1 Box of Marbles

Note 14

You are given two boxes: one of them containing 900 red marbles and 100 blue marbles, the other one contains 500 red marbles and 500 blue marbles.

A

B

(a) If we pick one of the boxes randomly, and pick a marble what is the probability that it is blue?

C

$$P(C) = P(A \cap C) + P(B \cap C)$$

$$= P(A) * P(C \mid A) + P(B) * P(C \mid B) = \frac{1}{2} \times \left(\frac{1}{10} + \frac{1}{2}\right) = \frac{3}{10}$$

(b) If we see that the marble is blue, what is the probability that it is chosen from box 1?

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{2} \times \frac{1}{10}}{\frac{3}{10}} = \frac{1}{6}$$

(c) Suppose we pick one marble from box 1 and without looking at its color we put it aside. Then we pick another marble from box 1. What is the probability that the second marble is blue?

first is blue: E D

$$P(D) = \frac{1}{10}$$

$$= P(E) * P(D \mid E) + P(\bar{E}) * P(D \mid \bar{E})$$

$$= \frac{1}{10} \times \frac{99}{999} + \frac{9}{10} \times \frac{100}{999}$$

$$= \frac{999}{9990} = \frac{1}{10}$$

2 Poisoned Smarties

Note 14

Supposed there are 3 people who are all owners of their own Smarties factories. Burr Kelly, being the brightest and most innovative of the owners, produces considerably more Smarties than her competitors and has a commanding 50% A of the market share. Yousef See, who inherited her riches, lags behind Burr and produces 40% C of the world's Smarties. Finally Stan Furd, brings up the rear with a measly 10%. B However, a recent string of Smarties related food poisoning has forced the FDA investigate these factories to find the root of the problem. Through her investigations, the inspector found that 2 Smarties out of every 100 at Kelly's factory was poisonous. At See's factory, 5% of Smarties produced were poisonous. And at Furd's factory, the probability a Smarty was poisonous was 0.1.

$\overline{D}$

(a) What is the probability that a randomly selected Smarty will be safe to eat?

$$P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$$
$$= P(A) * P(D|A) + P(B) * P(D|B) + P(C) * P(D|C)$$
$$= \frac{1}{2} * \frac{2}{100} + \frac{4}{10} \times \frac{5}{100} + \frac{1}{10} \times \frac{1}{10} = \frac{4}{100} = 4\%$$
$$P(\overline{D}) = 96\%$$

(b) If we know that a certain Smarty didn't come from Burr Kelly's factory, what is the probability that this Smarty is poisonous?

$$P(D|\bar{A}) = \frac{P(\bar{A} \cap D)}{P(\bar{A})} = \frac{8}{10} * \frac{5}{100} + \frac{2}{10} \times \frac{10}{100} = \frac{6}{100} = 6\%$$

$$P(\bar{A} \cap D) = P(\bar{A} \cap D \cap B) + P(\bar{A} \cap D \cap C)$$
$$= P(\bar{A}) * P(B|\bar{A}) * P(D|(B \cap \bar{A})) + \ldots$$
$$= \frac{1}{2} * \frac{8}{10} * \frac{5}{100} + \frac{1}{2} * \frac{2}{10} * \frac{10}{100}$$

(c) Given this information, if a randomly selected Smarty is poisonous, what is the probability it came from Stan Furd's Smarties Factory?

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{\frac{1}{10} \times \frac{1}{10}}{\frac{4}{100}} = \frac{1}{4} = 25\%$$

$$P(C \cap D) = P(C) * P(D|C)$$
$$= \frac{1}{10} \times \frac{1}{10}$$

3 Pairwise Independence

Note 14

Recall that the events A_1, A_2, and A_3 are *pairwise independent* if for all $i \neq j$, A_i is independent of A_j. However, pairwise independence is a weaker statement than *mutual independence*, which requires the additional condition that $\mathbb{P}[A_1 \cap A_2 \cap A_3] = \mathbb{P}[A_1]\mathbb{P}[A_2]\mathbb{P}[A_3]$.

Suppose you roll two fair six-sided dice. Let A_1 be the event that the first die lands on 1, let A_2 be the event that the second die lands on 6, and let A_3 be the event that the two dice sum to 7.

(a) Compute $\mathbb{P}[A_1]$, $\mathbb{P}[A_2]$, and $\mathbb{P}[A_3]$.

$P[A_1] = \frac{1}{6}$ $\quad P[A_3] = \frac{6}{36} = \frac{1}{6}$ $\quad$ 1, 6 / 2, 5 / 3, 4

$P[A_2] = \frac{1}{6}$

(b) Are A_1 and A_2 independent?

✓

(c) Are A_2 and A_3 independent?

✓ $\quad P(A_3 | A_2) = \frac{1}{6} = \frac{P(A_3 \cap A_2)}{P(A_2)}$

$P(A_3 \cap A_2) = \frac{1}{6} \times \frac{1}{6} = P(A_2) * P(A_3)$

(d) Are A_1, A_2, and A_3 pairwise independent?

Yes $\quad A_1, A_2$ ✓

A_1, A_3 same as A_2, A_3

(e) Are A_1, A_2, and A_3 mutually independent?

No

$P(A_1 \cap A_2 \cap A_3) = \frac{1}{36} \neq \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$