

Perdiz arrow points from Caddo burial contexts aid in defining discrete behavioral regions

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Abstract Recent research in the ancestral Caddo area has yielded evidence for distinct *behavioral regions*, across which material culture from Caddo burials—bottles and Gahagan bifaces—has been found to express significant morphological differences. This inquiry assesses whether Perdiz arrow points from Caddo burials, assumed to reflect design intent, may differ across the same geography, and extend the pattern of shape differences to a third category of Caddo material culture. Perdiz arrow points collected from the geographies of the northern and southern Caddo *behavioral regions* defined in a recent social network analysis were employed to test the hypothesis that morphological attributes differ, and are predictable, between the two communities. Results indicate significant between-community differences in maximum length, width, stem length, and stem width, but not thickness. Using the same traditional metrics combined with the tools of machine learning, a predictive model—support vector machine—was designed to assess the degree to which community differences could be predicted, achieving a receiver operator curve score of 97 percent, and an accuracy score of 94 percent. The subsequent geometric morphometric analysis identified significant differences in Perdiz arrow point shape, size, and allometry, coupled with significant results for modularity and morphological

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integration. These findings bolster recent arguments that established two discrete *behavioral regions* in the ancestral Caddo area, which are defined on the basis of discernible morphological differences across three categories of Caddo material culture.

Keywords American Southeast · Caddo · Texas · archaeoinformatics · computational archaeology · machine learning · museum studies · digital humanities · STEM ·

1 Introduction

Perdiz arrow points generally follow two distinct manufacturing trajectories; one that enlists flakes, and the other, blade flakes [1, 2, 3, 4]. Lithic tool stone in the Caddo area of northeast Texas is relatively sparse, consists primarily of chert, quartzite, and silicified wood characteristic of the local geological formations, which may contribute to local variation in both their shape and size [4, 5]. It has been demonstrated elsewhere that Perdiz arrow points from northeast Texas vary significantly by time, raw material, and burial context [4]. In outline, Perdiz arrow points possess a:

[t]riangular blade with edges usually quite straight but sometimes slightly convex or concave. Shoulders sometimes at right angles to stem but usually well barbed. Stem contracted, often quite sharp at base, but may be somewhat rounded. Occasionally, specimen may be worked on one face only or mainly on one face . . . [w]orkmanship generally good, sometimes exceedingly fine with minutely serrated blade edges [6, 504].

A social network analysis of diagnostic artifacts from Historic Caddo (post-CE 1680) sites in northeast Texas demonstrated two spatially distinct *behavioral regions* [7] (Fig. 1). The network analysis was limited to Historic Caddo types; however, Formative Early Caddo (CE 800 – 1200) Gahagan bifaces and Caddo bottle types have been found to express significantly different morphologies between the same two areas [8, 9, 10, 11], extending the temporal range of the *shape boundary* and the prehistoric longevity of distinct *behavioral regions*. Gahagan bifaces from the ancestral Caddo area also differ significantly in shape, size, and form from those recovered at central Texas sites [12], suggestive of a second *shape boundary* between the ancestral Caddo area and central Texas.

The goal of this exploratory endeavor was to assess whether traditional metrics collected for Perdiz arrow points support the *shape boundary* posited in recent social network and geometric morphometric analyses, to determine whether linear metrics might be useful predictors of regional membership, and—if so—to identify those morphological features that articulate with each *behavioral region* using geometric morphometrics. It is assumed that complete Perdiz arrow points included as offerings in Caddo burials represent the design intent of the maker. Should the analysis yield significant results, it would



Fig. 1 Historic Caddo network generated using ceramic and lithic types, which include Perdiz arrow points (DOI 10.17605/OSF.IO/WD2ZT), illustrating the two (north [blue] and south [red]) Caddo behavioral regions. The regions were identified using a modularity statistic to identify those nodes more densely connected to one another than to the rest of the network.

bolster the argument for at least two discrete Caddo *behavioral regions*; each empirically defined by discernible morphological differences across three discrete categories of Caddo material culture.

1.1 Caddo behavioral regions

In a June 18, 1937 Works Progress Administration interview with Lillian Cassaway, Sadie Bedoka—a Caddo-Delaware woman raised with the Caddo—stated that:

Each [Caddo] clan had its own shape to make its pottery. One clan never thought of making anything the same pattern of another clan.
You could tell who made the pottery by the shape [13, 395].

