Solution 4

Group A

1) Problem 1

Here we claim: for n people entered the room in order, the probability of the j_{th} people can actually sit on their own seat is $\frac{n-j+1}{n-j+2}$.

One each proof is from [1]. It is easy to prove by induction that before the j_{th} people entered the room, all the seat from number 2 to number $j-1$ has already been taken. Therefore, there is one more seat being take among the remaining $n-j+2$ available seats. Thus, the j_{th} people sitting in their own seat is $\frac{n-j+1}{n-j+2}$.

From the above claim, we know the second last people sit in his/her own seat with probability $\frac{2}{3}$ and the last people sit in his/her own seat with probability $\frac{1}{2}$. Since the second last people sit in his/her own seat doesn't provide any additional information about the remaining seats, thus the probability that last two people can take their own seats is $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

2) Problem 6

$$\begin{aligned} E[\frac{1}{S_T}] &= E[\frac{1}{S_0} e^{-(\mu - \frac{\sigma^2}{2})T - \sigma W_T}] \\ &= \frac{1}{S_0} e^{-(\mu T - \frac{\sigma^2}{2}T)} E[e^{-\sigma W_T}] \\ &= \frac{1}{S_0} e^{-(\mu T - \frac{\sigma^2}{2}T)} e^{\frac{\sigma^2}{2}T} \\ &= \frac{1}{S_0} e^{-\mu T + \sigma^2 T} \end{aligned}$$

3) Problem 11

Here is my solution for LeetCode problem 125:

```
class Solution {
public:
    bool isPalindrome(string s) {
        string temp = "";
        for (int i = 0; i < s.size(); i++) {
            if ((s[i] >= 'a' && s[i] <= 'z') || (s[i] >= 'A' && s[i] <= 'Z')
            || (s[i] >= '0' && s[i] <= '9')) {
                temp += s[i];
            }
        }
        transform(temp.begin(), temp.end(), temp.begin(), ::tolower);
        int pointer1 = 0;
        int pointer2 = temp.size() - 1;
```

```
        while (pointer1 < pointer2) {
            if (temp[pointer1] == temp[pointer2]) {
                pointer1++;
                pointer2--;
            } else {
                return false;
            }
        }
        return true;
    }
};
```

REFERENCES

[1] Yared Nigussie, Finding Your Seat Versus Tossing a Coin, The American Mathematical Monthly, Vol. 121, No. 6 (JuneJuly), pp. 545-546