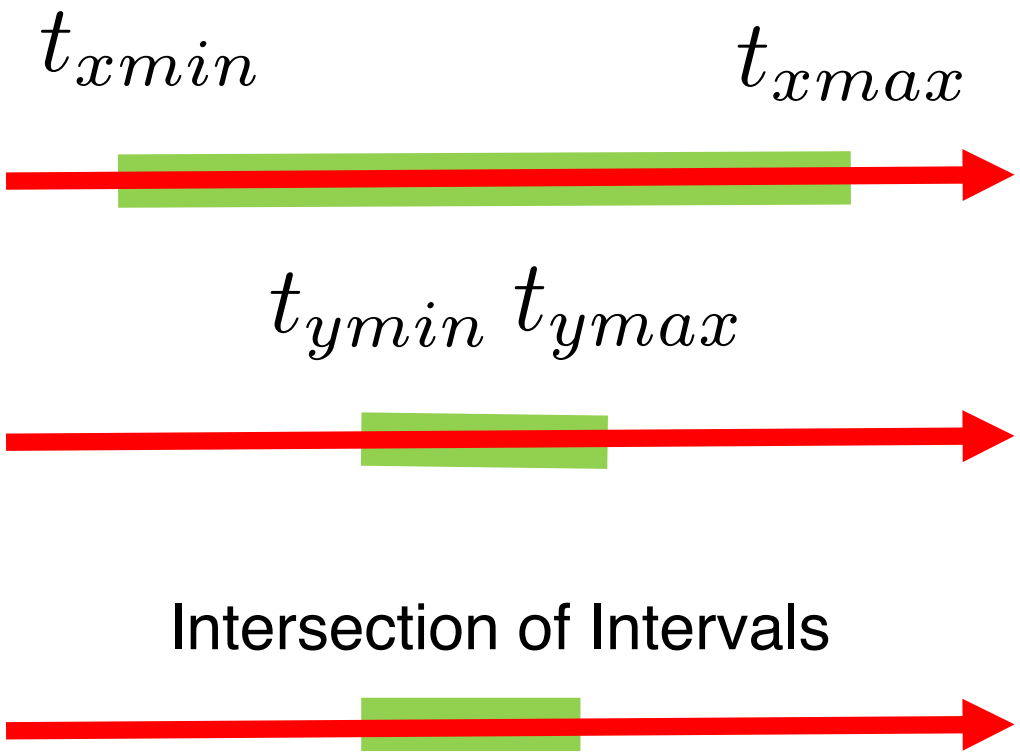
Ray-AABB Intersection



Ray-AABB Intersection

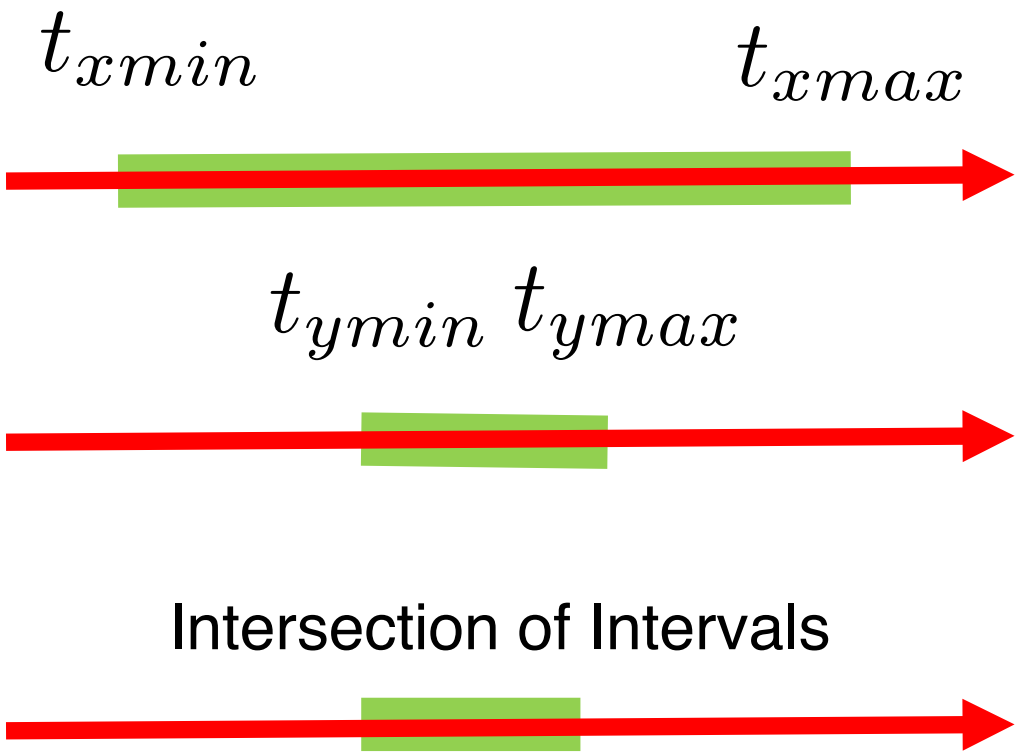


Ray-AABB Intersection



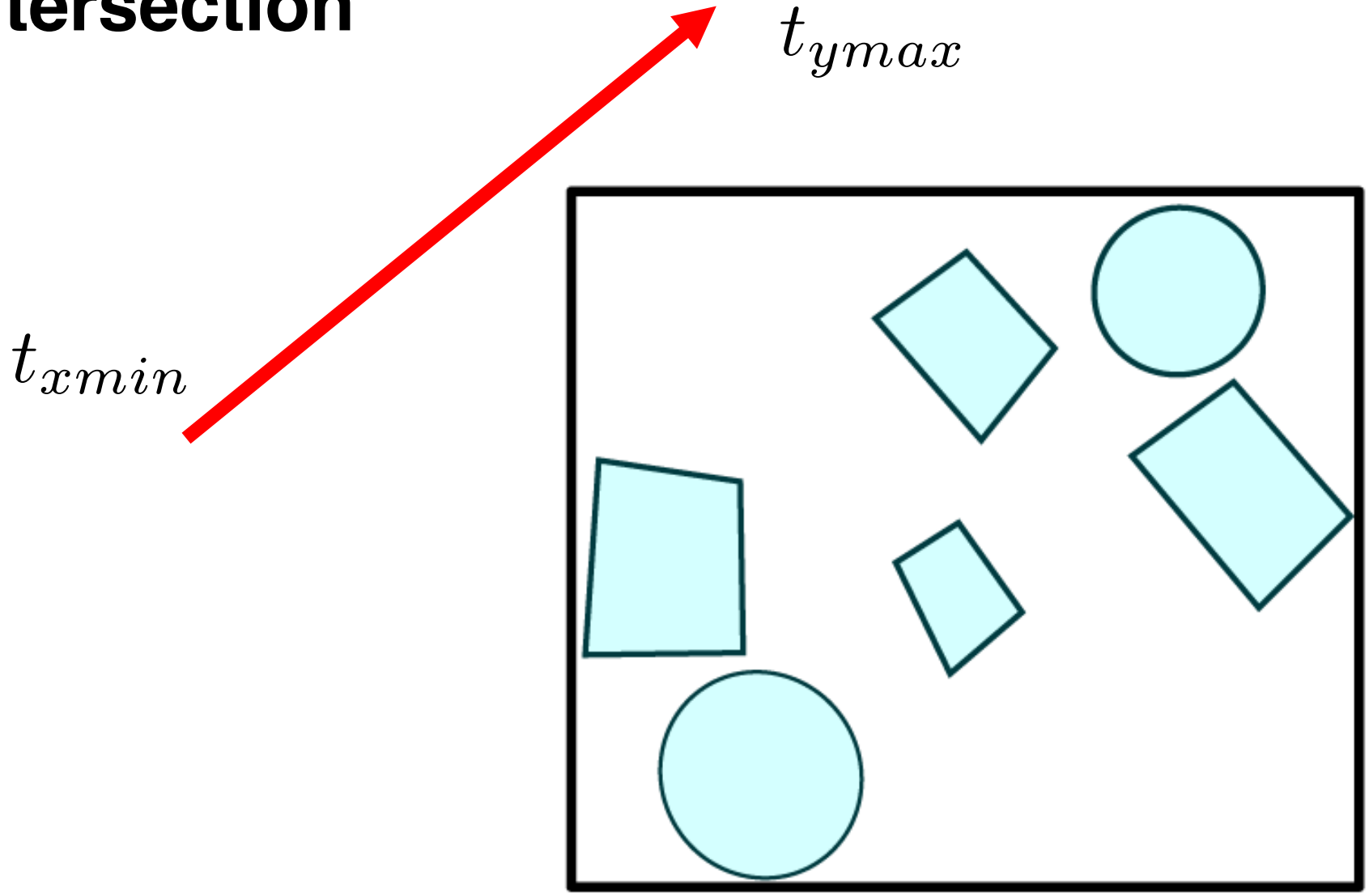
$$\max\left(t_{xmin}, t_{ymin}\right) \quad ? \quad \min\left(t_{xmax}, t_{ymax}\right)$$

Ray-AABB Intersection

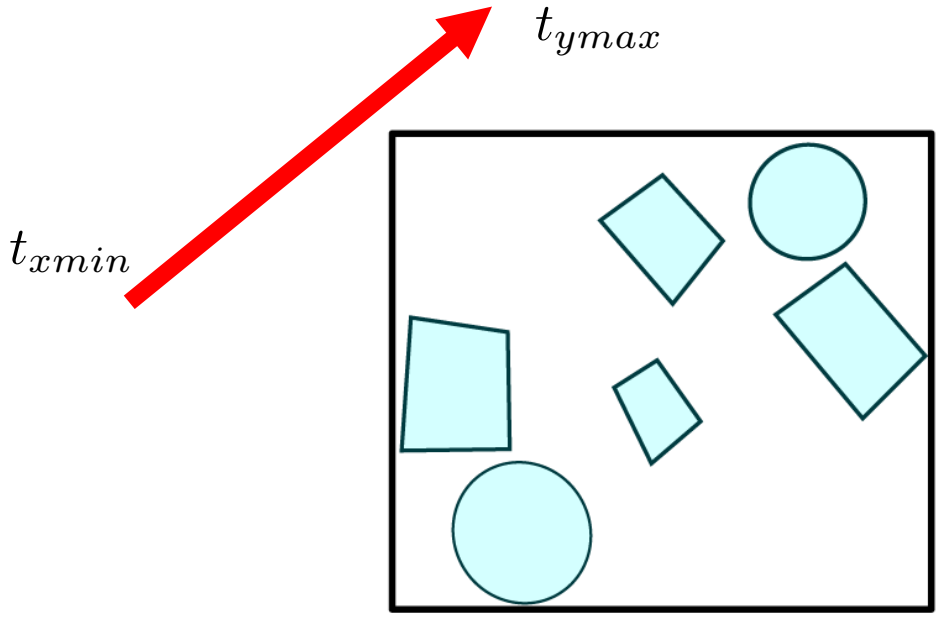


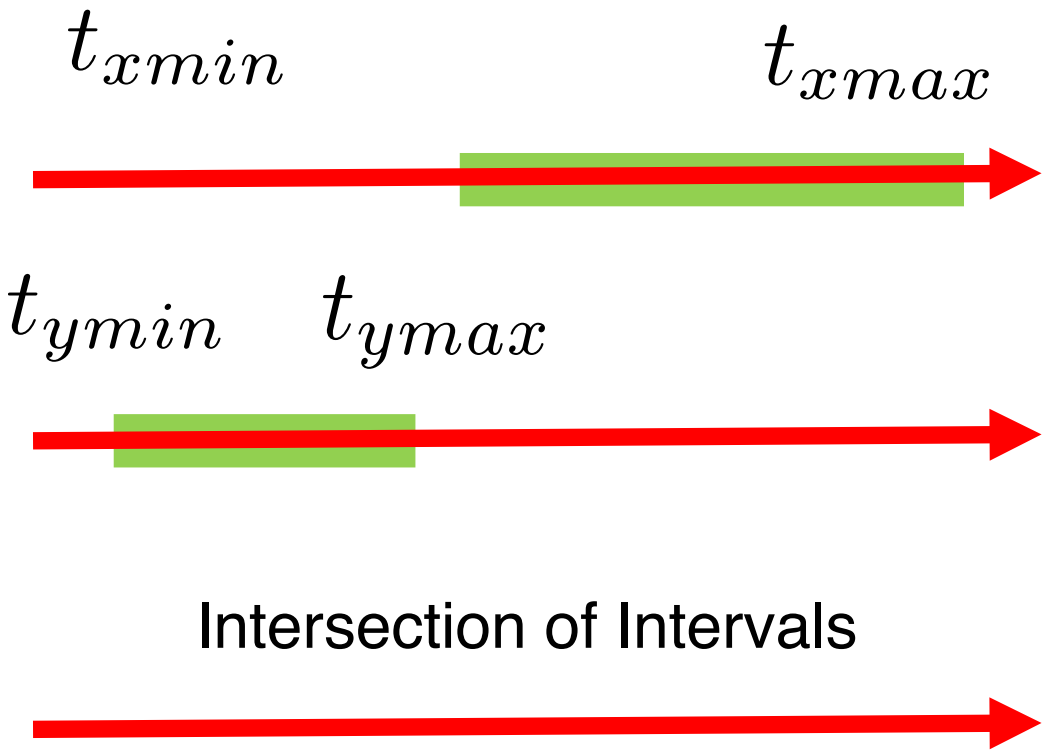
$$\max\left(t_{xmin}, t_{ymin}\right) \quad < \quad \min\left(t_{xmax}, t_{ymax}\right)$$

Ray-AABB Intersection



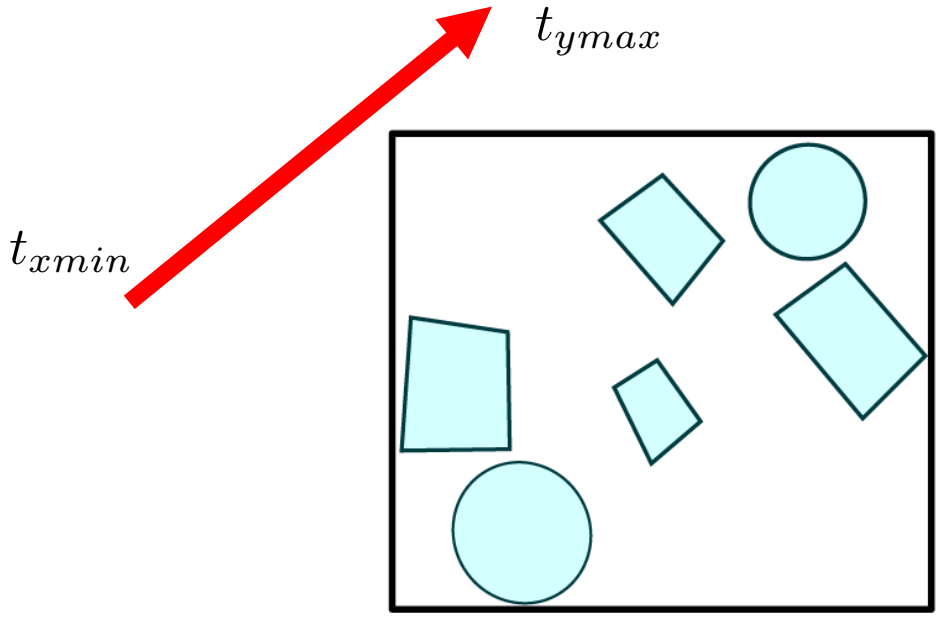
Ray-AABB Intersection

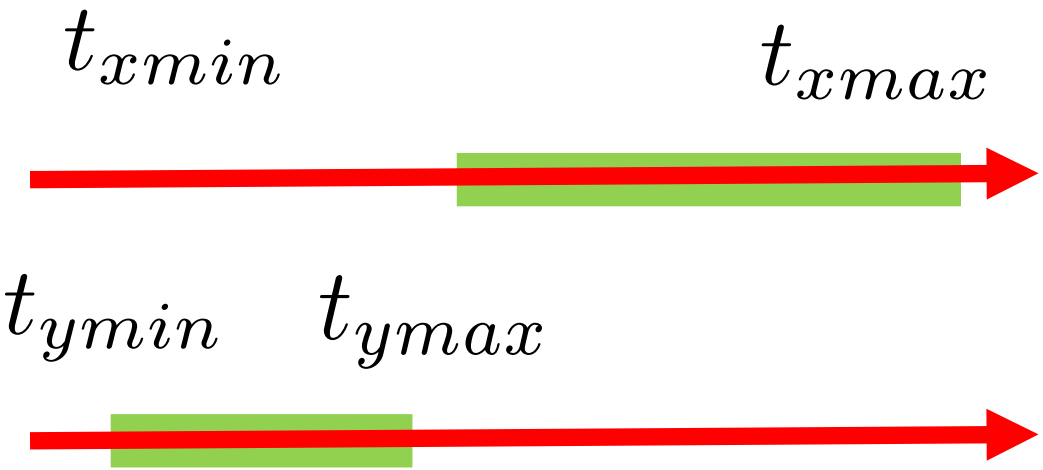




$$\max\left(t_{xmin}, t_{ymin}\right) \quad ? \quad \min\left(t_{xmax}, t_{ymax}\right)$$

Ray-AABB Intersection

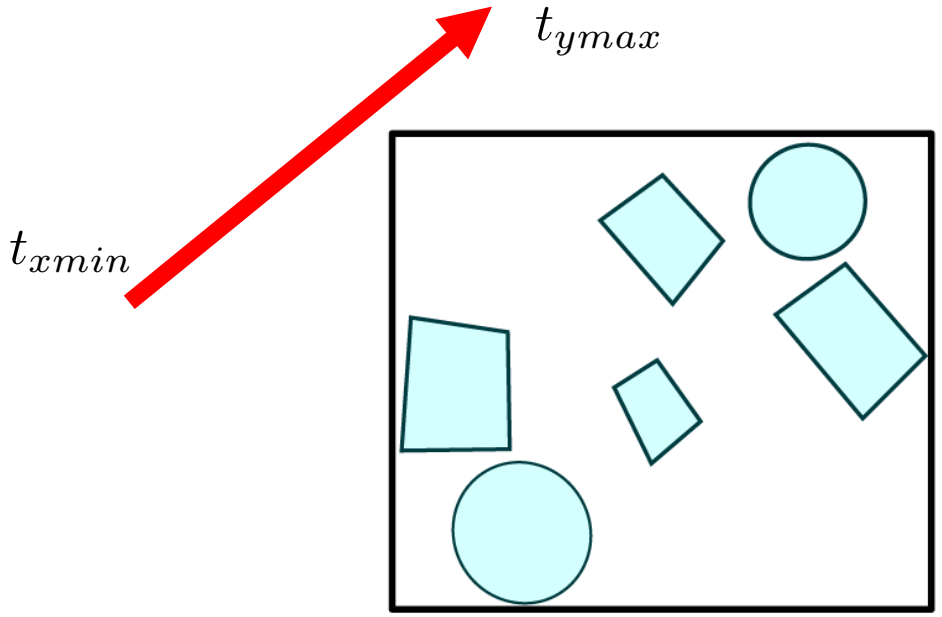


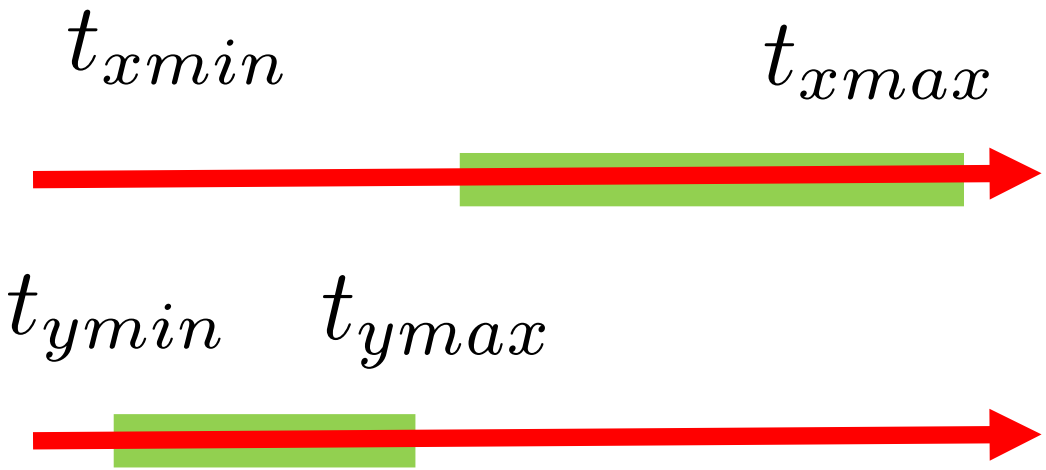


Intersection of Intervals

$$\max(t_{xmin}, t_{ymin}) \quad > \quad \min(t_{xmax}, t_{ymax})$$

Ray-AABB Intersection





Intersection of Intervals

$$\max(t_{xmin}, t_{ymin}) \quad > \quad \min(t_{xmax}, t_{ymax})$$ <-- CHECK

Building and Axis-Aligned Bounding Box (AABB)

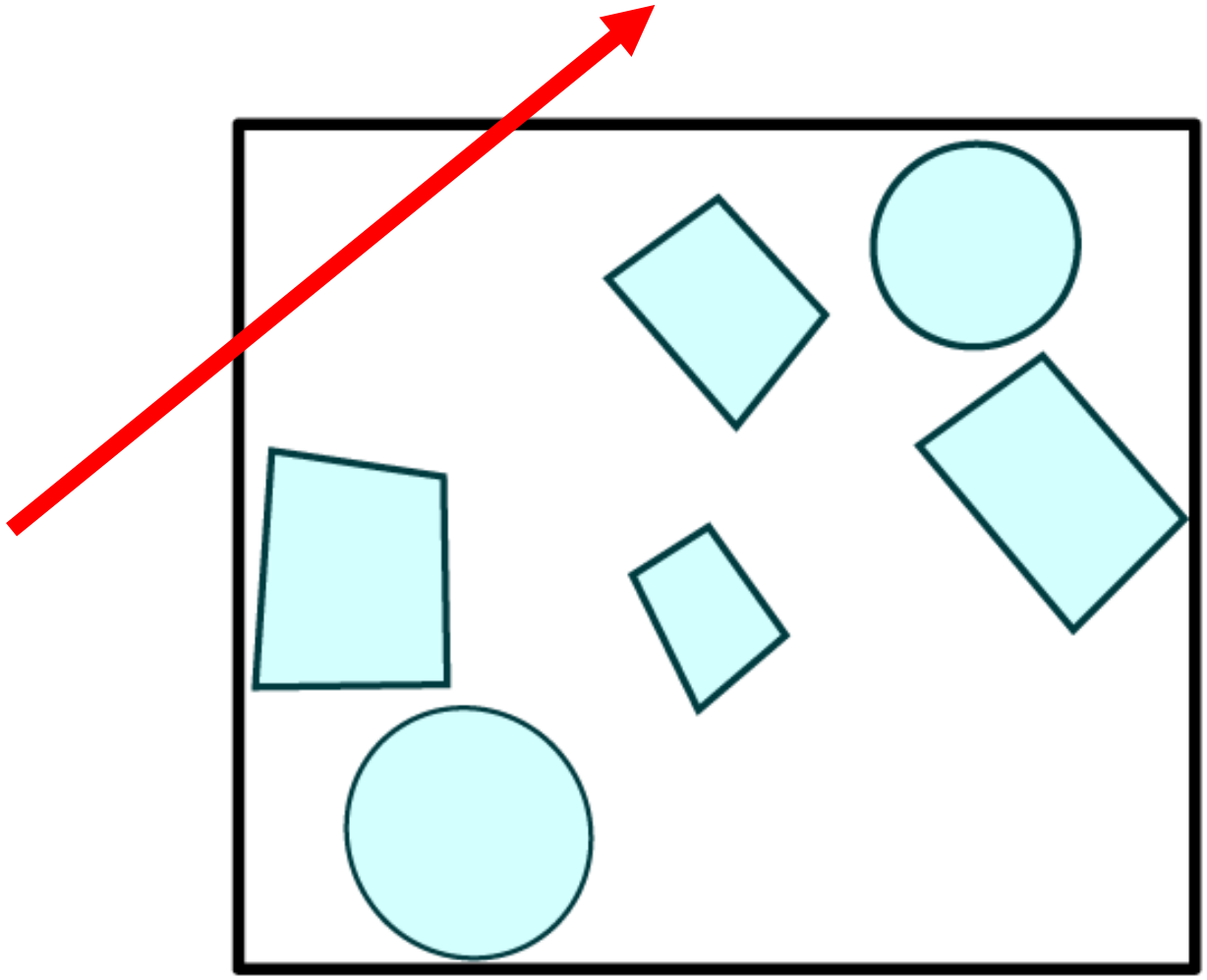
$$x_{min} = \min\left(v_x^i\right)$$

$$x_{max} = \max\left(v_x^i\right)$$

$$y_{min} = \min\left(v_y^i\right)$$

$$y_{max} = \max\left(v_y^i\right)$$

$$\mathbf{v}^i \in \text{Vertices}$$



Building and Axis-Aligned Bounding Box (AABB)

