DIS 11B

## 1 Probabilistic Bounds

CS 70, Spring 2024, DIS 11B

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Note 17

A random variable X has variance Var(X) = 9 and expectation  $\mathbb{E}[X] = 2$ . Furthermore, the value of X is never greater than 10. Given this information, provide either a proof or a counterexample for the following statements.

(a) 
$$\mathbb{E}[X^{2}] = 13$$
.  
 $Var(x) = \mathbb{E}[x^{2}] - (\mathbb{E}[x])^{2} = \mathbb{E}[x^{2}] - 4$   
 $\mathbb{E}[x^{2}] = 9 + 4 = 13$   
(b)  $\mathbb{P}[X = 2] > 0$ .  
 $P[X = 2] > 0$ .  
 $P[X = 0] = \frac{1}{4}$   
(c)  $\mathbb{P}[X \ge 2] = \mathbb{P}[X \le 2]$ .  
 $P[X \ge 2] = \mathbb{P}[X \le 2]$ .  
 $P[X \le 2] = \frac{1}{3} + \frac{1}{4} = \frac{1}{12}$   
(d)  $\mathbb{P}[X \le 1] \le 8/9$ .  
 $P[X \ge 2] = \frac{1}{3} + \frac{1}{4} = \frac{1}{12}$   
(e)  $\mathbb{P}[X \ge 6] \le 9/16$ .  
 $P[X \ge 2] = \frac{9}{16}$   
 $P[X \ge 2] \le \frac{9}{16}$ 

## 2 Vegas

Note 17

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction p of them cheat and carry a trick coin with heads on both sides. You want to estimate p with the following experiment: you pick a random sample of n people and ask each one to flip their coin. Assume that each person is independently likely to carry a fair or a trick coin.

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(a) Let 
$$X$$
 be the proportion of coin flips which are heads. Find  $\mathbb{E}[X]$ .

 $X_1 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1+p \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+p \\ 2 & 1 \end{bmatrix}$ 
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(b) Given the results of your experiment, how should you estimate p? (*Hint*: Construct an unbiased estimator for p using part (a). Recall that  $\hat{p}$  is an unbiased estimator if  $\mathbb{E}[\hat{p}] = p$ .)

$$2E[x] - 1 = P$$

$$P = E(2x - 1)$$

$$\hat{p} = 2x - 1$$

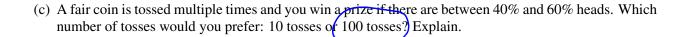
(c) How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?

## 3 Working with the Law of Large Numbers

Note 17

(a) A fair coin is tossed multiple times and you win a prize if there are more than 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(b) A fair coin is tossed multiple times and you win a prize if there are more than 40% heads. Which number of tosses would you prefer: 10 tosses of 100 tosses? Explain.



(d) A fair coin is tossed multiple times and you win a prize if there are exactly 50% heads. Which number of tosses would you prefer 10 tosses or 100 tosses? Explain.