General differences in Caddo ceramic forms have been noted elsewhere [14, 15]; however, the study of the Clarence H. Webb collection was the first to illustrate a significant north-south geographic shape difference among Hickory Engraved and Smithport Plain Caddo bottle types [10]. That preliminary observation was later confirmed using more robust samples of Hickory Engraved

and Smithport Plain bottles [8,9], then expanded to include a greater variety of Caddo bottle types across a larger spatial and temporal extent [11].

Co-presence of diagnostic artifact and attribute types has been used to define Caddo phases and periods, which serve as a heuristic tool that aids archaeologists in explaining the local cultural landscape, as well as regional differences between local landscapes. The Historic Caddo network expands those efforts, augmenting the previously defined phases and periods, and emphasizing the dynamic and manifold relational connections that reinforce and transcend the currently-defined categories [7]. This was achieved by enlisting a multi-scalar methodological approach [16,17], where the northern and southern communities were parsed into constituent groups using diagnostic types paired with a modularity statistic [18,19]. A number of the constituent groups identified in the network analysis were found to articulate with known Caddo polities, while others were not [7].

A subsequent analysis of Gahagan bifaces confirmed that a second category of Caddo material culture expressed significant morphological differences across the same geography as the Hickory Engraved and Smithport Plain bottles [20]. The morphology of Gahagan bifaces from sites in central Texas was later found to differ significantly when compared with those recovered from the Caddo region [12]. That Gahagan bifaces were found to differ across two spatial boundaries was noteworthy, particularly since it has regularly been assumed that these large bifaces were manufactured in central Texas and arrived in the ancestral Caddo area as products of trade and/or exchange [12, 20]. Further, that Gahagan bifaces were found to differ across the same geography as those communities posited in the Historic Caddo network analysis suggested that the temporal range of the shape boundary might extend to the Formative/Early Caddo period (CE 800 - 1250); a hypothesis that was later confirmed in a more comprehensive analysis of Caddo bottles [11].

2 Methods and results

Sixty seven intact Perdiz arrow points from Caddo burials in Camp, Nacogdoches, and Shelby counties were used for this study (supplementary materials). A standard suite of linear metrics was collected for each specimen, including maximum length, width, thickness, stem length, and stem width. Following collection, data were imported to R 4.1.1 [21] (supplementary materials), where boxplots were produced, along with a Principal Components Analysis (PCA) followed by analyses of variance (ANOVA) to test whether the morphology of Perdiz arrow points differs across the shape boundary (Fig.2).

Boxplots illustrate the distribution and mean for each of the five variables (Fig. 2a-e), and the PCA (Fig. 2f) illustrates over 92 percent of the variation in the sample among PC1 (84.65 percent) and PC2 (11.71 percent). The ANOVAs demonstrate significant differences in Perdiz arrow point morphology among four of the five variables (maximum length, width, stem length, and stem width) (supplementary materials). Maximum thickness does not dif-

fer significantly between the northern and southern communities, which led to the decision to conduct the subsequent geometric morphometric analysis as a two dimensional, rather than a three-dimensional, study (supplementary materials).

2.1 Predictive model

The utility of support vector machines to classify archaeological materials has increased greatly with the rise of big data [22,23,24,25]. All linear data were imported to Python and modeled using the `scikit-learn` package [26, 27] (supplementary materials). Data were subsequently split into training (75 percent) and testing (25 percent) subsets. A *standard scaler* was used to decrease the sensitivity of the algorithm to outliers by standardizing features, and a *nested cross validation* of the training set was used to achieve unbiased estimates of model performance, resulting in a mean cross validation score of 86 percent (supplementary materials). The model was subsequently fit on the training set, yielding a receiver operator curve score of 97 percent, and an accuracy score of 94 percent (supplementary materials).

2.2 Geometric morphometrics

Each of the arrow points was imaged using a flatbed scanner (HP Scanjet G4050) at 600 dpi. The landmarking protocol developed for this study (supplementary materials) includes six landmarks and 24 equidistant semilandmarks to characterize Perdiz arrow point shape, and were applied using the `StereoMorph` package in R [28]. The characteristic points and tangents used in the landmarking protocol were inspired by the work of Birkhoff [29].

