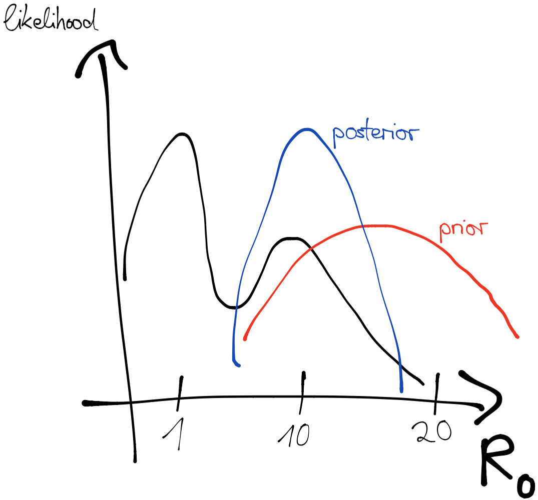
Revision

- posterior probabilities

$$p(\theta|\text{data}) = \frac{p(\text{data}|\theta)p(\theta)}{p(\text{data})}$$



Sampling from the posterior

We interpret $p(\theta|\mathrm{data})$ as the probability distribution of a random variable θ, from which we sample (via MCMC)

Why sample?

1. explore parameter space
2. samples can be useful
   - explore interventions, forecasts

MCMC: Sampling from a distribution

- We can calculate (in a deterministic model) $p(\theta|\mathrm{data})$ given any θ – how do we sample?

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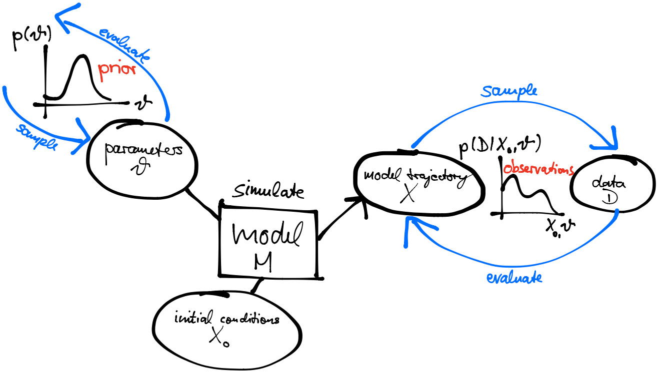
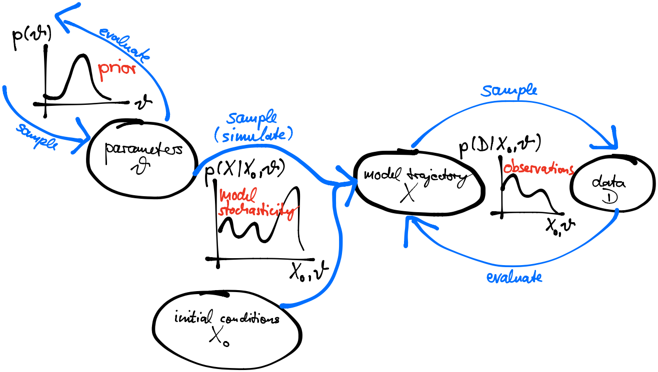
Deterministic process models



$$p(\theta|\text{data}) \propto p(\text{data}|\theta)p(\theta)$$

Use MCMC to get samples from it: θ_1, θ_2, θ_3, $\ldots$

Stochastic process models



Stochastic process models

- one θ can lead to many possible outcomes X
- we can
  1. sample from $p(X|\theta)$ (via simulation)
  2. evaluate the trajectory likelihood $p(\mathrm{data}|X,\theta)$
- we can't directly evaluate the likelihood $p(\mathrm{data}|\theta)$

$$p(\mathrm{data}|\theta) = \sum_X p(\mathrm{data}|X,\theta)p(X|\theta)$$

- The number of possible trajectories X for one value of θ is large (usually infinite)
- We replace the sum with a Monte Carlo (random) sample

