1 Perfect Square

Note 2

(a) Prove that if n^2 is odd, then n must also be odd.

contraposition n is even $\Rightarrow$ n^2 is even

$n = 2k$

$n^2 = 4k^2 = 2 * (2k^2)$ is even. proof

(b) Prove that if n^2 is odd, then n^2 can be written in the form $8k+1$ for some integer k.

direct proof

Use (a) n is odd

$n = 2q+1$ ($q \in N$)

$n^2 = 4q^2 + 4q + 1$

$\Rightarrow$ prove $8 \mid 4q(q+1)$ $k = \frac{4q(q+1)}{8} = \frac{q(q+1)}{2}$

~~case 1: $q=0$, $n=1$, $k=0$~~

do not need special case

~~$q > 0$~~

~~case 2: q, $q+1$~~

must have a even. $q(q+1)/2 \in N$

$8 \mid 4q(q+1) \Rightarrow \exists k$, $n^2 = 8k+1$

2 Numbers of Friends

Note 2

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party. Assume that friendships are always reciprocated: that is, if Alice is friends with Bob, then Bob is also friends with Alice.

(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

contradiction: None of them has the same number of friends.

$0 \sim n-1$

n people $\Rightarrow$ most have $n-1$ friendes

pigeon hole.

if $n-1$ people have friends. Can not have no friends.

must have two people at the same hole.

contradiction

x : one way of choosing.

3 Pebbles

$\forall x$

Note 2

Suppose you have a rectangular array of pebbles, where each pebble is either red or blue. Suppose that for every way of choosing one pebble from each column, there exists a red pebble among the chosen ones. P

P.

Prove that there must exist an all-red column.

Q

$\forall x (P) \Rightarrow Q$

contraposition. $\neg Q \Rightarrow \exists x (\neg P)$

doesn't exist all-red column

at least one blue pebble each column

exist one way, no red pebble among it