Integrating functions with MatLab[1]
MatLab exercise for MATH263B

While working through this MatLab assignment, keep a notepad handy to write down the answers to the problems listed in the text. These answers may be collected and graded.

In this exercise, you will learn how you can use MatLab to integrate functions; both symbolically and numerically. Along the way, you will learn about some of the pitfalls of using MatLab for these purposes.

In order to perform symbolic integration on functions, you need to declare the variables that occur in these functions as so-called *symbolic variables*. Let us declare two variables a and x this way.

```
>> syms x a
```

Now let us ask MatLab to find the indefinite integral $\int x^2 \, dx$

```
>> int(x^2)
```

The answer is almost as expected, but not quite: Note that MatLab does not add the constant of integration to the result (of course MatLab doesn't have to take any tests in my MATH263B class).

Now let us try to compute the integral $\int \frac{a}{x} \, dx$, where a is a constant. Enter

```
>> int(a/x)
```

Problem 1 *How does* MatLab*'s answer differ from the correct answer that you learned in class?* Hint: MatLab *uses* `log(x)` *for* $\ln x$*; this is* not *the problem.*

In the above example, although the dx-notation is not part of the MatLab command for symbolic integration, the program guessed (correctly) that x was supposed to be the variable of integration and a was supposed to be a constant.

Problem 2 *Which command should you enter in order to calculate* $\int \frac{a}{x} \, da$, *where x is treated as a constant?* Hint: *Enter* `>> help int` *to find out more about the syntax of the command* `int`.

Now let us use MatLab to find $\int \frac{x}{(x+3)(x-1)(x^2+1)} \, dx$. Enter

```
>> int(x/(x+3)/(x-1)/(x^2+1))
```

MATLAB's answer indicates that the program performed a partial fraction decomposition. You can easily find out what the decomposition is by letting the program symbolically differentiate the answer.

```
>> diff(ans)
```

Problem 3 *What is the partial fraction decomposition of* $\frac{x}{(x+3)(x-1)(x^2+1)}$?

You can recover the original form of the integrand by entering

```
>> simplify(ans)
```

Theoretically, if $f(x)$ is the simplest formula for any function that can be symbolically integrated, then entering the sequence of commands

```
>> int(f(x))
>> diff(ans)
>> simplify(ans)
```

should give you the original formula for $f(x)$. However, this works not always as expected. Enter

```
>> int(sin(x)^3*cos(x)^4)
>> diff(ans)
>> simplify(ans)
```

Problem 4 *Which simplified answer does* MATLAB *give you? How do you explain the discrepancy from what you expected?* Hint: *Recall that* $\sin^2 x = 1 - \cos^2 x$.

Some definite integrals just cannot be expressed in terms of the elementary functions that you have learned about in class. If you ask MATLAB to integrate such functions, you may get answers that contain symbols for exotic functions that are defined in terms of integrals. For example, when you enter

```
>> int(exp(x^2))
```

then your answer is expressed in terms of the so-called *error function* `erf`.

At other times, MATLAB will give you an incomplete answer that contains another integral.

Problem 5 *Try to symbolically integrate* $\sqrt{1 + x^3 - x^5}$. *Write down* MATLAB's *answer in ordinary mathematical notation.* Hint: MATLAB's *command for* $\sqrt{x}$ *is* `sqrt(x)`.

MATLAB has a built-in command `quad` for approximate integration using Simpson's Rule. As you learned in class, $\int_0^\pi \sin x \, dx = 2$. Let us see what we get from approximate integration:

```
>> quad('sin(x)', 0, pi)
```

Note that the expression for the function now needs to be enclosed in quotation marks. The reason is that for approximate integration, MATLAB needs to actually evaluate the function for may arguments rather than treating its formula as a symbolic expression. In fact, you can find out how many function evaluations MATLAB performed in order to compute the result by entering

```
>> [q, n] = quad('sin(x)', 0, pi)
```

The answer has two components: `q` is the approximation of the integral, and `n` is the number of function evaluations.

MATLAB's answer seems to be the exact value of the integral, but this is not quite true. Enter

```
>> format long
>> [q, n] = quad('sin(x)', 0, pi)
```

Now you can see that the result is accurate only up to eight decimal places (in fact, MATLAB's answer is *guaranteed* to be accurate only up to six decimal places, but in this particular example we get a somewhat better-than-hoped-for accuracy). You can force MATLAB to give you answers with greater precision, but this may slow down the calculation. In order to make sure that the answer is correct up to ten decimal places, enter

```
>> [q, n] = quad('sin(x)', 0, pi, 1.e-10)
```

Problem 6 *What is the error this time? How does it compare to the maximum allowed error tolerance of 10^{-10}? What happened to the number of function evaluations?*

When using the `quad` command, some extra care needs to be taken when entering the formula of the function. As mentioned above, the formula for $f(x)$ is not treated as a symbolic expression. Instead, it is treated as a *vector* of numbers. MATLAB uses slightly different notations for multiplication, division, and exponentiation for individual numbers and for numbers that are part of a vector. In the latter case, the symbols `.*`, `./`, `.^` are used. For example, in order to approximate $\int_1^3 \frac{x^2 \ln x}{x+1} \, dx$ you should enter

```
>> quad('x.^2.*log(x)./(x+1)', 1, 3)
```

Problem 7 *What is* MATLAB*'s answer?*

When using a computer algebra system to numerically integrate a function, you should always make sure beforehand that the integral that you are trying to evaluate is proper. For example, let us try to approximate the improper integral $\int_{-1}^{1} \frac{1}{x^2} dx$.

```
>> quad('1./x.^2', -1, 1)
```

MATLAB will complain that you asked it to divide by zero and give you the answer `NaN`, which stands for "not a number." Since $\int_{-1}^{1} \frac{1}{x^2} dx = \infty$, you might have expected an answer of "`inf`," but this is not what you get.

Now let us try to use `quad` for evaluating $\int_{-\pi}^{2\pi} \frac{1}{x^2} dx$. Let us also keep track of the number of function evaluations performed by MATLAB.

Problem 8 *What command do you enter for this? How many function evaluations does* MATLAB *go through? Why does* MATLAB *give you a numerical answer now; whereas it did not give you a numerical answer in the previous example?* Hint: *The last question requires some thought. You will need to remind yourself how Simpson's rule works and use your imagination to second-guess the steps taken by* MATLAB *in executing your commands.*

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