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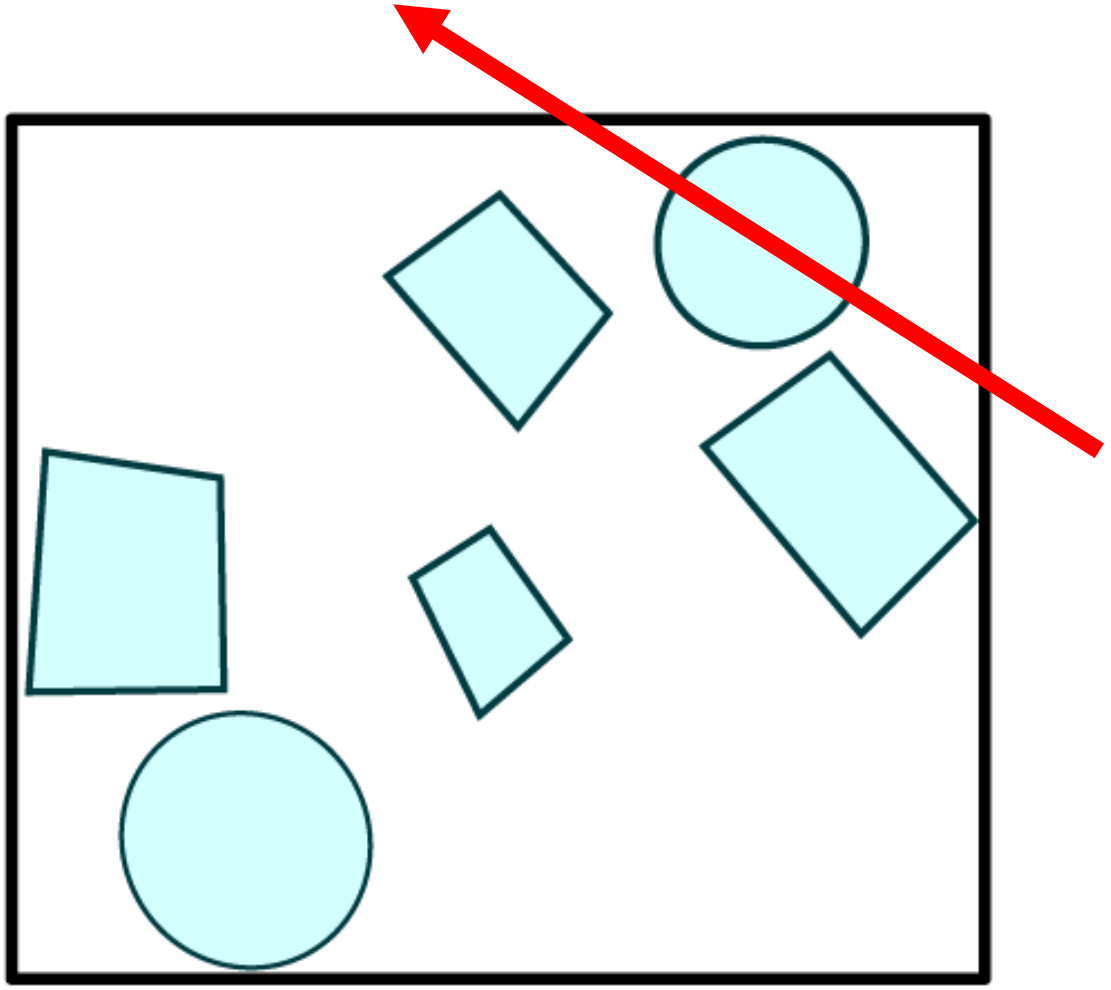

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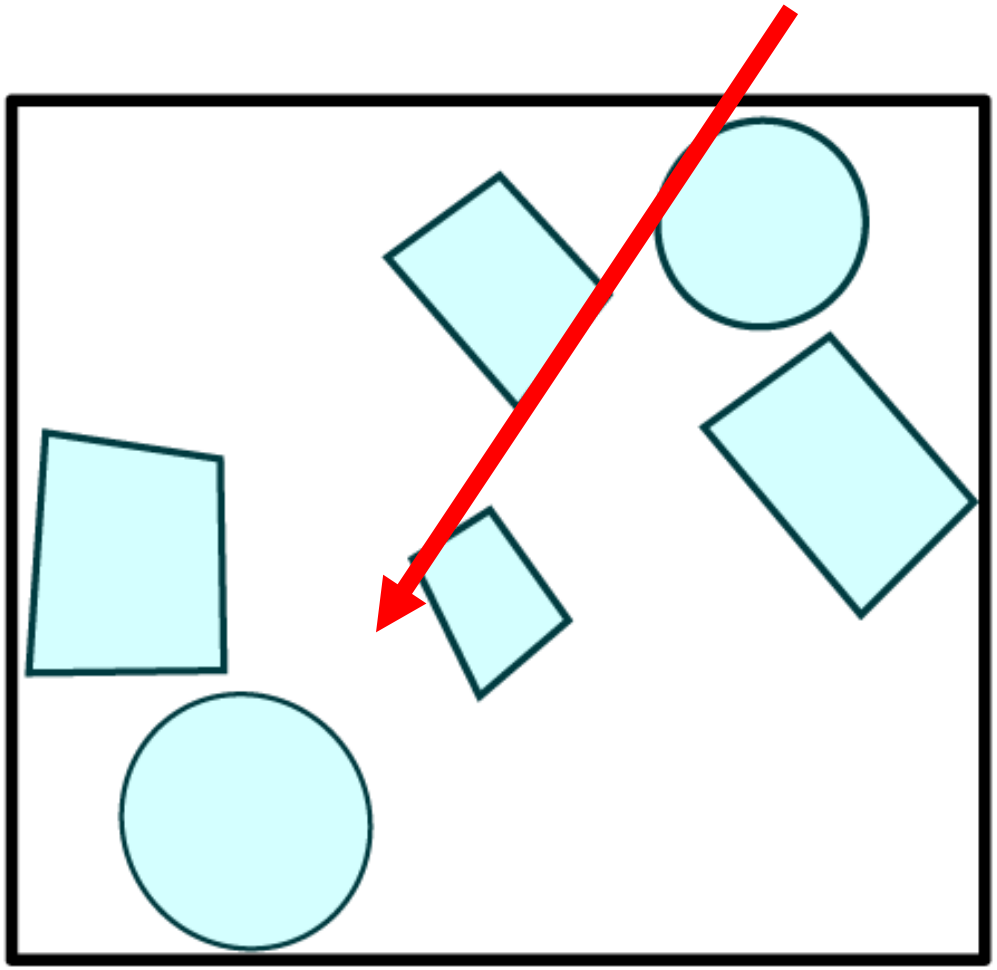


Ray-AABB Intersection

if $(x_d \geq 0)$ **then**

$$t_{\text{xmin}} = (x_{\text{min}} - x_e)/x_d$$

$$t_{\text{xmax}} = (x_{\text{max}} - x_e)/x_d$$

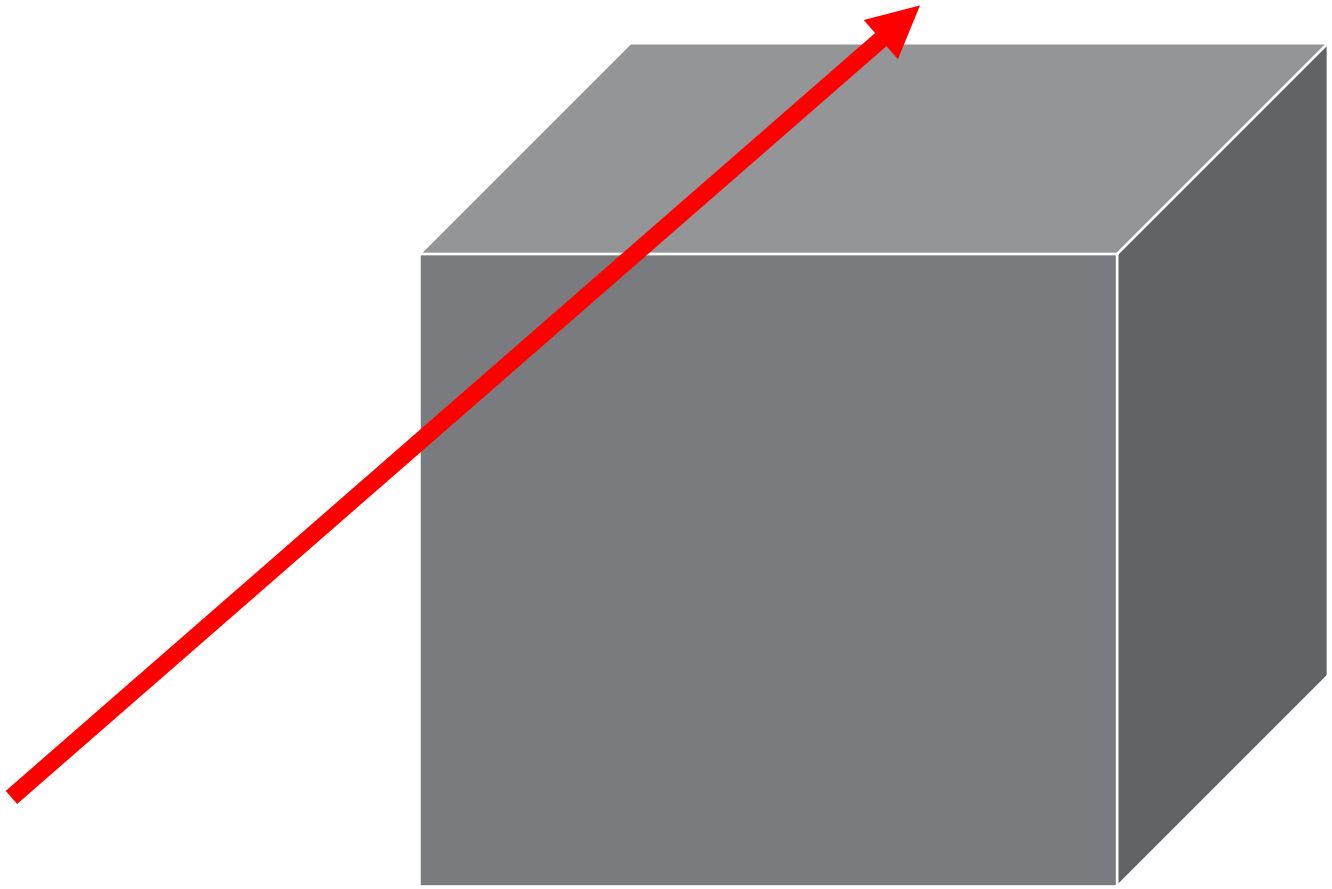
else

$$t_{\text{xmin}} = (x_{\text{max}} - x_e)/x_d$$

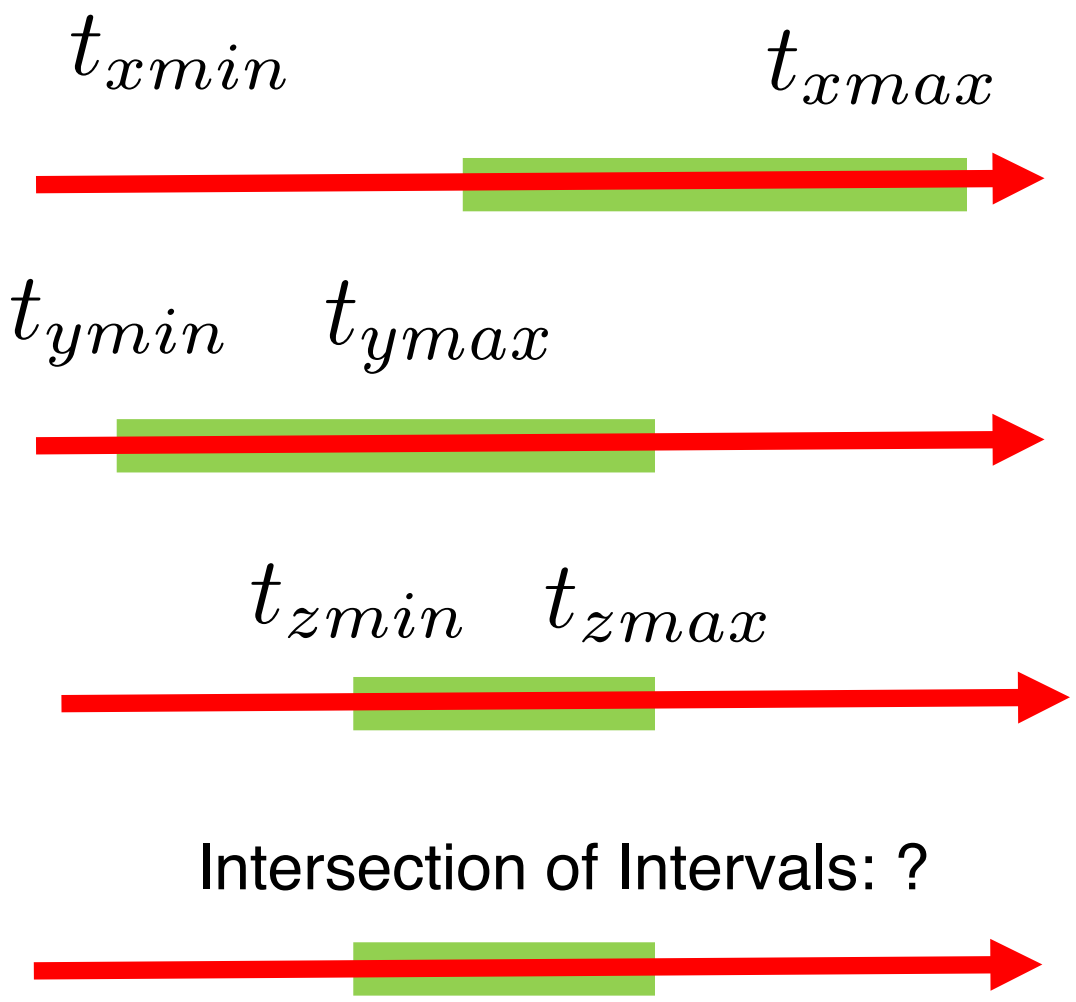
$$t_{\text{xmax}} = (x_{\text{min}} - x_e)/x_d$$

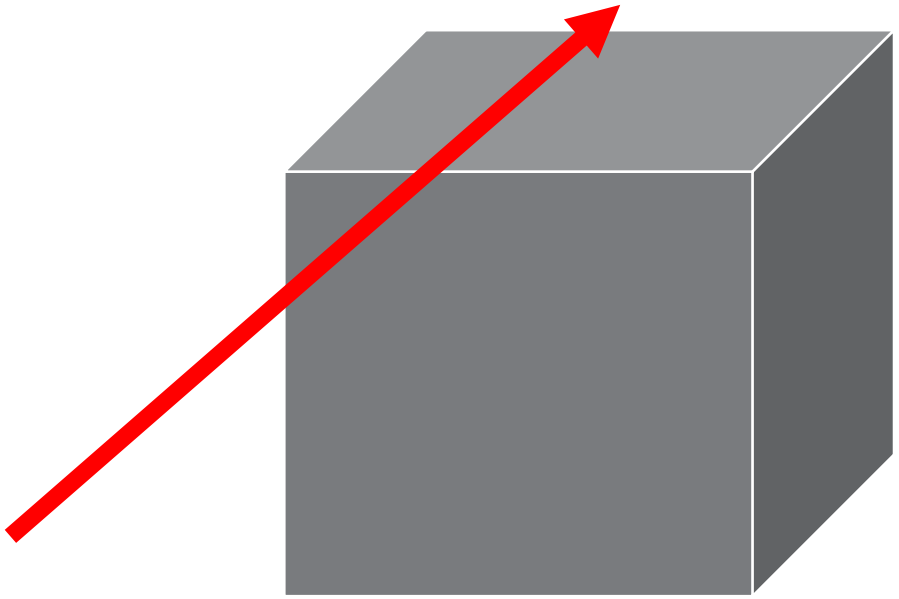
When does this fail ?

What happens in 3D ?

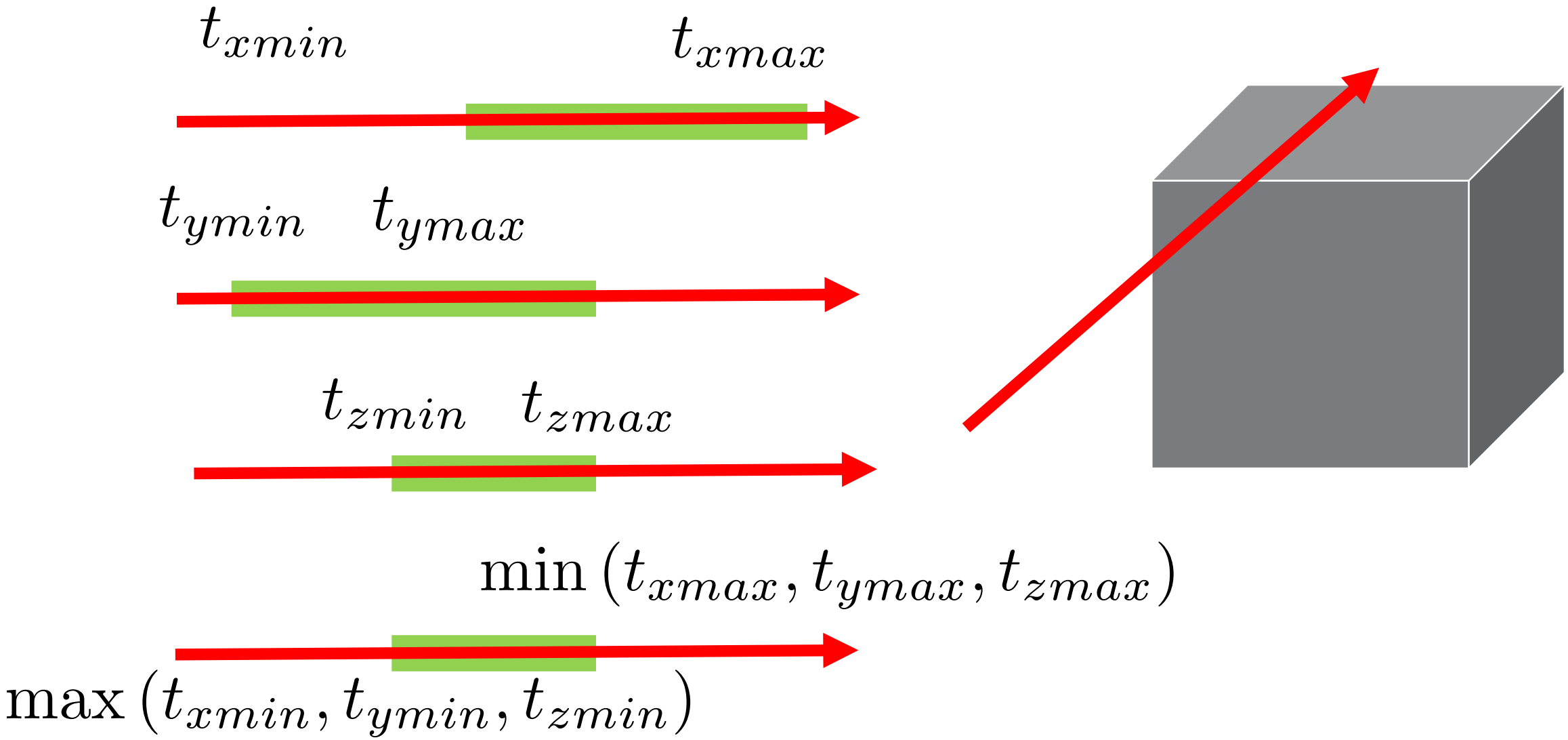


Ray-AABB Intersection





Ray-AABB Intersection



Ray-AABB Intersection

if $(x_d \geq 0)$ **then**

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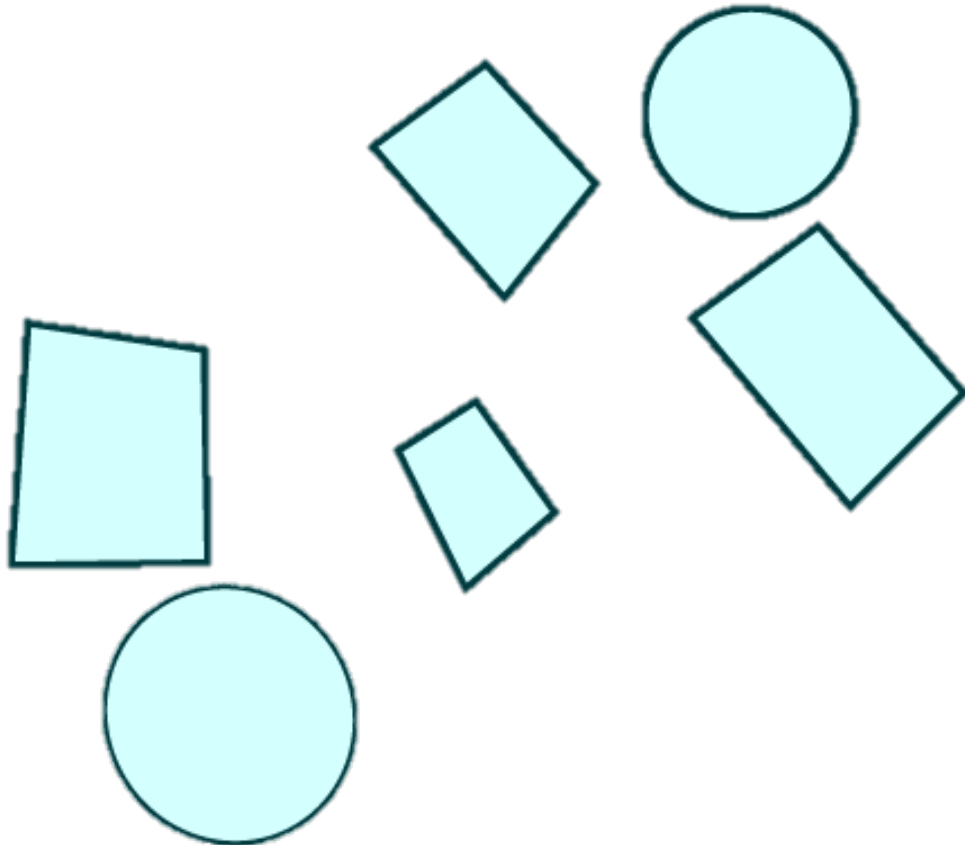
$$t_{\text{xmax}} = (x_{\text{max}} - x_e)/x_d$$

else

$$t_{\text{xmin}} = (x_{\text{max}} - x_e)/x_d$$

$$t_{\text{xmax}} = (x_{\text{min}} - x_e)/x_d$$

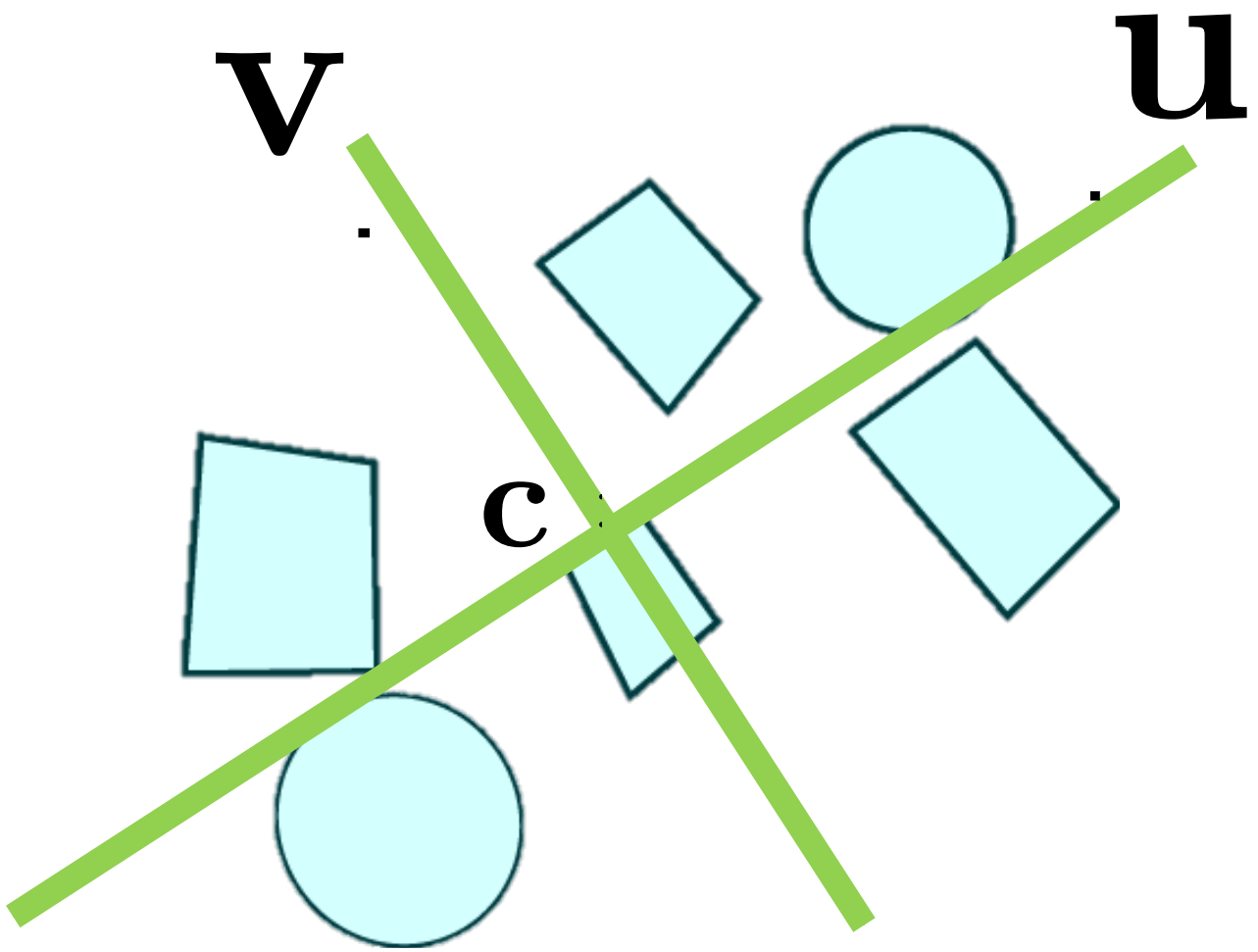
Building an Object-Oriented Bounding Box (OOBB)



Building an Object-Oriented Bounding Box (OOBB)

