Linear Algebra for Probability

Anonymous Beaker

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Key properties of Linear Algebra

Below I will denote matrices by capital letters and scalars by lowercase letters. I will denote the transpose of a matrix A as A^T and its inverse as A^{-1}.

Basic Properties of Matrices

- $c(A + B) = cA + cB$
- $A(BC) = (AB)C$
- $A(B + C) = AB + AC$
- $(B + C)A = BA + CA$

Determinants

- $det(AB) = det(A)det(B)$
- $det(A) = det(A^T)$
- $det(A^{-1}) = \frac{1}{det(A)}$
- $det(cA) = c^n det(A)$
- $det(I) = 1$

Inverse Matrices and Transposes

- A matrix A is invertible if and only if $det(A) \neq 0$ and we define its inverse by $A^{-1} = \frac{adj(A)}{det(A)}$. Check Berberian 7.4 for the definition of the adjoint.
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(AB)^T = B^T A^T$
- $(A^{-1})^T = (A^T)^{-1}$
- $(A + B)^T = A^T + B^T$
- $(A^T)^T = A$, $A^{-1}A = I = AA^{-1}$

Practical Things to know

- Determinant of a 2x2 matrix:
  If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $det(A) = ad - bc$
- Inverse of a 2x2 matrix:
  Let A be defined as above. Then $A^{-1} = \frac{1}{det(A)} \begin{bmatrix} d & -c \\ -b & a \end{bmatrix} = \frac{adj(A)}{det(A)}$