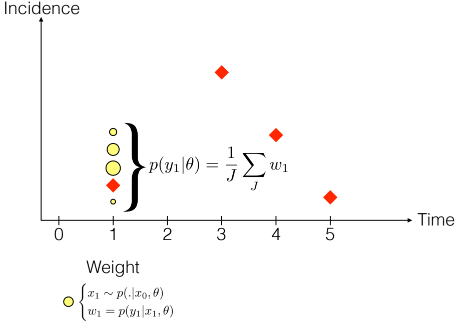
Sequential Monte Carlo (SMC) / Particle Filter I

We sample J trajectories $X_{J,1}$ from

$$p(X_{J,1}|\theta)$$

and average over

$$p(Y_1|X_{J,1},\theta)$$



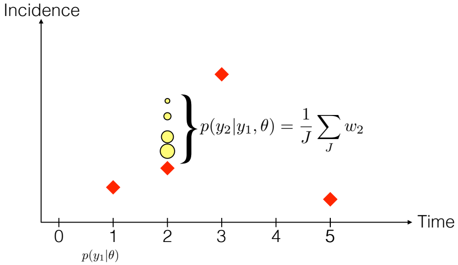
Sequential Monte Carlo (SMC) / Particle Filter II

We then sample J trajectories $X_{J,2}$ from

$$p(X_{J,2}|\,Y_1,\theta)$$

and average over

$$p(\,Y_2|\,Y_1,X_{J,2},\theta)$$

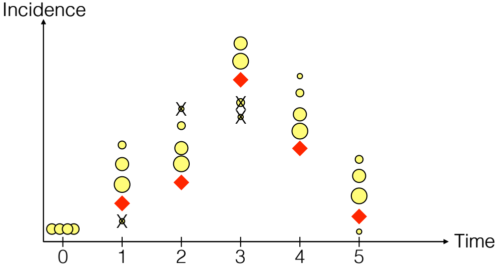


Sequential Monte Carlo (SMC) / Particle Filter III

The sum of all these (logged) values is

$$p(Y_{1:t}|\theta) \approx \prod tp(Y_t|Y_{1:(t-1)}, \theta)$$

which is a sample estimate of the likelihood.

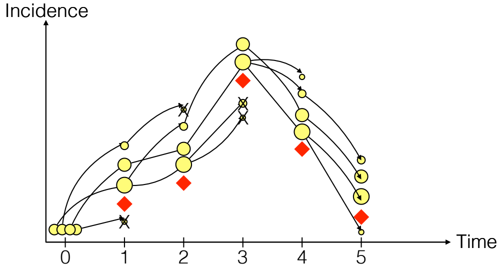


Sequential Monte Carlo (SMC) / Particle Filter IV

We can also retrieve filtered trajectories, that is samples from

$$p(X|\text{data})$$

by following the particles from the last point backwards.



pMCMC

- Once we can estimate $p(\mathrm{data}|\theta)$, we can combine this with the prior to evaluate the posterior $p(\theta|\mathrm{data})$ for any θ.
- We can then use MCMC to sample from this: pMCMC

Sampling from the posterior

Estimating the likelihood		
		MCMC
	SMC	pMCMC

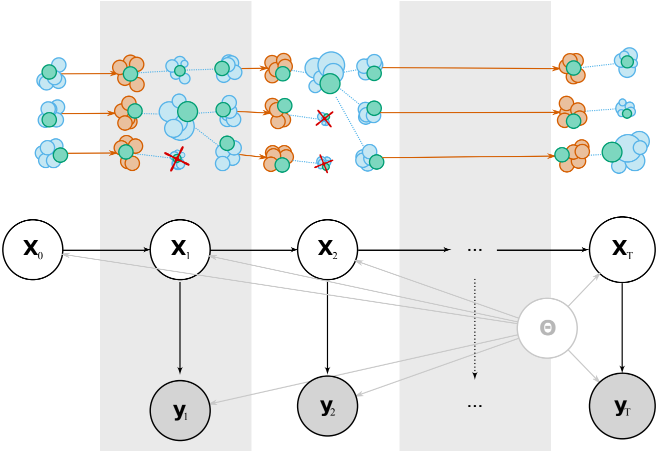
Sampling parameters/trajectories

Sampling from the posterior

Estimating the likelihood

	MCMC	SMC
SMC	pMCMC	SMC^2

SMC^2



Chopin et al. (2011)