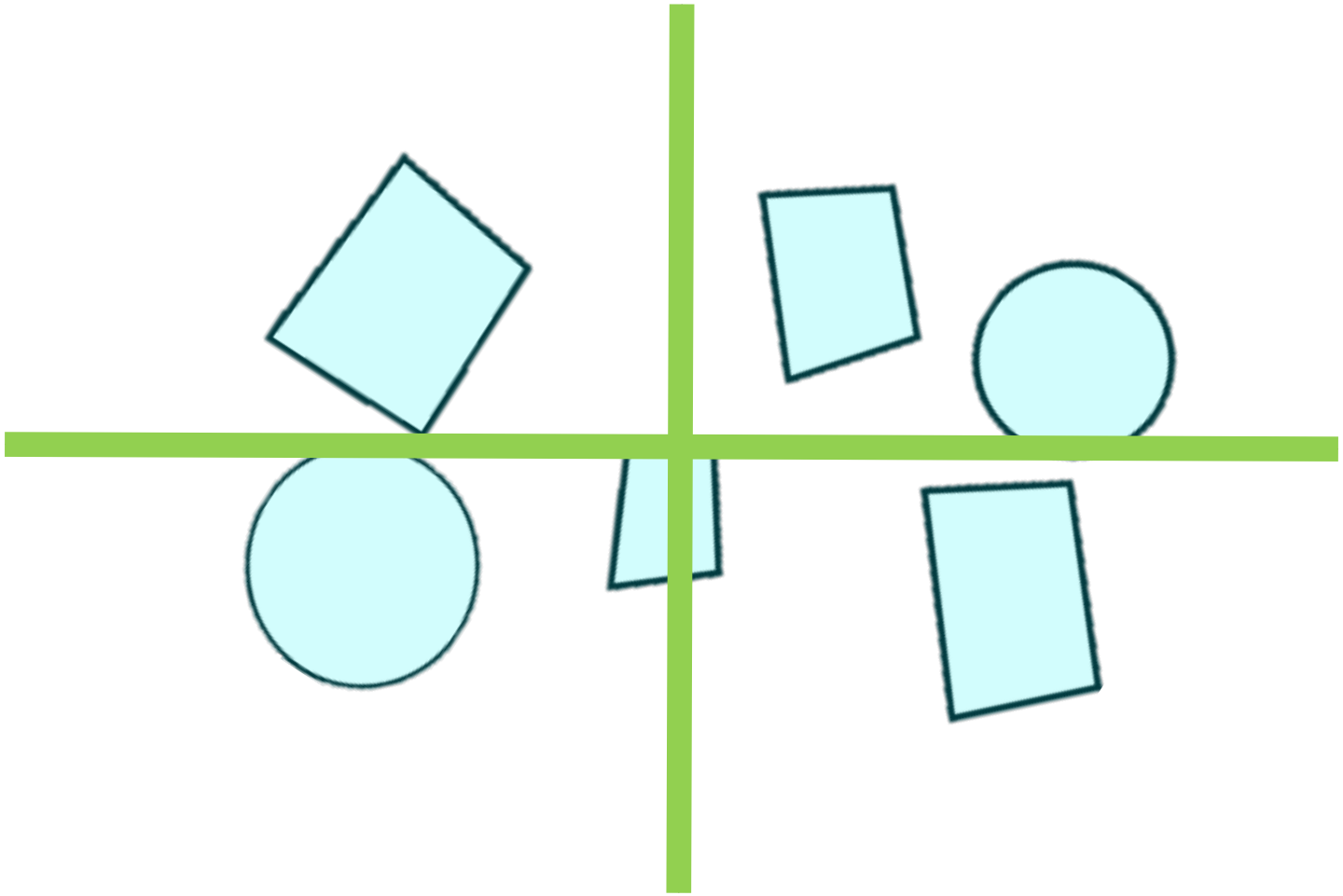
$$\mathbf{c} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{v}^i$$

$$\begin{bmatrix} \mathbf{u} & \mathbf{v} \end{bmatrix}$$



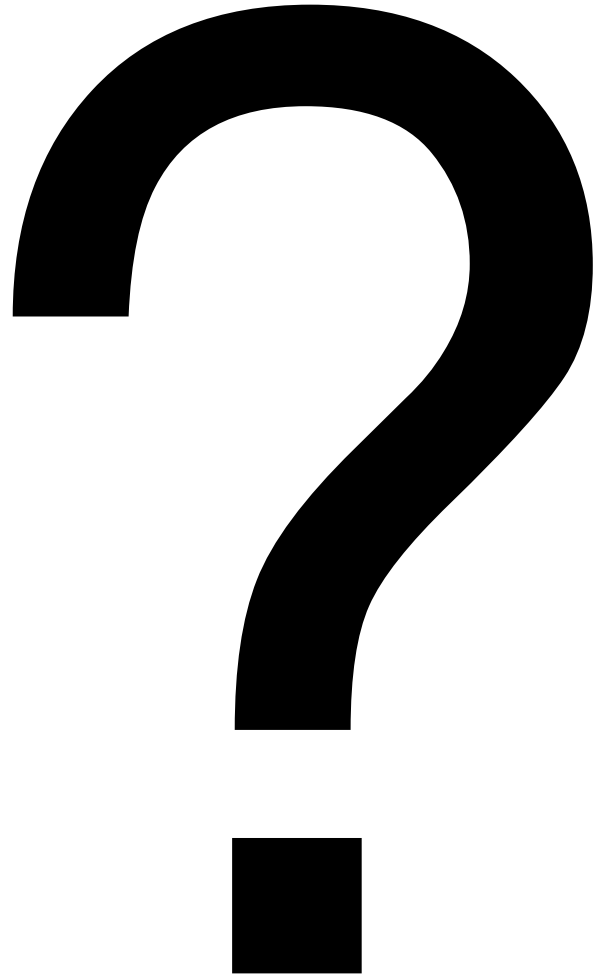
Find directions of maximum and minimum variance

Building an Object-Oriented Bounding Box (OOBB)



Build Rotation Matrix

Collision Query with Object-Oriented Bounding Box



Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

Two types of subdivisions – *object-based* and *spatial*

Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

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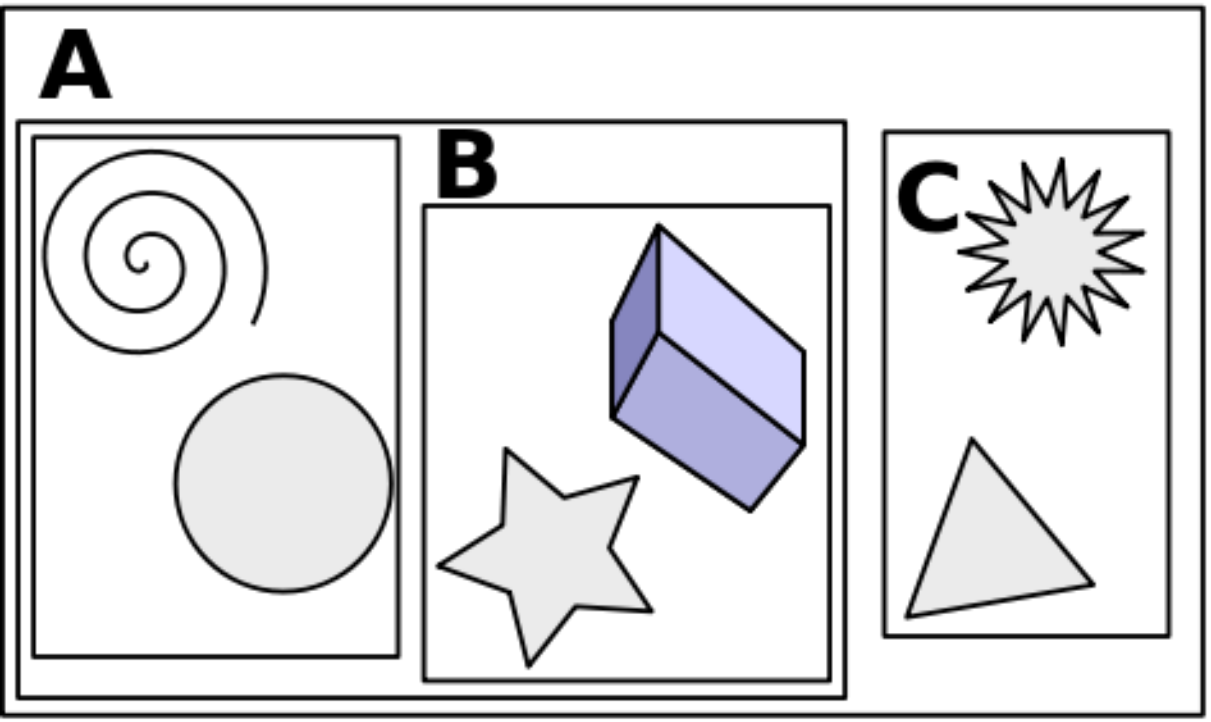
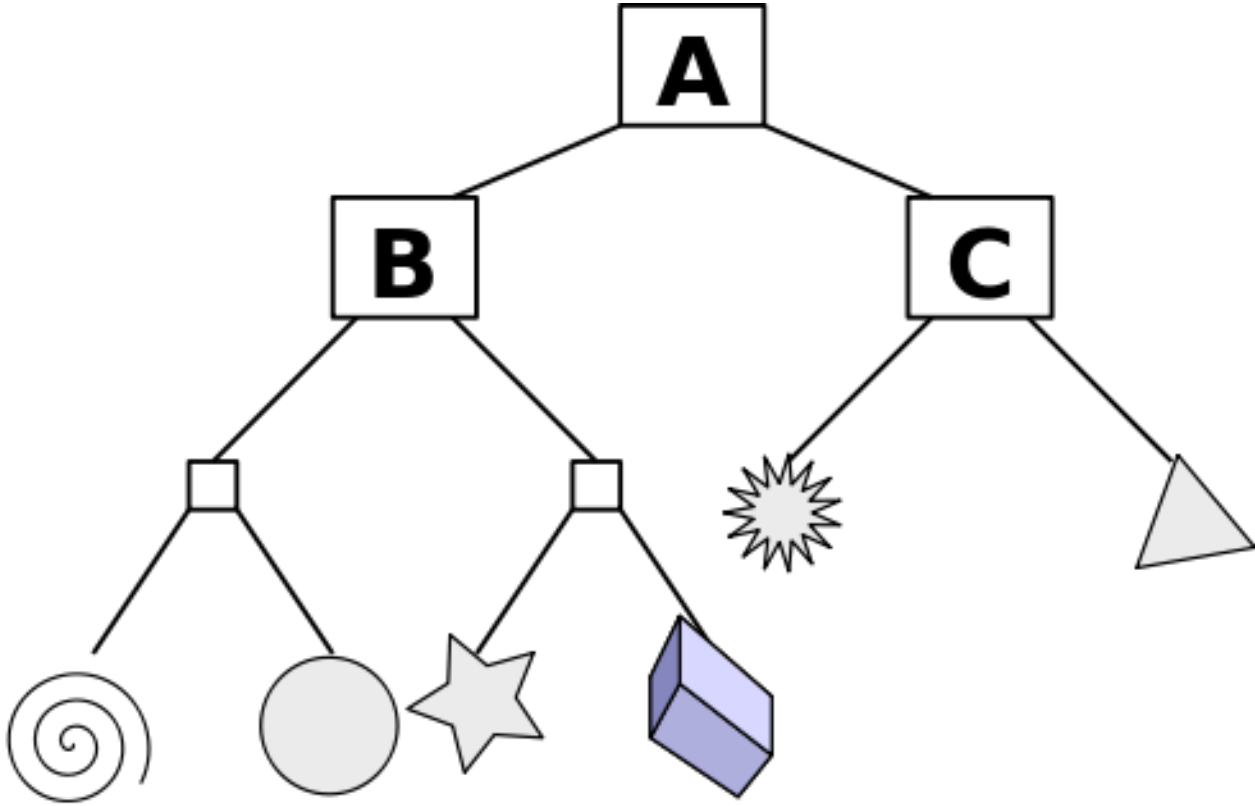
Our object-based data structures will be boundary volume hierarchies or BVHs.

BVHs are hierarchies of BVs represented by trees

Boundary Volume Hierarchy

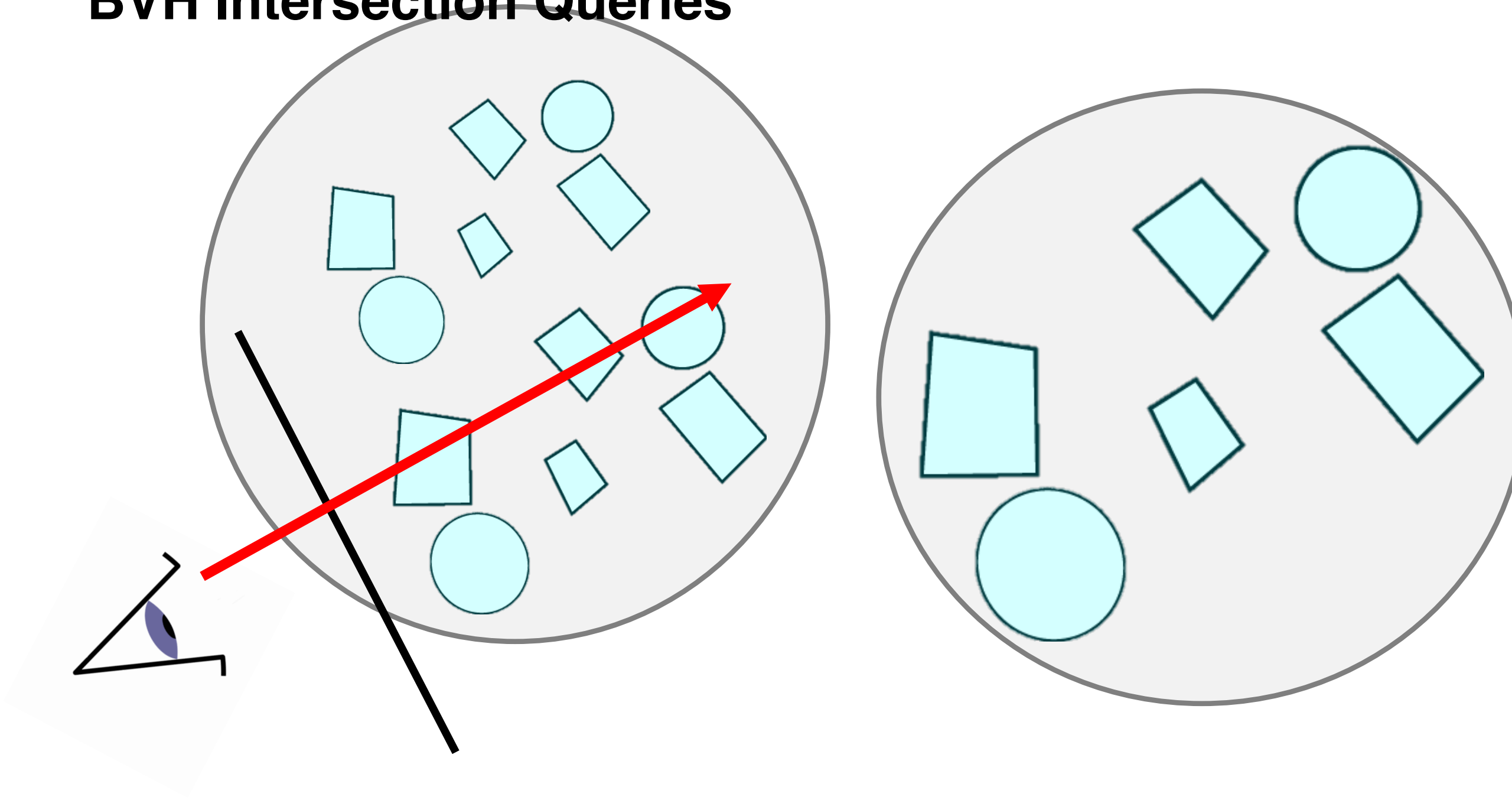




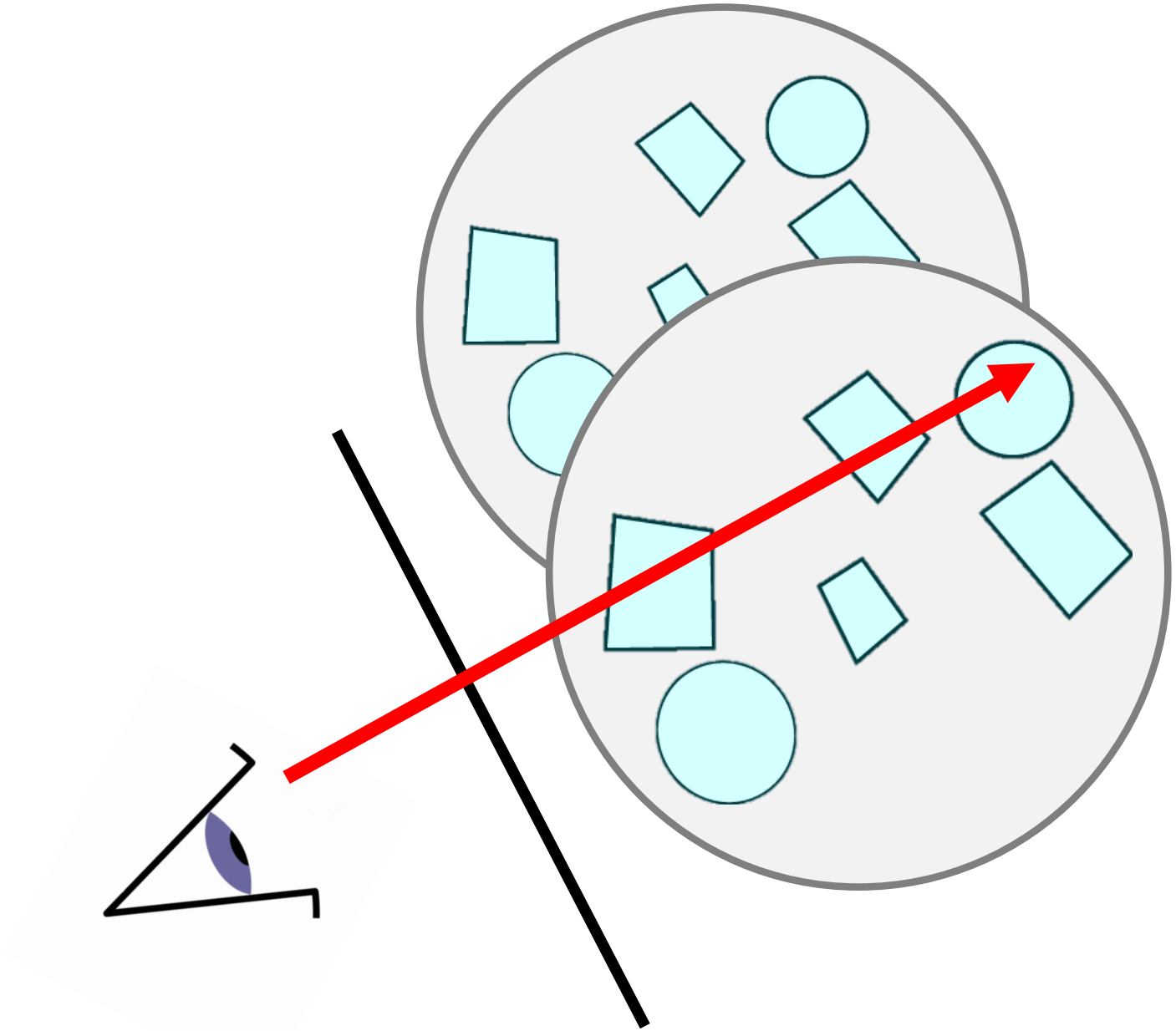
BVH Intersection Queries

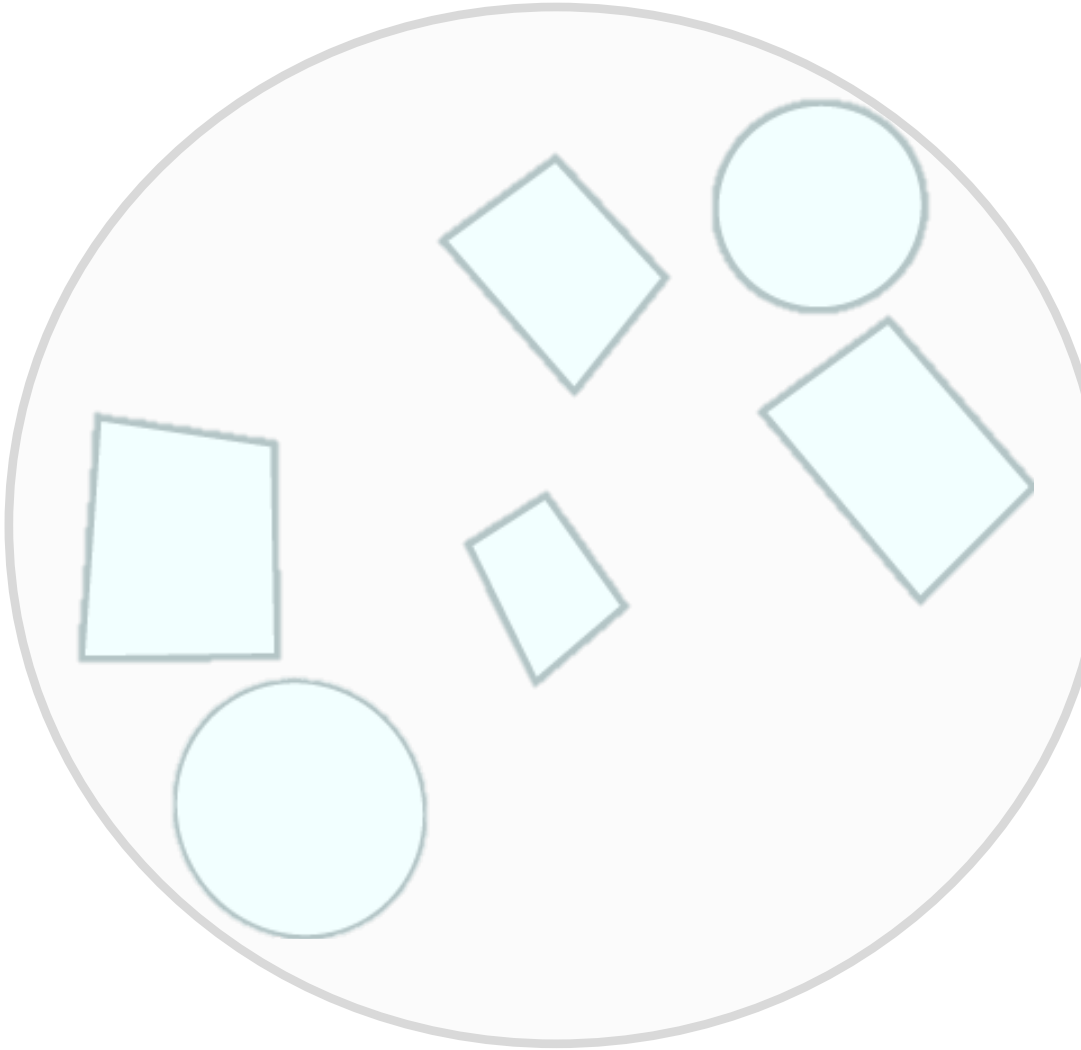
```
intersect(bvNode, ray,t)
{
    if (bvNode== null ll !bvNode.intersect(ray,t))
        return false;
    else
   {
      i1=intersect(bvNode.left, ray,t1);  //check left BV
      i2=intersect(bvNode.right ray,t2); //check right BC
      if (i1 && i2) {  t=min(t1,t2); return true;  }
      if (i1) {  t=t1; return true;  }
      if (i2) {  t=t2; return true;  }
      return false;
    }
}
```

