

Binary indexed tree (Fenwick tree)

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本文简单介绍Binary indexed tree (Fenwick tree)

Fenwick tree

它又叫 **Binary indexed tree** , 也叫树状数组。

能在 $\log(n)$ 查询区间和, 并且在 $\log(n)$ 时间内进行结点更新操作。

lowbit(x)函数

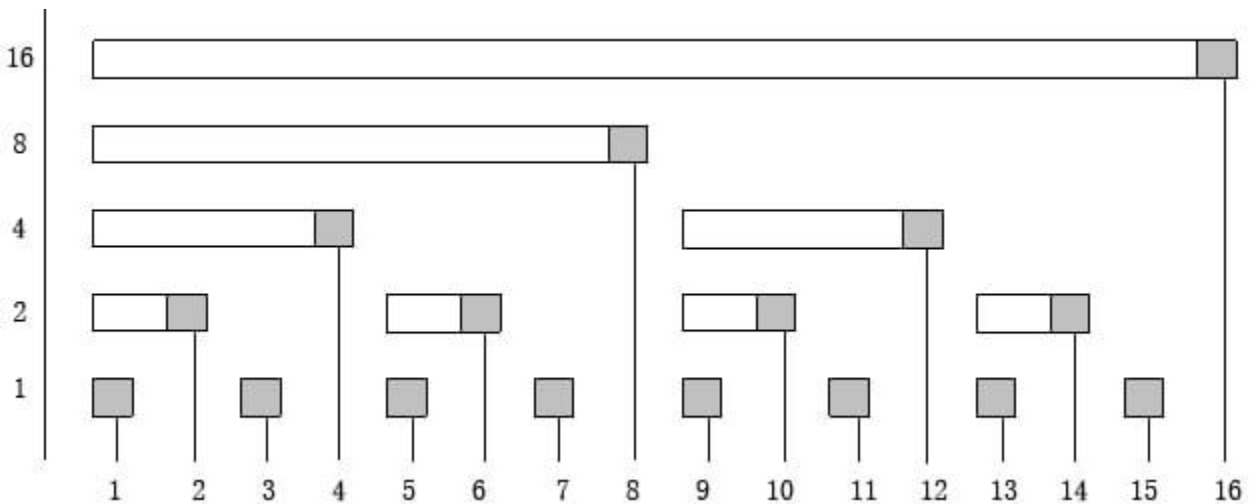
定义lowbit(x)为x的二进制表达式中最右边的1所对应的值。

比如, 1234的二进制是0100 1101 0010 lowbit(1234)=2, 在程序的实现中,

Lowbit(x)=x&-x; (为什么这样写呢? 因为计算机内部采用补码表示, -x是x按位取反, 1
数+1的结果)

树的结构图

让我们来看看图:横坐标是x, 纵坐标是lowbit(x)



(https://www.hrwhisper.me/wp-content/uploads/2015/11/fenwick_tree_binary_index_tree.jpg)

对于节点x,

- 为左子结点, 则父结点的编号是 $x + \text{lowbit}(x)$,
- 为右子结点, 则父结点的编号是 $x - \text{lowbit}(x)$

设 $C[i]$ 为以i结尾的水平长条内的元素之和, 如 $c[6]=a_5+a_6$ 。

- 顺着结点i往左走, 边走边往上爬, 沿途经过的 $c[i]$ 所对应的长条不重复不遗漏的包含所有需要累加的元素。
 - 如 $\text{sum}(6) = c[6] + c[4]$
- 如果修改了一个 $a[i]$, 那么从 $c[i]$ 往右走, 边走边往上爬, 沿途修改所有结点对应的值。
 - 如 $a[1] + 1$ 那么 $c[1] + 1, c[2]+1, c[4]+1, \dots$ 一直到最大值。

用C++ 的代码如下:

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```
1 inline int lowbit(int x) { return x&(-x) ; }
2
3 int sum(int x)
4 {
5     int ans=0;
6     while(x>0)
7     {
8         ans+=C[x];
9         x-=lowbit(x);
10    }
11    return ans;
12 }
13
14 void add(int x,int d)
15 {
16     while(x<=N)
17     {
18         C[x]+=d;
19         x+=lowbit(x);
20     }
21 }
```

实现代码

写成类的话：

C++

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```
1 class FenwickTree {
2     vector<int> sum_array;
3     int n;
4     inline int lowbit(int x) {
5         return x & -x;
6     }
7
8 public:
9     FenwickTree(int n) :n(n), sum_array(n + 1, 0) {}
10
11     void add(int x, int val) {
12         while (x <= n) {
13             sum_array[x] += val;
14             x += lowbit(x);
15         }
16     }
17
18     int sum(int x) {
19         int res = 0;
20         while (x > 0) {
21             res += sum_array[x];
22             x -= lowbit(x);
23         }
24         return res;
25     }
26 };
```

Python



```
1 class FenwickTree(object):
2     def __init__(self, n):
3         self.sum_array = [0] * (n + 1)
4         self.n = n
5
6     def lowbit(self, x):
7         return x & -x
8
9     def add(self, x, val):
10        while x <= self.n:
11            self.sum_array[x] += val
12            x += self.lowbit(x)
13
14    def sum(self, x):
15        res = 0
16        while x > 0:
17            res += self.sum_array[x]
18            x -= self.lowbit(x)
19        return res
```

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