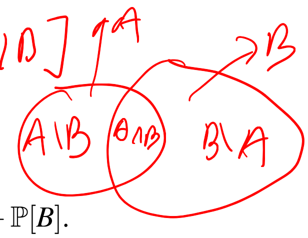
1 Probability Potpourri

is not $P[A \mid B]$



Note 13
Note 14

Provide brief justification for each part.

(a) For two events A and B in any probability space, show that $\mathbb{P}[A \setminus B] \geq \mathbb{P}[A] - \mathbb{P}[B]$.

if $P[A] \leq P[B] \rightarrow$ always true

$P[A] > P[B]$

$P(A \cap B) \geq P(A) * P(B) - P(B)^2$

$P[A \mid B] = \dfrac{P(A \cap B)}{P(B)} \geq P(A) - P(B)$

$P(A \setminus B)$

$P[A] - P[B] = P[A \setminus B] - P[B \setminus A]$

(b) Suppose $\mathbb{P}[D \mid C] = \mathbb{P}[D \mid \overline{C}]$, where $\overline{C}$ is the complement of C. Prove that D is independent of C.

↳ proof: $P[D \cap C] = P[C] * P[D]$

$P[D \mid C] = \dfrac{P[D \cap C]}{P[C]} = \dfrac{P[D \cap \bar{C}]}{P[\bar{C}]} = \dfrac{P[D] - P[D \cap C]}{1 - P[C]}$

$P[D] = P[D \cap C] + P[D \cap \bar{C}]$

(c) If A and B are disjoint, does that imply they're independent?

$P[D \cap C] - P[C] * P[D \cap C] = P[C] * P[D] - P(C) * P(D \cap C)$

NO

A: dice 1 $\frac{1}{6}$

B: dice 6 $\frac{1}{6}$

$P(A \cap B) = 0 \neq \frac{1}{6} * \frac{1}{6}$

2 Easter Eggs

Note 14

You made the trek to Soda for a Spring Break-themed homework party, and every attendee gets to leave with a party favor. You're given a bag with 20 chocolate eggs and 40 (empty) plastic eggs. You pick 5 eggs (uniformly) without replacement.

(a) What is the probability that the first egg you drew was a chocolate egg?

$$\frac{20}{60} = \frac{1}{3}$$

(b) What is the probability that the second egg you drew was a chocolate egg?

$$\frac{1}{3}$$

(c) Given that the first egg you drew was an empty plastic one, what is the probability that the fifth egg you drew was also an empty plastic egg?

$$\frac{39}{59}$$

3 Balls and Bins

Note 14

Suppose you throw n balls into n labeled bins one at a time.

(a) What is the probability that the first bin is empty?

$$\left(\frac{n-1}{n}\right)^n$$

(b) What is the probability that the first k bins are empty?

other bins may be empty too

$$\frac{(n-k)^n}{n^n}$$

$(n-k)$

(c) Let A be the event that at least k bins are empty. Let m be the number of subsets of k bins out of the total n bins. If we assume A_i is the event that the ith set of k bins is empty. Then we can write A as the union of A_i's:

$$A = \bigcup_{i=1}^{m} A_i.$$

k bins must be empty

Compute m in terms of n and k, and use the union bound to give an upper bound on the probability $\mathbb{P}[A]$.

$$m = \binom{n}{k} + \cancel{\binom{n}{k+1} + \cdots \binom{n}{n}}$$

$$m = \binom{n}{k}$$

$$P(A) \leq \sum_{i=1}^{m} A_i = m * \left(\frac{n-k}{k}\right)^n$$

$$= \binom{n}{k}\left(\frac{n-k}{k}\right)^n$$

(d) What is the probability that the second bin is empty given that the first one is empty?

n ball $\underline{n-1}$ $\underline{\text{(bin)}}$

$$\left(\frac{n-2}{n-1}\right)^n$$

A B

(e) Are the events that "the first bin is empty" and "the first two bins are empty" independent?

$$\left(\frac{n-1}{n}\right)^n \qquad \underline{\left(\frac{n-2}{n}\right)^n}$$

$$P(A \mid B) = 1 \neq P(A)$$ $\underline{\text{NO}}$

(f) Are the events that "the first bin is empty" and "the second bin is empty" independent?

A B

$$\left(\frac{n-1}{n}\right)^n \qquad \underline{\left(\frac{n-1}{n}\right)^n}$$

$$P(B \mid A) = \left(\frac{n-2}{n-1}\right)^n$$ $\underline{\text{no}}$.