3. Solution: Let b denote the number of black cards left
Let r denote the number of red cards left
Let $f(b,r)$ denote the random variable of the value of this game when there are b black cards and r red cards left

A recursive formula can be written as

$$\mathbb{E}[f(b,r)] = \max\left\{ r-b,\ \frac{b}{b+r}\mathbb{E}[f(b-1,r)] + \frac{r}{b+r}\mathbb{E}[f(b,r-1)] \right\}$$

The reason is

- If we chose to stop the game, we have drawn $(26-b)$ black cards and $(26-r)$ red cards. then we win $(26-b)-(26-r)$ $= (r-b)$ dollars.
- If we chose to continue the game, then we can compute the expectation using conditional probability formula.

The boundary conditions are

→ $f(b,0) = 0$, $b = 0, 1, 2, \cdots 26$.
i.e. when we have drawn all the red cards, the value of this game is 0, which means we keep on playing to the end and win \$0.

→ $f(0,r) = r$, $r = 0, 1, 2, \cdots, 26$
i.e. when we have drawn all the black cards, the value of this game is r which means we stop playing immediately and win \$r.

Runng this dynamic programming algorithm on computer gives

$$\mathbb{E}[f(26,26)] = \boxed{\$2.62}$$

So we would pay \$2.62 for this game.