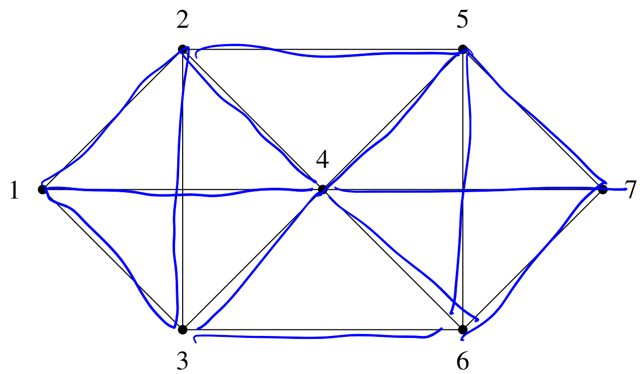
CS 70 Spring 2024 — Discrete Mathematics and Probability Theory — Seshia, Sinclair — DIS 2A

1 Eulerian Tour and Eulerian Walk

Note 5



(a) Is there an Eulerian tour in the graph above? If no, give justification. If yes, provide an example.

no, the graph is not even degree

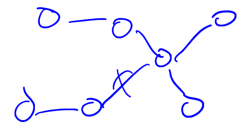
(b) Is there an Eulerian walk in the graph above? An Eulerian walk is a walk that uses each edge exactly once. If no, give justification. If yes, provide an example.

yes, 12 23 31 14 42 25 54
43 36 64 47 75 56 67

?

(c) What is the condition that there is an Eulerian walk in an undirected graph? Briefly justify your answer.

has at most two odd degree vertex



2 Coloring Trees

Note 5

(a) Prove that all trees with at least 2 vertices have at least two leaves. Recall that a leaf is defined as a node in a tree with degree exactly 1.

$n \to$ the size of the tree

$n=2$, $\to$ correct

$2 \le n \le k$, correct

$n=k+1$ remove the leaves.

remain will have at least two leaves.

$\downarrow$ at least one leaves.

V, $V-1$ Edges.

$2|V|-2 = \sum_{v \in V} \deg(v)$

$\ge L + 2(V-L)$

$= 2V - L \Rightarrow L \ge 2$

(b) Prove that all trees with at least 2 vertices are *bipartite*: the vertices can be partitioned into two groups so that every edge goes between the two groups.

[*Hint:* Use induction on the number of vertices.]

$n \to$ the size of the tree

$n=2 \to$ correct.

assume $2 \le n \le k$ correct.

$n=k+1$, choose one leaf.

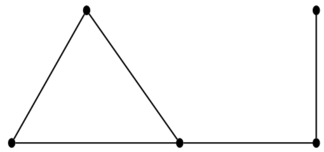
remain will be bipartite.

add the node back.

3 Degree Sequences

Note 5

The *degree sequence* of a graph is the sequence of the degrees of the vertices, arranged in descending order, with repetitions as needed. For example, the degree sequence of the following graph is $(3,2,2,2,1)$.



For each of the parts below, determine if there exists a simple undirected graph G (i.e. a graph without self-loops and multiple-edges) having the given degree sequence. Justify your claim.

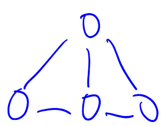
(a) $(3,3,2,2)$ ✓

(b) $(3,2,2,2,2,1,1)$

(c) $(6,2,2,2)$ X → only four vertices, can not have a degree of 6

(d) $(4,4,3,2,1)$

(a)



(b) the sum of degree is even X

(d) X, can not have degree 1,

Since there is two degree 4.