Landmark data were aligned to a global coordinate system [30,31,32], achieved through generalized Procrustes superimposition [33] performed in R 4.1.1 [21] using the `geomorph` package v4.0.0 [34]. Procrustes superimposition translates, scales, and rotates the coordinate data allowing for comparisons among objects [35,33]. The `geomorph` package uses a partial Procrustes superimposition that projects the aligned specimens into tangent space subsequent to alignment in preparation for the use of multivariate methods that assume linear space [36,32] (Fig.3).

Principal components analysis [37] was used to visualize shape variation among the arrow points (Fig. 4). The shape changes described by each principal axis are commonly visualized using thin-plate spline warping of a reference image or 3D mesh [38,39]. A residual randomization permutation procedure (RRPP; $n = 10,000$ permutations) was used for all Procrustes ANOVAs [40, 41], which has higher statistical power and a greater ability to identify patterns in the data should they be present [42]. To assess whether shape changes with size (allometry), and differs by group (region), Procrustes ANOVAs [43] were also run that enlist effect-sizes (z-scores) computed as standard deviates



Fig. 2 Boxplots for a, maximum length; b, maximum width; c, maximum thickness; d, stem length; e, stem width, and f, PCA for linear metrics associated with the Perdiz arrow points. Additional information related to the analysis, including data and code needed to reproduce these results, can be found in the supplemental materials at <https://seldenlab.github.io/perdiz3/>.

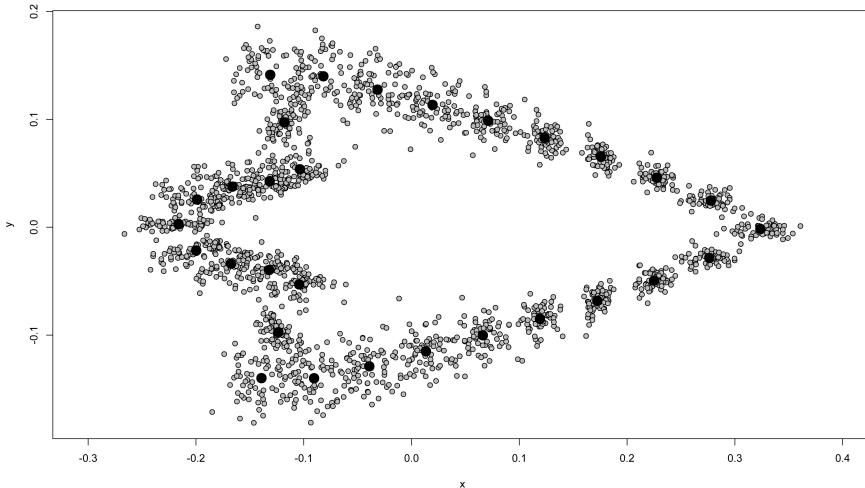


Fig. 3 Results of generalized Procrustes analysis, illustrating mean shape (black) and all specimens in the sample (gray). Additional information related to the GPA, including those data and code needed to reproduce these results, can be found in the supplemental materials at <https://seldenlab.github.io/perdiz3/>.

of the generated sampling distributions [44]. Procrustes variance was used to discriminate between regions and compare the amount of shape variation (morphological disparity) [45], estimated as the Procrustes variance using residuals of linear model fit [46]. A pairwise comparison of morphological integration was used to test the strength of integration between blade and basal morphology using a z-score [47].

The analysis of modularity, which compares within-module covariation of landmarks against between-module covariation was significant (Fig. 4 and supplementary materials), demonstrating that Perdiz arrow point blades and bases are, in fact, modular. The test for morphological integration was also significant (Fig. 4 and supplementary materials), indicating that the blades and bases of Perdiz arrow points are independent (modular), and integrated. These results demonstrate that blade and base shapes for Perdiz arrow points are predictable; a finding that would have great utility in studies of Perdiz arrow point morphology that incorporate fragmentary specimens.

