

## Chapter 6

# Assembler

These slides support chapter 6 of the book

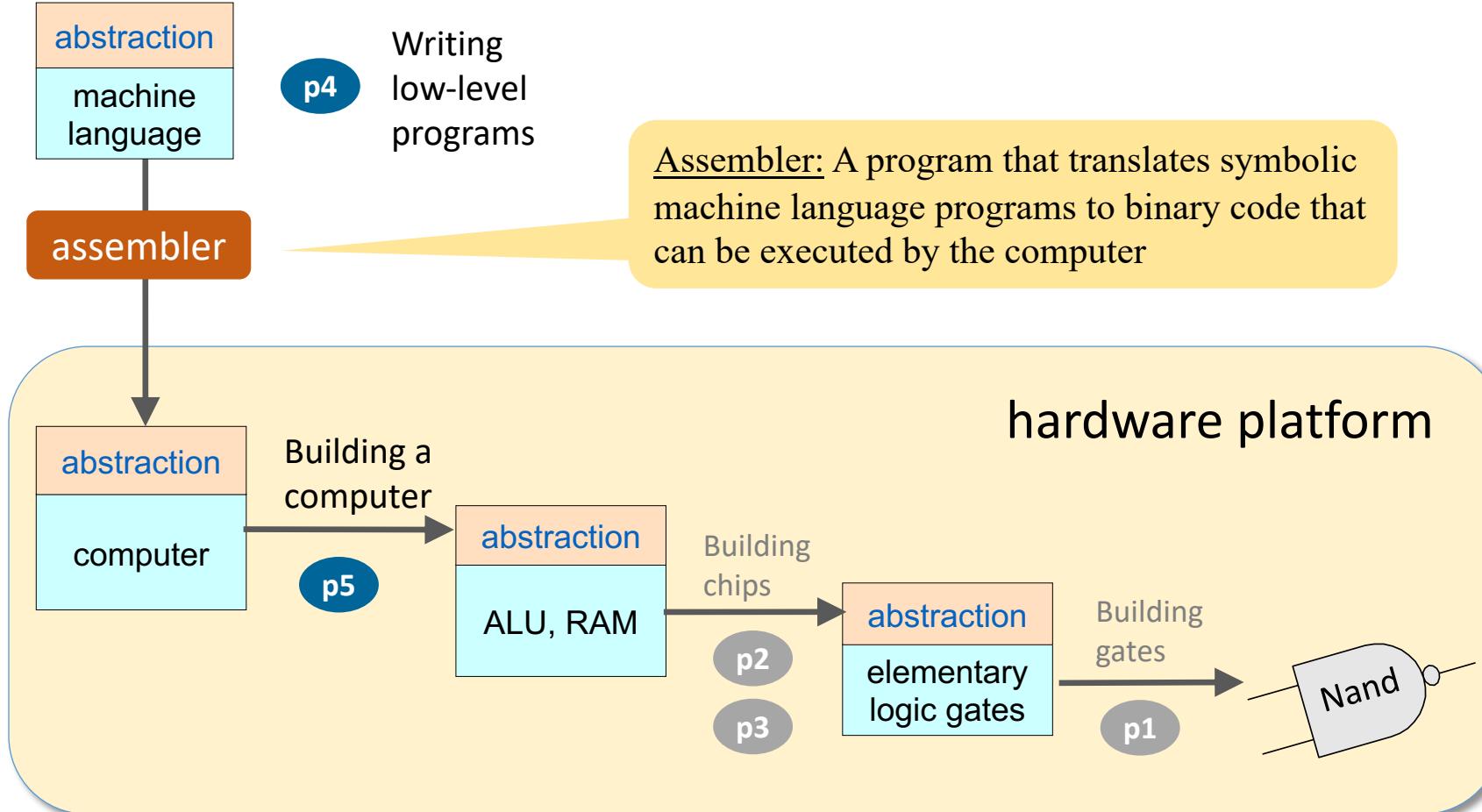
*The Elements of Computing Systems*

(1<sup>st</sup> and 2<sup>nd</sup> editions)

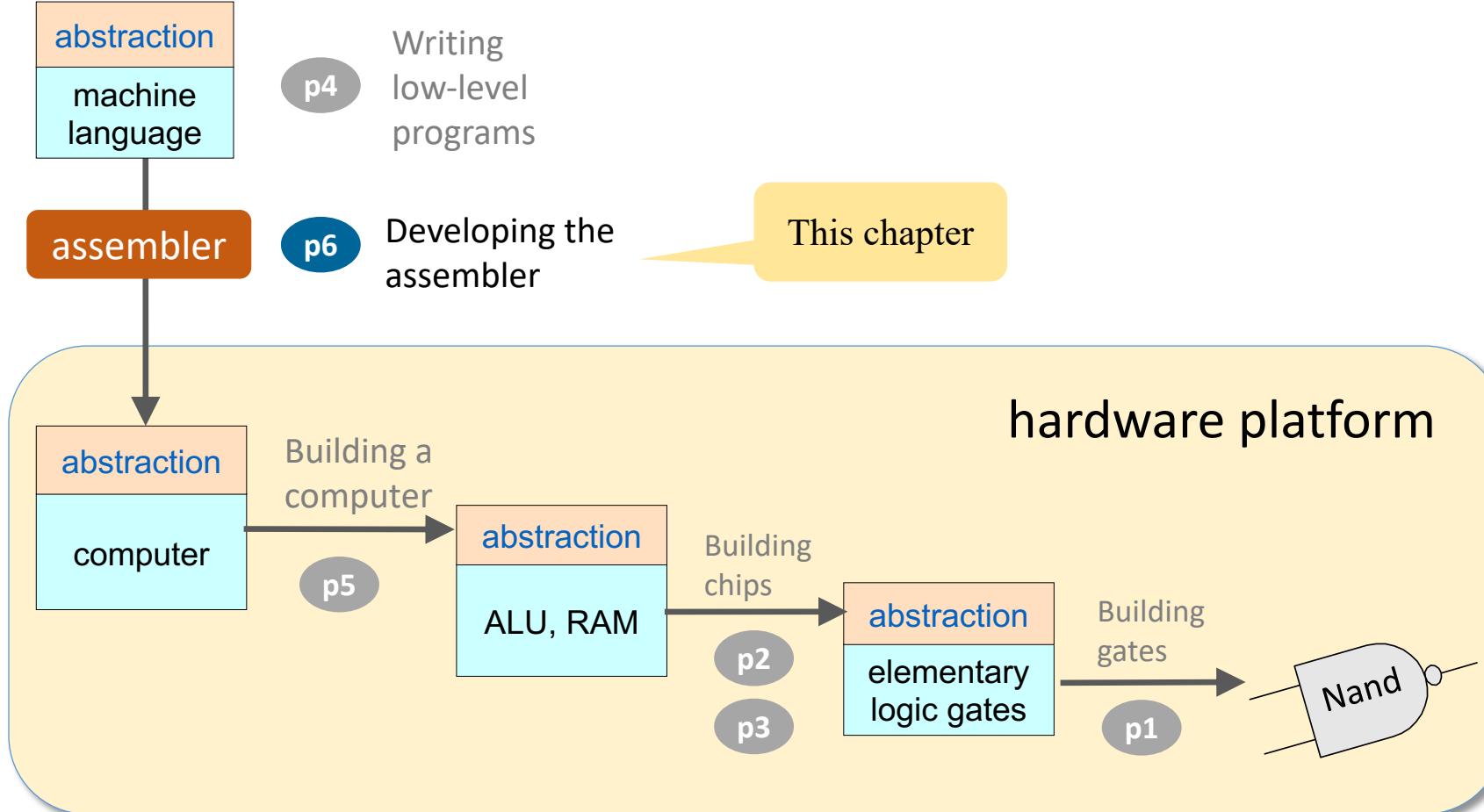
By Noam Nisan and Shimon Schocken

MIT Press

# Nand to Tetris Roadmap (Part I: Hardware)



# Nand to Tetris Roadmap (Part I: Hardware)



# The assembler

Symbolic low-level program

```
// Program: Sum1ToN (R0 represents N)
// Computes R1 = 1 + 2 + 3 + ... + R0
// Usage: put a value >= 1 in R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if(i > R0) goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum = sum + i
@sum
D=M
@i
D=D+M
@sum
M=D
// i = i + 1
@i
M=M+1
// goto LOOP
@LOOP
0;JMP
...
```

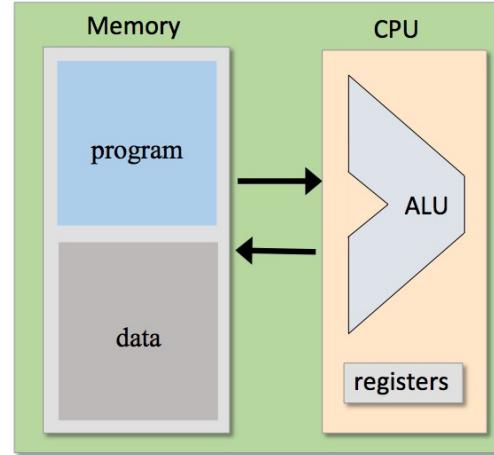
assembler

Binary code

```
010111100111100
1010101010101010
1100000010101010
1011000010000001
010111100111100
1010101010101010
1100000010101010
010111100111100
1010101010101010
1100000010101010
1011000010000001
010111100111100
1010101010101010
1100000010101010
010111100111100
1010101010101010
1100000010101010
010111100111100
1010101010101010
1100000010101010
1011000010000001
010111100111100
1010101010101010
1100000010101010
...
...
```

load and execute

Computer



## Why write an assembler?

- Because it is the “linchpin” that connects the hardware platform and the software hierarchy that sits on top of it
- Because it provides a simple example of key software engineering techniques (parsing, code generation, symbol tables, ...)

# Chapter 6: Assembler

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- Overview
- Assembler architecture
- Translating instructions
- Assembler API
- Translating programs
- Project 6
- Handling symbols
- Some history

# Chapter 6: Assembler

---

- Overview
  - Assembler architecture
  - Assembler API
  - Project 6
  - Some history
- 
- Translating instructions
    - A-instructions
    - C-instructions
  - Translating programs
  - Handling symbols

# Translating A-instructions

---

Symbolic syntax:

$@xxx$



Binary syntax:

$\theta vvvvvvvvvvvvvvvvv$

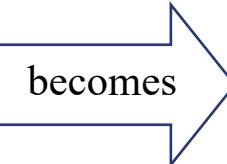
Where  $xxx$  is a non-negative decimal value, or a symbol bound to such a value

Where:

$\theta$  is the A-instruction op-code, and  $v v v \dots v$  is the value in binary

Example:

$@17$



$0000000000010001$

## Implementation

Simple: Translate the decimal value into its 16-bit representation.

