CS 70

Discrete Mathematics and Probability Theory Seshia, Sinclair

Spring 2024

Countability: True or False

Note 11

(a) The set of all irrational numbers $\mathbb{R}\setminus\mathbb{Q}$ (i.e. real numbers that are not rational) is uncountable.

True

(b) The set of integers x that solve the equation $3x \equiv 2 \pmod{10}$ is countably infinite.

(c) The set of real solutions for the equation
$$x + y = 1$$
 is countable. $\left\{ \begin{array}{c} -1 \\ 1 \end{array} \right\} \left\{ \begin{array}{c} -1$

For any two functions $f: Y \to Z$ and $g: X \to Y$, let their composition $f \circ g: X \to Z$ be given by $(f \circ g)(x) =$ f(g(x)) for all $x \in X$. Determine if the following statements are true or false.

(d) f and g are injective (one-to-one) $\implies f \circ g$ is injective (one-to-one).

(e) f is surjective (onto) $\implies f \circ g$ is surjective (onto).



Counting Cartesian Products

For two sets *A* and *B*, define the cartesian product as $A \times B = \{(a,b) : a \in A, b \in B\}$.

Note 11

(a) Given two countable sets A and B, prove that $A \times B$ is countable.

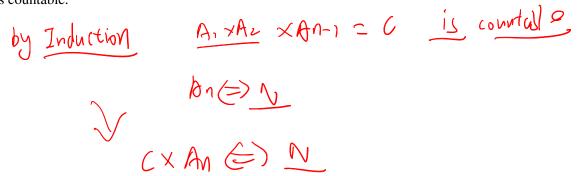
Sort by (a+b) NXN is court all a $f(a) \rightarrow N$

$$+(b) \rightarrow N$$
 $(0,0)(1,0)$ $NXM (=) A \times B$ $(0,0)(1,0)(2,0)(1,0)$

(b) Given a finite number of countable sets A_1, A_2, \dots, A_n , prove that

$$A_1 \times A_2 \times \cdots \times A_n$$

is countable.



(c) Consider a countably infinite number of finite sets: B_1, B_2, \ldots for which each set has at least 2 elements. Prove that $B_1 \times B_2 \times \cdots$ is uncountable.

diagonalization. 对角线化

3 Hello World!

Note 12

Determine the computability of the following tasks. If it's not computable, write a reduction or self-reference proof. If it is, write the program.

(a) You want to determine whether a program *P* on input *x* prints "Hello World!". Is there a computer program that can perform this task? Justify your answer.

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is Test print hellow (P, 7):

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(b) You want to determine whether a program *P* prints "Hello World!" before running the *k*th line in the program. Is there a computer program that can perform this task? Justify your answer.

NO ?? Print ("Hello world")

for i in ronge (lon (P)):

if Print; Hwby & CP, x, i):

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(c) You want to determine whether a program *P* prints "Hello World!" in the first *k* steps of its execution. Is there a computer program that can perform this task? Justify your answer.

No? Yes?? 执行程序的哪步是不可确定的。 今但是注射机已抽行的指线数 在是可以确定的