CS 170 HW 1

Due **1/27/2020, at 10:00 pm (grace period until 10:30pm)**

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write none.

2 Course Policies

(a) What dates and times are the exams for CS170 this semester?

If you have any conflicts, please fill out the **exam conflicts form**:

`https://forms.gle/oFtgYRU3u1iTF84u5`

Filling out the exam conflict form does not guarantee you will be granted accommodation.

(b) Homework is due at 10:00pm, with a late deadline at 10:30pm. At what time do we recommend you have your homework finished?

(c) We provide 2 homework drops for cases of emergency or technical issues that may arise due to homework submission. If you miss the Gradescope late deadline (even by a few minutes) and need to submit the homework, what should you do?

(d) What is the primary source of communication for CS170 to reach students? We will email out all important deadlines through this medium, and you are responsible for checking your emails and reading each announcement fully.

(e) Please read all of the following:

(i) **Syllabus and Policies:** `https://cs170.org/syllabus/`

(ii) **Homework Guidelines:** `https://cs170.org/resources/homework-guidelines/`

(iii) **Regrade Etiquette:** `https://cs170.org/resources/regrade-etiquette/`

(iv) **Piazza Etiquette:** `https://cs170.org/resources/piazza-etiquette/`

Once you have read them, copy and sign the following sentence on your homework submission.

"I have read and understood the course syllabus and policies."

3 Understanding Academic Dishonesty

Before you answer any of the following questions, make sure you have read over the syllabus and course policies (`https://cs170.org/syllabus/`) carefully. For each statement below, write *OK* if it is allowed by the course policies and *Not OK* otherwise.

(a) You ask a friend who took CS 170 previously for their homework solutions, some of which overlap with this semester's problem sets. You look at their solutions, then later write them down in your own words.

(b) You had 5 midterms on the same day and are behind on your homework. You decide to ask your classmate, who's already done the homework, for help. They tell you how to do the first three problems.

(c) You look up a homework problem online and find the exact solution. You then write it in your words and cite the source.

(d) You were looking up Dijkstra's on the internet, and run into a website with a problem very similar to one on your homework. You read it, including the solution, and then you close the website, write up your solution, and cite the website URL in your homework writeup.

4 Asymptotic Complexity Comparisons

(a) Order the following functions so that for all i, j, if f_i comes before f_j in the order then $f_i = O(f_j)$. Do not justify your answers.

- $f_1(n) = 3^n$
- $f_2(n) = n^{\frac{1}{3}}$
- $f_3(n) = 12$
- $f_4(n) = 2^{\log_2 n}$
- $f_5(n) = \sqrt{n}$ $\rightarrow n^{\frac{1}{2}}$
- $f_6(n) = 2^n$
- $f_7(n) = \log_2 n$
- $f_8(n) = 2^{\sqrt{n}}$
- $f_9(n) = n^3$

$f_3\ f_7\ f_2\ f_5\ f_4\ f_9\ f_8\ f_6\ f_1$

As an answer you may just write the functions as a list, e.g. $f_8, f_9, f_1, \ldots$

(b) In each of the following, indicate whether $f = O(g)$, $f = \Omega(g)$, or both (in which case $f = \Theta(g)$). ***Briefly*** justify each of your answers. Recall that in terms of asymptotic growth rate, logarithmic < polynomial < exponential.

	$f(n)$	$g(n)$
(i)	$\log_3 n$	$\log_4(n)$
(ii)	$n \log(n^4)$	$n^2 \log(n^3)$
(iii)	$\sqrt{n}$	$(\log n)^3$
(iv)	$n + \log n$	$n + (\log n)^2$

(i) Θ $\rightarrow$ $\lim_{n\to\infty} \frac{\log_3 n}{\log_4 n} = \frac{\lg 4}{\lg 3} = C$

(ii) $f(n) = 4n\log(n)$ $\quad g(n) = 3n^2\log(n)$

$f(n) = O(g(n))$ $\quad \lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{4}{3n} = 0$

5 Computing Factorials

Consider the problem of computing $N! = 1 \times 2 \times \cdots \times N$.