# Translating C-instructions

Symbolic syntax:  $dest = comp ; jump$

Binary syntax:

1 1 1 a c c c c c d d d j j j

comp	c	c	c	c	c	c
0		1 0	1 0	1 0	1 0	0
1		1 1	1 1	1 1	1 1	1
-1		1 1	1 1	0 1	1 0	0
D		0 0	1 1	1 0	0 0	0
A	M	1 1	0 0	0 0	0 0	0
!D		0 0	1 1	0 1	0 0	1
!A	!M	1 1	0 0	0 0	0 0	1
-D		0 0	1 1	1 1	1 1	1
-A	-M	1 1	0 0	1 1	0 1	1
D+1		0 1	1 1	1 1	1 1	1
A+1	M+1	1 1	0 1	1 1	1 1	1
D-1		0 0	1 1	1 1	1 0	0
A-1	M-1	1 1	0 0	0 1	0 0	0
D+A	D+M	0 0	0 0	0 0	1 0	0
D-A	D-M	0 1	0 0	0 1	1 1	1
A-D	M-D	0 0	0 0	1 1	1 1	1
D&A	D&M	0 0	0 0	0 0	0 0	0
D A	D M	0 1	0 1	1 0	0 1	1

$a == 0$   $a == 1$

dest d d d effect: the value is stored in:

null	0 0 0	the value is not stored
M	0 0 1	RAM[A]
D	0 1 0	D register
DM	0 1 1	D register and RAM[A]
A	1 0 0	A register
AM	1 0 1	A register and RAM[A]
AD	1 1 0	A register and D register
ADM	1 1 1	A register, D register, and RAM[A]

jump j j j effect:

null	0 0 0	no jump
JGT	0 0 1	if $comp > 0$ jump
JEQ	0 1 0	if $comp = 0$ jump
JGE	0 1 1	if $comp \geq 0$ jump
JLT	1 0 0	if $comp < 0$ jump
JNE	1 0 1	if $comp \neq 0$ jump
JLE	1 1 0	if $comp \leq 0$ jump
JMP	1 1 1	Unconditional jump

Implementation: Simple: Translate each field of the symbolic instruction ( $dest, comp, jump$ ) into its binary code, and assemble the codes into a 16-bit instruction.

# Translating C-instructions

Symbolic syntax:  $dest = comp ; jump$

Binary syntax:

1 1 1 a c c c c c d d d j j j

comp	c	c	c	c	c	c
0		1 0	1 0	1 0	1 0	0
1		1 1	1 1	1 1	1 1	1
-1		1 1	1 1	0 1	1 0	0
D		0 0	1 1	1 0	0 0	0
A	M	1 1	0 0	0 0	0 0	0
!D		0 0	1 1	0 1	0 0	1
!A	!M	1 1	0 0	0 0	0 0	1
-D		0 0	1 1	1 1	1 1	1
-A	-M	1 1	0 0	1 0	1 1	1
D+1		0 1	1 1	1 1	1 1	1
A+1	M+1	1 1	0 1	1 1	1 1	1
D-1		0 0	1 1	1 1	1 0	0
A-1	M-1	1 1	0 0	0 1	0 0	0
D+A	D+M	0 0	0 0	0 0	1 0	0
D-A	D-M	0 1	0 0	0 1	1 1	1
A-D	M-D	0 0	0 0	1 1	1 1	1
D&A	D&M	0 0	0 0	0 0	0 0	0
D A	D M	0 1	0 1	1 0	0 0	1

$a == 0$   $a == 1$

dest d d d effect: the value is stored in:

null	0 0 0	the value is not stored
M	0 0 1	RAM[A]
D	0 1 0	D register
DM	0 1 1	D register and RAM[A]
A	1 0 0	A register
AM	1 0 1	A register and RAM[A]
AD	1 1 0	A register and D register
ADM	1 1 1	A register, D register, and RAM[A]

jump j j j effect:

null	0 0 0	no jump
JGT	0 0 1	if $comp > 0$ jump
JEQ	0 1 0	if $comp = 0$ jump
JGE	0 1 1	if $comp \geq 0$ jump
JLT	1 0 0	if $comp < 0$ jump
JNE	1 0 1	if $comp \neq 0$ jump
JLE	1 1 0	if $comp \leq 0$ jump
JMP	1 1 1	Unconditional jump

Binary:

Example:  $D = D+1 ; JLE$

1110011111010110

# Translating C-instructions

Symbolic syntax:  $dest = comp ; jump$

Binary syntax:

1 1 1 a c c c c c d d d j j j

comp	c	c	c	c	c	c
0		1	0	1	0	1
1		1	1	1	1	1
-1		1	1	1	0	1
D		0	0	1	1	0
A	M	1	1	0	0	0
!D		0	0	1	1	0
!A	!M	1	1	0	0	0
-D		0	0	1	1	1
-A	-M	1	1	0	0	1
D+1		0	1	1	1	1
A+1	M+1	1	1	0	1	1
D-1		0	0	1	1	1
A-1	M-1	1	1	0	0	1
D+A	D+M	0	0	0	0	1
D-A	D-M	0	1	0	0	1
A-D	M-D	0	0	0	1	1
D&A	D&M	0	0	0	0	0
D A	D M	0	1	0	1	0

a == 0 a == 1

Example: A = - 1

dest d d d effect: the value is stored in:

null	0	0	0	the value is not stored
M	0	0	1	RAM[A]
D	0	1	0	D register
DM	0	1	1	D register and RAM[A]
A	1	0	0	A register
AM	1	0	1	A register and RAM[A]
AD	1	1	0	A register and D register
ADM	1	1	1	A register, D register, and RAM[A]

jump j j j effect:

null	0	0	0	no jump
JGT	0	0	1	if $comp > 0$ jump
JEQ	0	1	0	if $comp = 0$ jump
JGE	0	1	1	if $comp \geq 0$ jump
JLT	1	0	0	if $comp < 0$ jump
JNE	1	0	1	if $comp \neq 0$ jump
JLE	1	1	0	if $comp \leq 0$ jump
JMP	1	1	1	Unconditional jump

Binary:

1 1 1 0 1 1 1 0 1 0 1 0 0 0 0 0



# Chapter 6: Assembler

---

- Overview
- Translating instructions
- Translating programs
- Handling symbols
- Assembler architecture
- Assembler API
- Project 6
- Some history

# Program translation

Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```



Binary code

```
000000000010000
1110111111001000
000000000010001
1110101010001000
000000000010000
1111110000010000
0000000000000000
1111010011010000
000000000010010
1110001100000001
000000000010000
1111110000010000
000000000010001
1111000010001000
000000000010000
111110111001000
0000000000000100
1110101010000111
000000000010001
1111110000010000
...
```

Need to handle:

- White space
- Instructions
- Symbols

We'll start with programs  
that have no symbols,  
and handle symbols later

# Program translation

Symbolic code

```
// Computes R1=1 + ... + R0  
// i = 1  
@16  
M=1  
// sum = 0  
@17  
M=0  
  
// if i>R0 goto STOP  
@16  
D=M  
@0  
D=D-M  
@18  
D;JGT  
// sum += i  
@16  
D=M  
@17  
M=D+M  
// i++  
@16  
M=M+1  
@4  
0;JMP  
@17  
D=M  
...  
no symbols
```



Binary code

Need to handle:

- White space
- Instructions
- Symbols (later)

# Program translation

Symbolic code

```
// Computes R1=1 + ... + R0  
// i = 1  
@16  
M=1  
// sum = 0  
@17  
M=0  
  
// if i>R0 goto STOP  
@16  
D=M  
@0  
D=D-M  
@18  
D;JGT  
// sum += i  
@16  
D=M  
@17  
M=D+M  
// i++  
@16  
M=M+1  
@4  
0;JMP  
@17  
D=M  
...  
no symbols
```



Binary code

Need to handle:

- White space
- Instructions
- Symbols (later)

# Program translation

Symbolic code

```
// Computes R1=1 + ... + R0  
// i = 1  
@16  
M=1  
// sum = 0  
@17  
M=0  
  
// if i>R0 goto STOP  
@16  
D=M  
@0  
D=D-M  
@18  
D;JGT  
// sum += i  
@16  
D=M  
@17  
M=D+M  
// i++  
@16  
M=M+1  
@4  
0;JMP  
@17  
D=M  
...  
no symbols
```



Binary code

Need to handle:

- White space
- Instructions
- Symbols (later)

Ignore it

White space:

- Empty lines,
- Comments,
- Indentation

# Program translation

---

Symbolic code

```
@16  
M=1  
@17  
M=0  
@16  
D=M  
@0  
D=D-M  
@18  
D;JGT  
@16  
D=M  
@17  
M=D+M  
@16  
M=M+1  
@4  
0;JMP  
@17  
D=M  
...
```

Translate

Binary code

Need to handle:

- ✓ White space
- Instructions
- Symbols (later)

# Program translation

---

Symbolic code

```
@16  
M=1  
@17  
M=0  
@16  
D=M  
@0  
D=D-M  
@18  
D;JGT  
@16  
D=M  
@17  
M=D+M  
@16  
M=M+1  
@4  
0;JMP  
@17  
D=M  
...
```



Binary code

Need to handle:

- White space
- Instructions
- Symbols (later)

Translate,  
one by one

# Program translation

Symbolic code

```
@16  
M=1  
@17  
M=0  
@16  
D=M  
@0  
D=D-M  
@18  
D;JGT  
@16  
D=M  
@17  
M=D+M  
@16  
M=M+1  
@4  
0;JMP  
@17  
D=M  
...
```

Translate

Binary code

```
000000000010000  
1110111111001000  
000000000010001  
1110101010001000  
000000000010000  
1111110000010000  
0000000000000000  
1111010011010000  
000000000010010  
1110001100000001  
000000000010000  
1111110000010000  
000000000010001  
1111000010001000  
000000000010000  
1111110111001000  
0000000000000100  
1110101010000111  
000000000010001  
1111110000010000  
...
```

Need to handle:

- White space
- Instructions
- Symbols (later)

Translate,  
one by one

# Program translation

Symbolic code

```
@16  
M=1  
@17  
M=0  
@16  
D=M  
@0  
D=D-M  
@18  
D;JGT  
@16  
D=M  
@17  
M=D+M  
@16  
M=M+1  
@4  
0;JMP  
@17  
D=M  
...
```

Translate

Binary code

```
000000000010000  
1110111111001000  
000000000010001  
1110101010001000  
000000000010000  
1111110000010000  
0000000000000000  
1111010011010000  
000000000010010  
1110001100000001  
000000000010000  
1111110000010000  
000000000010001  
1111000010001000  
000000000010000  
1111110111001000  
0000000000000100  
1110101010000111  
000000000010001  
1111110000010000  
...
```

Need to handle:

- ✓ White space
- ✓ Instructions
- Symbols

# Program translation

Symbolic code

```
@16  
M=1  
@17  
M=0  
@16  
D=M  
@0  
D=D-M  
@18  
D;JGT  
@16  
D=M  
@17  
M=D+M  
@16  
M=M+1  
@4  
0;JMP  
@17  
D=M  
...
```

Translate

Binary code

```
000000000010000  
1110111111001000  
000000000010001  
1110101010001000  
000000000010000  
1111110000010000  
0000000000000000  
1111010011010000  
000000000010010  
1110001100000001  
000000000010000  
1111110000010000  
000000000010001  
1111000010001000  
000000000010000  
1111110111001000  
0000000000000100  
1110101010000111  
000000000010001  
1111110000010000  
...
```

Need to handle:

- White space
- Instructions
- Symbols

# Program translation

Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@stop
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

Translate

Binary code

Need to handle:

- White space
- Instructions
- Symbols

Original program,  
with symbols

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- Predefined symbols
- Label symbols
- Variable symbols

Original program,  
with symbols

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@stop
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- Predefined symbols
- Label symbols
- Variable symbols

Original program,  
with symbols

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- Predefined symbols
- Label symbols
- Variable symbols

This particular program uses  
one predefined symbol: R0

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

The Hack language features  
23 *predefined symbols*:

<i>symbol</i>	<i>value</i>
R0	0
R1	1
R2	2
...	...
R15	15
SCREEN	16384
KBD	24576
SP	0
LCL	1
ARG	2
THIS	3
THAT	4