A Procrustes ANOVA was used to test whether a significant difference exists in Perdiz arrow point (centroid) size, and results indicate a significant difference ($RRPP = 10,000$; $Rsq = 0.30681$; $Pr(>F) = 1e-04$). A second Procrustes ANOVA was used to test whether a significant difference exists in arrow point shape by region (northern vs. southern communities), and results indicate a significant difference ($RRPP = 10,000$; $Rsq = 0.0536$; $Pr(>F) = 0.0161$). A comparison of mean consensus configurations was used to characterize intraspecific shape variation of Perdiz arrow points from the northern

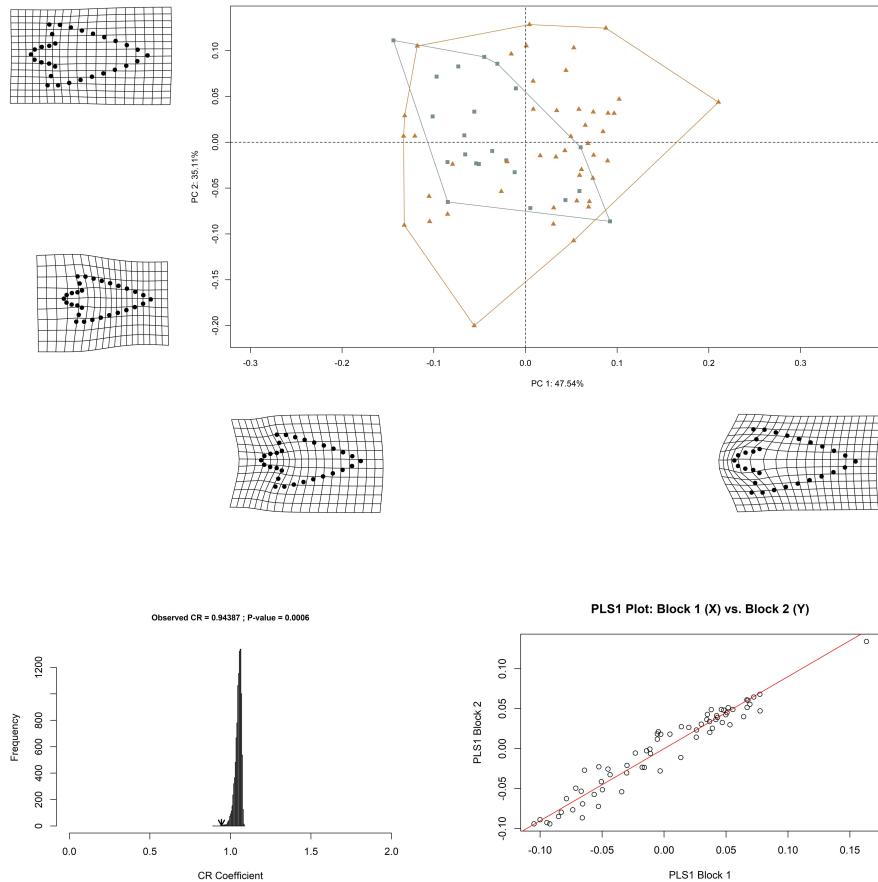


Fig. 4 Principal components analysis plot (PC1/PC2) for Perdiz arrow points by behavioral region/community (top; gray squares, north; orange triangles, south), and results of modularity (bottom left) and blade/base morphological integration (bottom right) analyses. Additional information related to the PCA, including the full listing of results and those data and code needed to reproduce these results, can be found in the supplemental materials at <https://seldenlab.github.io/perdiz3/>.

and southern *behavioral regions*. Differential morphology occurs primarily in the basal area, where the angle between the shoulder and base is more acute, with a base that is generally shorter and narrower in the southern *behavioral region* than it is in the north (Fig. 5 and supplementary materials).

3 Discussion

The *shape boundary* empirically delineates two discrete *behavioral regions* in the ancestral Caddo area. That the Perdiz arrow points recovered from Caddo



Fig. 5 Mean shapes for Perdiz arrow points from the northern (left), and southern (center) behavioral region. In the comparison of the two (right), the northern behavioral region is represented by gray circles, and the southern, by black. Additional information related to the mean shapes, including those data and code needed to reproduce these results, can be found in the supplemental materials at <https://seldenlab.github.io/perdiz3/>.

burials north and south of the *shape boundary* were found to differ significantly, expands the scope of the *behavioral regions* to include a third class of artifacts (Caddo bottles, bifaces, and—now—arrow points) [8,9,10,11,12, 20]. This study clearly illustrates that those morphological differences among Perdiz arrow points found in the northern and southern *behavioral regions* are predictable (supplementary materials), and can be identified using the standard suite of linear metrics regularly collected in the context of cultural resource management endeavors.