BVH Intersection Queries

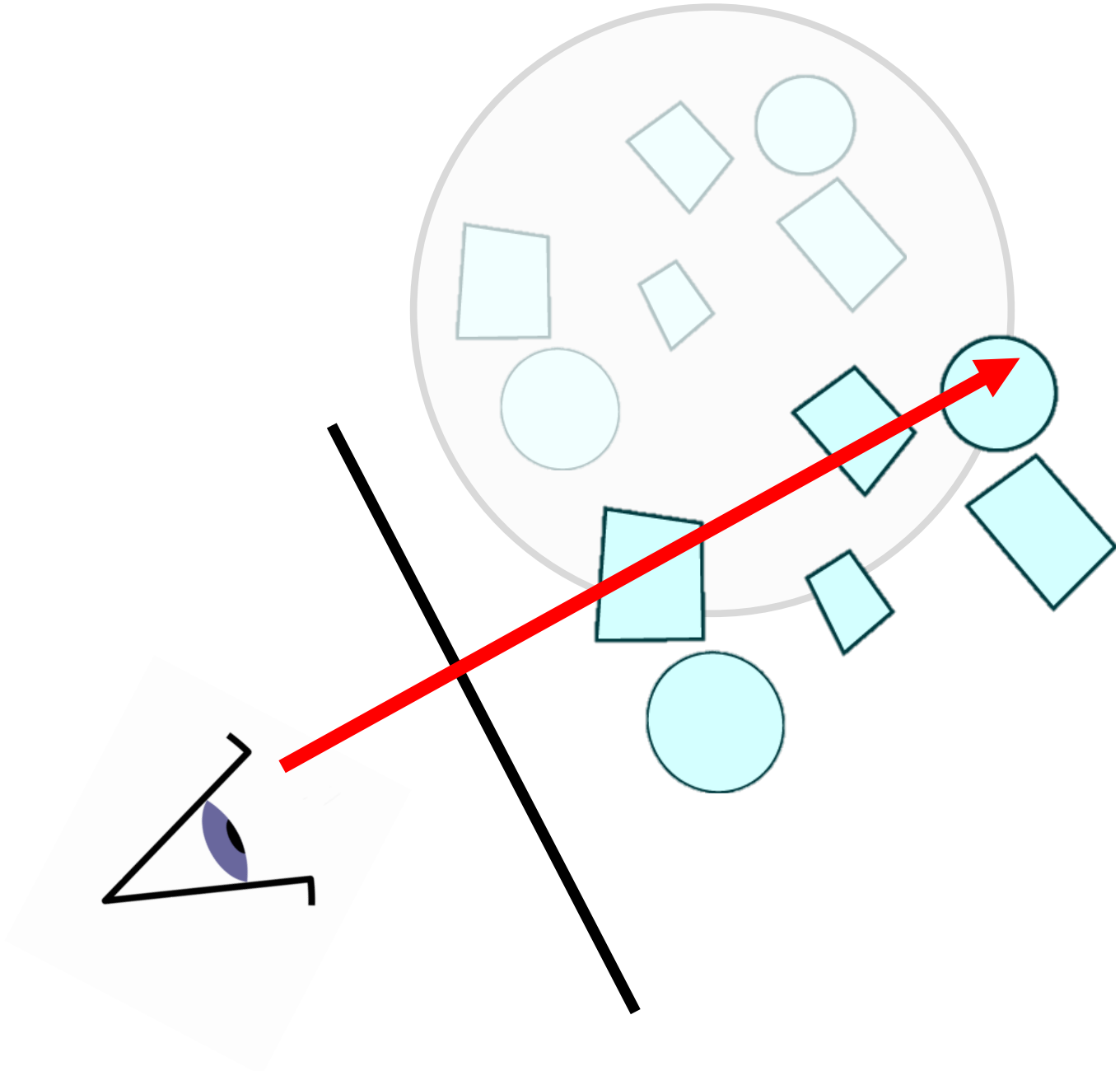


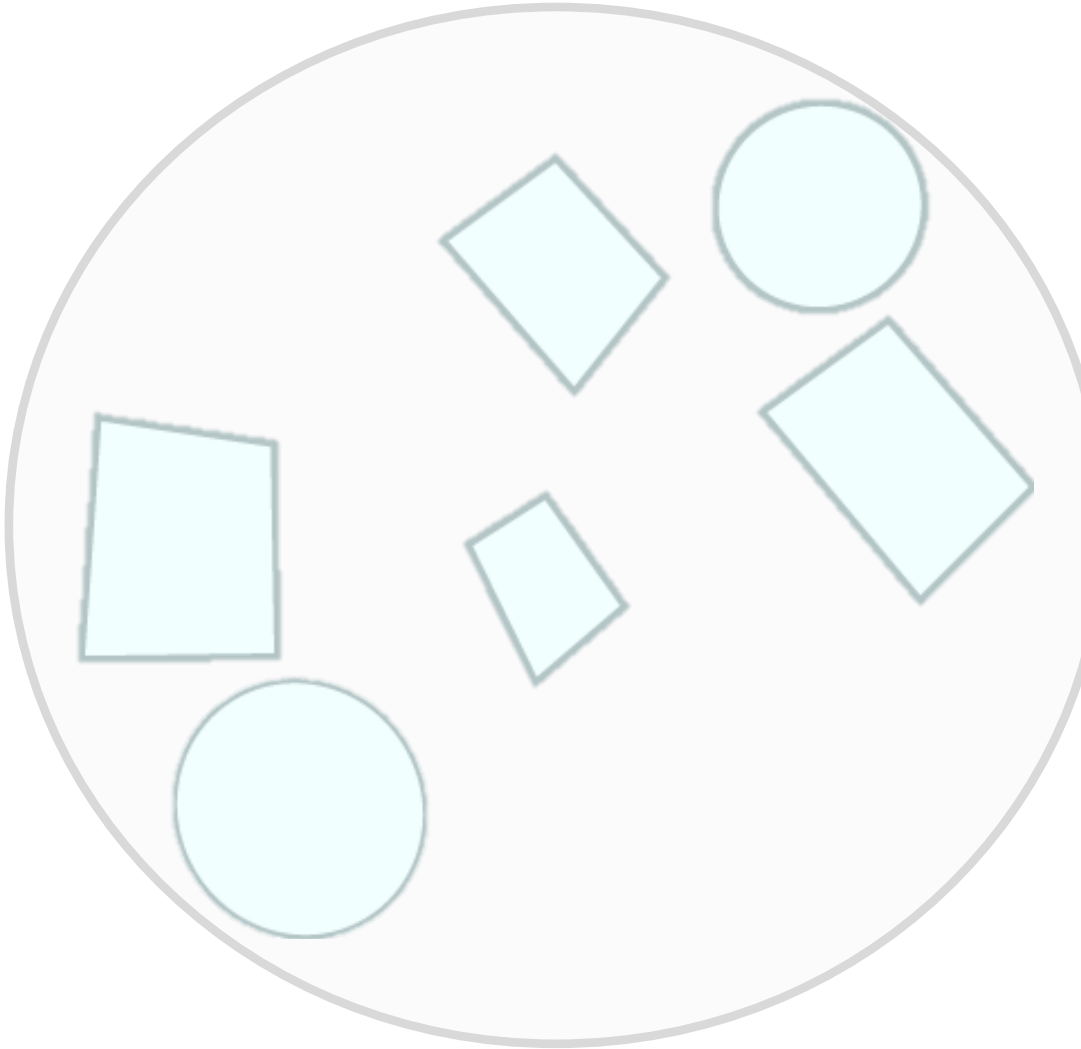
BVH Intersection Queries





BVH Intersection Queries

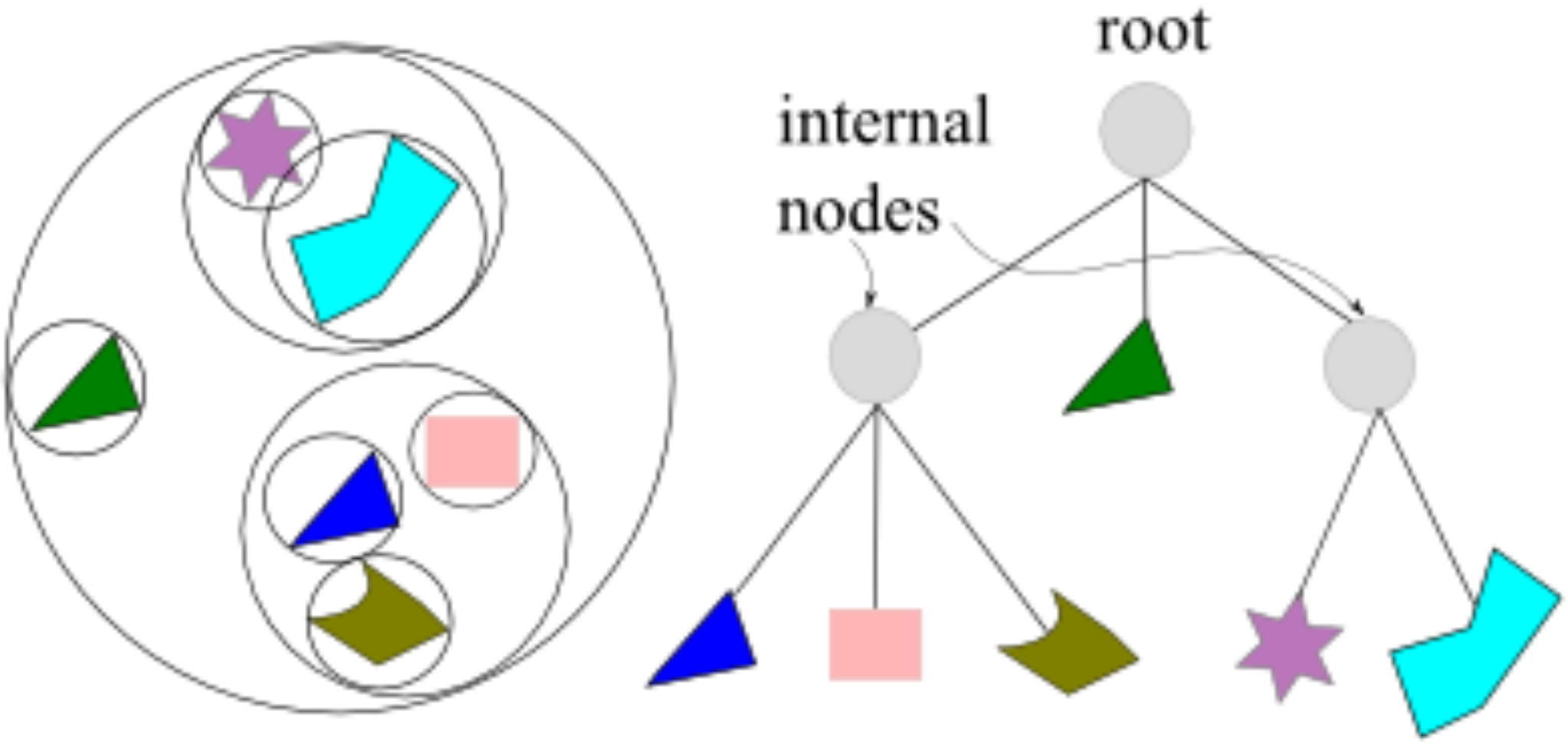




BVH Distance Queries

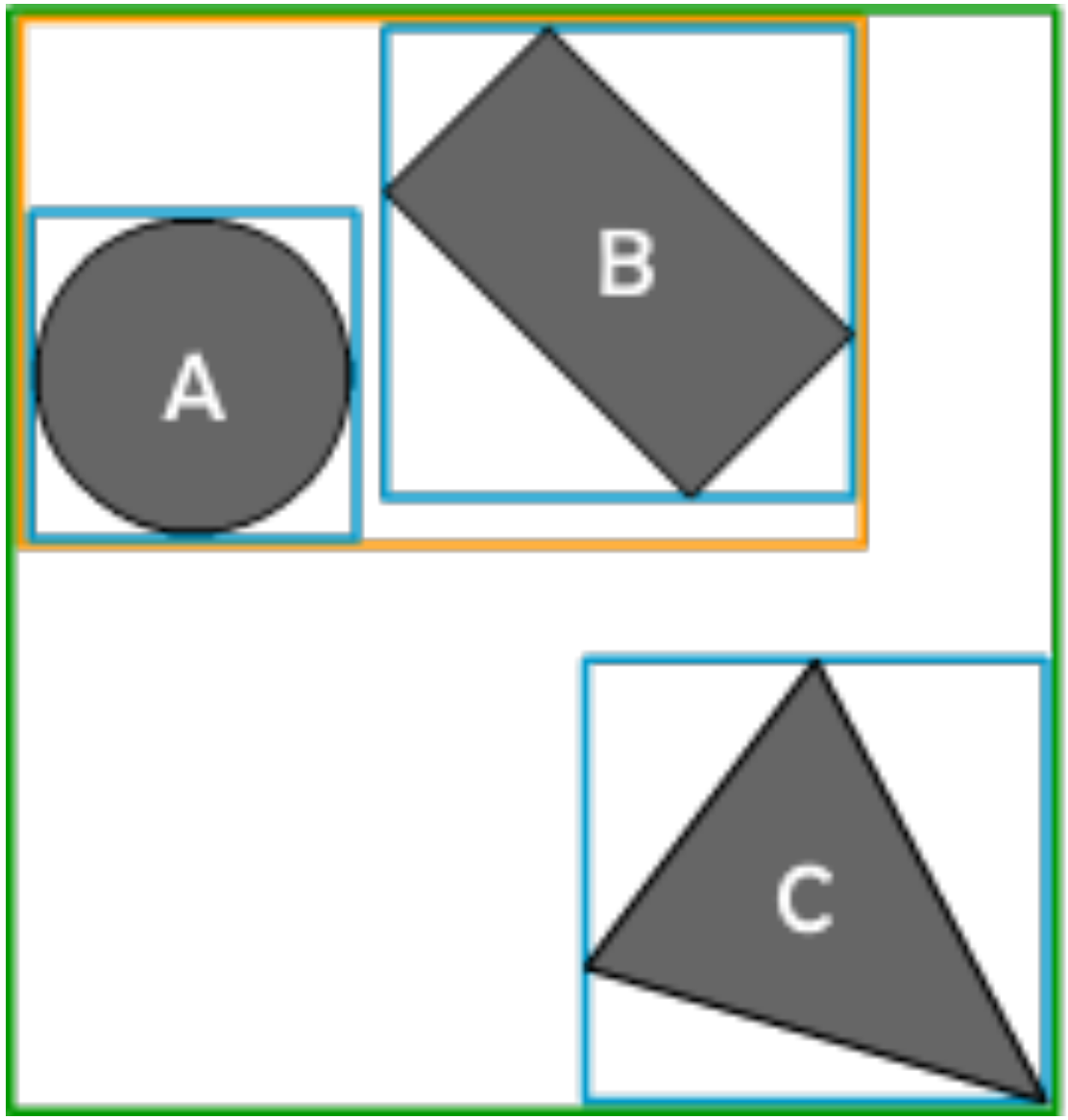
```
minDistance(bvNode, point, currentMin)
{
   d1=minDistance(bvNode.left, point, currentMin);
   d2=minDistance(bvNode.right, point, currentMin);

    if(min(d1,d2) > currentMin) {
       return currentMin
    }

    return min(d1,d2)
}
```