This particular program uses  
one predefined symbol: R0

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

The Hack language features  
23 *predefined symbols*:

<i>symbol</i>	<i>value</i>
R0	0
R1	1
R2	2
...	...
R15	15
SCREEN	16384
KBD	24576
SP	0
LCL	1
ARG	2
THIS	3
THAT	4

Translating *@preDefinedSymbol* :

Replace *preDefinedSymbol* with its *value*

Example: @R15 → 0000000000001111

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- ✓ Predefined symbols
- Label symbols
  - Variable symbols

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- Predefined symbols
- Label symbols
- Variable symbols

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- Predefined symbols
- Label symbols
- Variable symbols

This particular program uses two label symbols: LOOP, STOP

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Label symbols

- Used to label destinations of goto instructions
- Declared by the pseudo-instruction (*label*)
- The (*label*) directive defines the symbol *label* to refer to the memory location holding the next instruction in the program,
- Which corresponds to the instruction's *line number*

This particular program uses two label symbols: LOOP, STOP

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
...
```

## Label symbols

- Used to label destinations of goto instructions
- Declared by the pseudo-instruction (*label*)
- The (*label*) directive defines the symbol *label* to refer to the memory location holding the next instruction in the program,
- Which corresponds to the instruction's *line number*

Example: symbol    value

LOOP	4
STOP	18

This particular program uses two  
label symbols: LOOP, STOP

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
...
```

## Label symbols

- Used to label destinations of goto instructions
- Declared by the pseudo-instruction (*label*)
- The (*label*) directive defines the symbol *label* to refer to the memory location holding the next instruction in the program,
- Which corresponds to the instruction's *line number*

Example: symbol    value

LOOP	4
STOP	18

## Translating @*labelSymbol* :

Replace *labelSymbol* with its *value*

Example: @LOOP →



000000000000100
-----------------

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- ✓ Predefined symbols
- ✓ Label symbols
  - Variable symbols

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- Predefined symbols
- Label symbols
- Variable symbols

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbols

- Predefined symbols
- Label symbols
- Variable symbols

This particular program uses two variable symbols: i, sum

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Variable symbols

- Any symbol *xxx* which is neither predefined, nor defined elsewhere using an (*xxx*) label declaration, is treated as a *variable*
- Each variable is bound to a running memory address, starting at 16

Example: *symbol*    *value*

i	16
sum	17

This particular program uses two variable symbols: *i*, *sum*

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Variable symbols

- Any symbol *xxx* which is neither predefined, nor defined elsewhere using an (*xxx*) label declaration, is treated as a *variable*
- Each variable is bound to a running memory address, starting at 16

Example: *symbol*    *value*  
              i            16  
              sum          17

## Translating @variableSymbol :

1. If *variableSymbol* is seen for the first time, bind to it a *value*, from 16 onward  
Else, it has a *value*
2. Replace *variableSymbol* with its *value*.

Example: @sum →  0000000000010001

# Handling symbols

---

Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

Symbol table

symbol	value
R0	0
R1	1
R2	2
...	...
R15	15
SCREEN	16384
KBD	24576
SP	0
LCL	1
ARG	2
THIS	3
THAT	4
LOOP	4
STOP	18
i	16
sum	17

A data structure that the assembler creates and uses during the program translation

Contains the predefined symbols,  
label symbols,  
variable symbols,  
And their bindings.

# Handling symbols

---

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@stop
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbol table

<i>symbol</i>	<i>value</i>

A data structure that the assembler creates and uses during the program translation

# Handling symbols

## Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@stop
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```

## Symbol table

symbol	value
R0	0
R1	1
R2	2
...	...
R15	15
SCREEN	16384
KBD	24576
SP	0
LCL	1
ARG	2
THIS	3
THAT	4

A data structure that the assembler creates and uses during the program translation

### Initialization:

Creates the table and adds all the predefined symbols

# Handling symbols

Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@stop
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
@stop
@sum
D=M
...
...
```

Symbol table

symbol	value
R0	0
R1	1
R2	2
...	...
R15	15
SCREEN	16384
KBD	24576
SP	0
LCL	1
ARG	2
THIS	3
THAT	4
LOOP	4
STOP	18

A data structure that the assembler creates and uses during the program translation

**Initialization:**

Creates the table and adds all the predefined symbols

**First pass:** Counts lines and adds the label symbols

# Handling symbols

Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@stop
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@loop
0;JMP
@stop
@sum
D=M
...
```

Symbol table

symbol	value
R0	0
R1	1
R2	2
...	...
R15	15
SCREEN	16384
KBD	24576
SP	0
LCL	1
ARG	2
THIS	3
THAT	4
LOOP	4
STOP	18
i	16
sum	17

A data structure that the assembler creates and uses during the program translation

**Initialization:**

Creates the table and adds all the predefined symbols

**First pass:** Counts lines and adds the label symbols

**Second pass:** Generates binary code;  
In the process, adds the variable symbols

(details, soon)

# Translating programs

Symbolic code

```
// Computes R1=1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i>R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
// sum += i
@i
D=M
@sum
M=D+M
// i++
@i
M=M+1
@LOOP
0;JMP
(STOP)
@sum
D=M
...
```



Binary code

```
000000000010000
1110111111001000
000000000010001
1110101010001000
000000000010000
1111110000010000
0000000000000000
1111010011010000
000000000010010
1110001100000001
000000000010000
1111110000010000
000000000010001
1111000010001000
000000000010000
1111110111001000
000000000000100
1110101010000111
000000000010001
1111110000010000
...
```

Need to handle:

- ✓ White space
- ✓ Instructions
- ✓ Symbols

# Chapter 6: Assembler

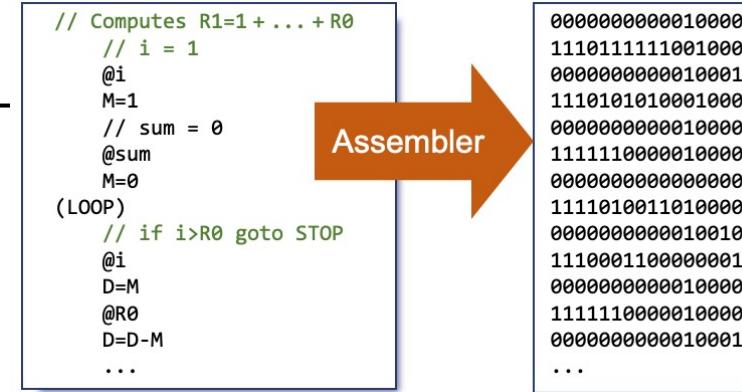
---

- Overview
  - Translating instructions
  - Translating programs
  - Handling symbols
- 
- Assembler API
  - Project 6
  - Some history

# Assembler: Usage

Input (*Prog.asm*): a text file containing a sequence of lines, each being a comment, an A instruction, or a C-instruction

Output (*Prog.hack*): a text file containing a sequence of lines, each being a string of sixteen 0 and 1 characters



Usage: (if the assembler is implemented in Java)

```
$ java HackAssembler Prog.asm
```

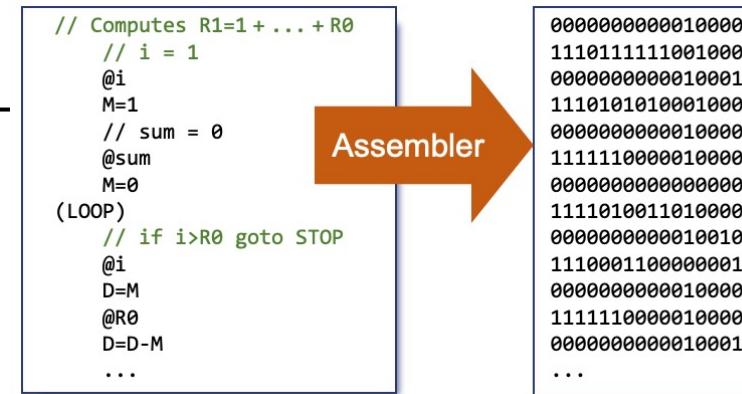
Action: Creates a *Prog.hack* file, containing the translated Hack program

# Assembler: Algorithm

## Initialize:

Opens the input file (*Prog.asm*),  
and gets ready to process it

Constructs a symbol table,  
and adds to it all the predefined symbols



## First pass:

Reads the program lines, one by one,  
focusing only on (*label*) declarations.  
Adds the found labels to the symbol table

## Second pass (main loop):

(starts again from the beginning of the file)

While there are more lines to process:

    Gets the next instruction, and parses it

    If the instruction is *@symbol*

        If *symbol* is not in the symbol table, adds it

        Translates the *symbol* to its binary value

    If the instruction is *dest=comp;jump*

        Translates each of the three fields into its binary value

Assembles the binary values into a string of sixteen 0's and 1's

Writes the string to the output file.

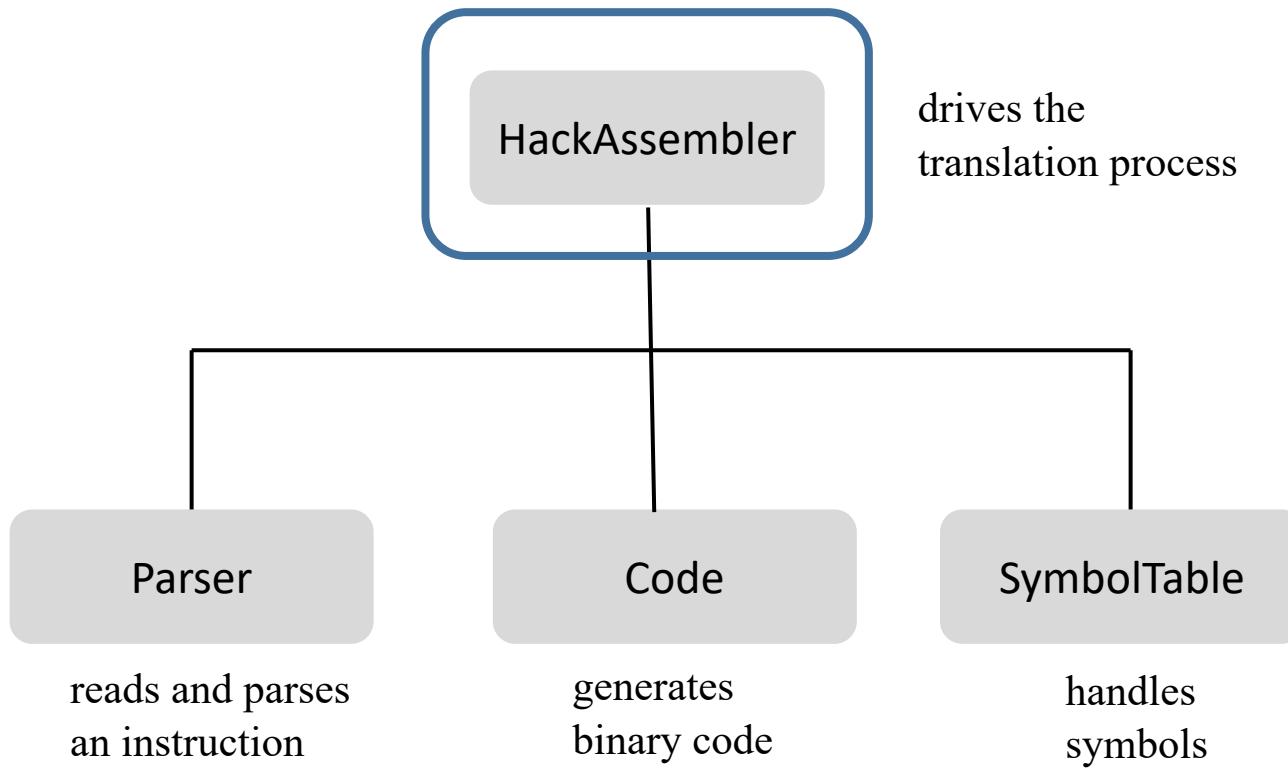
## Assembler implementation options

- Manual



# Assembler: Architecture

---



## Proposed architecture

- Four software modules
- Can be realized in any programming language

# HackAssembler

---

## Initialize:

Opens the input file (*Prog.asm*) and gets ready to process it

Constructs a symbol table, and adds to it all the predefined symbols

## First pass:

Reads the program lines, one by one focusing only on (*label*) declarations.

