Epidemic impact study based on SIRU high-order model

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***Abstract*—So far, most of those vaccinated have developed resistance to the novel coronavirus, one step closer to achieving immunity step. In this paper, factors of vaccine and virus variation were newly considered in the original SIR Model, and a SIRU primary model was established to illustrate the influence of the control of vaccine coverage rate on the establishment of immune barrier. Combined with the vaccine efficiency and virus mutation rate, the primary model was improved to obtain a SIRU higher-order model. Finally, the relevant data were used for processing and analysis, and it was concluded that the higher the vaccination rate, the higher the vaccine effectiveness, the lower the virus mutation rate and the stronger the immune barrier effect. Finally, by introducing the determination of epidemic-economy correlation coefficient and information entropy method to determine the weight and rank of each economic index, the weighted rank sum ratio was calculated to quantitatively analyze and compare the economic level of the implementation of physical isolation and the establishment of immune barrier, and corresponding conclusions were drawn.**

Keywords—SIRU higher order differential equation, information entropy method,integral rank method

I. Introduction

The outbreak of the novel coronavirus in 2020, due to its strong infectivity, caused a large number of susceptible people to be diagnosed and even lost their lives in a short period of time. A year later, researchers developed a vaccine against the novel coronavirus, reducing the risk of infection and social panic at the same time [1].

Based on the existing SIR Model of infectious diseases, this paper first establishes a primary model with basic differential equations to illustrate the effect of the control of vaccination rate on the establishment of immune barrier. Combined with the vaccine efficiency and virus mutation rate, the flow and transformation of the four populations as the core, the primary model was improved to obtain a higher-order model. Finally, relevant data released by the authorities after the outbreak of the epidemic were used for processing and analysis, and it was verified that the higher the vaccination rate, the higher the effectiveness of the vaccine, the lower the mutation rate of the virus and the stronger the effect of the immune barrier. While discussing the differences in the costs and benefits of physical isolation and immune barrier to the whole society, we introduced the determination of epidece-economy correlation coefficient, the information entropy method to determine the weight and rank of each economic index, calculated the weighted rank sum ratio, respectively, to conduct quantitative analysis and comparison of the economic level of the implementation of physical isolation and the establishment of immune barrier, and drew corresponding conclusions.

II. Establishment of SIRU model

A. The establishment of elementary model of SIRU

Based on the existing SIR[2] model related to infectious diseases, a new model was established in combination with vaccination. The model divides the population during the outbreak into four categories: susceptible(s), infected(i), removed(r), and vaccinated(u). Among them, the remover refers to the group with strong immunity after recovery. Assuming that the total number of people in the surveyed area remains unchanged, then at time t, the proportions of the four in the total number of people are denoted as s(t), i(t), r(t) and u(t) respectively, and there should be:

$$s(t)+i(t)+r(t)+u(t)=1$$

Let the number of effective contact per patient every day be λ, λ is also known as the contact rate,and when the healthy person is effectively exposed to become a patient. From this can be obtained each patient daily effective contact with the number of health is λs(t), all patients Ni(t), daily effective contact with the number of health is Nλs(t)i(t), elimination of N, can be obtained:

$$\frac{di}{dt}=\lambda si$$

Considering the actual situation, the patient will be cured and the identity will change from i to r, we can get:

$$\frac{di}{dt}=\lambda si-\mu i$$

Where, μrepresents the proportion of patients who are cured per day, and μincludes the proportion of deaths.

Then, this paper discusses the variation trend of s(t). Considering that the main impact of the introduction of vaccine is the susceptible population, the following formula can be obtained:

$$\frac{ds}{dt}=-\lambda si-\beta s$$

Beta represents the proportion of people who are susceptible to vaccination

Similarly, the change of r(t) is analyzed. The source of r(t) is i, so we can get:

$$\frac{dr}{dt}=\mu i$$

Finally, consider the change of u(t). The source of u(t) is s, so there are:

$$\frac{\mathrm{du}}{\mathrm{dt}} = \beta \mathrm{s}$$

B. The establishment of higher-order model of SIRU

Two new variables, vaccine inefficiency α and virus mutation rate γ [3], were introduced in this paper. On this basis, we modified the above model:

The source of i added the number of infections in patients who did not respond to the vaccine[4], and the number of infections in patients who did respond to the vaccine after the virus mutated and those who did not die. So the change of i is as follows:

$$\frac{\mathrm{di}}{\mathrm{dt}} = \lambda(\alpha \mathrm{u} + \mathrm{s})\mathrm{i} - \mu \mathrm{i} + \lambda \gamma \mathrm{i}[\mathrm{r} - \sigma + (1-\alpha)\mathrm{u}]$$

