The long time Vlasov-Poisson equation with a strong external magnetic field in 2d

$$\partial_t f^{\epsilon}(t,\mathbf{x},\mathbf{v}) + \frac{\mathbf{v}}{\epsilon}\cdot\nabla_{\mathbf{x}} f^{\epsilon}(t,\mathbf{x},\mathbf{v}) + \frac{1}{\epsilon}\left(\mathbf{E}^{\epsilon}(t,x) + \frac{1}{\epsilon}\mathbf{v}^{\perp}\right)\cdot\nabla_{\mathbf{v}} f^{\epsilon}(t,\mathbf{x},\mathbf{v}) = 0, \qquad t>0, \mathbf{x}, \mathbf{v}\in\mathbb{R}^2$$

$$\mathbf{E}^{\epsilon} = -\nabla_{\mathbf{x}}\phi^{\epsilon}(t,\mathbf{x}), \qquad -\Delta\phi^{\epsilon}(t,\mathbf{x}) = \rho^{\epsilon}(t,\mathbf{x}) - n_i^{\epsilon}, \qquad \rho^{\epsilon}(t,\mathbf{x}) := \int_{\mathbb{R}} f^{\epsilon}(t,\mathbf{x},\mathbf{v})d\mathbf{v}$$

The initial ddata is the Kelvin-Helmholtz instability type:

$$f_0(\mathbf{x},\mathbf{v}) = \frac{1}{2\pi}(1+\sin(x_2)+\eta\cos(kx_1))\exp\left(-|\mathbf{v}|^2/2\right)$$