QISHI QUIZ2	**Due on 11:59PM OCT 12, 2015**
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Math

Problem 5

Choose two random variables X and Y that uniformly lie in [0,1]. Let $A = \min(A,B)$ and $B = \max(A,B)$. Without loss of generality, we divide the stick into three pieces and let the length of the first piece be A, the second piece has length B-A and the third piece has length $1-B$. We first calculate the expected length for the smallest one. Suppose $S = \min(A, B-A, 1-B)$, then the cdf of S is the following:

$$\begin{aligned}
F(a) &= 1 - P(A \geq a, B-A \geq a, 1-B \geq a) \\
&= 1 - P(x \geq a, y \geq x, y-x \geq a, 1-y \geq a) - P(y \geq a, x \geq y, x-y \geq a, 1-x \geq a) \\
&= 1 - 2P(x \geq a, y \geq x, y-x \geq a, 1-y \geq a) \\
&= 1 - 2 * P(x \geq a, y \geq x+a, y \leq 1-a) \\
&= 1 - (1-3a)^2,
\end{aligned}$$

for $a \leq \frac{1}{3}$. Thus we know the pdf is 6(1-3a),

$$E[S] = \int_0^{\frac{1}{3}} 6a(1-3a)da = \frac{1}{9}.$$

Similarly, if we denote the length of the largest one as $L = \max(A, B-A, 1-B)$, then the cdf of L is the following:

$$\begin{aligned}
F(a) &= P(A \leq a, B-A \leq a, 1-B \leq a) \\
&= P(x \leq a, y \geq x, y-x \leq a, 1-y \leq a) - P(y \leq a, x \geq y, x-y \leq a, 1-x \leq a) \\
&= 2P(x \leq a, y \geq x, y-x \leq a, 1-y \leq a) \\
&= 2 * P(x \leq a, 1-a \leq y \leq a+x, y \geq x) \\
&= \begin{cases} -3a^2+6a-2 & a \geq \frac{1}{2} \\ (3a-1)^2 & \frac{1}{3} \leq a \leq \frac{1}{2} \end{cases}.
\end{aligned}$$

Thus, we know the pdf is

$$f(a) = \begin{cases} 6-6a & a \geq \frac{1}{2} \\ 6(3a-1) & \frac{1}{3} \leq a \leq \frac{1}{2} \end{cases}.$$

$$E[L] = \int_{\frac{1}{3}}^{\frac{1}{2}} 6a(3a-1)da + \int_{\frac{1}{2}}^{1} 6a - 6a^2 da = \frac{1}{9} + \frac{1}{2} = \frac{11}{18}.$$

Since the all three pieces sum up to 1, we know the expected length of the middle size one is $1 - \frac{11}{18} - \frac{1}{9} = \frac{1}{3}$.

If we divide the stick into n segments then the k_{th} largest segment will have expected length $\frac{1}{n}(\frac{1}{n} + \cdots + \frac{1}{k})$[2][3].

Problem 7

Based on Theorem 3.7 in [1], we know when a new variable is added to the original regression, the change in R^2 followed the following formula:

$$R_{1,2}^2 = R_1^2 + (1 - R_1^2)r^2, \tag{1}$$

where the explicit formula for r^2 can be found on page 44 in [1]. Since$0 \leq r^2 \leq 1$, thus we know

$$0.2 \leq R_{1,2} \leq 1. \tag{2}$$

Programming

Problem 11

```
/**
 * Definition for singly-linked list.
 * struct ListNode {
 *     int val;
 *     ListNode *next;
 *     ListNode(int x) : val(x), next(NULL) {}
 * };
 */
class Solution {
public:
    void deleteNode(ListNode* node) {
        while (node->next->next != NULL) {
            node->val = node->next->val;
            node = node->next;
        }
        node->val = node->next->val;
        node->next = NULL;
    }
};
```

Problem 17

```
class Solution {
public:
    int maxProfit(vector<int> &prices) {
        if (prices.size() < 2) {
            return 0;
        }
        vector<int> forward;
        vector<int> backward;
        forward.push_back(0);
        backward.push_back(0);

```

```
        int valley = prices[0];
        for (int i = 1; i < prices.size(); i++) {
            forward.push_back(max(forward[forward.size() - 1], prices[i] - valley));
            valley = min(valley, prices[i]);
        }

        int top = prices[prices.size() - 1];
        for (int i = prices.size() - 2; i >= 0; i--) {
            backward.insert(backward.begin(), max(backward[0], top - prices[i]));
            top = max(top, prices[i]);
        }

        int profit = 0;
        for (int i = 0; i < prices.size(); i++) {
            profit = max(profit, forward[i] + backward[i]);
        }
        return profit;
    }
};
```

References

[1] H. G. William, *LaTeX: Econometric Analysis*, 7th Edition, 2010

[2] H.A. David and H.N.Nagaraja *LaTeX: Order Statistics*, Addison Wesley, Massachusetts, 3rd edition, 2003.

[3] http://math.stackexchange.com/questions/13959/if-a-1-meter-rope-is-cut-at-two-uniformly-randomly-chosen-points-what-is-the-av