Then, the changes of s were analyzed. s added the number of people who were not infected with the vaccine, the number of people who were not infected with the vaccine and the number of people who were not killed in the case of virus mutation, and the number of people who were cured of the infection in the case of virus mutation (excluding the death part). But it also reduces the number of people who can effectively vaccinate if the virus does not mutate. Thus:

$$\frac{\mathrm{ds}}{\mathrm{dt}} = -\lambda \mathrm{si} + \lambda(1-\lambda \mathrm{i})\alpha \mathrm{u} + (1-\lambda \mathrm{i})$$
$$[\mathrm{r} - \sigma + (1-\alpha)\mathrm{u}] - \beta \mathrm{s}(1-\alpha)(1-\gamma) + \gamma(\mu - \varepsilon)\mathrm{i}$$

Later, r can be changed to the proportion of people who are cured (excluding those who die) when the new virus does not mutate, and the proportion of people who die. But at the same time with the mutation of the virus the total number of removals is reduced. Thus:

$$\frac{\mathrm{dr}}{\mathrm{dt}} = (1-\gamma)(\mu - \varepsilon)\mathrm{i} - \gamma(\mathrm{r} - \sigma) + \varepsilon \mathrm{i}$$

Finally, in u, the number of ineffective vaccinators and the number of effective vaccinators with virus mutation are reduced, but the number of effective vaccinators without virus mutation is also increased, as shown in the following equation:

$$\frac{\mathrm{du}}{\mathrm{dt}} = -\alpha \mathrm{u} + \beta \mathrm{s}(1-\alpha)(1-\gamma) - \gamma(1-\alpha)\mathrm{u}$$

C. *The solution of SIRU model*

Based on the high-order model of SIRU, this paper uses MATLAB to calculate the influence of the values of β, α and γ respectively on s, i, u and r, so as to obtain the effect of the control of vaccine coverage rate on the construction of immune barrier, and the effect of vaccine effectiveness and virus variability on immune barrier [5].

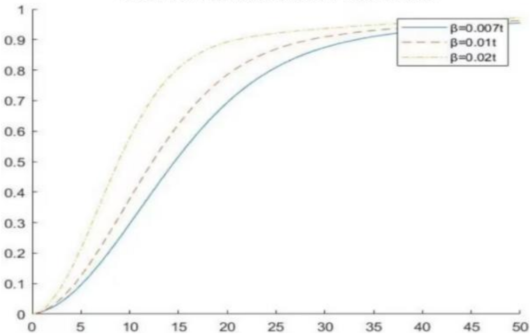

Fig. 1. SIRU model - inoculation ratio + removal ratio chart

FIG. 1 shows the relationship between u(t)+r(t) with β and time t, regardless of virus variation and assuming full vaccine effectiveness (i.e., vaccine inefficiency α=0). Compared with s and i, u and r can represent relatively safe populations, so we can use u+r as an indicator of the degree of immune barrier construction. As can be seen from Figure 1, when the value of β increases, that is, the vaccination rate of susceptible people increases, the efficiency of the construction of the immune barrier is faster, the time required is shorter, and the effect of the immune barrier is stronger.

III. Economic impact assessment model of the pandemic

A. The economic impact of the pandemic

Through data collection, this paper sorted out the monthly number of new COVID-19 patients and various economic indicators from March to May 2020 and February and March 2021, as shown in the following table:

TABLE I. EPIDEMIC - ECONOMIC INDICATORS FOR MARCH, APRIL AND MAY 2020

Index	2020.3	2020.4	2020.5
Cumulative number of confirmed cases	82631	84385	84588
Newly confirmed cases	2663	1754	203
Consumer Price index %	104.3	103.3	102.4
Output price index of industrial producers %	98.5	96.9	96.3
Total retail sales of consumer goods / 100 million yuan	26450	28178	31973
Import and export value /thousand US dollars	350359336	355134209	365234622
Cargo volume / 10,000 tons	325647	384525	406969
Passenger volume /10,000 people	38573	57320	73748