The geometric morphometric analysis demonstrated significant morphological differences for Perdiz arrow points from the two *behavioral regions*. The most pronounced difference in shape occurs in basal morphology (see Fig. 5). Allometry was also found to be significant, demonstrating that the shape of Perdiz arrow points differ with size. Those arrow points used in this study are considered to represent *design intent*, and are not thought to exhibit retouch or resharpening. This finding provides additional support for the argument that Perdiz arrow point morphology is highly variable [4].

Whether—and to what extent—it may be possible to identify specific groups of makers using unmodified arrow points from similar contexts remains unknown. Additional complexity is added when considering that those Perdiz points included in Caddo burials may also have been manufactured and transported to the ancestral Caddo territory by Toyah (or other) groups from central and southwest Texas. The notion that all cultural material recovered from Caddo burials was manufactured and placed with the deceased *only* by members of a Caddo polity or group was discarded long ago. The blades and bases of the arrow points were found to be modular, as well as integrated. This indicates that each module functions independently, and that base shape is a

predictor of blade shape (and vice versa). More work is warranted to assess whether Perdiz arrow points from constituent groups posited in the network analysis—which operated within each *behavioral region*—may express unique morphologies, and aid in further delimiting the local boundaries of Caddo polities.

3.1 Morphologically distinct behavioral regions

general:

- morphological attributes are representative of *intentional attributes* (Costin 2005) related to morphological characteristics
- dimensional standardization has utility in identifying the range of variation and overlap of product morphology in and between communities (Arnold 1991)
- relative dimensional standardization may imply a smaller number of production units (Costin 1991) contrasted with larger production units
- might there have been limits of technological/aesthetic social tolerance within/between the *behavioral regions*?
- ceramic production (women - assumed) = local; stone tool production (men- assumed) = non-local? if so, might that mean that women (again, old assumption) may have experienced more restricted social interactions? Caddo women were/are the community leaders; necessary to remain local for critical decisions (thus = protected)? Example of European contact - males met Europeans away from the villages to protect the leadership. Some Europeans (Spanish) assumed male leadership—to their own detriment.

bottles:

- bottles as a complete system; stone tools as partial/components
- similarities/differences in bottle shapes transcend type assignments
- bottles from the northern *behavioral region* express a significantly greater diversity in size; potential (size) standardization in the southern *behavioral region*?
- production activities more likely to be localized than exchange systems, and assumed to leave a clearer signature (Costin 1991)

bifaces:

- bifaces from the Caddo area express a significantly greater diversity in size when compared to central Texas

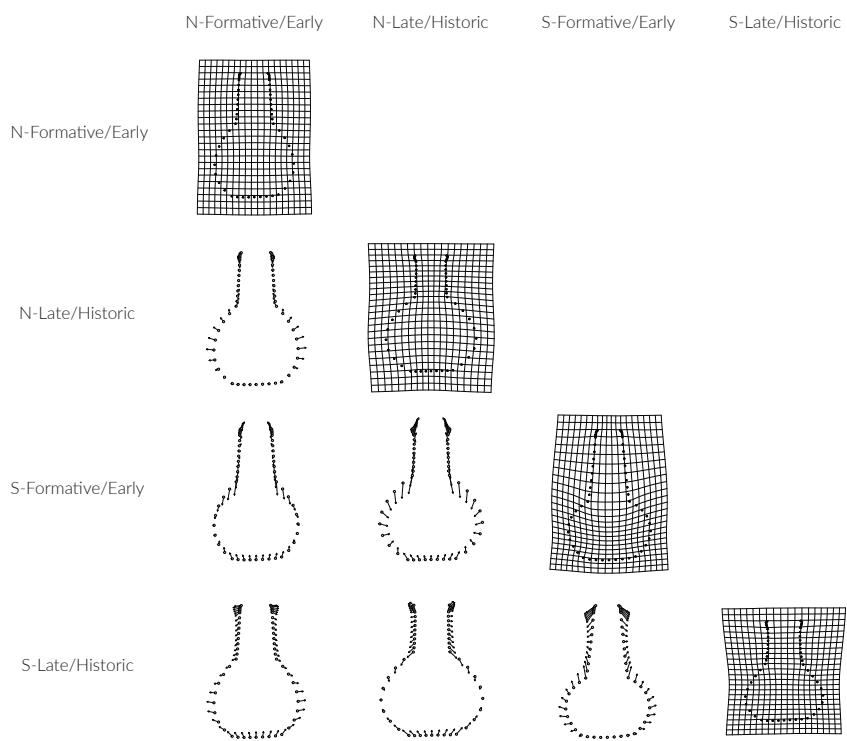


Fig. 6 Mean shapes and comparisons of morphological differences in Caddo bottles from the northern and southern behavioral regions (DOI 10.17605/OSF.IO/8SPWC).