Adds the found labels to the symbol table

## Second pass (main loop):

(starts again from the beginning of the file)

While there are more lines to process:

    Gets the next instruction, and parses it

    If the instruction is `@symbol`

        If *symbol* is not in the symbol table, adds it

        Translates the *symbol* into its binary value

    If the instruction is `dest=comp ; jump`

        Translates each of the three fields into its binary value

Assembles the binary values into a string of sixteen 0's and 1's

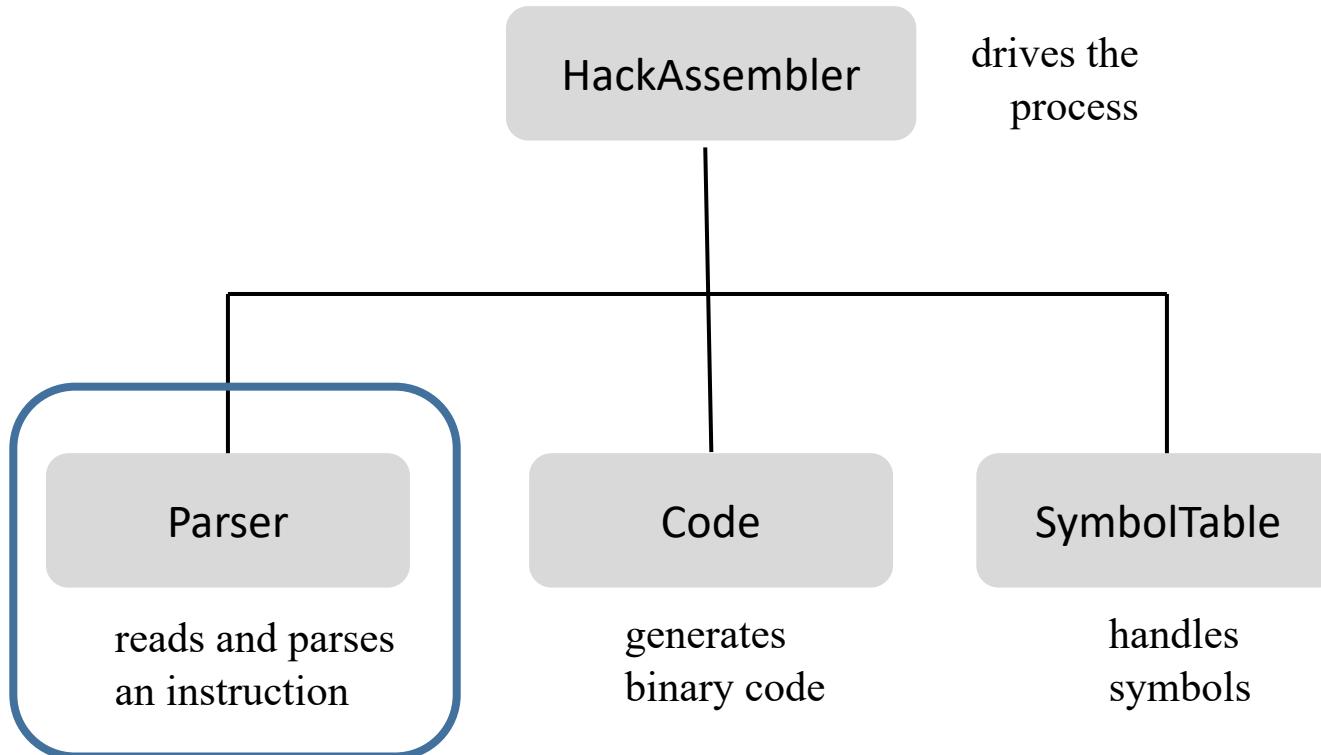
Writes the string to the output file.

The HackAssembler implements the assembly algorithm, using the services of:

- Parser
- Code
- SymbolTable

# Assembler API

---



# Parser API

---

## Routines:

- Constructor / initializer: Creates a Parser and opens the source text file
- Getting the current instruction:
  - hasMoreLines()**: Checks if there is more work to do (boolean)
  - advance()**: Gets the next instruction and makes it the *current instruction* (string)

- Parsing the *current instruction*:
  - instructionType()**: Returns the current instruction type (constant):
    - A\_INSTRUCTION for @xxx, where xxx is either a decimal number or a symbol
    - C\_INSTRUCTION for dest=comp;jump
    - L\_INSTRUCTION for (label)

current instruction

Examples:	@17	instructionType() returns A_INSTRUCTION
	@sum	instructionType() returns A_INSTRUCTION
	D=0	instructionType() returns C_INSTRUCTION
	(END)	instructionType() returns L_INSTRUCTION

# Parser API

---

## Routines:

- Constructor / initializer: Creates a Parser and opens the source text file

- Getting the current instruction:

**hasMoreLines()**: Checks if there is more work to do

**advance()**: Gets the next instruction and makes it the *current instruction*

- Parsing the *current instruction*:

**instructionType()**: Returns the instruction type

**symbol()**: Returns the instruction's *symbol* (string)

current instruction

Used if the current instruction is  
@*symbol* or (*symbol*)

Examples:

@sum

symbol() returns "sum"

(LOOP)

symbol() returns "LOOP"

# Parser API

---

## Routines:

- Constructor / initializer: Creates a Parser and opens the source text file

- Getting the current instruction:

**hasMoreLines()**: Checks if there is more work to do

**advance()**: Gets the next instruction and makes it the *current instruction*

- Parsing the *current instruction*:

**instructionType()**: Returns the instruction type

**symbol()**: Returns the instruction's *symbol* (string)

**dest()**: Returns the instruction's *dest* field (string)

**comp()**: Returns the instruction's *comp* field (string)

**jump()**: Returns the instruction's *jump* field (string)

}

Used if the current instruction is  
*dest=comp ; jump*

current instruction

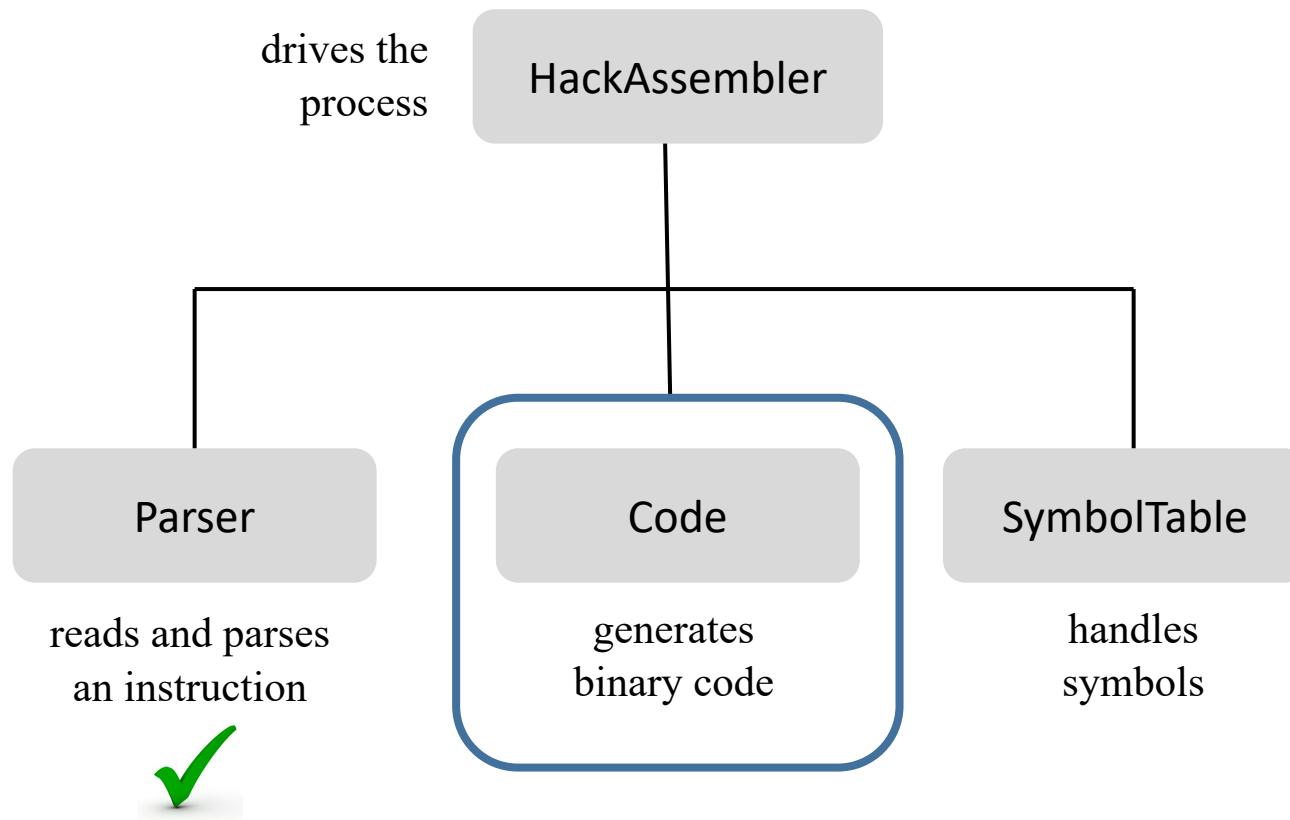
Examples:    D=D+1;JLE    dest() returns "D"    comp() returns "D+1"    jump() returns "JLE"

M=-1

dest() returns "M"    comp() returns "-1"    jump() returns null

# Implementation

---



# Code API

---

Deals only with C-instructions:  $dest = comp ; jump$

## Routines:

`dest(string)`: Returns the binary representation of the parsed *dest* field (string)

`comp(string)`: Returns the binary representation of the parsed *comp* field (string)

`jump(string)`: Returns the binary representation of the parsed *jump* field (string)

According to the language specification:

		comp									dest			jump		
		c	c	c	c	c	c	d	d	d	j	j	j	j	j	
0		1	0	1	0	1	0	0	0	0	0	0	0	0	0	
1		1	1	1	1	1	1	0	0	0	0	0	0	0	0	
-1		1	1	1	0	1	0	0	1	0	0	0	0	0	0	
D		0	0	1	1	0	0	0	1	0	0	0	0	0	0	
A	M	1	1	0	0	0	0	1	0	0	0	0	0	0	0	
!D		0	0	1	1	0	1	0	0	1	0	0	0	0	0	
!A	!M	1	1	0	0	0	0	1	0	0	0	0	0	0	0	
-D		0	0	1	1	1	1	0	0	0	0	0	0	0	0	
-A	-M	1	1	0	0	1	1	1	0	0	0	0	0	0	0	
D+1		0	1	1	1	1	1	1	1	1	0	0	0	0	0	
A+1	M+1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	
D-1		0	0	1	1	1	0	0	0	0	0	0	0	0	0	
A-1	M-1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	
D+A	D+M	0	0	0	0	1	0	0	0	0	0	0	0	0	0	
D-A	D-M	0	1	0	0	1	1	0	0	0	0	0	0	0	0	
A-D	M-D	0	0	0	1	1	1	0	0	0	0	0	0	0	0	
D&A	D&M	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
D A	D M	0	1	0	1	0	1	0	1	0	0	0	0	0	0	

a == 0      a == 1

## Examples:

`dest("DM")` returns "011"