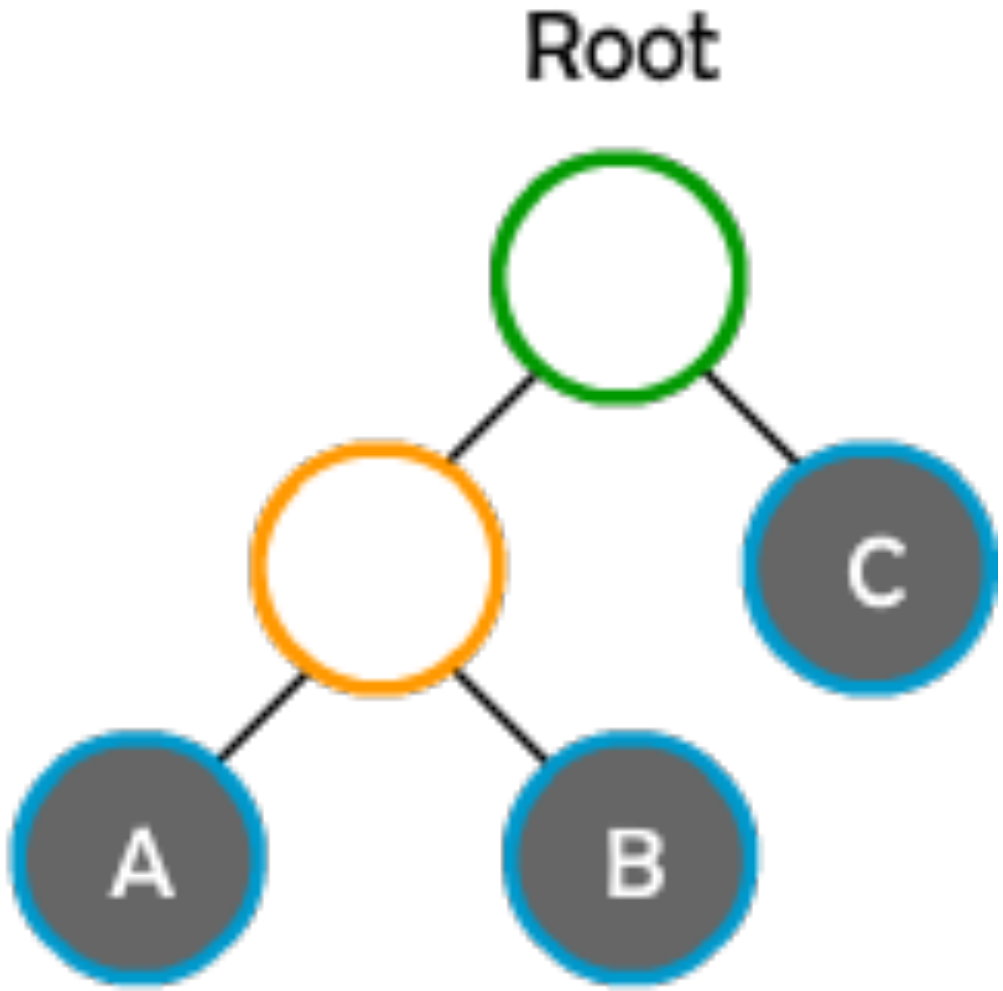
Sphere Trees



https://www.bogotobogo.com/Games/spatialdatastructure.php

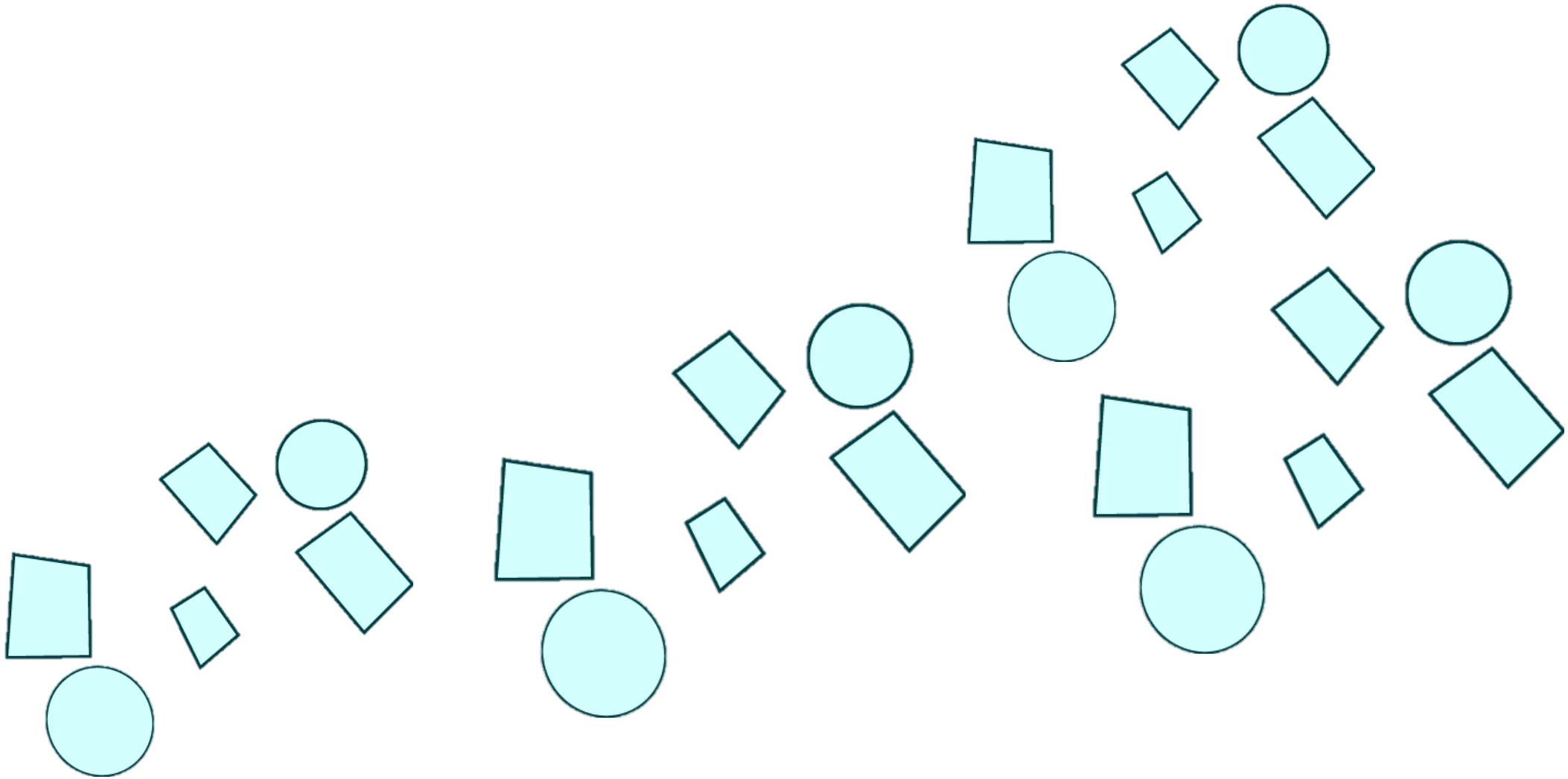
AABB Trees



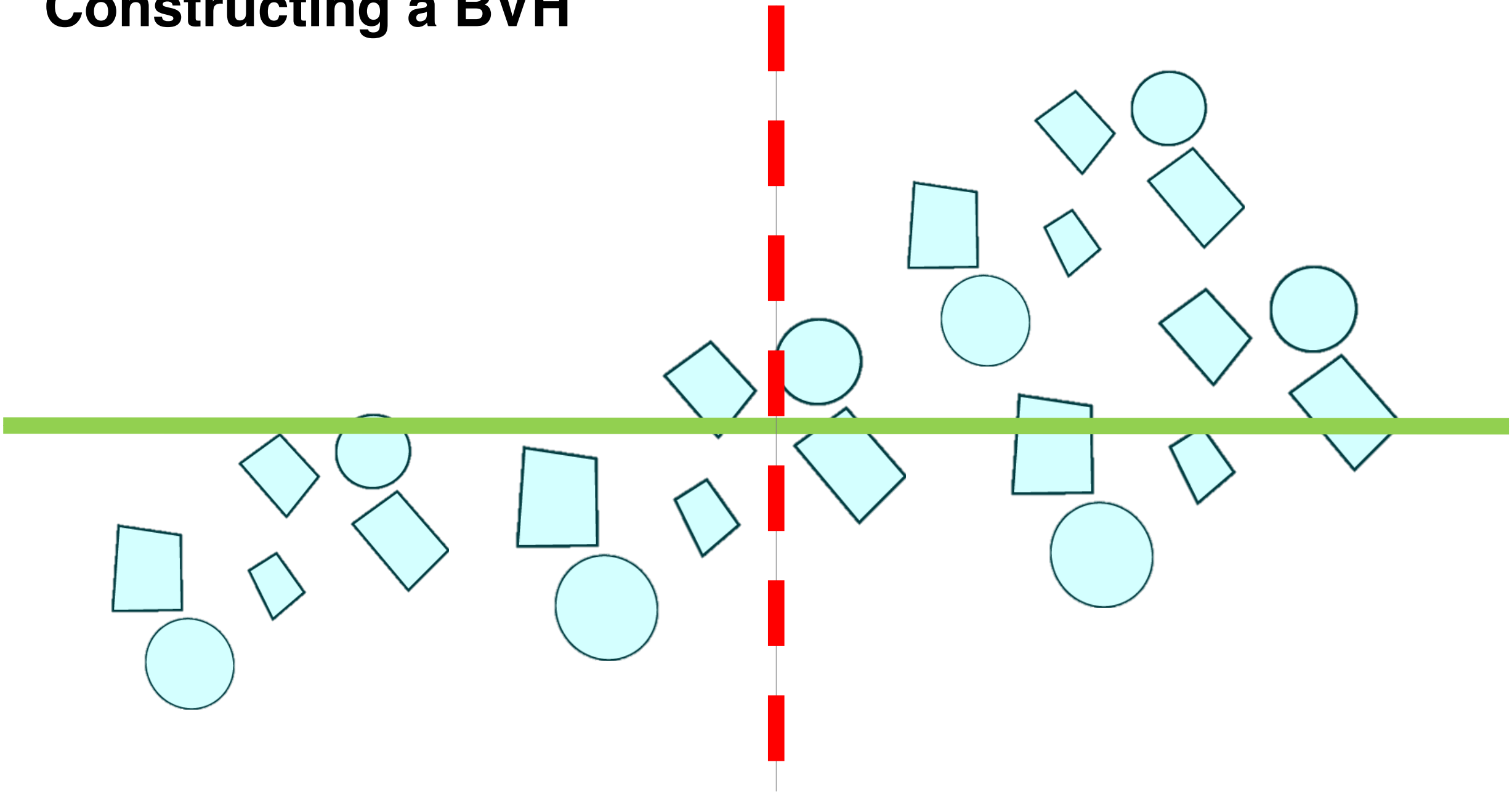


http://allenchou.net/2014/02/game-physics-broadphase-dynamic-aabb-tree/

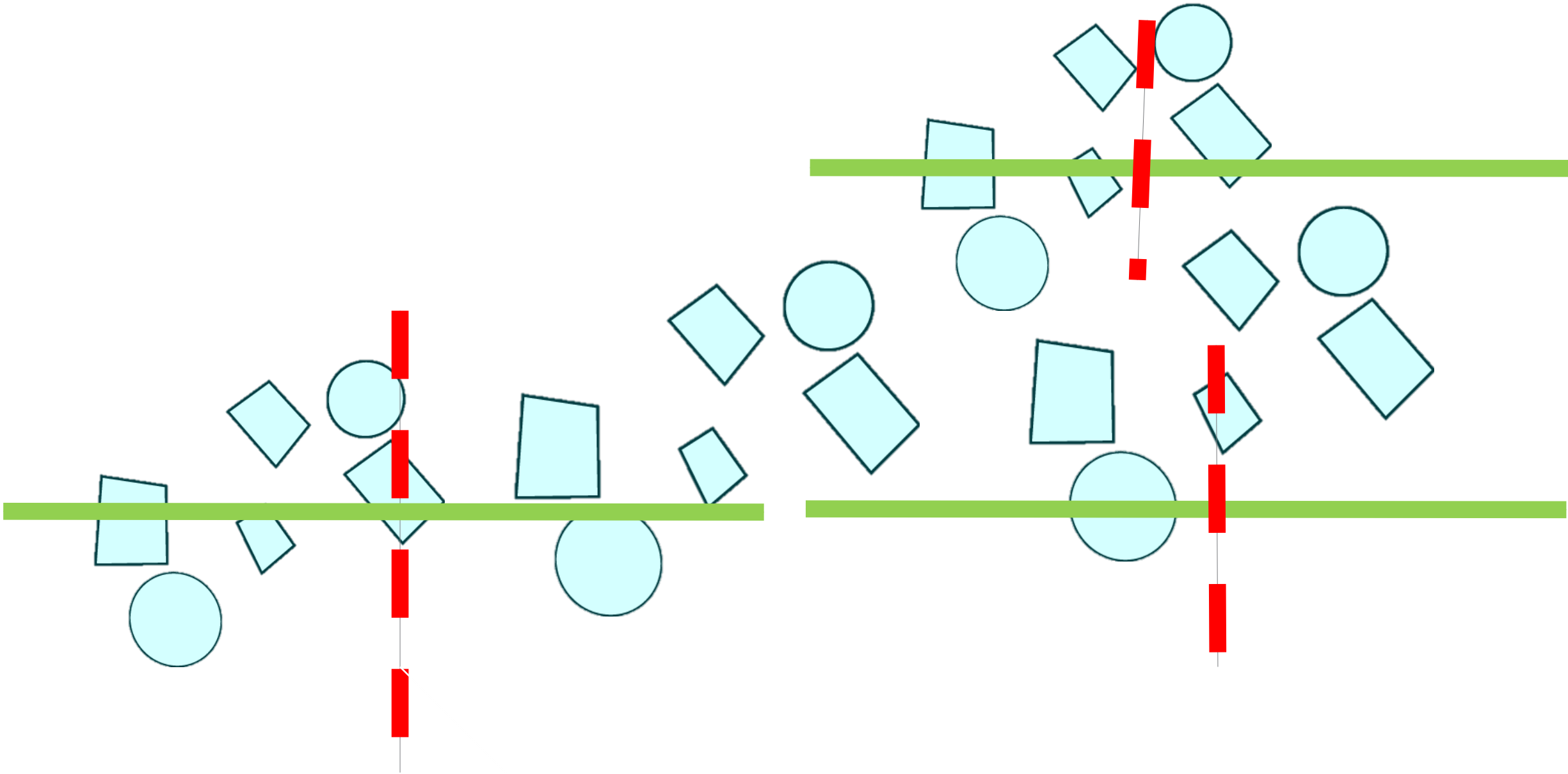
Constructing a BVH



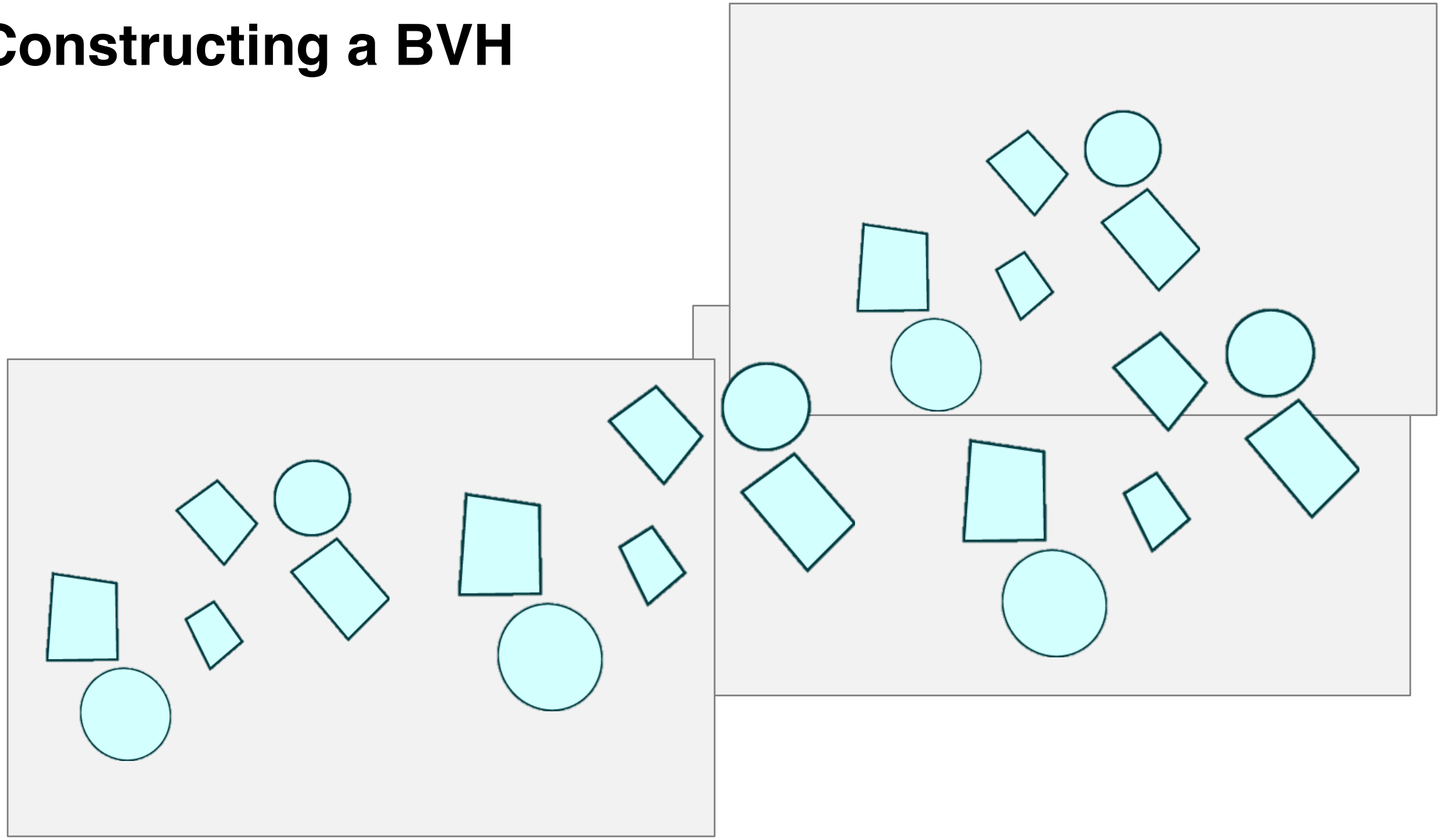
Constructing a BVH



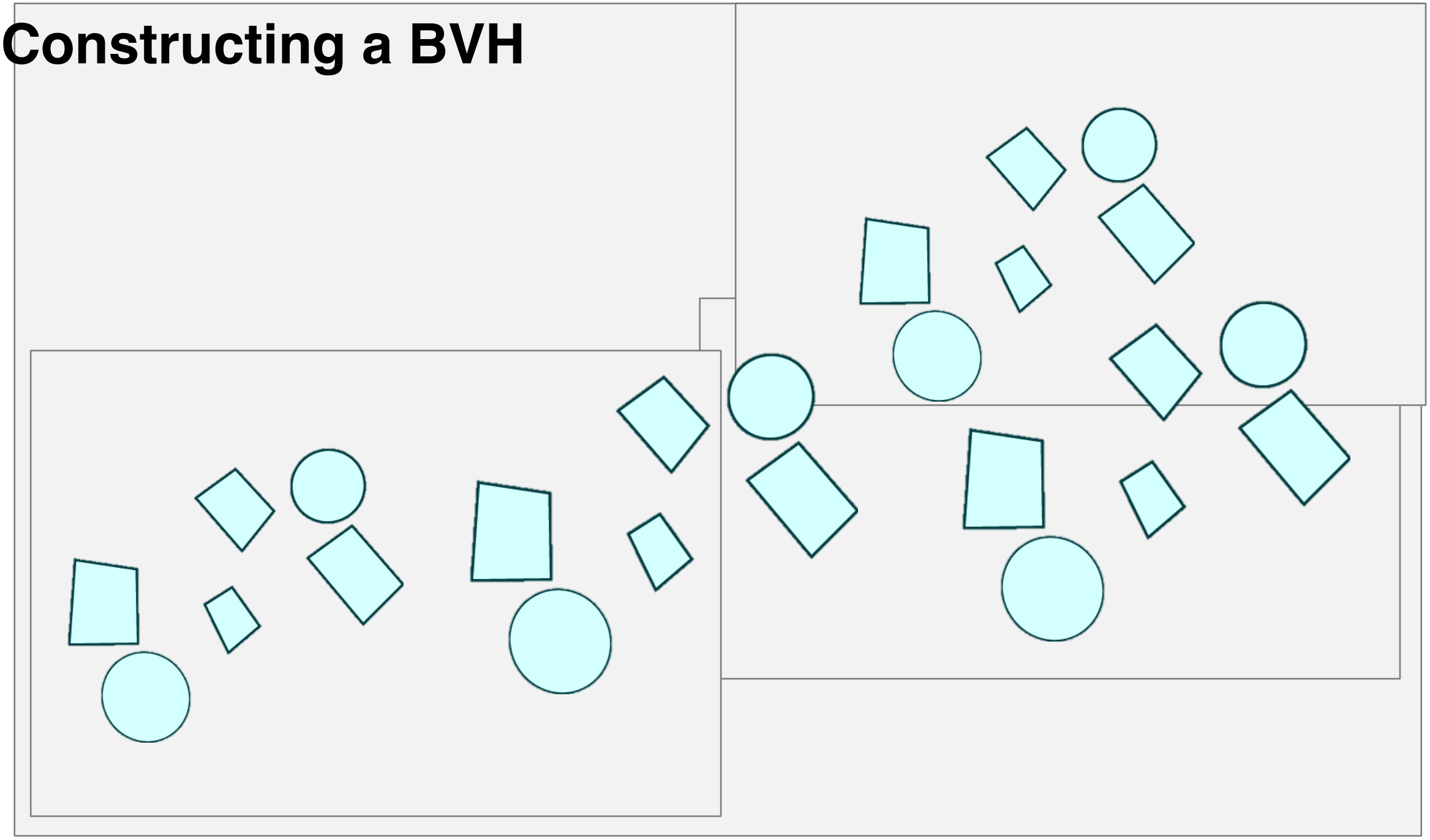
Constructing a BVH



Constructing a BVH



Constructing a BVH



Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

Two types of subdivisions – ***object-based*** and *spatial*

Our object-based data structures will be boundary volume hierarchies or BVHs.

BVHs are hierarchies of BVs represented by trees

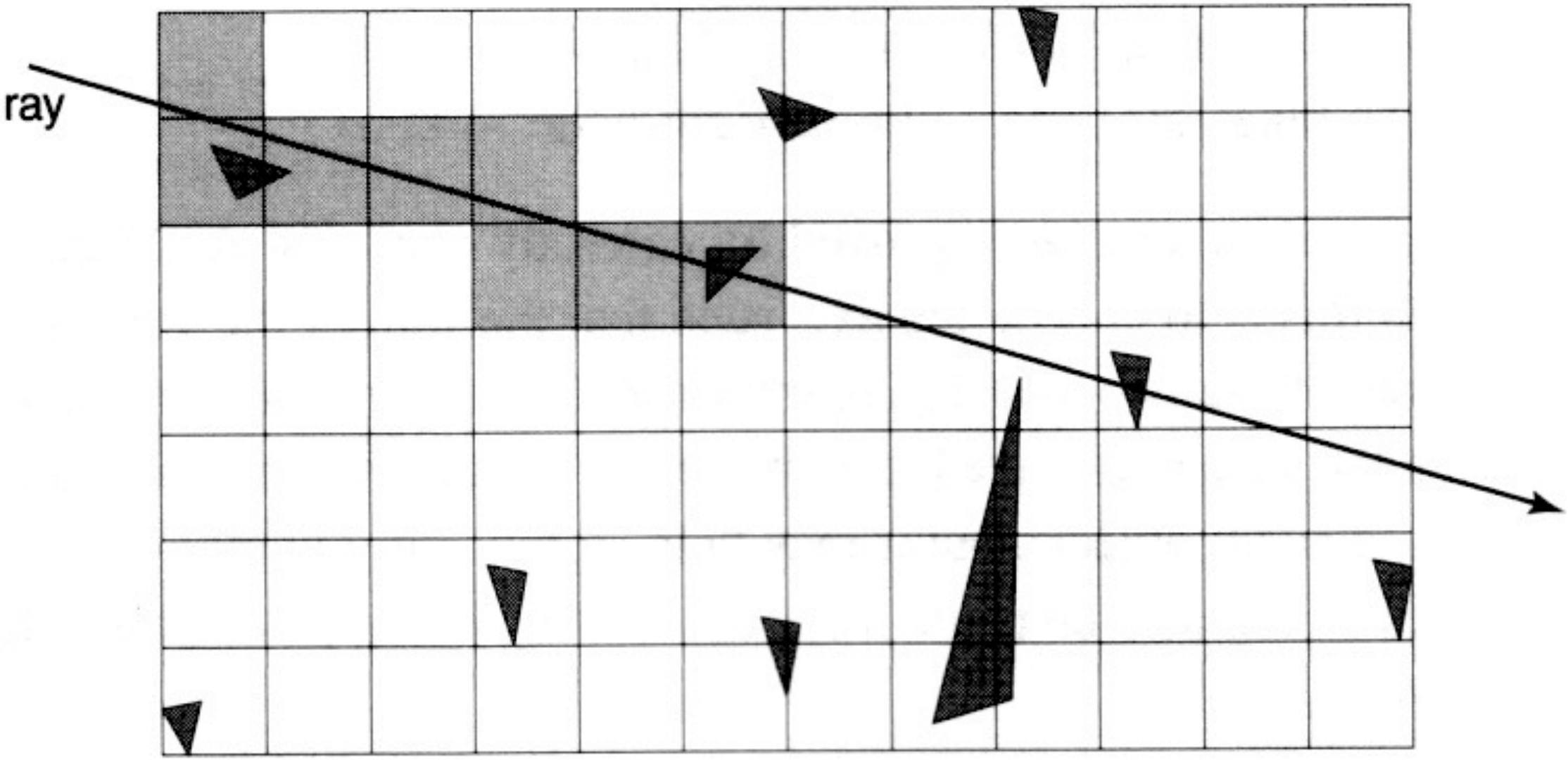
Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

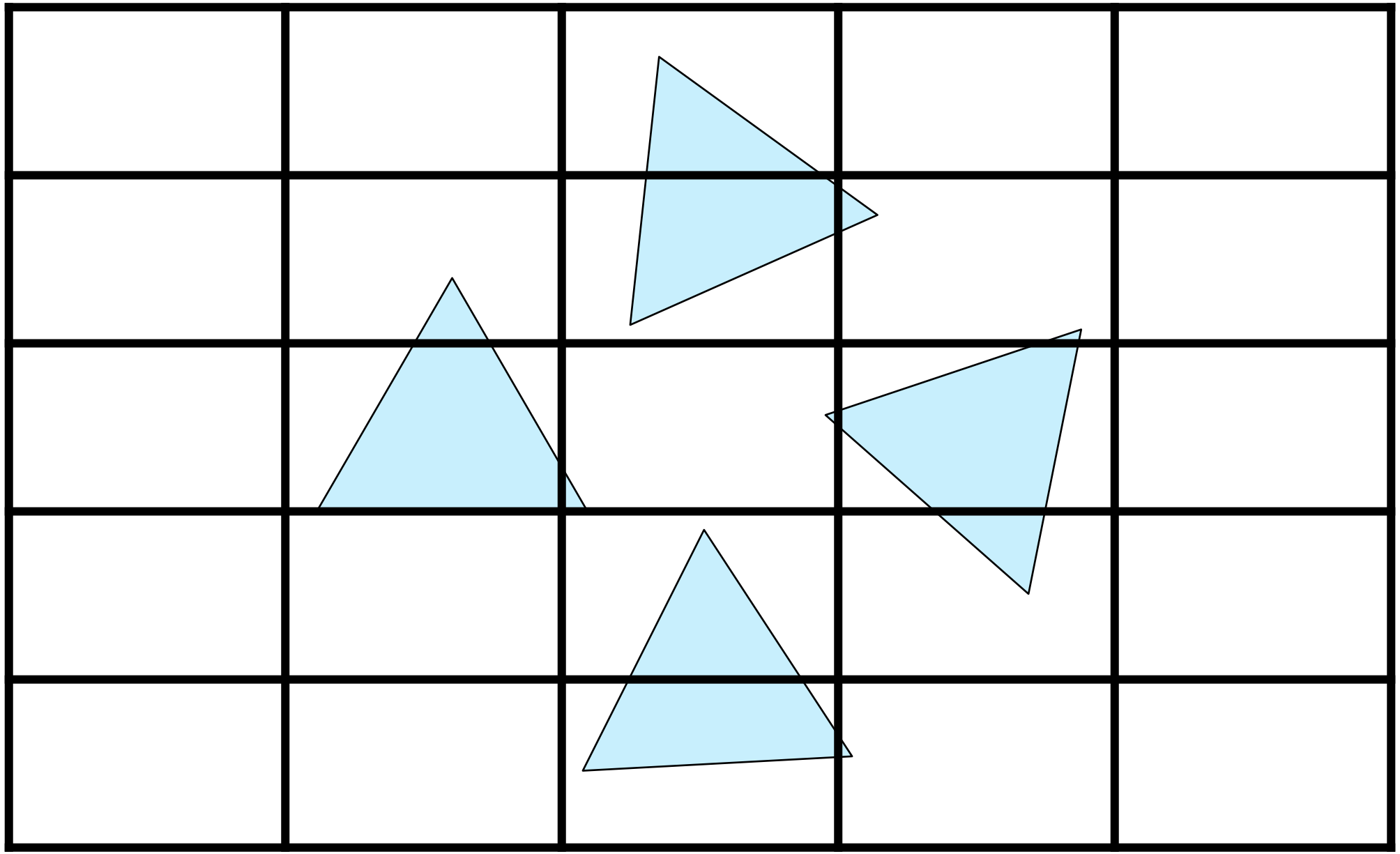
Two types of subdivisions – ***object-based*** and *spatial*

Spatial subdivision divides space hierarchically and represents this as a tree.

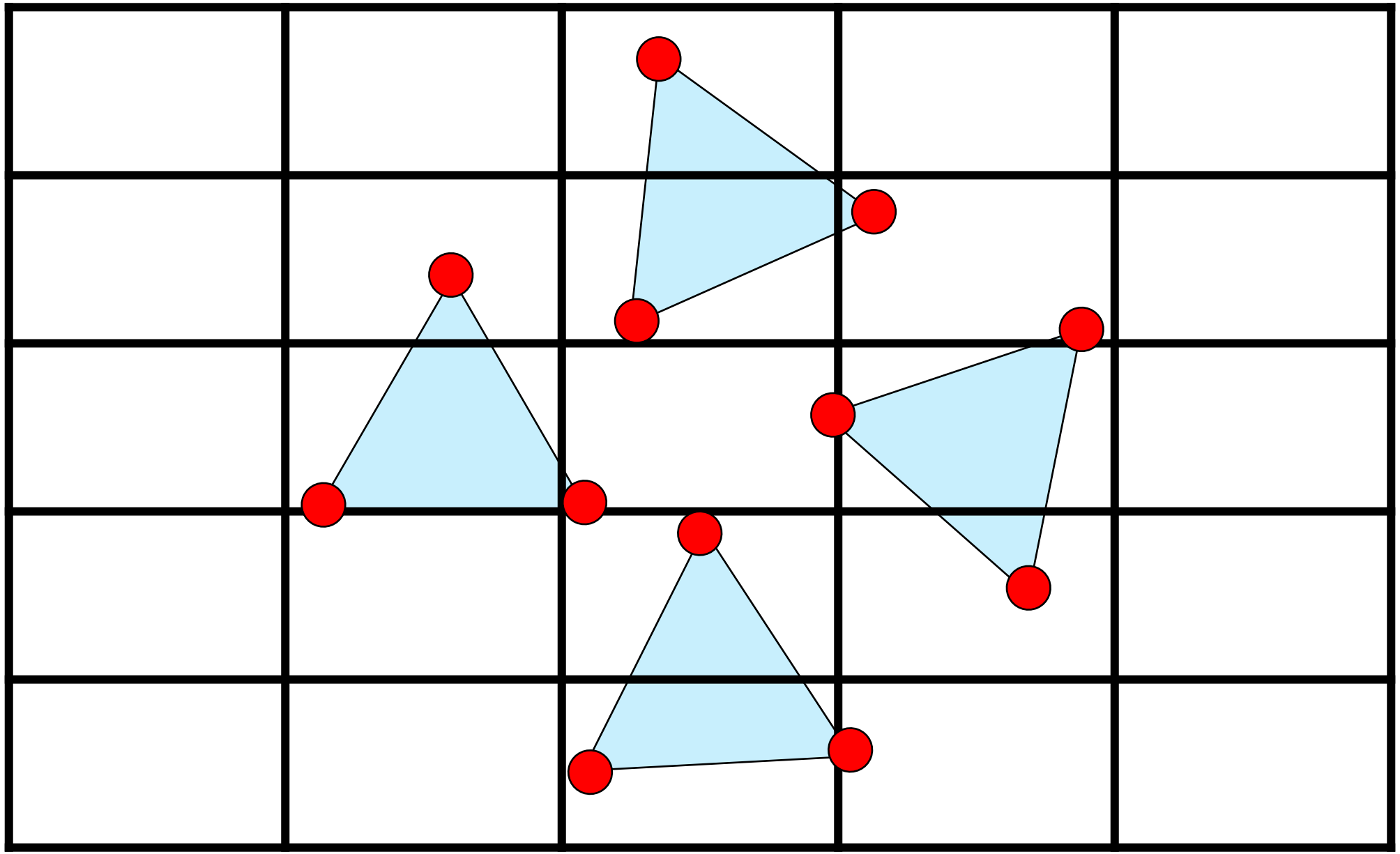
Axis-Aligned Spatial Subdivision (Uniform)



