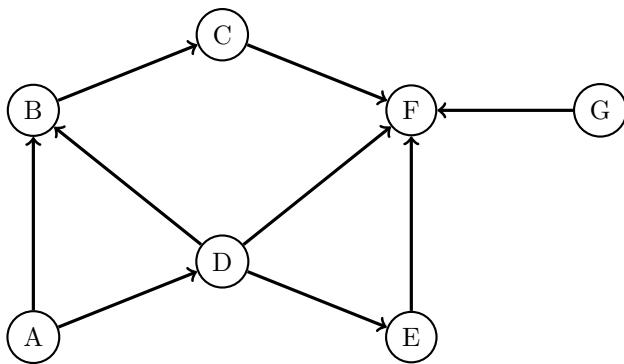


## Graphs



	A	B	C	D	E	F	G
A	0	1	0	1	0	0	0
B	0	0	1	0	0	0	0
C	0	0	0	0	0	1	0
D	0	1	0	0	1	1	0
E	0	0	0	0	0	1	0
F	0	0	0	0	0	0	0
G	0	0	0	0	0	1	0

- 1.1 Write the graph above as an adjacency matrix, then as an adjacency list. What would be different if the graph were undirected instead?

→ 沿对角线对称，对称的话则将0改为1

A B C F D E

F C B E D A

A B P C E F

- 1.2 Give the DFS preorder, DFS postorder, and BFS order of the graph traversals starting from vertex A. Break ties alphabetically.

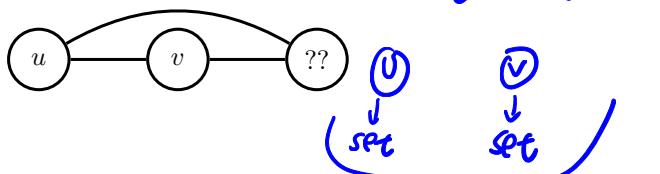
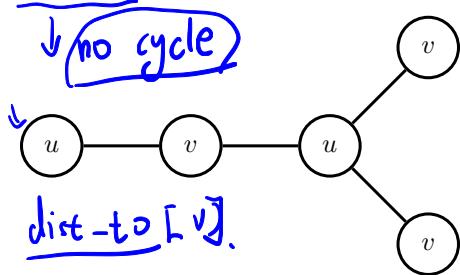
A D E B C G F

- 1.3 Give a valid topological sort of the graph. (Hint: Consider the reverse postorder of the whole graph.)

## Graph Algorithm Design

two mark, U, and V.  
every connected component

- 2.1 An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets  $U$  and  $V$  such that every edge connects an item in  $U$  to an item in  $V$ . For example below, the graph on the left is bipartite, whereas on the graph on the right is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?



BFS → enqueue → mark it, set dist-to.

if vertex after dequeue:

neighbor is marked. and not dist-to[v] then there is a loop.

$O(V+E)$

so, false:  
true

- 2.2 Provide an algorithm that finds the shortest cycle (in terms of the number of edges used) in a directed graph in  $O(EV)$  time and  $O(E)$  space, assuming  $E > V$ .

BFS. → source, → every vertex first find a loop  
moves[v], record how many times go to here.

loop all dists  
[C]

$O(V(E+V)) = O(EV)$        $O(1V) = O(E)$

- 2.3 Consider the following implementation of DFS, which contains a crucial error:

create the fringe, which is an empty Stack

push the start vertex onto the fringe and mark it → circle

while the fringe is not empty:

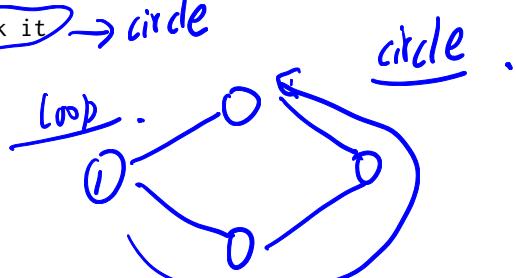
pop a vertex off the fringe and visit it

for each neighbor of the vertex:

if neighbor not marked:

push neighbor onto the fringe

mark neighbor



Give an example of a graph where this algorithm may not traverse in DFS order.

