5. Solution: Let $X_1, \dots, X_5$ be i.i.d. $\exp(\lambda_X = \frac{1}{100})$

Let $Y_1, \dots, Y_5$ be i.i.d. $\exp(\lambda_Y = \frac{1}{200})$

We want to compute $\mathbb{E}[\min(X_1, \dots, X_5, Y_1, \dots, Y_5)]$

First we evaluate $\mathbb{P}(\min(X_1, \dots, X_5, Y_1, \dots, Y_5) > x)$.

$$\mathbb{P}(\min(X_1, \dots, X_5, Y_1, \dots, Y_5) > x) = \prod_{i=1}^{5} \mathbb{P}(X_i > x) \cdot \mathbb{P}(Y_i > x)$$

$$= \left(\int_x^\infty \frac{1}{100} e^{-\frac{t}{100}} dt\right)^5 \cdot \left(\int_x^\infty \frac{1}{200} e^{-\frac{t}{200}} dt\right)^5$$

$$= e^{-\frac{3}{40}x}.$$

Hence $\mathbb{E}[\min(X_1, \dots, X_5, Y_1, \dots, Y_5)] = \int_0^\infty \mathbb{P}(\min(X_1, \dots, X_5, Y_1, \dots, Y_5) > x)\, dx$

$$= \int_0^\infty e^{-\frac{3}{40}x} dx = \boxed{\frac{40}{3}}$$

6. Solution: The coin is tossed $n = 60 \times 60 \times 24 \times 365 \times 100$ times

Let $P(n)$ be the probability of tossing 100 consecutive heads

We have $P(1) = P(2) = \dots = P(99) = 0$ $\quad P(100) = \frac{1}{2^{100}}$

In general if there exists 100 consecutive heads, then

- Either they appear at the end, i.e.

$\underbrace{\text{NO 100 Consecutive heads !!!}} \; T \; \overbrace{HH \cdots H}^{100 \text{ heads}}$

This is with probability $2^{-101} \cdot [1 - P(n-101)]$

- Or they appear somewhere before the end. i.e.

$\underbrace{\text{Appears here}}$ ← last toss.

This is with probability $P(n-1)$

So. $\underline{P(n) = P(n-1) + 2^{-101} \cdot [1 - P(n-101)]}$, $\quad n > 100$.

Now our question is $P(n) < 0.01\%$???