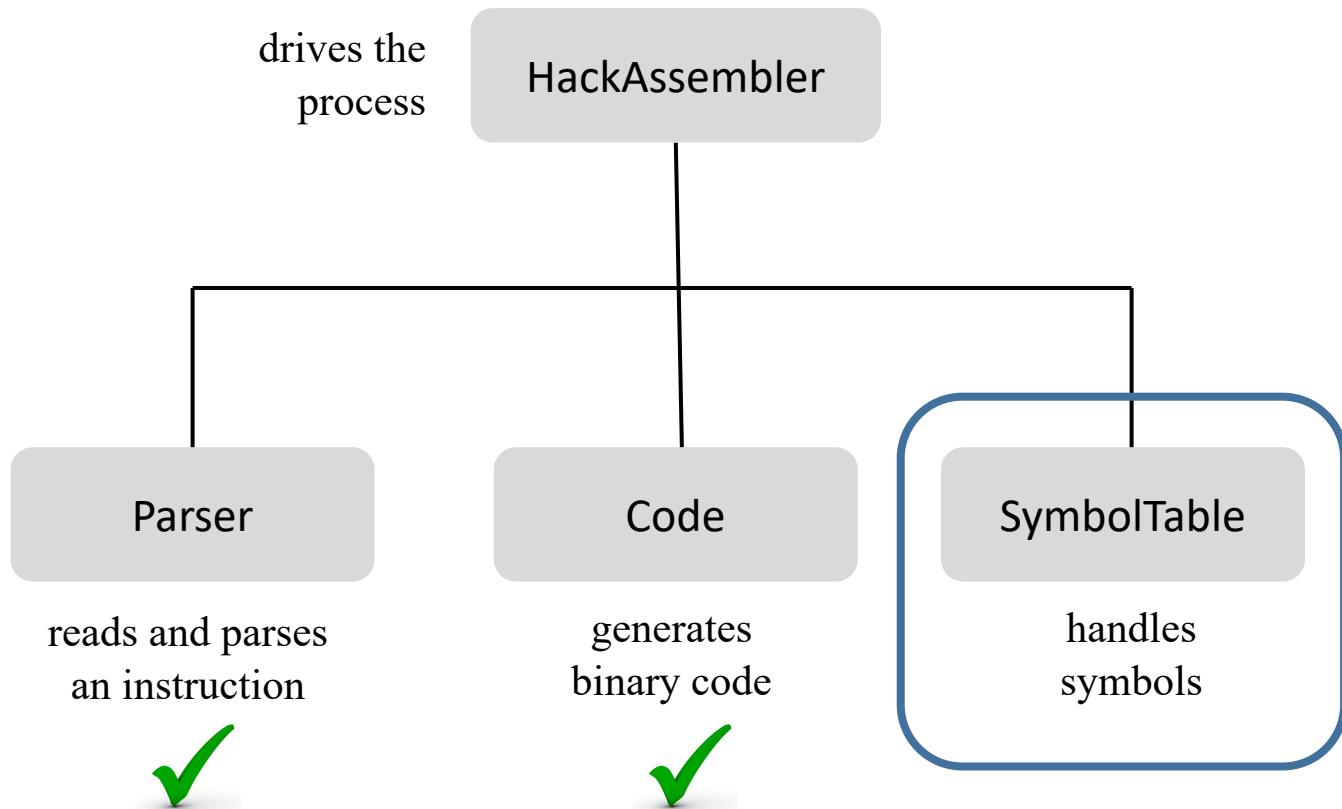
`comp("A+1")` returns "0110111"

`comp("D&M")` returns "1000000"

`jump("JNE")` returns "101"

# Implementation

---



# SymbolTable API

---

## Routines

**Constructor / initializer:** Creates and initializes a SymbolTable

**addEntry(symbol (string), address (int)):** Adds <symbol, address> to the table (void)

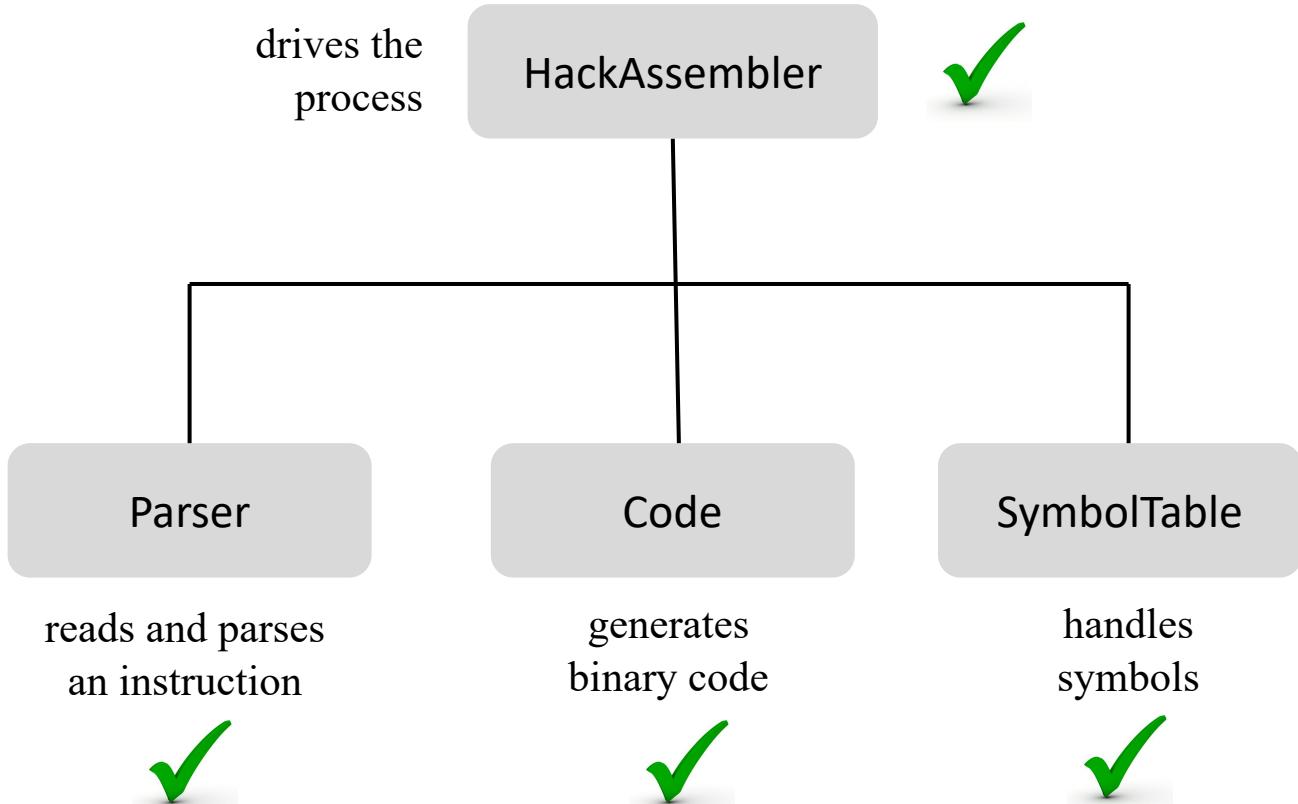
**contains(symbol (string)):** Checks if symbol exists in the table (boolean)

**getAddress(symbol (string)):** Returns the address (int) associated with symbol

Symbol table: (example)	<i>symbol</i>	<i>address</i>
	R0	0
	R1	1
	R2	2
	...	...
	R15	15
	SCREEN	16384
	KBD	24576
	SP	0
	LCL	1
	ARG	2
	THIS	3
	THAT	4
	LOOP	4
	STOP	18
	i	16
	sum	17

# HackAssembler: Drives the translation process

---



# Assembler API (detailed)

---

## Parser module:

Routine	Arguments	Returns	Function
Constructor / initializer	Input file or stream	—	Opens the input file/stream and gets ready to parse it.
hasMoreLines	—	boolean	Are there more lines in the input?
advance	—	—	Skips over whitespace and comments, if necessary. Reads the next instruction from the input, and makes it the current instruction. This method should be called only if hasMoreLines is true. Initially there is no current instruction.
instructionType	—	A_INSTRUCTION, C_INSTRUCTION, L_INSTRUCTION (constants)	Returns the type of the current instruction:  A_INSTRUCTION for @xxx, where xxx is either a decimal number or a symbol.  C_INSTRUCTION for dest=comp;jump  L_INSTRUCTION for (xxx), where xxx is a symbol.
symbol	—	string	If the current instruction is (xxx), returns the symbol xxx. If the current instruction is @xxx, returns the symbol or decimal xxx (as a string).  Should be called only if instructionType is A_INSTRUCTION or L_INSTRUCTION.
dest	—	string	Returns the symbolic dest part of the current C-instruction (8 possibilities).  Should be called only if instructionType is C_INSTRUCTION.
comp	—	string	Returns the symbolic comp part of the current C-instruction (28 possibilities). Should be called only if instructionType is C_INSTRUCTION.
jump	—	string	Returns the symbolic jump part of the current C-instruction (8 possibilities). Should be called only if instructionType is C_INSTRUCTION.

# Assembler API (detailed)

---

## Code module:

<b>Routine</b>	<b>Arguments</b>	<b>Returns</b>	<b>Function</b>
dest	string	3 bits, as a string	Returns the binary code of the <i>dest</i> mnemonic.
comp	string	7 bits, as a string	Returns the binary code of the <i>comp</i> mnemonic.
jump	string	3 bits, as a string	Returns the binary code of the <i>jump</i> mnemonic.

## SymbolTable module:

<b>Routine</b>	<b>Arguments</b>	<b>Returns</b>	<b>Function</b>
Constructor	—	—	Creates a new empty symbol table.
addEntry	symbol (string), address (int)	—	Adds <symbol, address> to the table.
contains	symbol (string)	boolean	Does the symbol table contain the given symbol?
getAddress	symbol (string)	int	Returns the address associated with the symbol.

## HackAssembler module:

We propose no API; Implement as you see fit.