(a) N is $\log N$ bits long (this is how many bits are needed to store a number the size of N). Find an $f(N)$ so that $N!$ is $\Theta(f(N))$) bits long. Simplify your answer as much as possible, and give an argument for why it is true.

Hint: You may use Stirling's formula:

$$\sum_{k=1}^{n} \log k \sim \log\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right)$$

Where $f(n) \sim g(n)$ means that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$.

Note: There are ways to solve this problem without using Stirling's formula. Any answer that adequately proves a theta bound will receive full points.

(b) Give a simple (naive) algorithm to compute $N!$. You may assume that multiplying two bits together (e.g. $0 \times 0, 0 \times 1$) takes 1 unit of time.

Please give a 3-part solution.

4. (b) (iii) $f(n) = \Omega(g(n))$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(\log n)^3} = \lim_{n\to\infty} \frac{\frac{1}{2} n^{-\frac{1}{2}}}{3(\log n)^2 \times \frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{2} n^{\frac{1}{2}}}{3(\log n)^2}$$

$$= \frac{1}{6} \lim_{n\to\infty} \frac{\frac{1}{2} n^{-\frac{1}{2}}}{2\log n \times \frac{1}{n}}$$

$$= \frac{1}{24} \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\log n}$$

$$= \frac{1}{24} \lim_{n\to\infty} \frac{\frac{1}{2} n^{-\frac{1}{2}}}{\frac{1}{n}}$$

$$= \frac{1}{48} \lim_{n\to\infty} n^{\frac{1}{2}} = +\infty$$

(iv) $f(n) = \Theta(g(n))$

$$\lim_{n\to\infty} \frac{n + \log n}{n + (\log n)^2}$$

$$\lim_{n\to\infty} \frac{1 + \frac{1}{n}}{1 + 2\log n \times \frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n+1}{n + 2\log n}$$

$$= \lim_{n\to\infty} \frac{1}{1 + \frac{2}{n}} = 1$$

5 (a) $\log(n!) \to n \log N$

$$\log n! = \sum_{i=k}^{N} \log k = \Theta\left(\log \sqrt{2\pi n} \left(\frac{n}{e}\right)^n\right)$$

$$\log\sqrt{2\pi n}) + \log\left(\frac{n}{e}\right)^n = \Theta(n \log n)$$

$$= \Theta\left(\cancel{n \log \sqrt{2\pi n}\left(\frac{n}{e}\right)}\right)$$

$$= \Theta\left[\cancel{n \log \frac{\sqrt{2\pi}}{e} * n^{\frac{3}{2}}}\right]$$

$$= \cancel{\Theta\left[n^{\frac{3}{2}} \log n\right]}$$

$$= \cancel{\Theta\left[n \log n\right]}$$

(b) Iteractiv

factorial (N)

f = 1

for i = 2 : n

f = $\underline{f}$ * $\underline{i}$

return i

Run time

log N bit

N log N bit

$N \log^2 N$

$O(N^2 \log^2 N)$

6 Polynomial Evaluation

Given coefficients $a_0, \ldots, a_n \in \mathbb{N}$, consider the polynomial:

$$p(x) = \sum_{k=0}^{n} a_k x^k$$

For this problem, assume that addition and multiplication of two natural numbers takes only $\mathcal{O}(1)$ time (regardless of how large they are).

(a) Describe a naive algorithm that, given $[a_0, \ldots, a_n]$ and x, computes $p(x)$. Give an analysis of its runtime as a function of the degree of the polynomial n.

(b) As an alternative, we can compute the following expression:

$$p(x) = a_0 + x(a_1 + x(a_2 + \ldots + x(a_{n-1} + x \cdot a_n) \ldots))$$

Describe and analyze an algorithm that evaluates the polynomial using the above expression. The runtime should be a function of n as well.

Give a 3-part solution.

(a) $P_1(a_0 \ldots a_n, x)$:

sum = a_0
for i = 1 to n :
b = a_i
for j = 1 to i :
b = b * x
sum += b
return sum

$\Theta(n^2)$

② Proof of correctness.

↓ invariants use

① Algorithm description.

(b) $P_2(a_0 \ldots a_n, x)$

sum = a_n
for i = n−1 ... 0
sum += a_i + x * sum
return sum

③

$\Theta(n)$ ← Run time analysis