TABLE II. EPIDEMIC - ECONOMIC INDICATORS FOR JANUARY, FEBRUARY AND MARCH 2021

Index	2020.3	2020.4	2020.5
Cumulative number of confirmed cases	89564	89912	90217
Newly confirmed cases	2493	348	305
Consumer Price index %	101	100.6	99.5
Output price index of industrial producers %	100.3	101.7	104.4
Total retail sales of consumer goods / 100 million yuan	34868.5	35495.8	35484.1
Import and export value /thousand US dollars	462659051	371830507	468470355
Cargo volume / 10,000 tons	415485	261829	432865
Passenger volume /10,000 people	67112	57293	74544

Based on the above table and the actual situation, from March to May 2020, China mainly adopted early detection and quarantine measures at the initial stage of the outbreak, such as lockdown measures in Wuhan and cancellation of multi-line flights[6]. Since no vaccine or other means of vaccine control were available at that time, it can be assumed that the socio-economic impact of the epidemic during this period was primarily due to physical isolation. From January to March 2021, mass production of COVID-19 vaccine will be realized[7], and more and more people will be vaccinated with COVID-19 vaccine. The intensity and extent of physical isolation has been significantly weakened compared with last year. At this stage, China is on the road of building an immune barrier, so it can be estimated that the social and economic impact of COVID-19 during this period will mainly come from the establishment of an immune barrier.

B. The correlation coefficient between epidemic and economy was determined

Based on the above data, the Person correlation coefficient is calculated with the following formula:

$$r = \frac{\sum XY - \frac{\sum X \sum Y}{N}}{\sqrt{\left(\sum X^2 - \frac{\left(\sum X\right)^2}{N}\right)\left(\sum Y^2 - \frac{\left(\sum Y\right)^2}{N}\right)}}$$

After calculation, the correlation coefficients between the number of new epidemic cases in the physical isolation period and the immune barrier period and various economic indicators are shown in Table 3 and 4:

TABLE III. PHYSICAL ISOLATION CORRELATION COEFFICIENT

Index	Coefficient of correlation
Consumer Price index	0.9838
Output price index of industrial producers	0.9186
Total retail sales of consumer goods	-0.9986
Import and export value	-0.9985
Cargo volume	-0.9200
Passenger volume	-0.9824

TABLE IV. IMMUNE BARRIER CORRELATION COEFFICIENT

Index	Coefficient of correlation
Consumer Price index	0.7182
Output price index of industrial producers	-0.7728
Total retail sales of consumer goods	-0.9994
Import and export value	0.4374
Cargo volume	0.4022
Passenger volume	0.0625

It can be seen that during the physical quarantine period last year, the epidemic had the greatest impact on the retail sales of consumer goods and the total value of imports and exports, followed by the volume of freight. During the period of establishing the immunity barrier this year, the epidemic will have a great impact on the output prices of industrial producers and the retail sales of consumer goods.

C. Information entropy method to determine the weight of each economic index

The decision matrix is obtained from Table 1 and Table 2: $D = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{18} \\ a_{21} & a_{22} & \cdots & a_{28} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{m8} \end{pmatrix}$

Using information entropy method[8], the decision matrix of 8 economic indicators is further processed. According to the definition of entropy, the quantity index given by Shannon[9], the entropy of attributes of each scheme X_j is

$$E_j = -k\sum_{i=1}^{m} r_{ij}\ln r_{ij}, \quad k = 1/\ln m, \quad j = 1,2,\cdots, \quad n$$

Define: $F_j = 1 - E_j, 0 \le F_j \le 1$

X_j is the differentiation degree of the attribute. Further, the normalized differentiation degree is taken as the weight ω_j of the attribute X_j, $\omega_j = \frac{F_j}{\sum_{j=1}^{n} F_j}$, $j = 1,2,\cdots, \quad n$

D. Compile rank and calculate weighted rank sum ratio

The rank method [10], this paper set the $x_1, x_2, \cdots, x_n$ from yuan overall extraction capacity of n samples, and follow the order since the childhood, set up the statistics for the $x_1, x_2, \cdots, x_n$. If $x_i = x_k$ is called k rank in the samples of the x_i, denoted by R, for each i= 1, 2, ----, n, said R_i is the ith a rank statistics. $R_1, R_2, \cdots, R_n$ always called rank statistics. So, the weighted rank ratio:

$$WRSR_i = \frac{1}{n}\sum_{j=1}^{m} \omega_j R_{ij}, \quad i = 1,2,\cdots,n$$

After calculation, the economic level in the two periods can be obtained as shown in Figure 2 and 3 below:

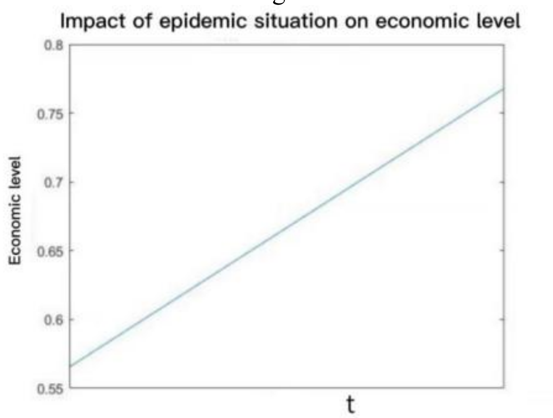

Fig. 2. Impact of epidemic situation on economic level in physical isolation

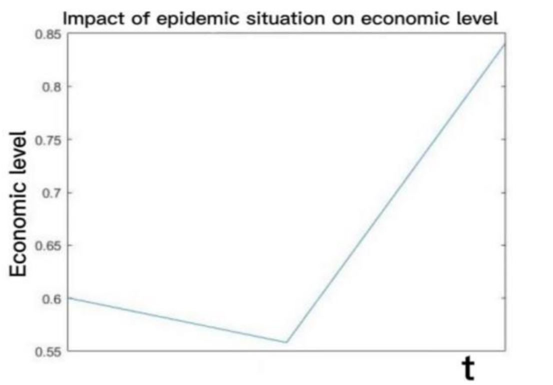

Fig. 3. Impact of epidemic situation on economic level in immune barrierof epidemic situation on economic level in physical isolation

E. Results of Evaluation

Combined with the above analysis and actual life, during the period of physical isolation, due to the currency circulation blockage, the production cost of manufacturers increases, and the output price index of industrial producers increases, which makes the price of daily necessities rise, thus making the consumer price index higher than the normal level, which will have a negative impact on inflation, the stock market and the exchange rate. At the same time, the adoption of physical isolation measures will inevitably reduce the import and export of goods as well as the volume of cargo and passenger traffic, causing considerable economic losses.

In the process of building an immune barrier, the cost to society as a whole, first of all, is the cost of health care, the cost of vaccines. Each vaccine costs between 20 and 100 yuan, plus transportation costs, setting up temporary vaccination sites and so on. As mentioned above, the vaccination rate would need to reach 80% to build up the immunity barrier. Assuming China's population is about 1.4 billion, it would need to vaccinate about 1.12 billion people, which is a considerable national financial expenditure.

Overall, the economic costs of physical isolation to society as a whole are in all respects greater than those of immune barriers. However, in the early stage of the outbreak, the best way to control the epidemic is physical isolation, which can quickly bring the epidemic under good control. In addition, although the establishment of immune barrier is relatively inexpensive to the society, it still takes a long period to extract antibodies and produce vaccines, so it is impossible to start the establishment of immune barrier at the very beginning.

In addition to the above mentioned, both physical isolation and immune barriers can make the epidemic progress in a good direction, which is the common denominator of both. But physical isolation can be crucial in the early stages of an outbreak to cut off immediate transmission; However, the immune barrier mainly plays a role in preventing the second outbreak and preventing the future, which is the difference between the two benefits.

IV. CONCLUSION

In this paper, the extent of the immune barrier is measured as $r(t)+u(t)-\sigma(t)$, the proportion of removers plus vaccinators. According to the data, this paper defines the successful establishment of immune barrier when $r(t)+u(t)-\sigma(t)\geq 0$. Based on this, the population is divided into four categories, namely susceptible population s, infected person i, remover r and vaccinator u. The differential equation model is established with the changes of the four groups of people over time and the transformation relationship between them as the core, and the numerical solution is obtained by Runge Kutta function in MATLAB.

In order to consider the effect of vaccine effectiveness and virus variation on immune barrier in the original model. In this paper, two new factors of vaccine effectiveness and virus mutation rate were added, and the original model was further improved to observe the corresponding changes in the four groups of population.

Considering the different costs of physical isolation and immune barrier to the whole society, this paper processed the data of economic indicators during physical isolation and immune process to obtain the corresponding economic level. By comparison, the similarities and differences of benefits brought by physical isolation and immune barrier were obtained.

After considering the two conditions of vaccine effectiveness and virus variation, we made a detailed analysis of the flow changes of the four groups of population, for example, the remover group was divided into the cured group and the dead group, the vaccinated group was divided into the ineffective group and the effective group, and so on. While these increase the difficulty of the model, they also make the model more detailed and more realistic.

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