CS 70 Discrete Mathematics and Probability Theory
Spring 2024 Seshia, Sinclair

DIS 6A

1 Hello World!

Same as disc 05B

Note 12

Determine the computability of the following tasks. If it's not computable, write a reduction or self-reference proof. If it is, write the program.

(a) You want to determine whether a program P on input x prints "Hello World!". Is there a computer program that can perform this task? Justify your answer.

P'(x):
run P(x) and suppress all print.
print("Hello world")

reduction

→ halt problem

(b) You want to determine whether a program P prints "Hello World!" before running the kth line in the program. Is there a computer program that can perform this task? Justify your answer.

No

(c) You want to determine whether a program P prints "Hello World!" in the first k steps of its execution. Is there a computer program that can perform this task? Justify your answer.

Yes

2 Code Reachability

Note 12

Consider triplets (M, x, L) where

- `M` is a Java program
- `x` is some input
- `L` is an integer

and the question of: if we execute $M(x)$, do we ever hit line L?

Prove this problem is undecidable.

reduction to halt problem

testHalt(P, x)

reachable(M, x, L)

P'(x):

run P(x) → first line?

return true → second line?

if reachable(P', x, 2):

return reachable(P', x, 2)

return true.

else:

return false

3 Strings

Note 10

What is the number of strings consisting of:

(a) n ones, and m zeroes?

$$\binom{n+m}{n} = \frac{(n+m)!}{m! * n!}$$

(b) n_1 A's, n_2 B's and n_3 C's?

$$\binom{n_1+n_2+n_3}{n_1}\binom{n_2+n_3}{n_2} = \frac{(n_1+n_2+n_3)!}{(n_2+n_3)!\, n_1!} \times \frac{(n_2+n_3)!}{n_3! * n_2!}$$

$$= \frac{(n_1+n_2+n_3)!}{n_1!\, n_2!\, n_3!}$$

(c) $n_1, n_2, \ldots, n_k$ respectively of k different letters?

$$\frac{(n_1+n_2+\ldots+n_k)!}{n_1! * n_2! * \ldots * n_k!}$$

4 You'll Never Count Alone

Note 10

(a) An anagram of LIVERPOOL is any re-ordering of the letters of LIVERPOOL, i.e., any string made up of the letters L, I, V, E, R, P, O, O, L in any order. For example, IVLERPOOL and POLIVOLRE are anagrams of LIVERPOOL but PIVEOLR and CHELSEA are not. The anagram does not have to be an English word.

How many different anagrams of LIVERPOOL are there?

use !

$\frac{(5+2+2)!}{2! * 2!}$ $\frac{9!}{2! \cdot 2!} = \frac{9!}{4} = 90720$

(b) How many solutions does $y_0 + y_1 + \cdots + y_k = n$ have, if each y must be a non-negative integer?

k bucket ⊔ ⊔ ⊔ ⊔ ⊔ ⊔

n ball → k

$\binom{n+k}{k}$

(c) How many solutions does $y_0 + y_1 + \cdots + y_k = n$ have, if each y must be a positive integer?

k ⊔ ⊔ ⊔ ⊔

$n-k$ ball

$n-k-1$

$\binom{n-k-1+k}{k} = \binom{n-1}{k}$