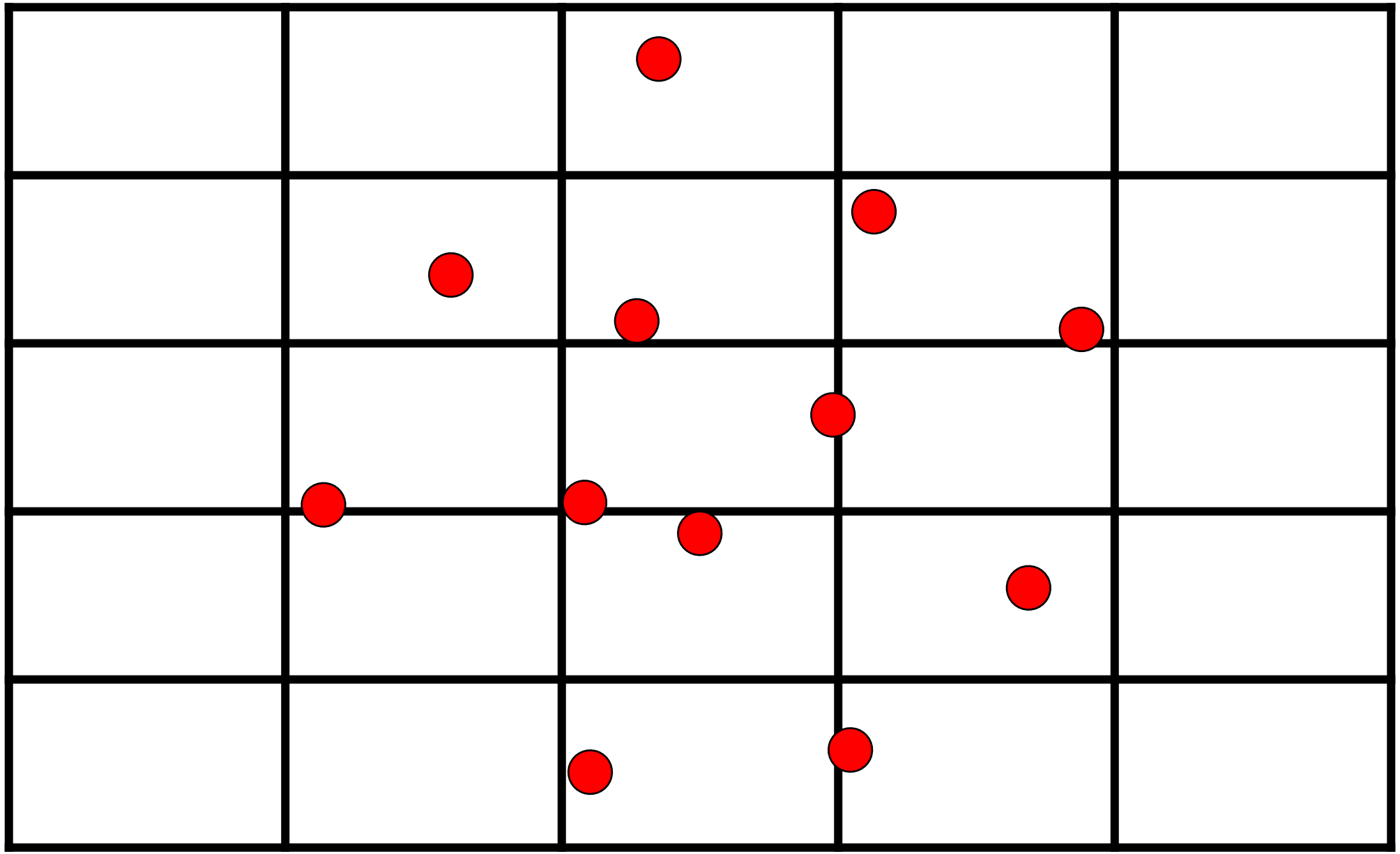
Construction



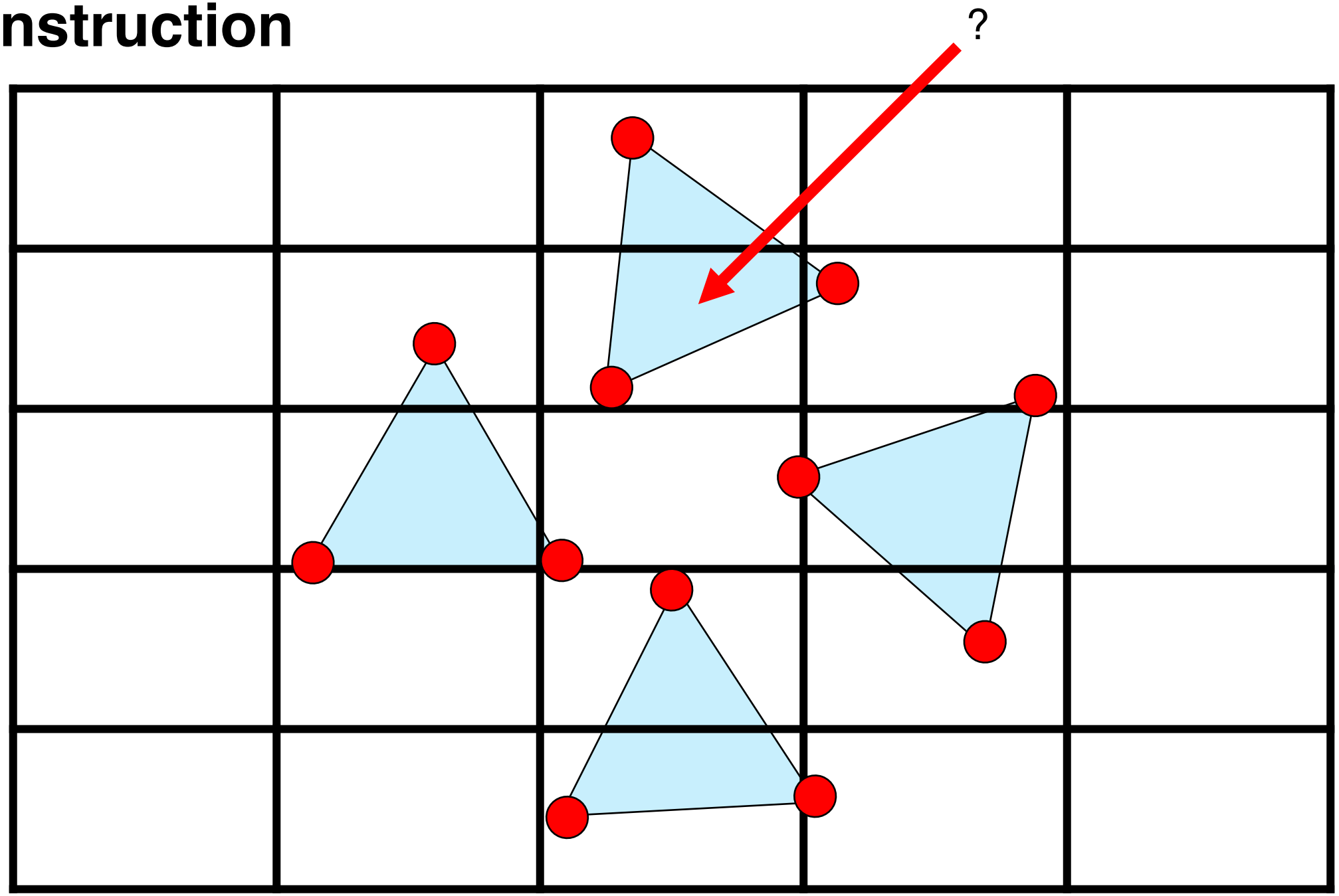
Construction



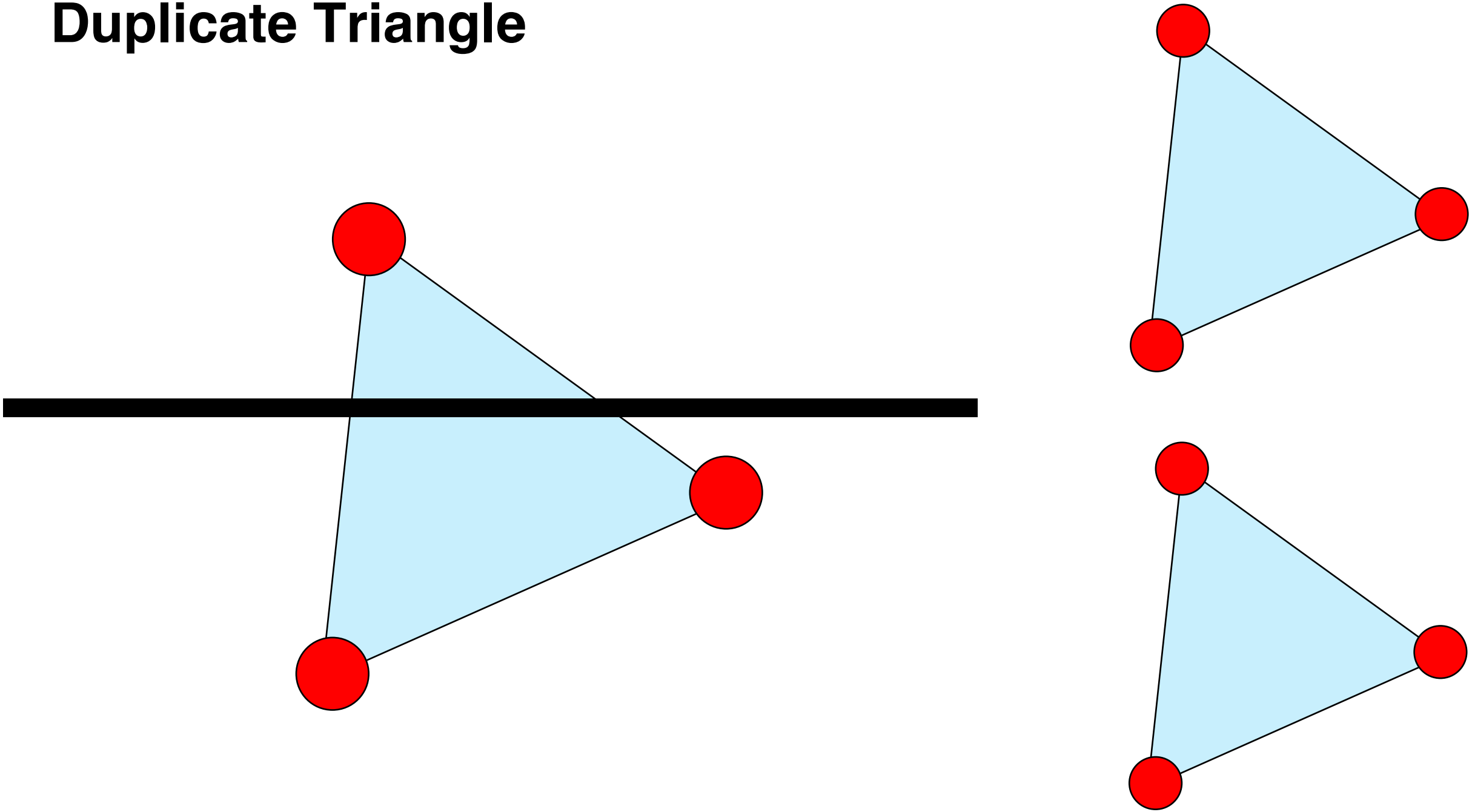
Construction



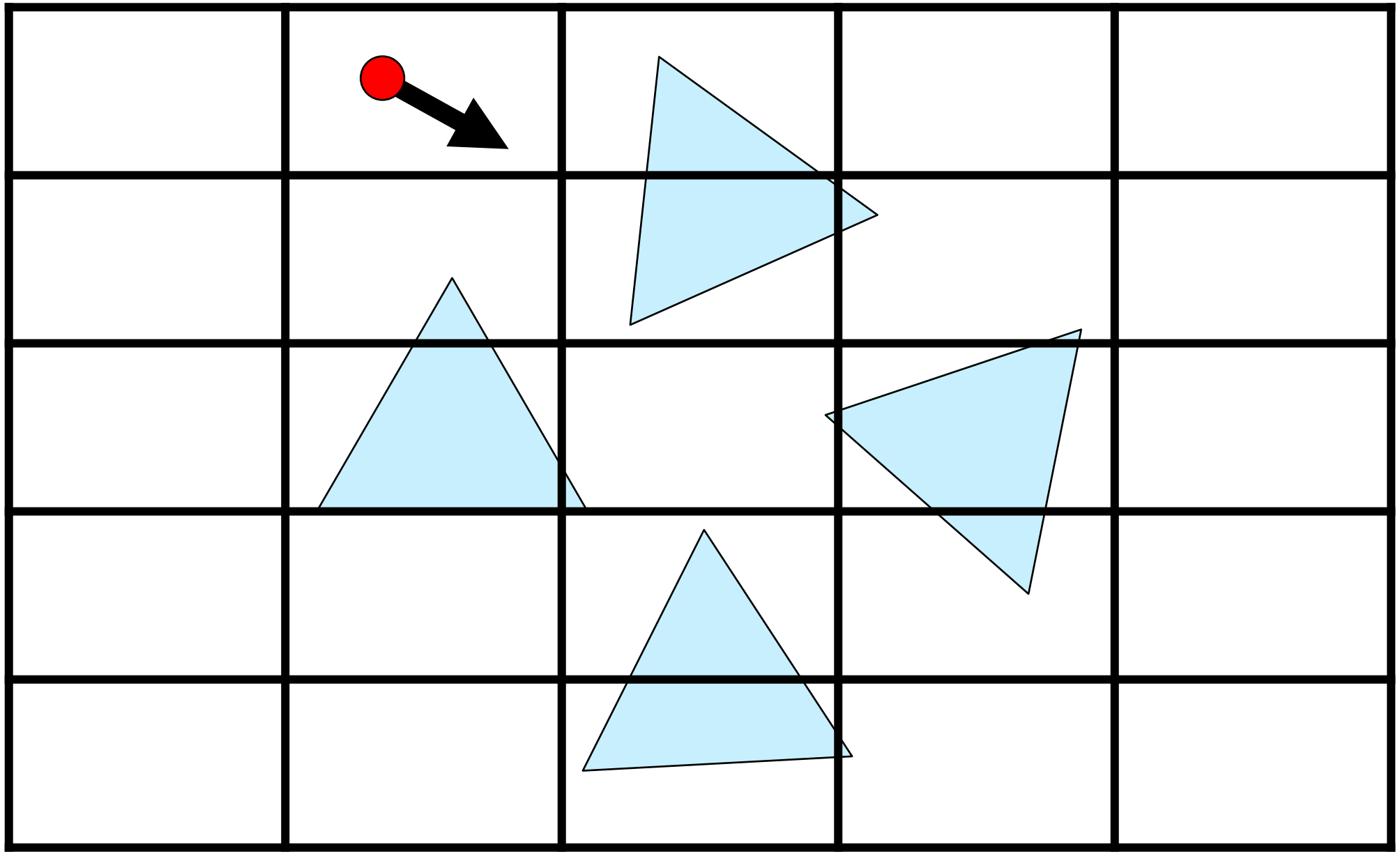
Construction



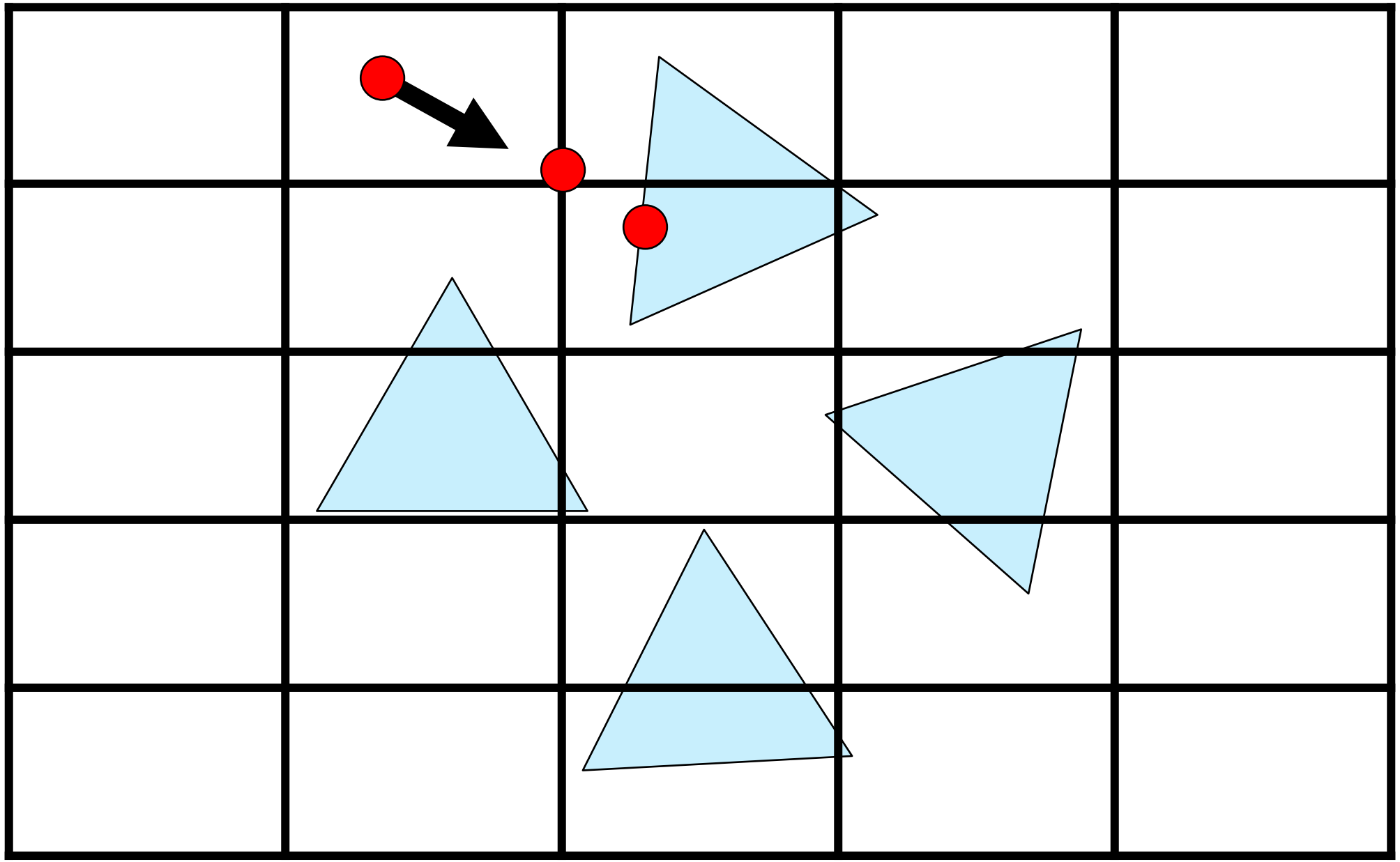
Duplicate Triangle



Intersection Tests

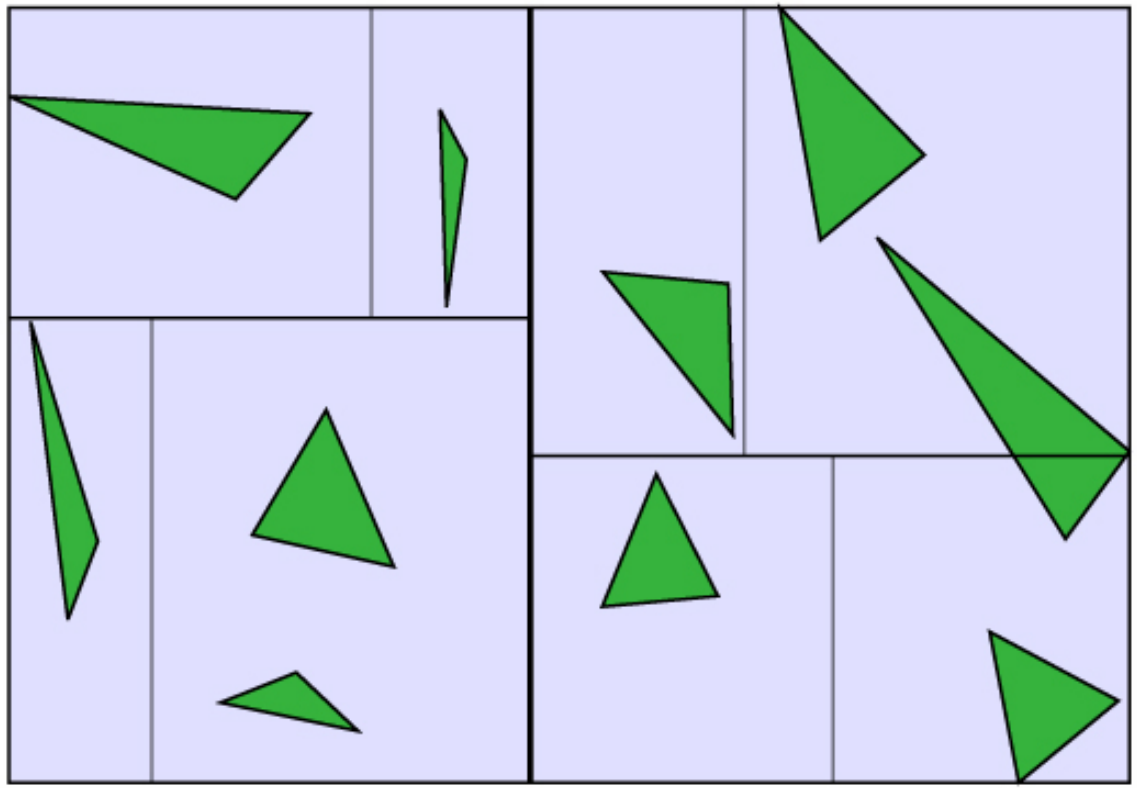


Intersection Tests

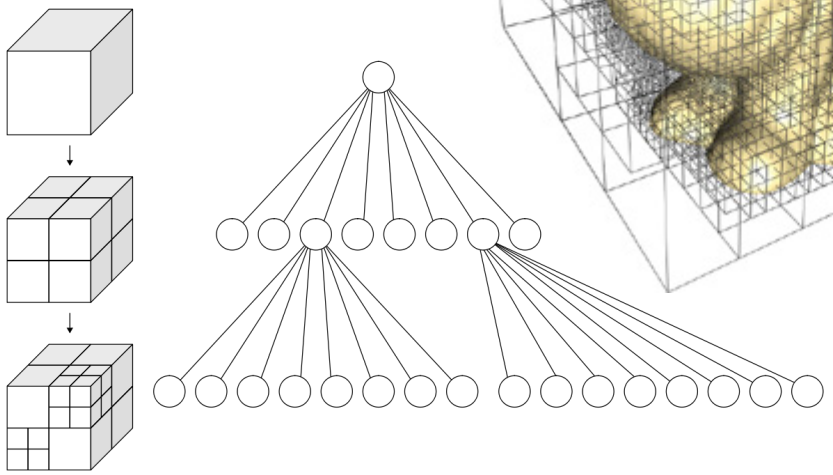


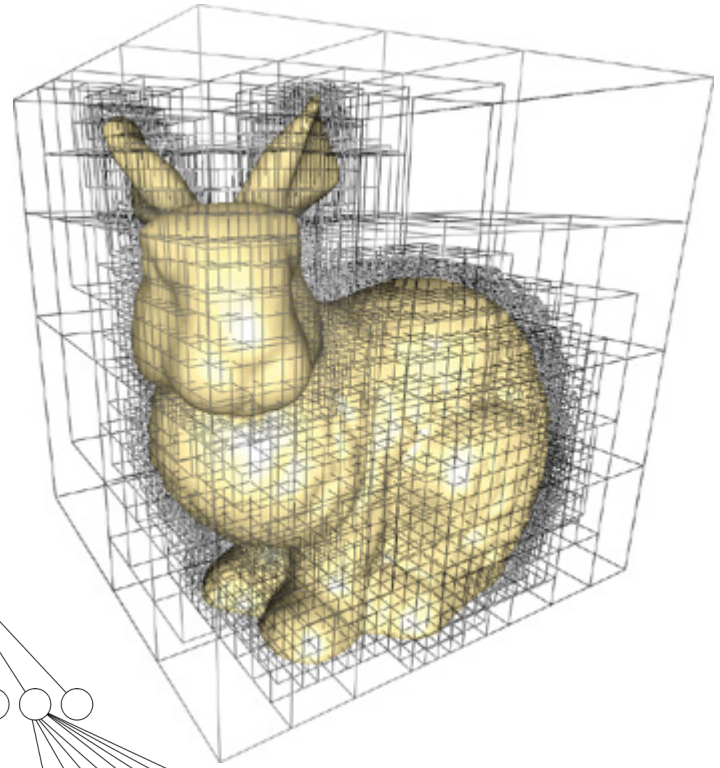
Axis-Aligned Spatial Subdivision (Non-Uniform)