# Chapter 6: Assembler

---

- Overview
- Translating instructions
- Translating programs
- Handling symbols
- Assembler architecture
- Assembler API
- Some history



# Developing a Hack Assembler

---

## Contract

- Develop a program that translates symbolic Hack programs into binary Hack instructions
- The source program (input) is supplied as a text file named *Prog.asm*
- The generated code (output) is written into a text file named *Prog.hack*
- Assumption: *Prog.asm* is error-free

Usage (if the assembler is implemented in Java):

```
$ java HackAssembler Prog.asm
```

## Staged development plan

1. Develop a basic assembler that translates programs that have no symbols
2. Develop an ability to handle symbols
3. Morph the basic assembler into an assembler that translates any program

# Testing

Prog.asm

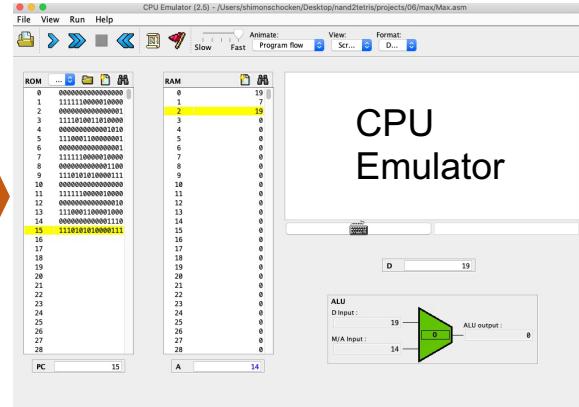
```
// Computes R1 = 1 + ... + R0
// i = 1
@i
M=1
// sum = 0
@sum
M=0
(LOOP)
// if i > R0 goto STOP
@i
D=M
@R0
D=D-M
@STOP
D;JGT
...
```

Prog.hack

```
0000000000010000
1110111111001000
0000000000010001
1110101010001000
0000000000010000
1111110000010000
0000000000000000
1111010011010000
0000000000010010
1110001100000001
0000000000010000
1111110000010000
0000000000010001
1110000100010000
0000000000010000
...
```

Your assembler

Load / Run



Or use a supplied test script that loads Prog.hack into the CPU emulator and tests it using pre-defined testing scenarios

## Test programs

- Add.asm
  - Max.asm
  - Rect.asm
  - Pong.asm
- MaxL.asm
  - RectL.asm
  - PongL.asm

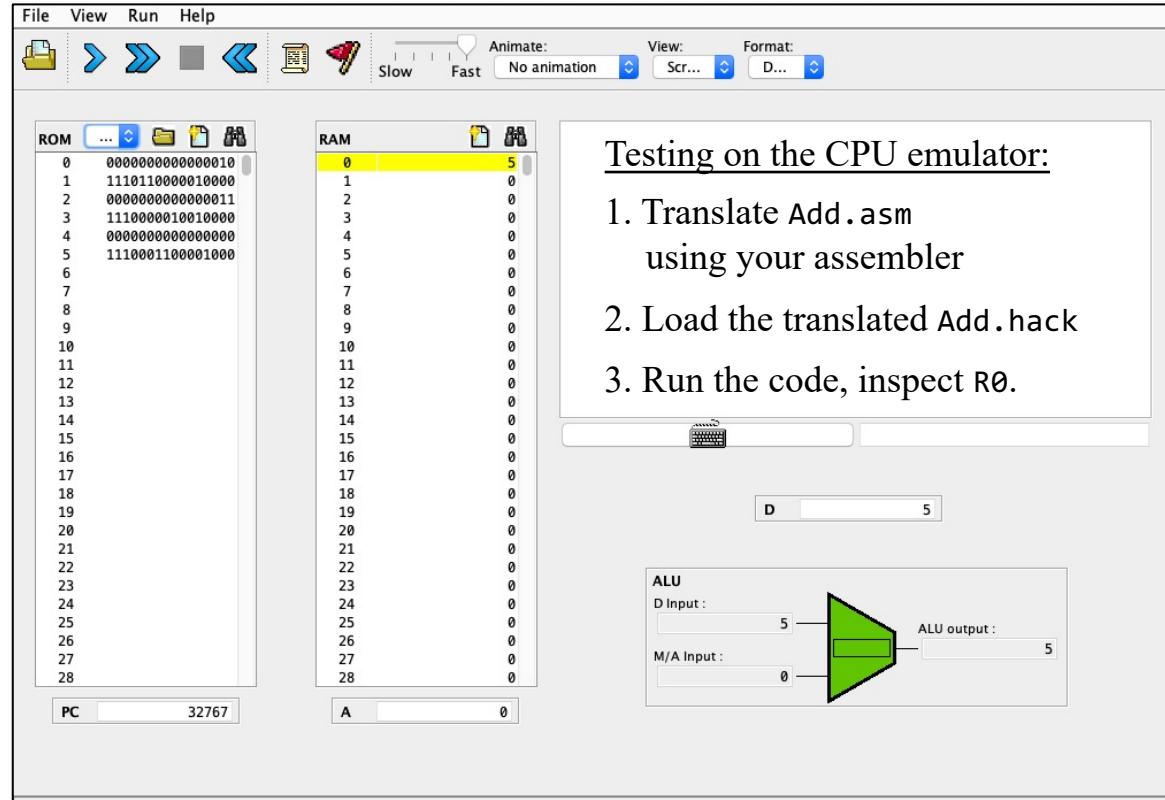
(with symbols)

(same programs, without symbols,  
for unit-testing the basic assembler)

# Testing

Add.asm

```
// Computes RAM[0] =  
//      2 + 3  
  
@2  
D=A  
  
@3  
D=D+A  
  
@0  
M=D
```



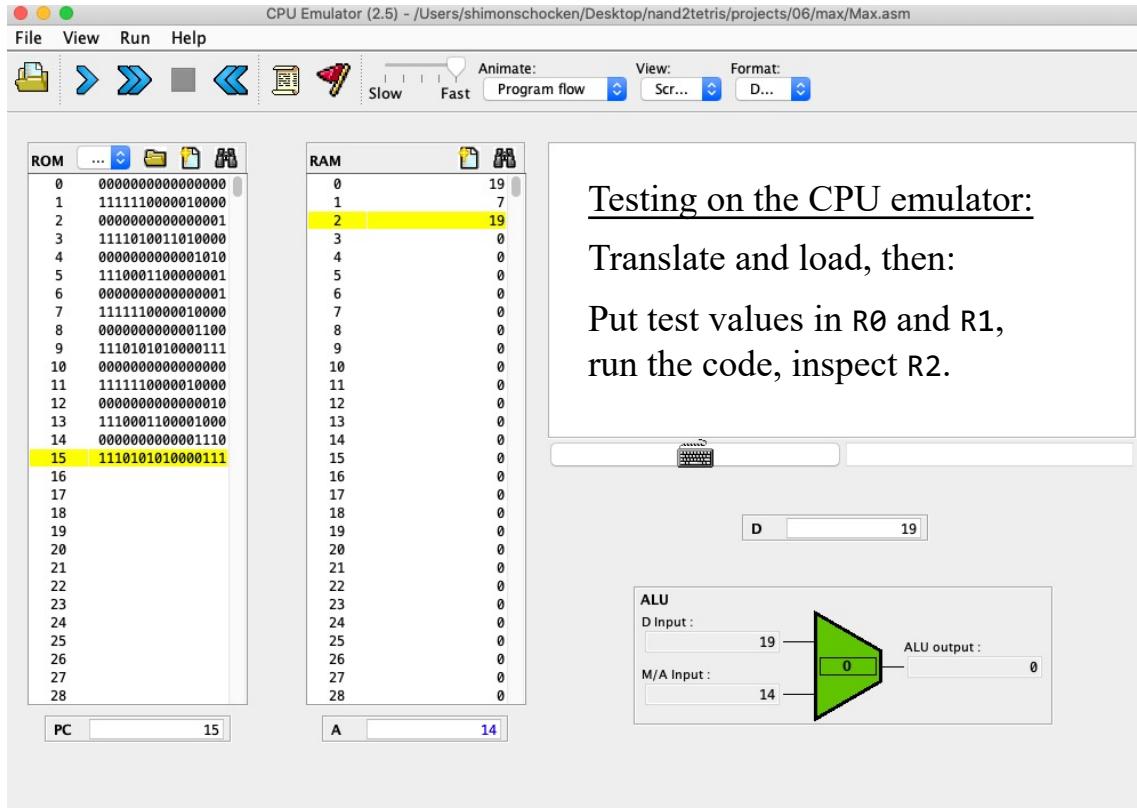
Note: When loading a binary `Prog.hack` file into the CPU emulator, the emulator may translate it back to symbolic code (depending on the emulator's version).

To inspect the binary code, select this option from the ROM menu.

# Testing

Max.asm

```
// Computes RAM[2] =  
// max(RAM[0],RAM[1])  
  
@R0  
D=M  
  
@R1  
D=D-M  
  
@OUTPUT_RAM0  
D;JGT  
  
// Output RAM[1]  
@R1  
D=M  
  
@R2  
M=D  
  
@END  
0;JMP  
  
(OUTPUT_RAM0)  
@R0  
D=M  
  
@R2  
M=D  
  
(END)  
@END  
0;JMP
```



Testing on the CPU emulator:

Translate and load, then:

Put test values in R0 and R1,  
run the code, inspect R2.

# Testing

---

Max.asm

```
// Computes RAM[2] =  
// max(RAM[0],RAM[1])  
  
@R0  
D=M  
@R1  
D=D-M  
@OUTPUT_RAM0  
D;JGT  
  
// Output RAM[1]  
@R1  
D=M  
@R2  
M=D  
@END  
0;JMP  
  
(OUTPUT_RAM0)  
@R0  
D=M  
@R2  
M=D  
  
(END)  
@END  
0;JMP
```

with symbols

MaxL.asm

```
// Computes RAM[2] =  
// max(RAM[0],RAM[1])  
  
@0  
D=M  
@1  
D=D-M  
@12  
D;JGT  
  
// Output RAM[1]  
@1  
D=M  
@2  
M=D  
@16  
0;JMP  
  
@0  
D=M  
@2  
M=D  
  
@16  
0;JMP
```

without symbols

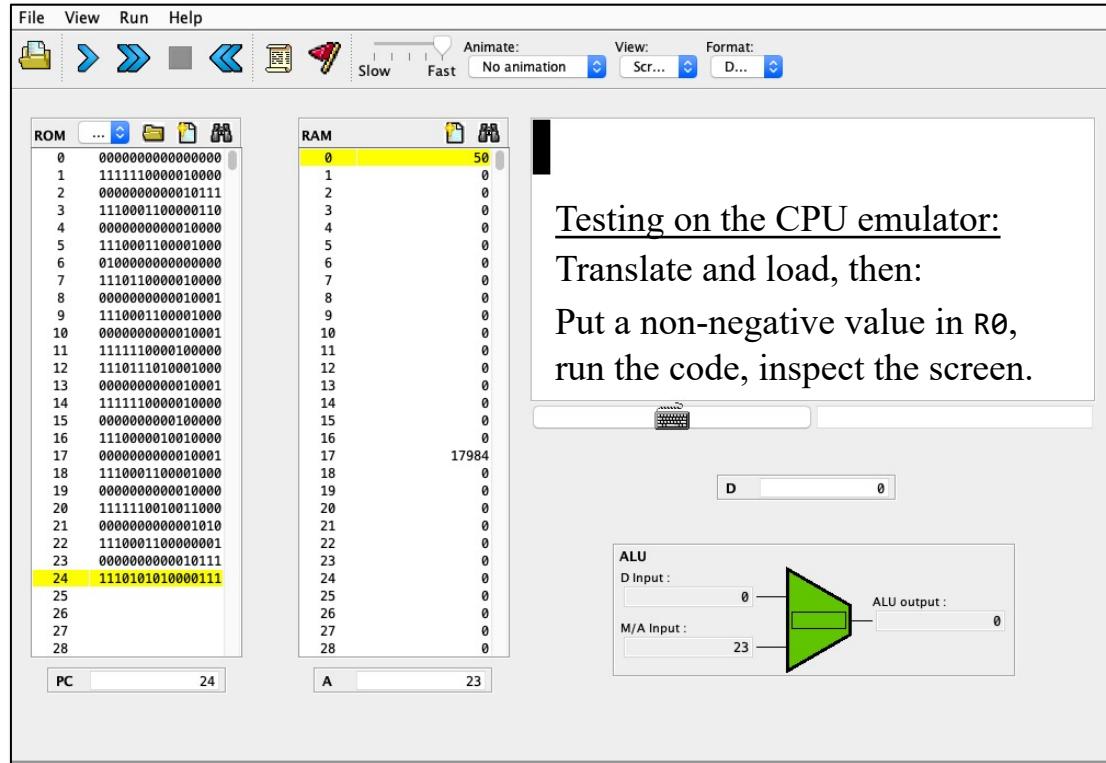
For unit-testing the  
basic assembler

(Each symbol was  
replaced with its value)

