CS 70 Discrete Mathematics and Probability Theory
Spring 2024 Seshia, Sinclair **DIS 0A**

1 Propositional Practice

Note 1

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

(b) All integers are natural numbers or are negative, but not both.

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

(d) $(\forall x \in \mathbb{Z})\ (x \in \mathbb{Q})$

(e) $(\forall x \in \mathbb{Z})\ (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$

(f) $(\forall x \in \mathbb{N})\ ((x > 7) \implies ((\exists a, b \in \mathbb{N})\ (a + b = x)))$

(a) $(\exists x \in R)(x \notin Q)$ true . pi

(b) $(\forall x \in Z)((x \in N \wedge x \geq 0) \vee (x \notin N \wedge x < 0))$

$(x \in N \vee x < 0)$. true

(c) $(\forall x \in Z)(6 | x) \Rightarrow (2 | x) \vee (3 | x)$. true

(d) ~~every~~ All integar number is a rational number. true

(e) ~~It a number~~ Any Integar that is divisible by 2 or ~~it is divisible~~ by 3, ~~then~~ ~~it~~ is also divisible by 6. fake 4

(f) It a nature number is greater than 7, then there exists two nature number whose sum equals to it. true

2 Truth Tables

Note 1

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \wedge (Q \vee P) \cancel{\equiv} P \wedge Q$

P	Q	$P \wedge (Q \vee P)$	$P \wedge Q$	equivalent
F	F	F	F	✓
F	T	F	F	✓
T	F	T	F	✗
T	T	T	T	✓

(b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

P	Q	R	$(P \vee Q) \wedge R$	$(P \wedge R) \vee (Q \wedge R)$	
F	F	F	F	F	✓
F	F	T	F	F	✓
F	T	F	F	F	✓
F	T	T	T	T	✓
T	F	F	F	F	✓
T	F	T	T	T	✓
T	T	F	F	F	✓
T	T	T	T	T	✓

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

P	Q	R	$(P \wedge Q \vee R)$	$(P \vee R) \wedge (Q \vee R)$

✓

3 Implication

Note 0
Note 1

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

true

(b) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

$(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x+y=0)$

$\Rightarrow (\exists y \in \mathbb{Z})(\forall x \in \mathbb{Z})(x+y=0)$

false

(c) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.

true