4 Conclusion

Acknowledgments

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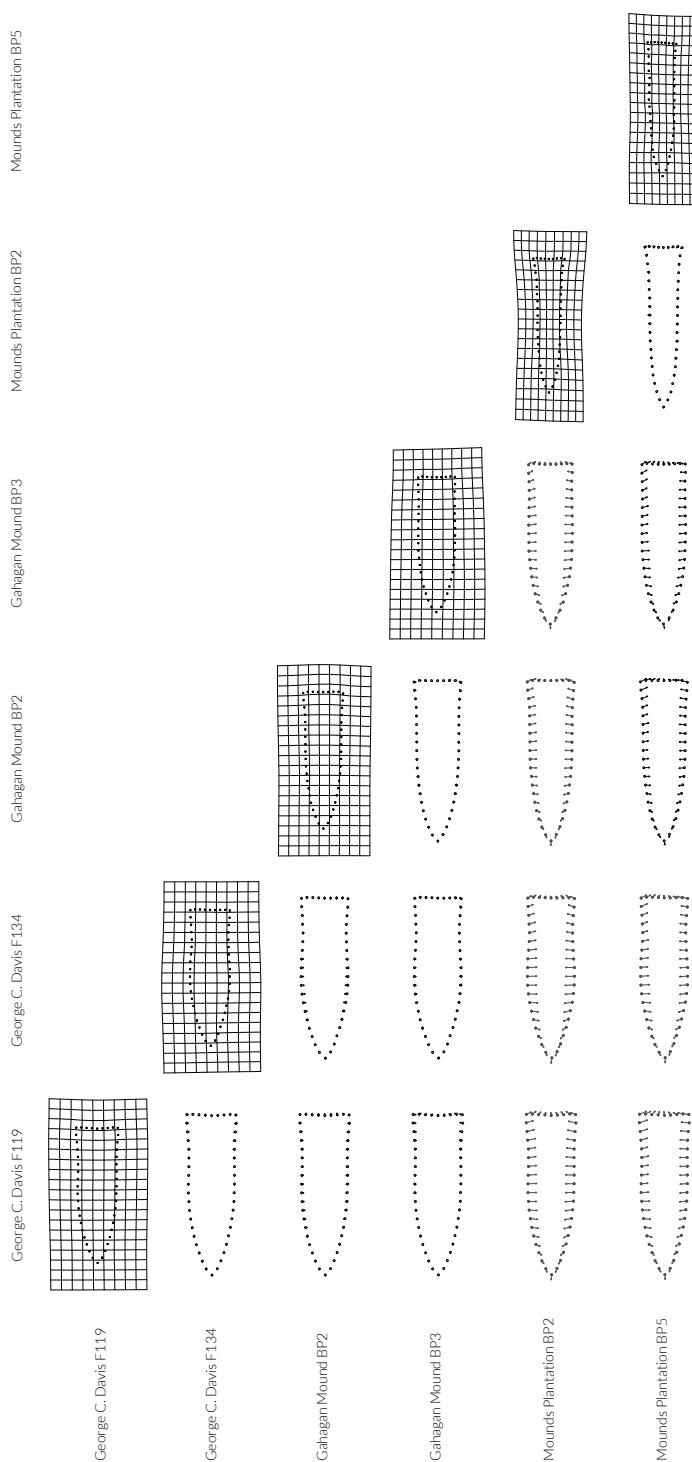


Fig. 7 Mean shapes and comparisons of morphological differences in Gahagan biface shape by site from burial contexts at the Gahagan Mound, George C. Davis, and Mounds Plantation sites, where comparisons in red reflect significant differences (DOI 10.17605/OSF.IO/2G95W).

Data Management

The data and analysis code associated with this project can be accessed through the GitHub repository (<https://github.com/seldenlab/perdiz3>) or the supplementary materials (<https://seldenlab.github.io/perdiz3/>); which are digitally curated on the Open Science Framework at DOI: 10.17605/OSF.IO/UK9ZD.

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