# Testing

Rect.asm

```
// Draws a rectangle.  
@R0  
D=M  
@n  
M=D  
@i  
M=0  
@SCREEN  
D=A  
@address  
M=D  
  
(LOOP)  
    @i  
    D=M  
    @n  
    D=D-M  
    @END  
    D;JGT  
    ...
```



Draws a rectangle, 16 pixels wide and R0 lines high

# Testing

---

Rect.asm

```
// Draws a rectangle.  
@R0  
D=M  
@n  
M=D  
@i  
M=0  
@SCREEN  
D=A  
@address  
M=D  
(LOOP)  
@i  
D=M  
@n  
D=D-M  
@END  
D;JGT  
...
```

with symbols

RectL.asm

```
// Draws a rectangle.  
@0  
D=M  
@16  
M=D  
@17  
M=0  
@16384  
D=A  
@18  
M=D  
(LOOP)  
@17  
D=M  
@16  
D=D-M  
@27  
D;JGT  
...
```

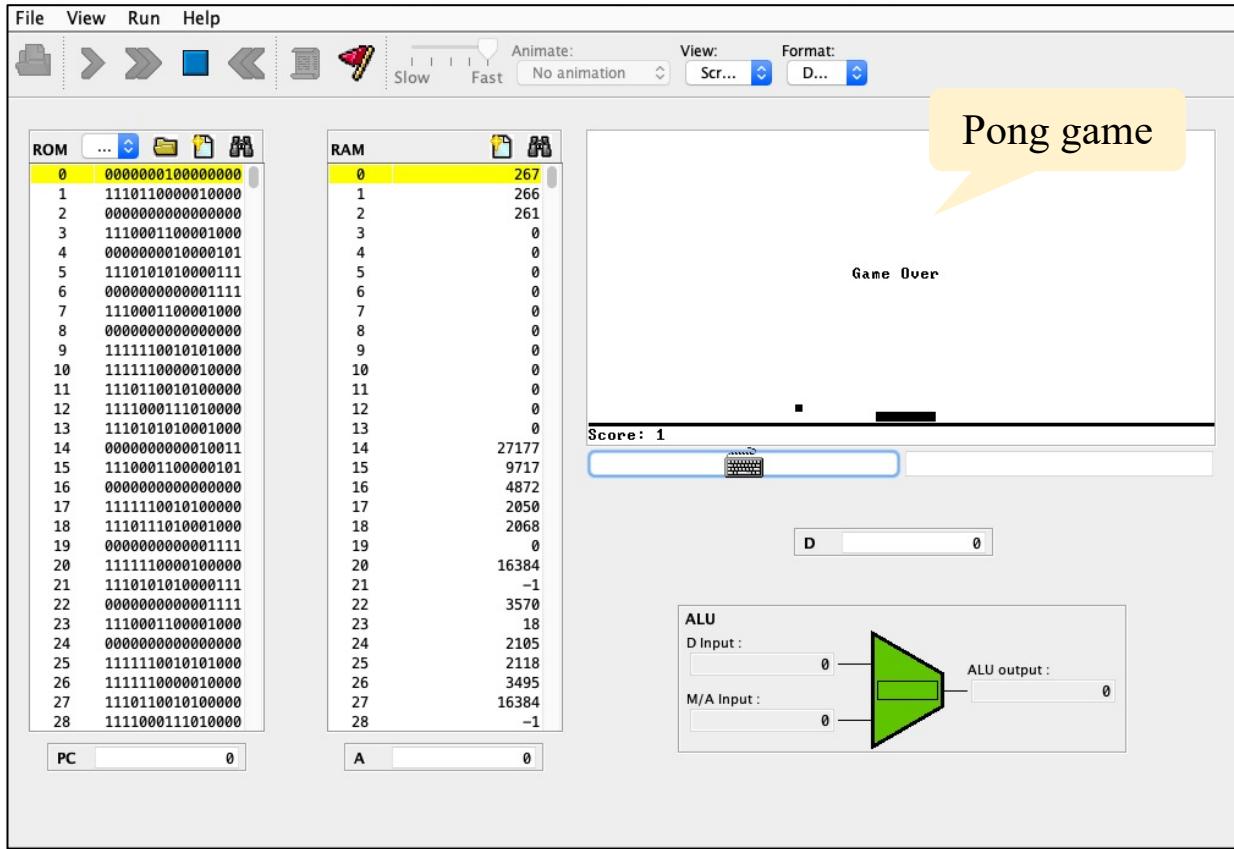
without symbols

For unit-testing the  
basic assembler

# Testing

Pong.asm

```
// Pong.asm
@256
D=A
@SP
M=D
@133
0;JMP
@R15
M=D
@SP
AM=M-1
D=M
A=A-1
D=M-D
M=0
@END_EQ
D;JNE
@SP
A=M-1
M=-1
(END_EQ)
@R15
A=M
...
```



Translate and load, and then play the game:

Select “no animation”, set the speed slider to “fast”, and run the code.

Move the paddle using the left- and right-arrow keys.

# Testing

---

Pong.asm

```
// Pong.asm
@256
D=A
@SP
M=D
@133
0;JMP
@R15
M=D
@SP
AM=M-1
D=M
A=A-1
D=M-D
M=0
@END_EQ
D;JNE
@SP
A=M-1
M=-1
(END_EQ)
@R15
A=M
...
```

## Background

- The original Pong program was written in the high-level Jack language
- The computer's operating system is also written in Jack
- The Pong code + the OS code were compiled by the Jack compiler, creating a single Pong.asm file

The compiled code (Pong.asm) has:

No white space, and

Compiler-generated addresses and symbols (which may look strange)

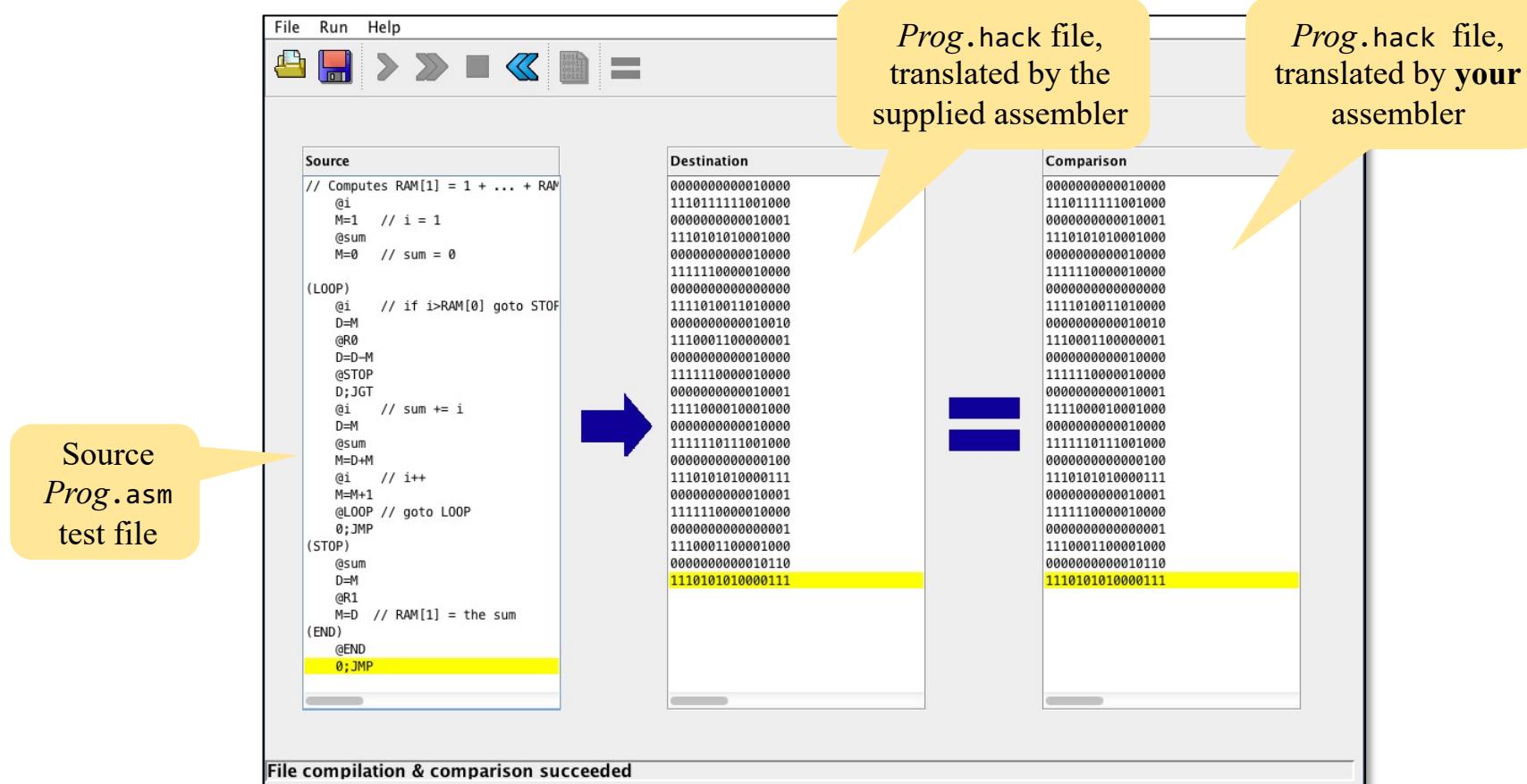
28,374 instructions

## Testing option II: Using the hardware simulator

---

1. Use your assembler to translate *Prog.asm*, generating the executable file *Prog.hack*
2. Put the *Prog.hack* file in a folder containing the chips that you developed in project 5:  
`Computer.hdl`, `CPU.hdl`, and `Memory.hdl`
3. Load `Computer.hdl` into the Hardware Simulator
4. Load *Prog.hack* into the ROM32K chip-part
5. Run the clock to execute the program.

# Testing option III: Using the supplied assembler



1. Use your assembler to translate *Prog.asm*, generating the executable file *Prog.hack*
2. Load *Prog.asm* into the supplied assembler, and load *Prog.hack* as a compare file
3. Translate *Prog.hack*, and inspect the comparison feedback messages.

