MAD 3400 Lab 3: Interpolation **Fall 2015**

DUE: November 6, 2015

•Turn in well-written, clear, concise answers. Include a `diary` of commands, all functions and scripts created/used, and tables of output or graphs in your answers.

•Pay particular attention to open-ended questions, such as those beginning with *explain* or *describe*.

•You are encouraged to discuss problems with other students in the class.

•However, each student should individually submit answers in his/her own words.

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[1] The following table contains data on the total population (in millions) of the United States over the last century.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
Population	76.2	92.2	106.0	123.2	132.2	151.3	179.3	203.2	226.5	248.7	281.4	308.7

Let x be the number of years **after** 1900 and y be the population (in millions).

i) Find the polynomial p which interpolates all the data points.
•What is the degree of p?
•What are $p(95)$ and $p(115)$?
•Plot p on the interval 1890 to 2020, and plot the data points on the same graph. Do this by:

```
≫ z=[-10:0.1:120];
≫ plot(z+1900,polyval(p,z))
≫ hold on
≫ plot(x+1900,y,'*');
```

To get a better look at p on the interval $[1900, 2010]$: ≫ `axis([1900 2010 0 340])`

ii) Find and plot (as above) the polynomial p which interpolates all of the data points *except* for the year 1950. What degree is it?

iii) What is the absolute error in the value of the population in 1950?
Recall that the absolute error=| actual population in 1950 $-p(50)$|

iv) Compare this to the error in the linear interpolation using data from 1940 and 1960 only.
Which gives a closer estimate, linear interpolation or interpolation using all the points (except 1950)?

v) Hand-in on the same plot, a graph of p, a graph of the linear interpolant, and the data points.

vi) Repeat all the above calculations *(ii)–(v)* for the year 2000 instead of 1950.
Which gives a closer estimate, linear interpolation or interpolation using all the points (except 2000)?

vii) One might think the value of $p(115)$ would be a good prediction of the population in 2015. How does it compare with the actual 2015 population of 321.7 million? Why did this happen?

In question *(vii)* we are attempting *extra*polation not *inter*polation ("extra" – outside, "inter" – between).
Moral: using interpolating polynomials outside the data range is not a good idea.

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[2] Problem 3 on page 206 of the textbook.

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[3] Let $f(x) = e^x$ on $[-1, 1]$.

[a] Suppose f is interpolated by a polynomial using 2 equispaced points. What is the theoretical error bound from Theorem 4.4 on page 213? How does this compare to the actual error (you can use Matlab to approximate it)?

[b] Suppose f is interpolated by a polynomial using 3 equispaced points. What is the theoretical error bound from Theorem 4.4 on page 213? How does this compare to the actual error (you can use Matlab to approximate it)?

[c] Suppose f is interpolated by a polynomial using 4 equispaced points. What is the theoretical error bound from Theorem 4.4 on page 213? How does this compare to the actual error (you can use Matlab to approximate it)?

[d] Suppose f is interpolated by a polynomial using 4 Chebyshev points. What is the theoretical error bound from Theorem 4.3 on page 213 and Theorem 4.6 on page 233? How does this compare to the actual error (you can use Matlab to approximate it)?

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[4] Use the `rand` command to sample 50 x-values in the interval $[0, 1]$ and then evaluate the function $y = 3x + 7$ on these values with a random "measurement error" of at most 0.25. Find the best fit line through these 50 points.

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[5] Complete the Matlab script `http://math.fau.edu/kalies/mad3400/regressionexp.m` and use it to find the best fit function of the form $y = Ce^{Ax}$ to the data points in Problem [1]. Plot the function and the data points on the same graph. What are the values of A and C? What is the maximum and average errors at the data points?