BSP Tree

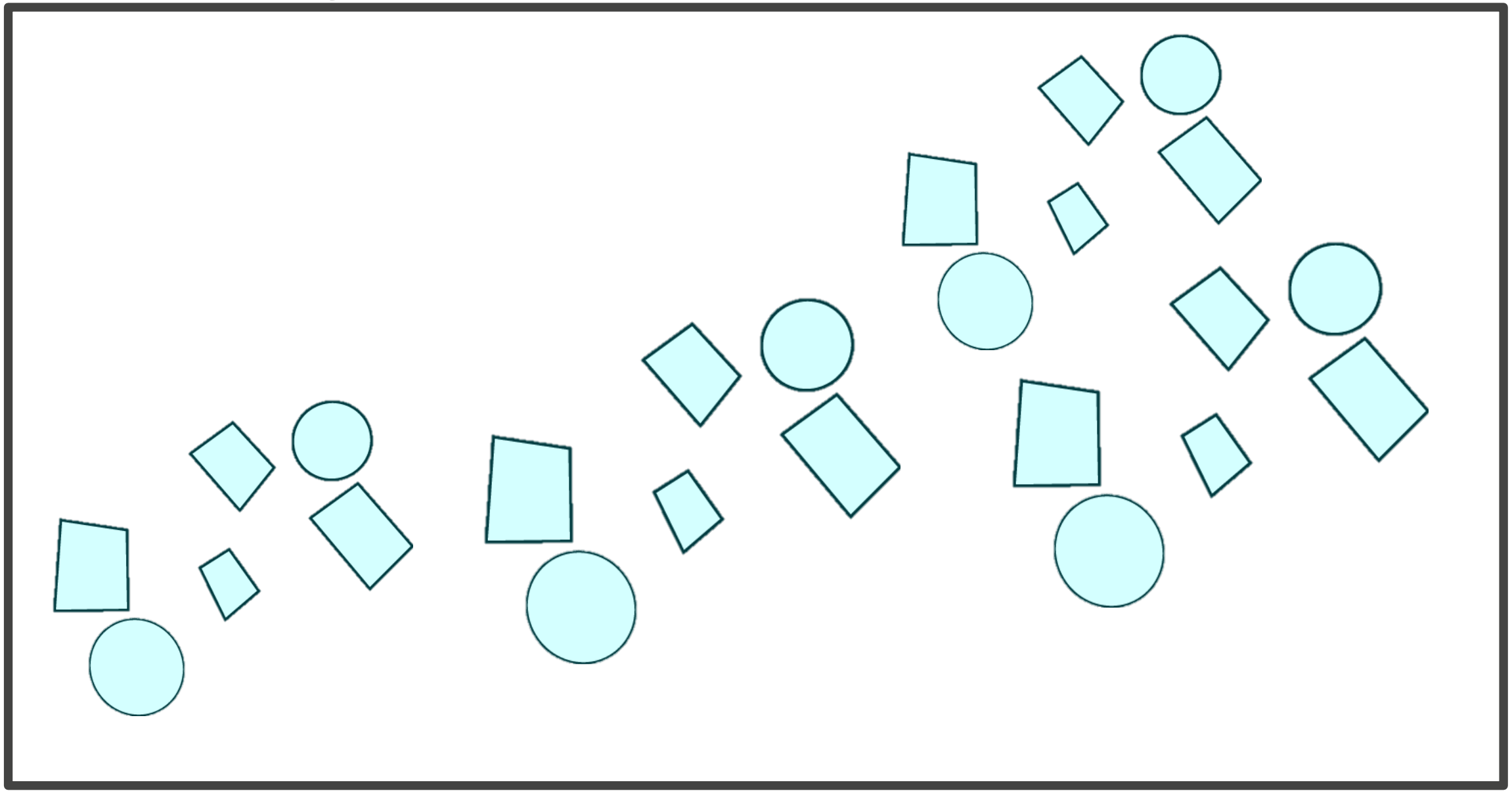


Octree

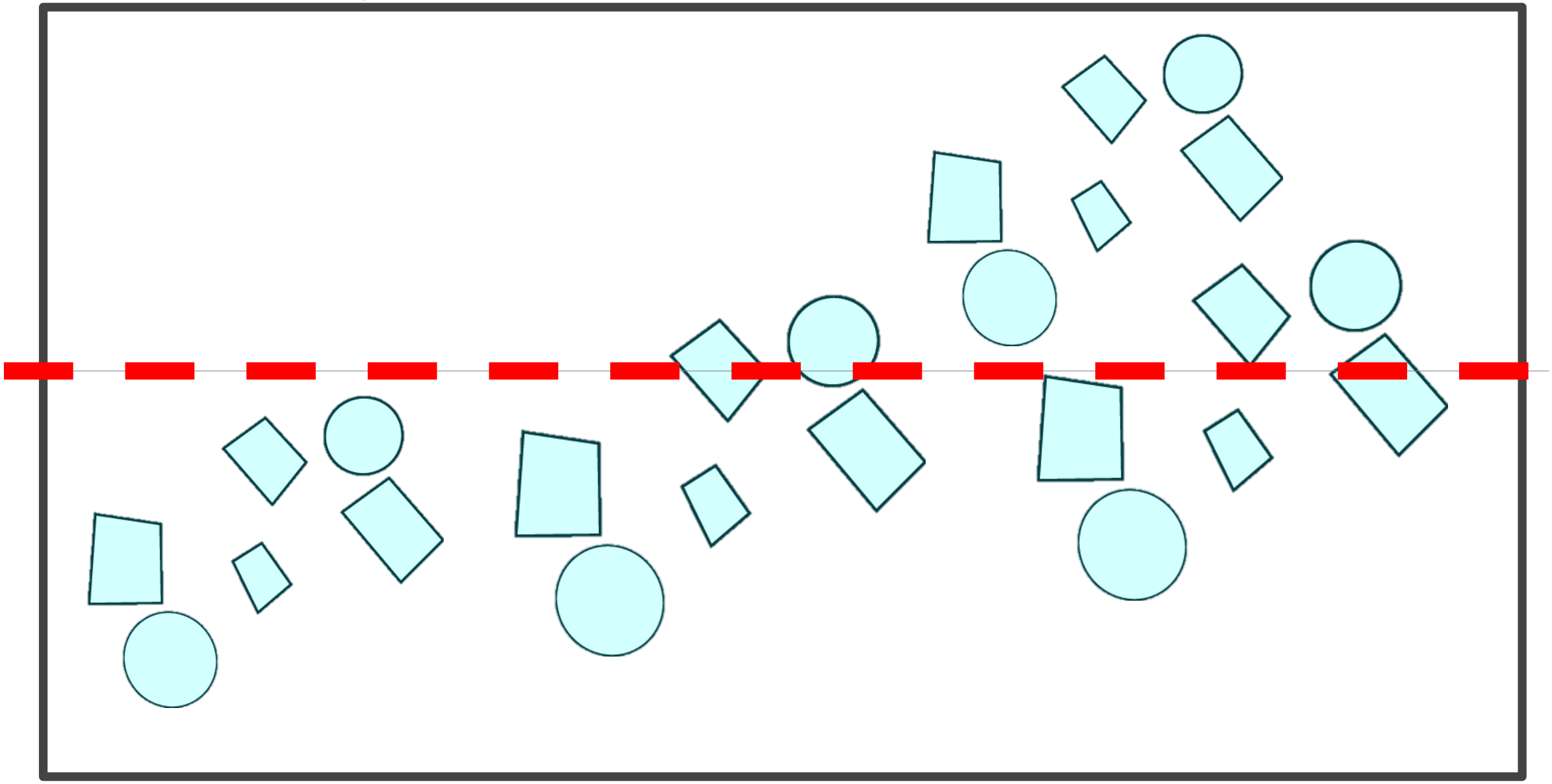




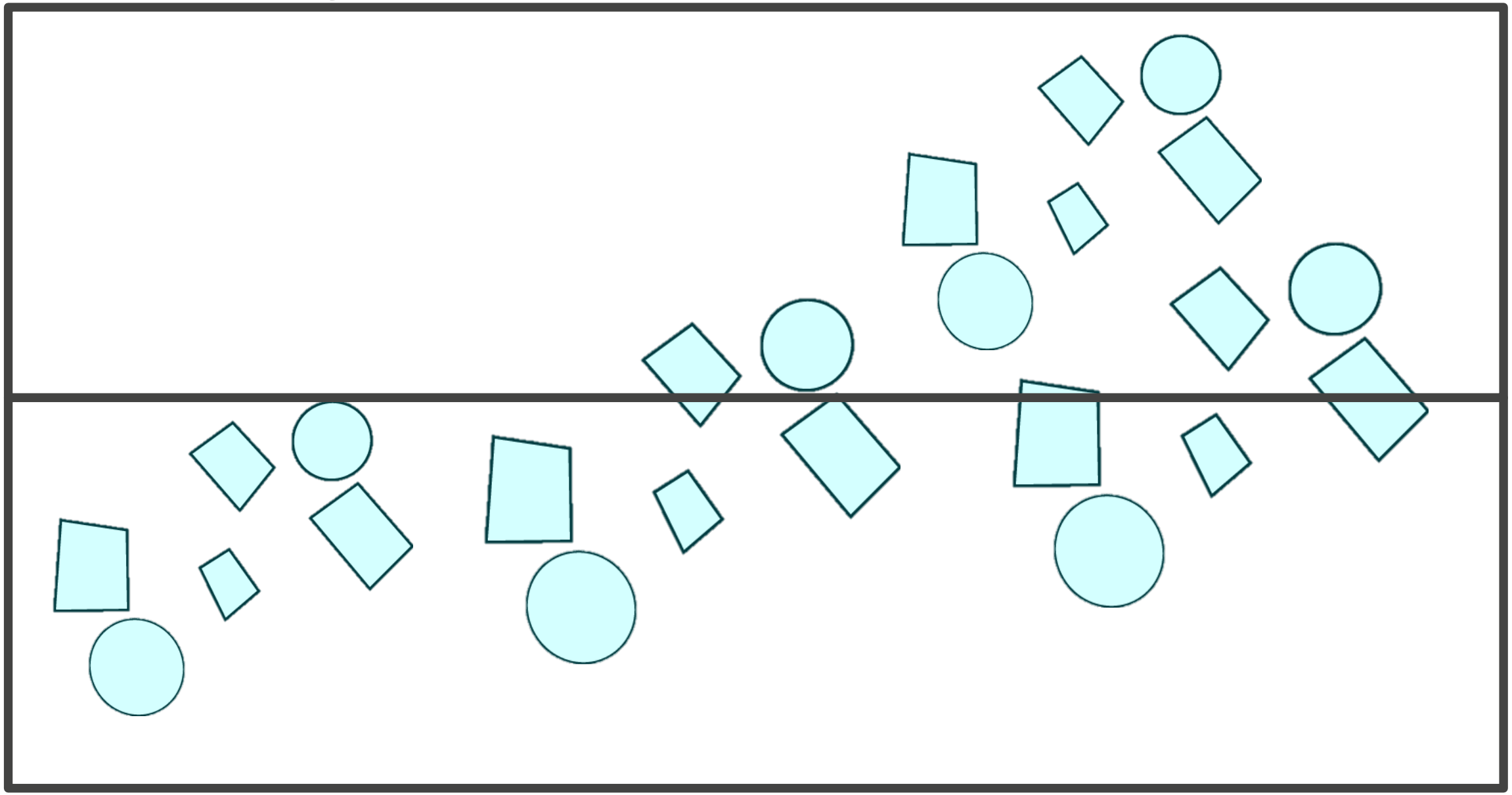
Constructing a k-d Tree



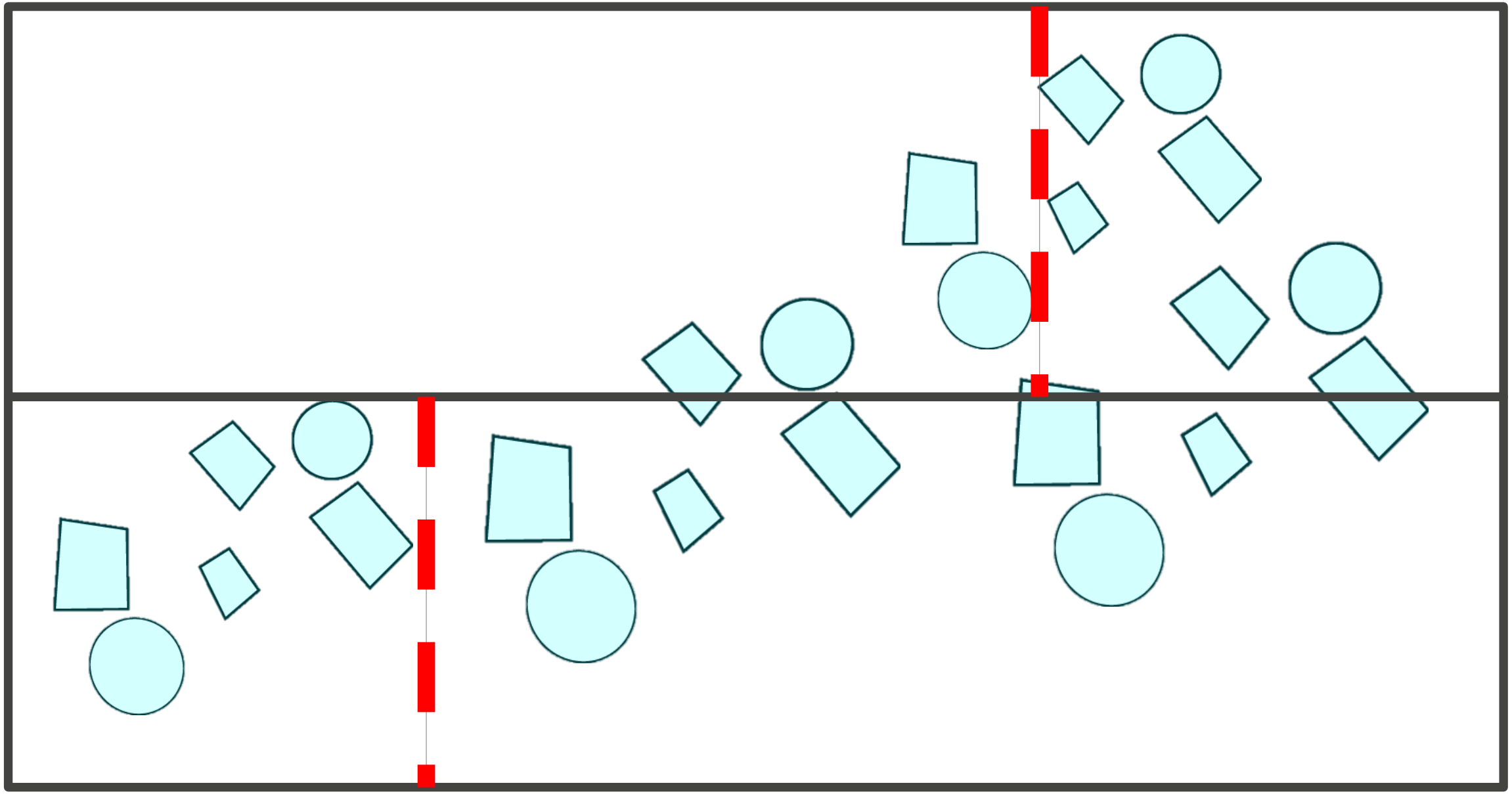
Constructing a k-d Tree



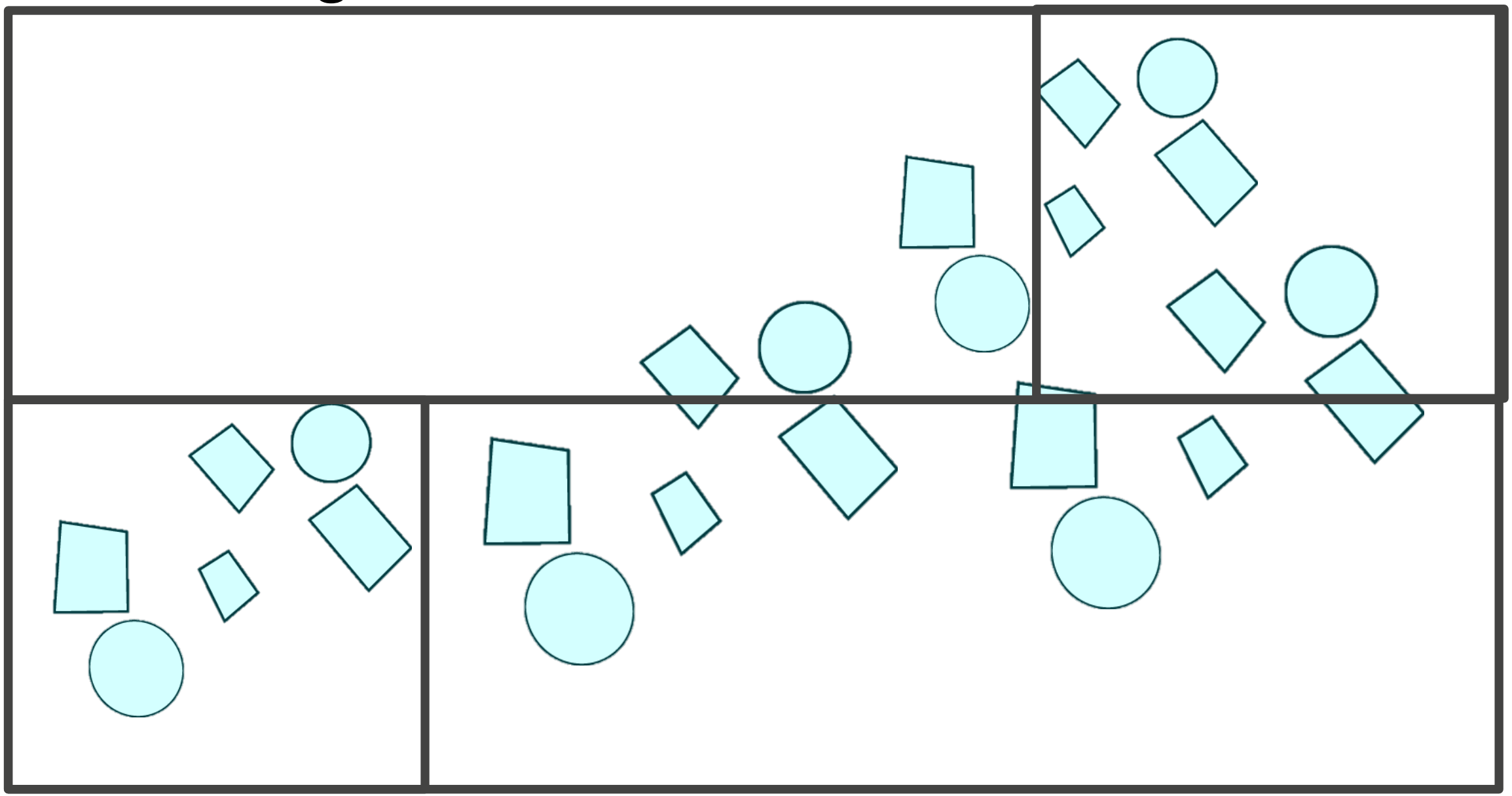
Constructing a k-d Tree



Constructing a k-d Tree



Constructing a k-d Tree



Ray Intersection Tests

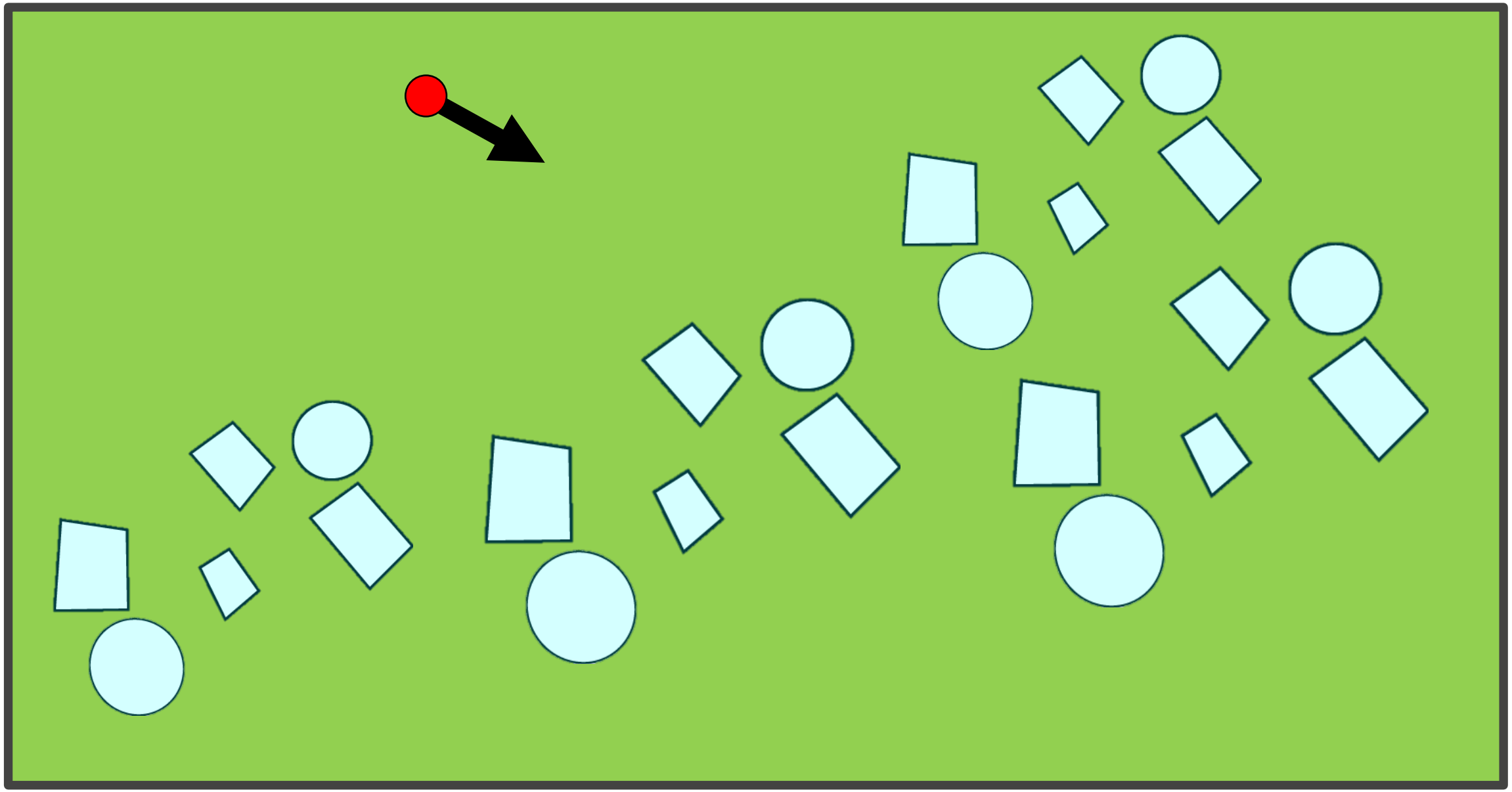
Depth First Search Again

If ray interacts with child node then recurse

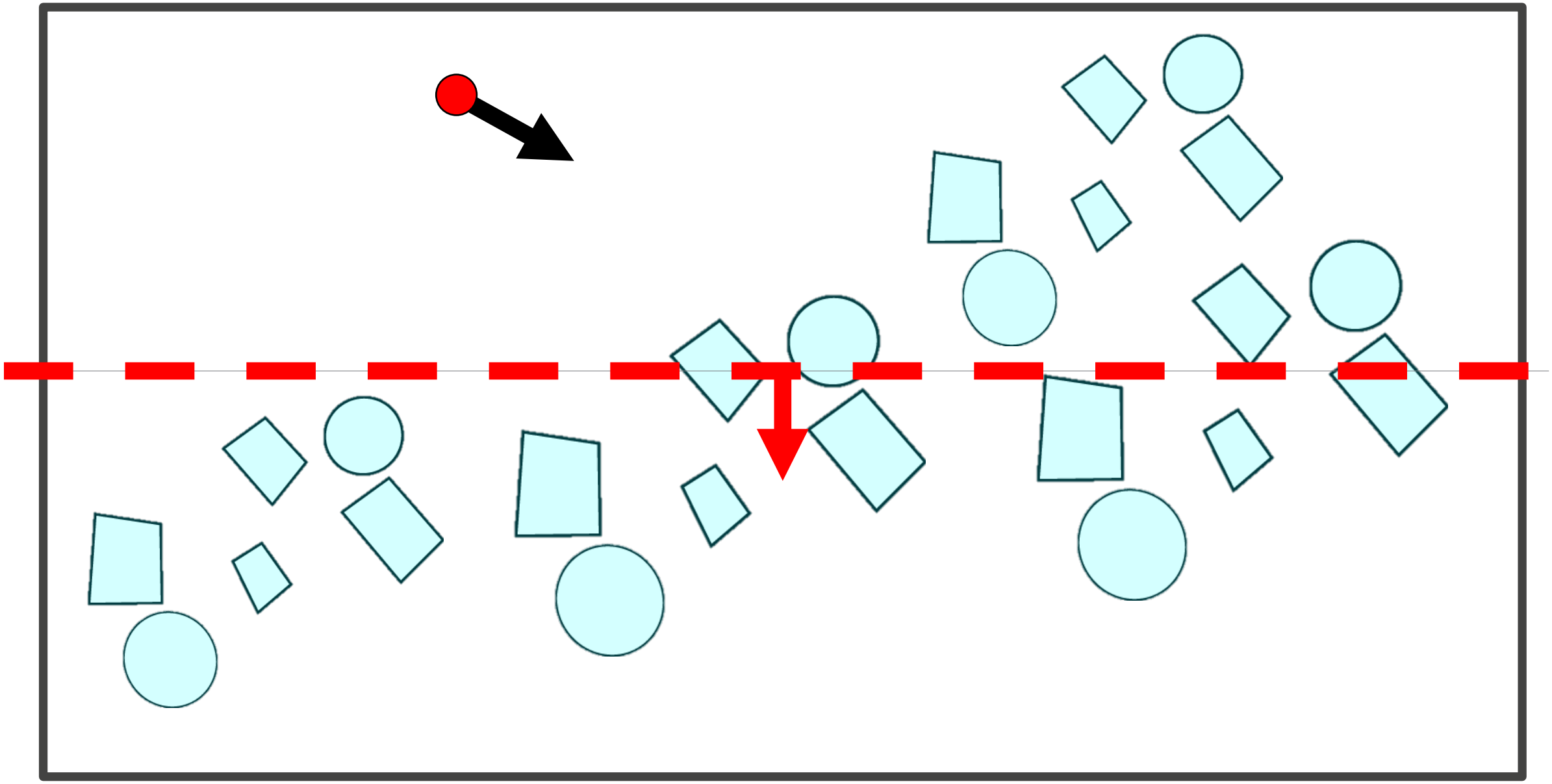
Interactions are

- Child contains ray origin point
- Ray crossed into child node

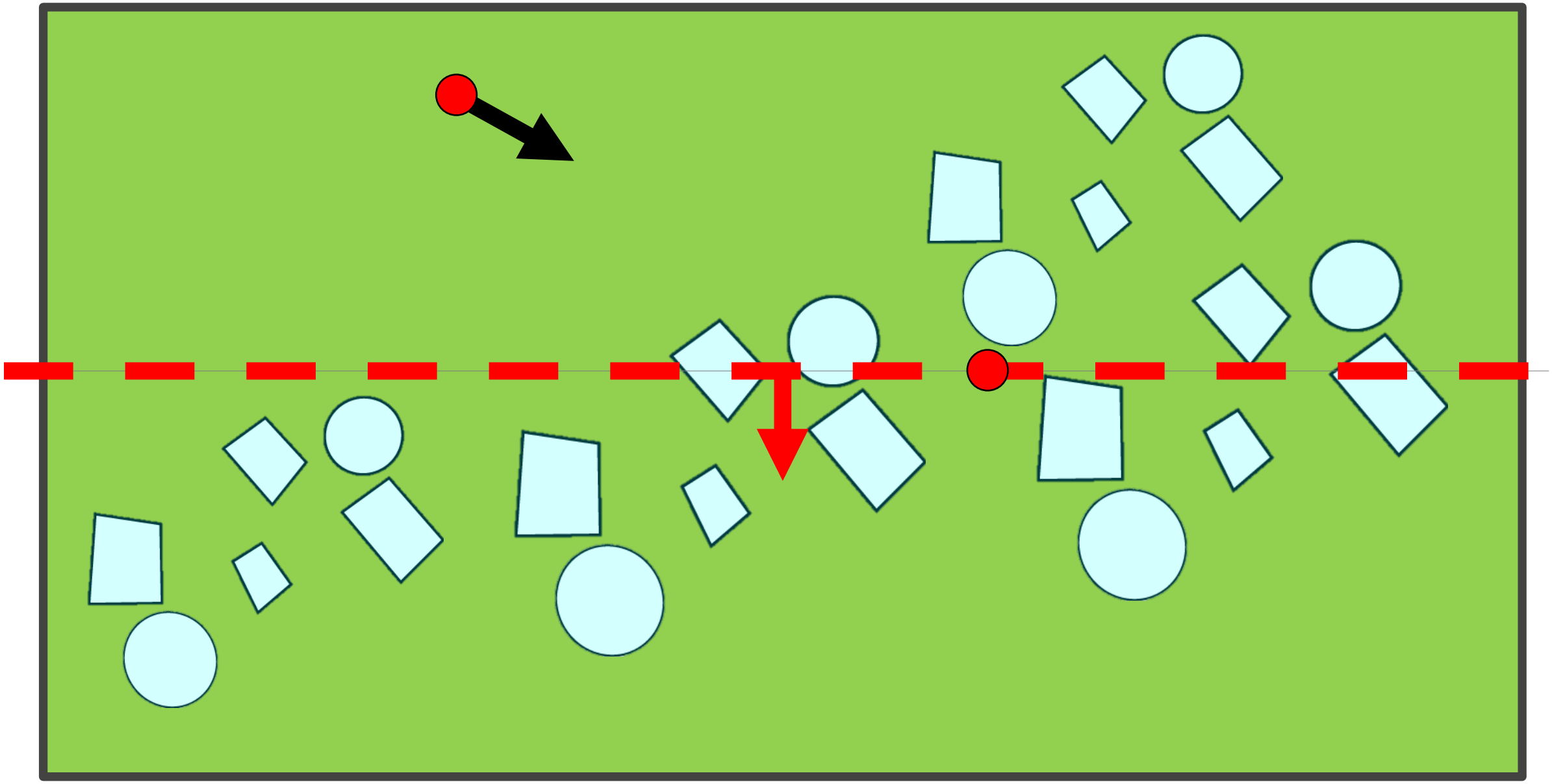
Ray Intersection Tests