# Project 6

---

Guidelines: [www.nand2tetris.org](http://www.nand2tetris.org) (projects section)

Files: nand2tetris/projects/06 (on your PC)

## Tools

- The programming language in which you develop your assembler
- CPU emulator (for testing the translated programs)
- Assembler (if you plan to use it)

## Guides

- [CPU emulator tutorial](#)
- [Assembler tutorial](#)

# Chapter 6: Assembler

---

- Overview
  - Translating instructions
  - Translating programs
  - Handling symbols
  - Assembler architecture
  - Assembler API
  - Project 6
- 
- Some history

# The Industrial Revolution (1760 – 1840) / Case in Point: Textile

---



Cotton picking



cleaning



spinning

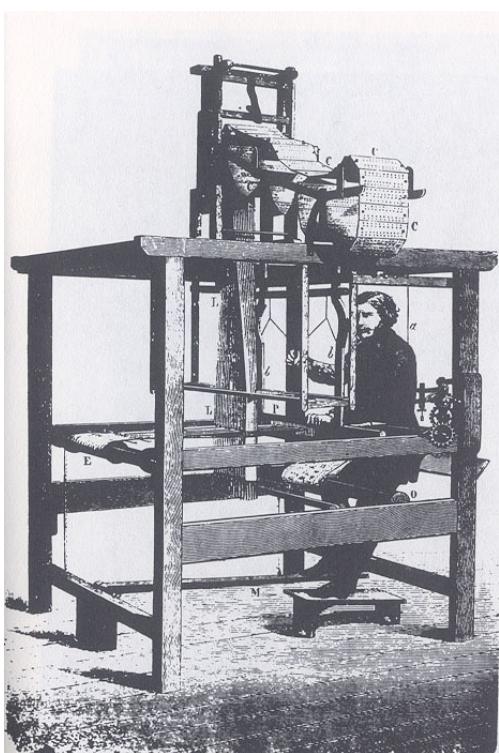
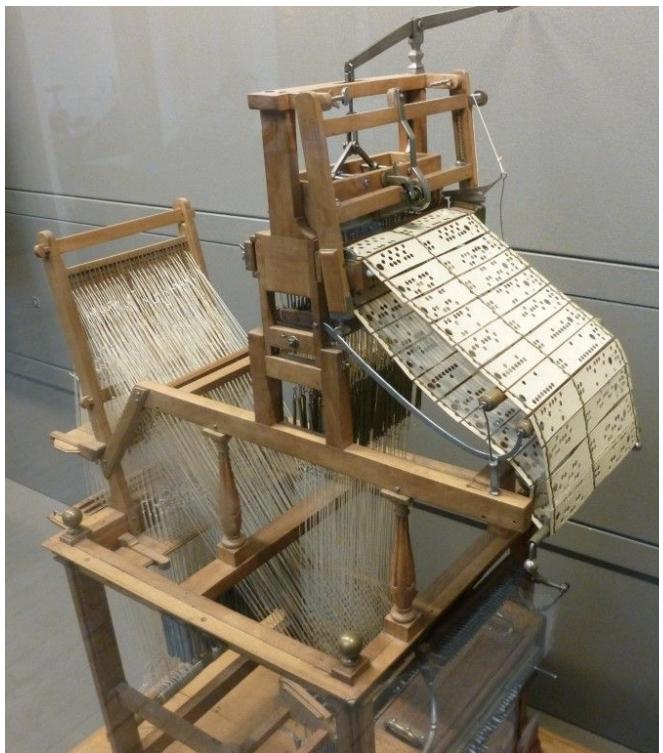


weaving

- Until 1800: labor intensive / slavery / child labor
- Industrial revolution: all key processes are mechanized.

# The Industrial Revolution (1760 – 1840) / Case in Point: Textile

---



Jacquard loom (1801)

- The weaving instructions were programmed by punched cards
- The cards coded instructions for the loom's hardware.

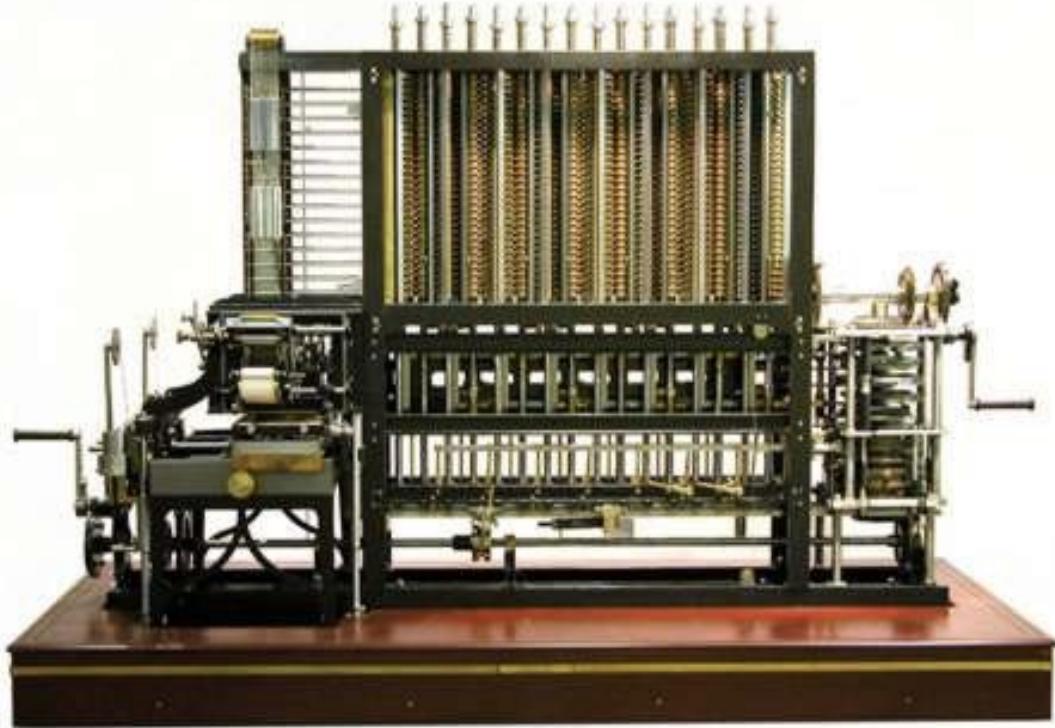
# The Industrial Revolution (1760 – 1840) / Case in Point: Textile

---



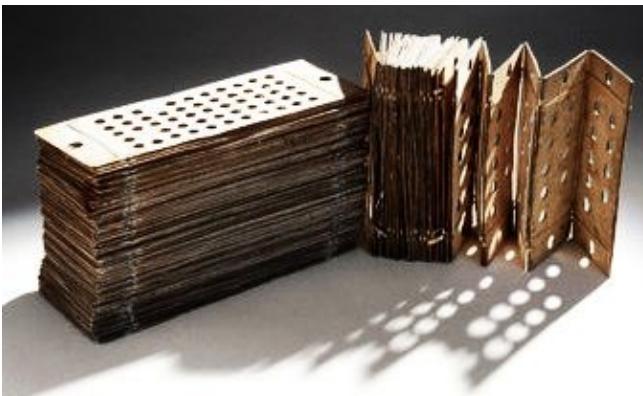
- The weaving instructions were programmed by punched cards
- The cards coded instructions for the loom's hardware.

# The Analytic Engine (1837)



- An early mechanical computer, designed by Charles Babbage
- Designed to tabulate data collected in a UK national census

- Featured a simple programming model with conditional branching
- Software = punched cards



# Ada Lovelace (1815 – 1852)

---



## Ada's insight:

If you want to give a computer instructions, don't start by punching cards ("binary code")

Instead, write the instructions on paper, using a *symbolic language*

When convinced that the symbolic program is error-free, translate it into punched cards.



- Ada Augusta King-Noel, Countess of Lovelace
- Gifted mathematician and writer
- Worked closely with Babbage on the Analytic Engine.

# Ada Lovelace (1815 – 1852)



Number of Operation.			Nature of Operation.			Variables acted upon.			Variables receiving results.			Indication of change in the value on any Variable.			Statement of Results.			Data.			Working Variables.					
1	$\times$	$IV_2 \times IV_3$				$IV_4, IV_5, IV_6$							$\begin{cases} IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2n$												
2	$-$		$IV_4 - IV_1$			$IV_4$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2n - 1$												
3	$+$		$IV_1 + IV_1$			$IV_5$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2n + 1$												
4	$+$		$IV_2 + 2IV_4$			$IV_{11}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2n - 1$												
5	$+$		$IV_{11} + IV_2$			$IV_{11}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$\frac{1}{2} \cdot 2n - 1$												
6	$-$		$IV_{12} - 2IV_11$			$IV_{12}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= \frac{1}{2} \cdot 2n - 1 = \Lambda_0$												
7	$-$		$IV_2 - IV_1$			$IV_{10}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= n - 1 (= 3)$												
8	$+$		$IV_2 + 0IV_2$			$IV_7$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2 + 0 = 2$												
9	$+$		$IV_6 + IV_7$			$IV_{11}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= \frac{2n}{2} = \Lambda_1$												
10	$\times$		$IV_{21} \times 3IV_{11}$			$IV_{12}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= B_1 \cdot \frac{2n}{2} = B_1 \Lambda_1$												
11	$+$		$IV_{12} + IV_{13}$			$IV_{13}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= \frac{1}{2} \cdot 2n - 1 + B_1 \cdot \frac{2n}{2}$												
12	$-$		$IV_{10} - IV_1$			$IV_{10}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= n - 2 (= 2)$												
13	$-$		$IV_6 - IV_1$			$IV_6$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2n - 1$												
14	$+$		$IV_1 + IV_7$			$IV_7$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2 + 1 = 3$												
15	$+$		$+IV_6 - 2IV_6$			$IV_6$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2n - 1$												
16	$\times$		$IV_8 \times 3IV_{11}$			$IV_{11}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3} = \frac{2n(2n-1)}{3}$												
17	$-$		$-IV_6 - IV_1$			$IV_6$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2n - 2$												
18	$+$		$+IV_1 + IV_7$			$IV_7$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2 + 1 = 4$												
19	$+$		$+IV_6 - 2IV_6$			$IV_6$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= 2n - 2$												
20	$\times$		$IV_9 \times IV_{11}$			$IV_{11}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = \frac{n(n-1)(n-2)}{3}$												
21	$\times$		$IV_{22} \times 3IV_{11}$			$IV_{12}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= B_2 \cdot \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = B_2 \Lambda_2$												
22	$+$		$+IV_{12} + 2IV_{12}$			$IV_{12}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= \Lambda_0 + B_1 \Lambda_1 + B_2 \Lambda_2$												
23	$-$		$-IV_{10} - IV_1$			$IV_{10}$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= n - 3 (= 1)$												
24	$+$		$IV_{13} + 0IV_{24}$			$IV_{24}$							$\begin{cases} IV_{12} = IV_{12} \\ IV_{24} = IV_{24} \end{cases}$	$= B_2$												
25	$+$		$+IV_1 + IV_3$			$V_3$							$\begin{cases} IV_1 = IV_1 \\ IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \\ IV_5 = IV_5 \\ IV_6 = IV_6 \end{cases}$	$= n + 1 = 4 + 1 = 5$												
													$\begin{cases} IV_6 = IV_6 \\ IV_7 = IV_7 \end{cases}$	by a Variable-card.												
													$\begin{cases} IV_6 = IV_6 \\ IV_7 = IV_7 \end{cases}$	by a Variable-card.												

Early program, written by Ada,  
to compute Bernoulli numbers on  
the Analytic Engine

- Often described as “the first programmer”
- The programming language Ada is named after her.

# Ada Lovelace (1815 – 1852)

---

“The Analytical Engine might act upon other things besides numbers...

Suppose, for instance, that the fundamental relations of pitched sounds in the science of harmony and of musical composition were susceptible of such expression and adaptations...

If so, the engine might compose elaborate and scientific pieces of music of any degree of complexity.”

(1840!)