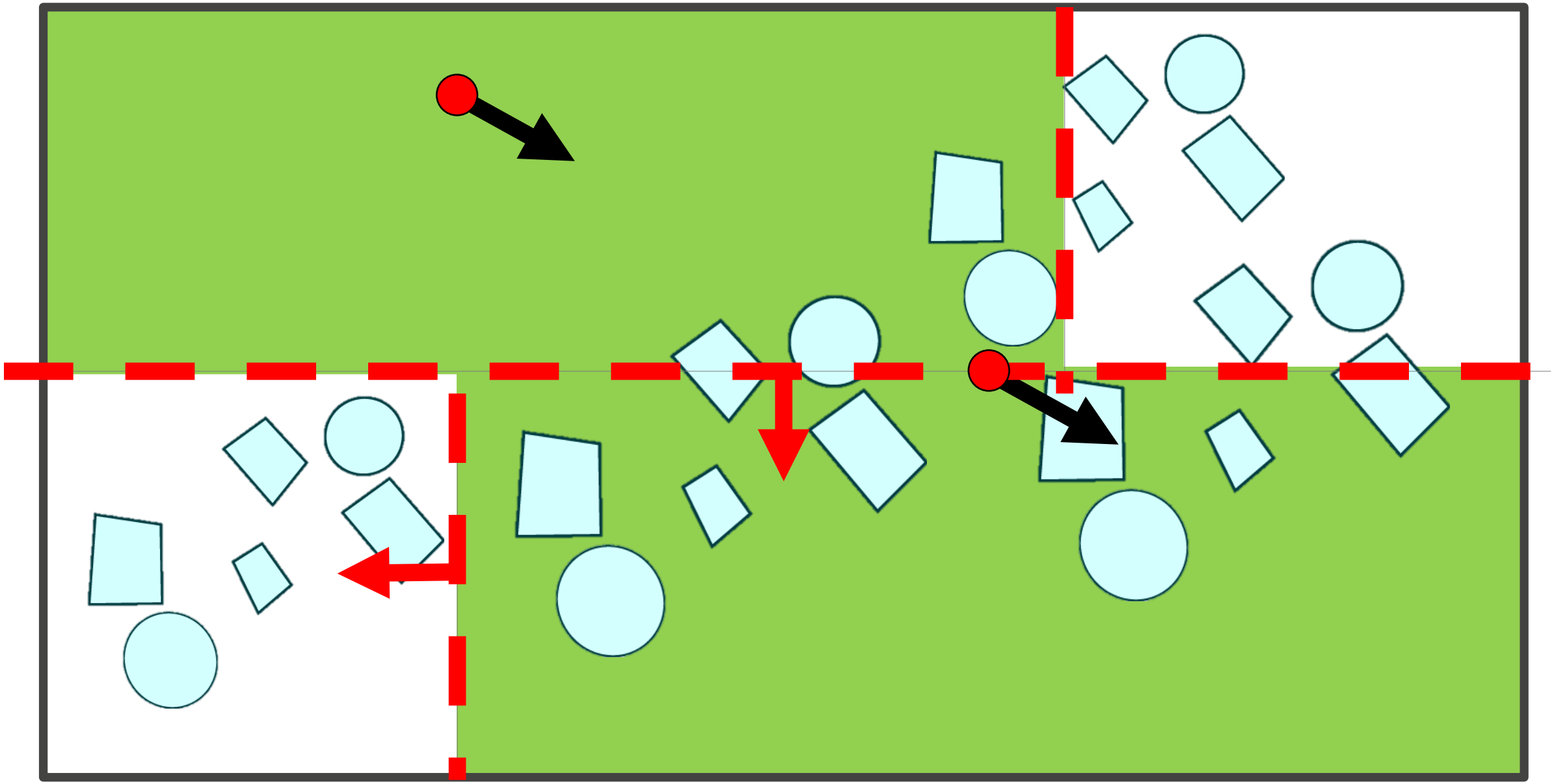
Ray Intersection Tests



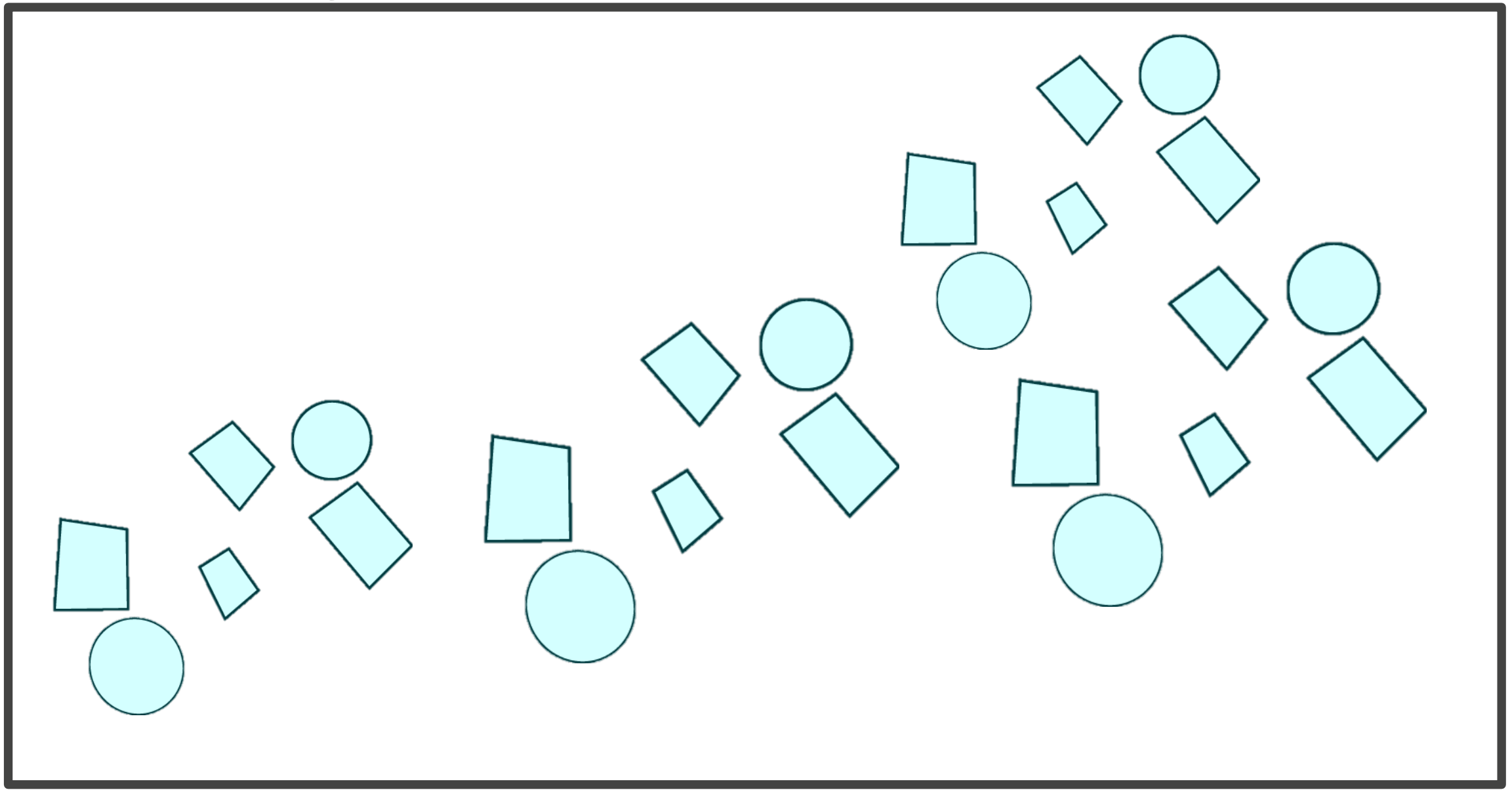
Ray Intersection Tests



Ray Intersection Tests



Constructing an Quadtree Tree



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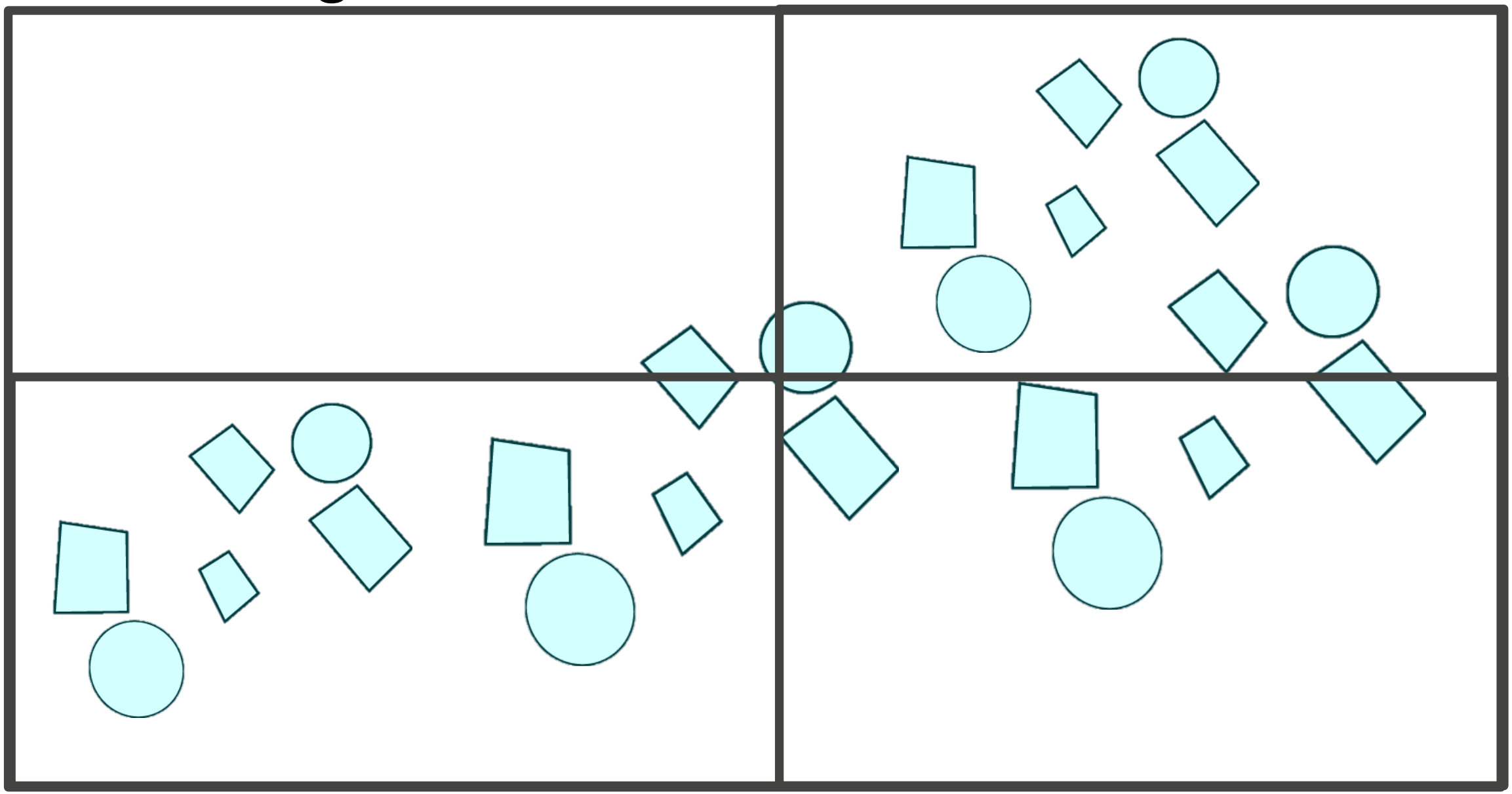
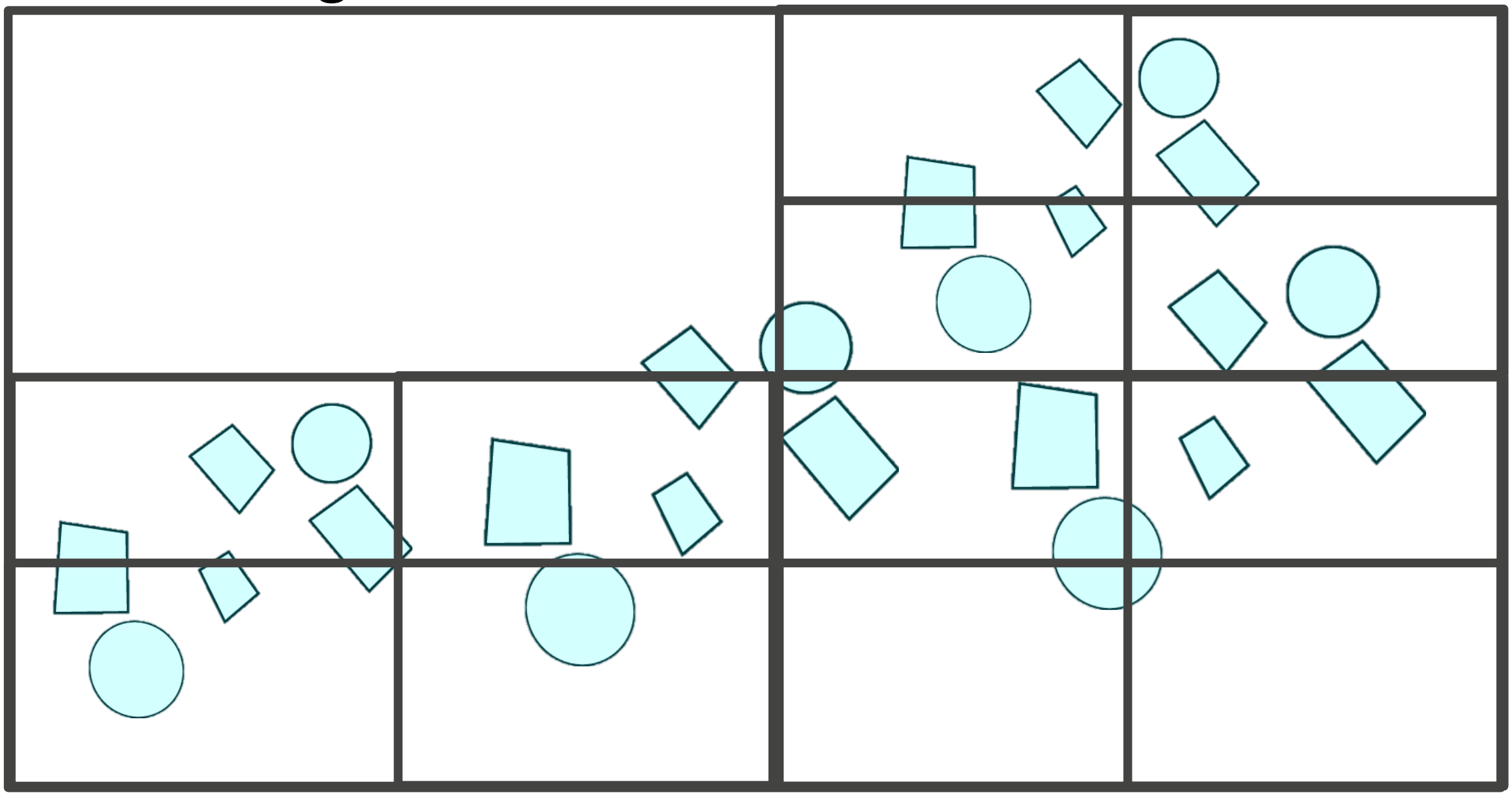
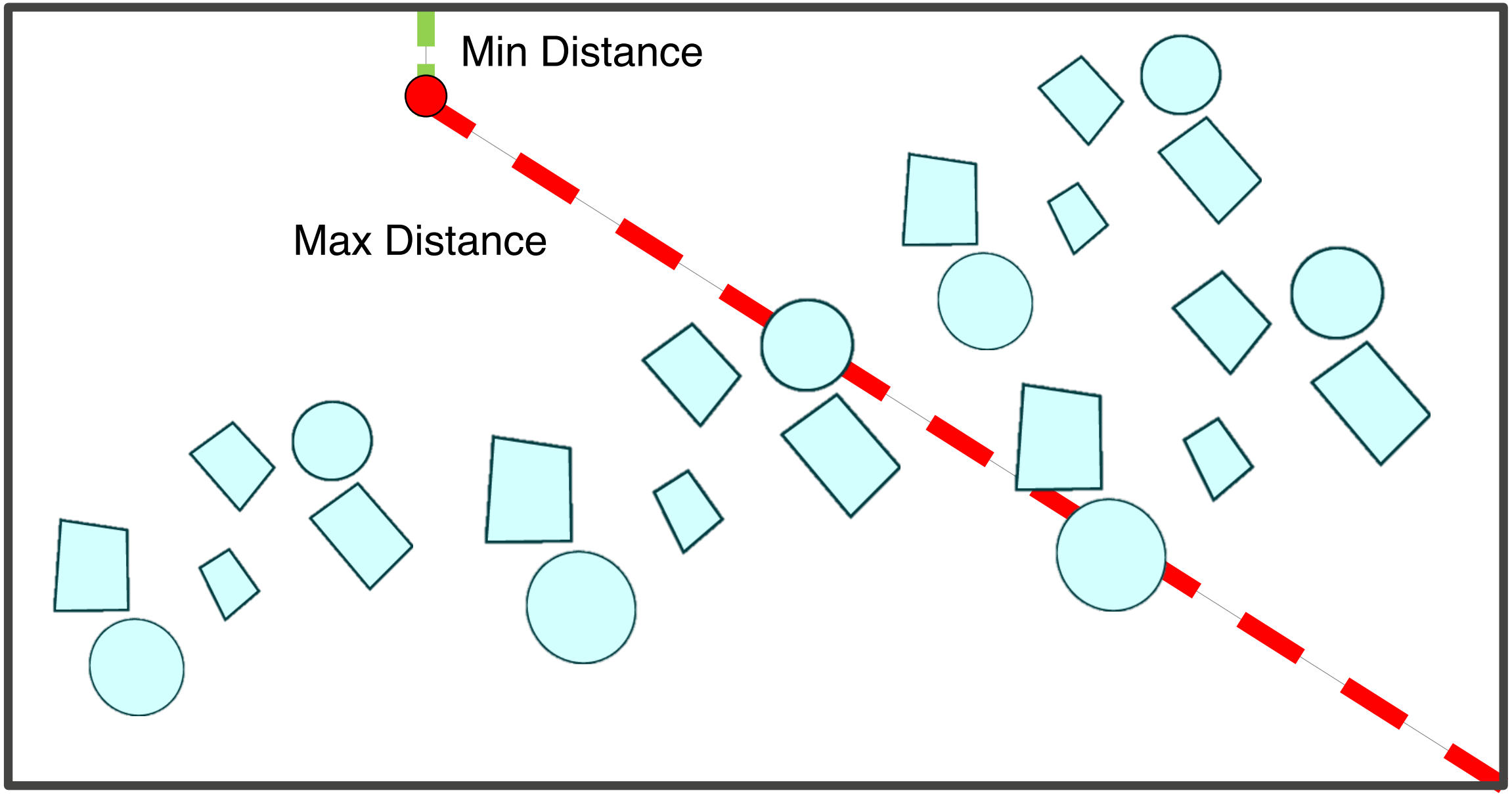


Constructing an Quadtree Tree



Distance Query



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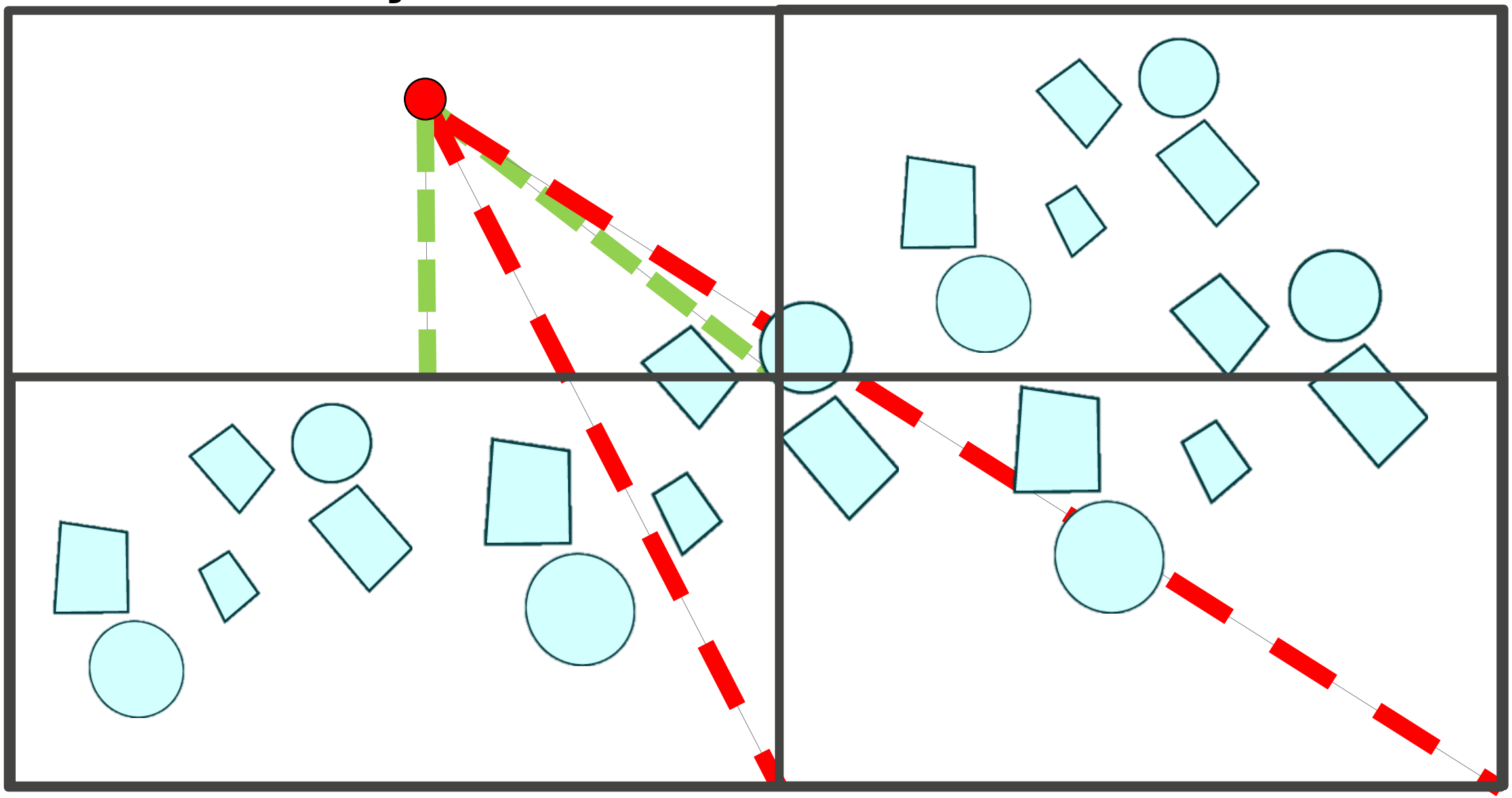
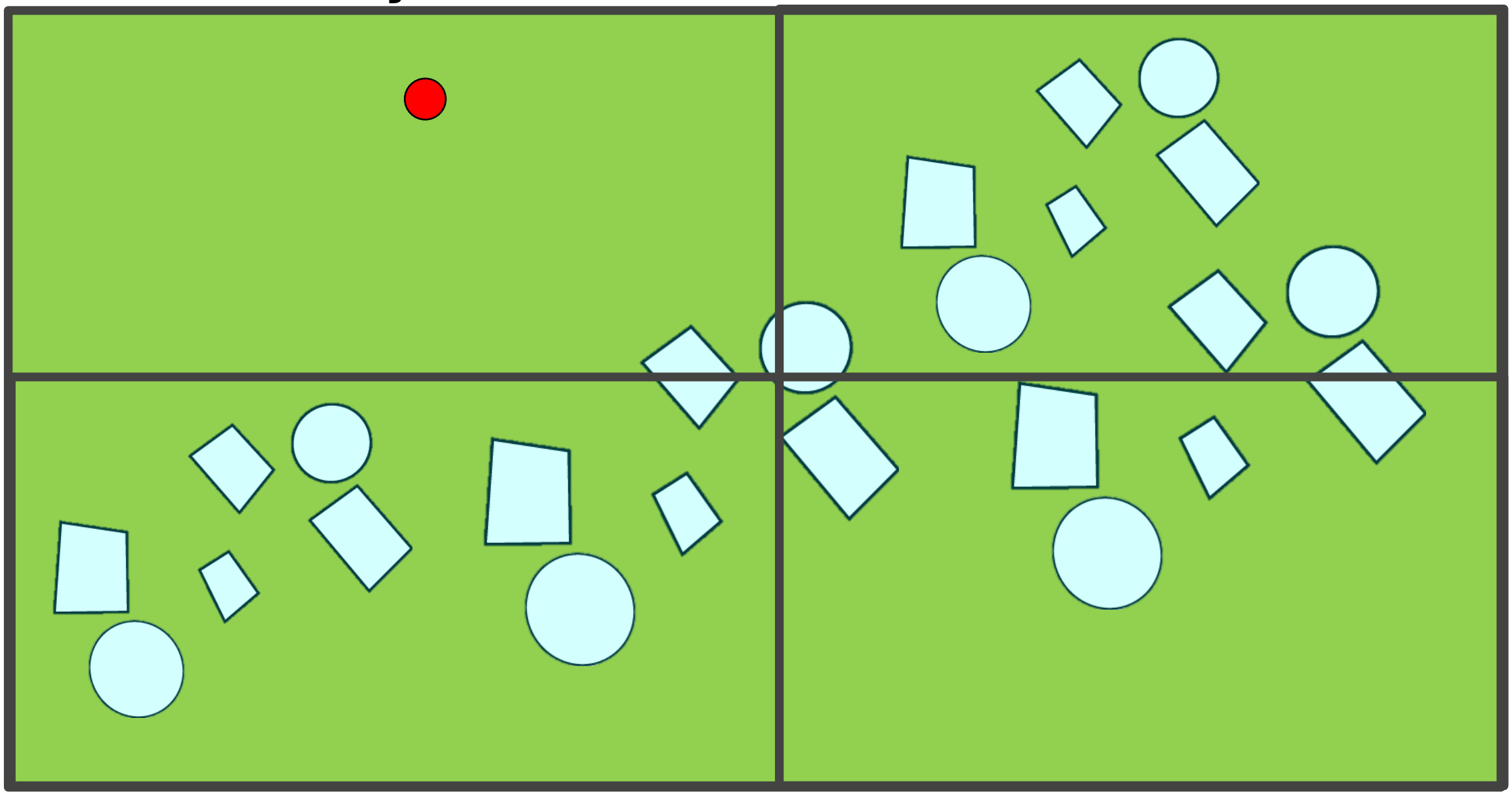
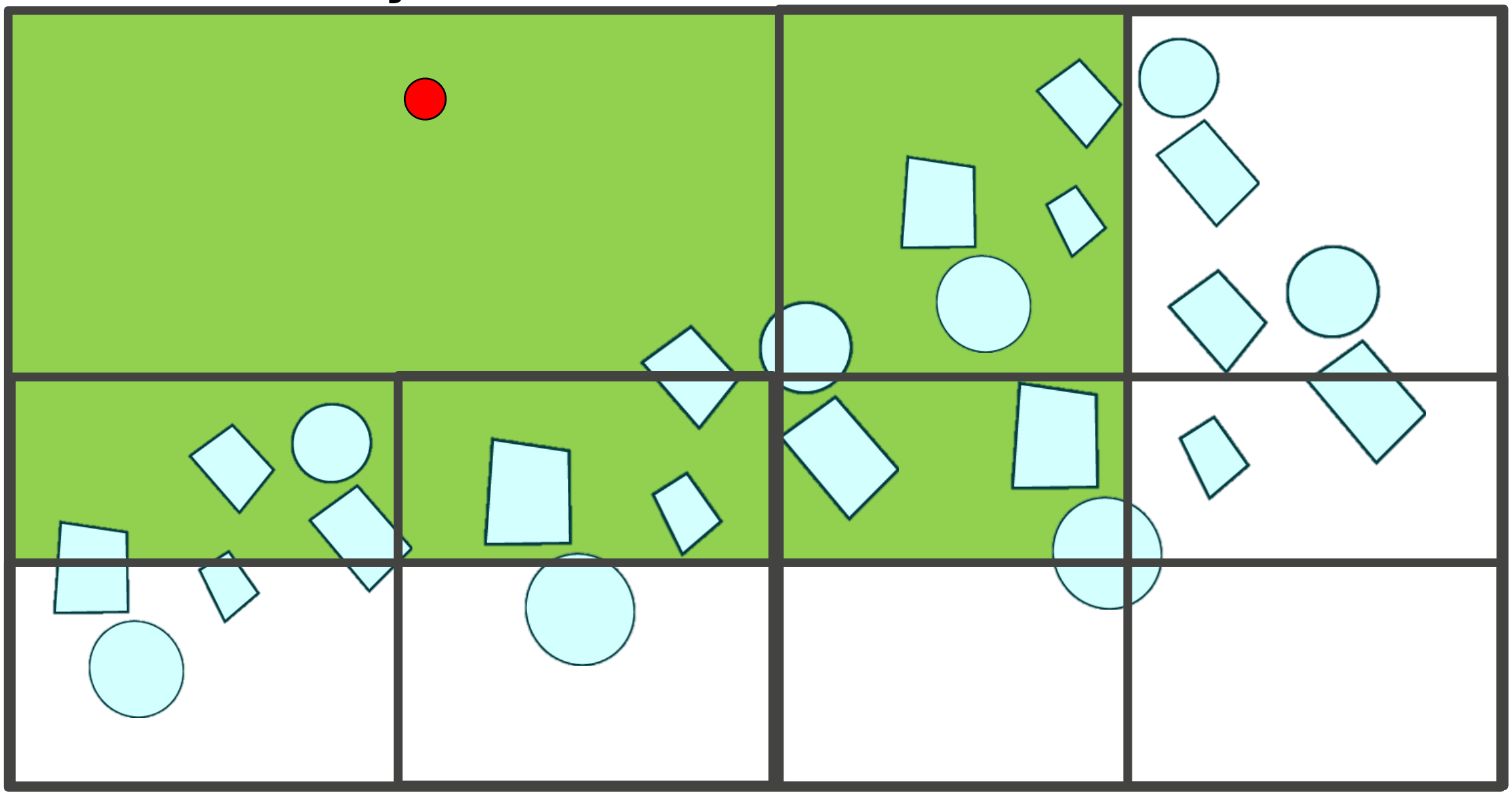


Distance Query



Distance Query



Done

Assignment 3 due on Friday

Assignment 4 out now

Office hours now BA5268