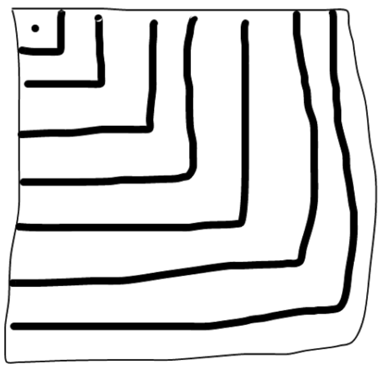
By PY 26/09/2015

1.Draw 2 balls from n balls which are labeled 1 to n. How many different combinations can be formed? The pair is not ordered (so, for example, (2; 6) = (6; 2)).
Solution:
Method 1: There are C(n,2) combinations that two balls are different, there are n combinations that two balls are the same. So the total is C(n,2)+n=n(n+1)/2.
Method 2: Unordered with replacement. C(n+2-1,2)=n(n+1)/2.
http://www.probabilitycourse.com/chapter2/2_1_4_unordered_with_replacement.php

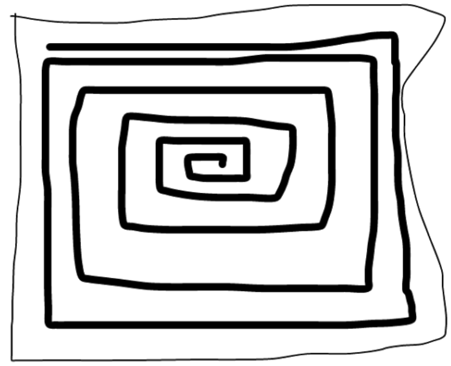
2.What's the property for the virtual function in C++? And can C++ has virtual constructor.
Solution:
Virtual functions basically provide polymorphic behavior. That is, when you work with an object whose dynamic type is different than the static (compile time) type with which it is referred to, it provides behavior that is appropriate for the actual type of object instead of the static type of the object.
To create an object you need complete information. In particular, you need to know the exact type of what you want to create. Consequently, a "call to a constructor" cannot be virtual.
To construct an object, a constructor needs the exact type of the object it is to create.
Furthermore you cannot have a pointer to a constructor.

3. Counting islands. Given a 2d grid map of '1's (land) and '0's (water), count the number of islands. An island is surrounded by water and is formed by connecting adjacent lands horizontally or vertically. You may assume all four edges of the grid are all surrounded by water.
https://leetcode.com/problems/number-of-islands/
Solution:
DFS or BFS. Time complexity is O(n^2) since each element will be visited at most by a constant number.

4. For the above problem, assume the matrix contains only 1. Which one is prefered with respect to the space complexity? BFS or DFS?
Solution:
The size of the queue in BFS is at most 2n while the size of the stack in DFS can be n^2, so BFS is prefered. Which is drawn in the following.



BFS



DFS

4. You know the joint distribution of random variables (X,Y). Assume you know X, what is the best guess of Y?
EY|X is the best guess with respect to the minimum mean square error(MMSE).

5. Give me an example that (X,Y) are individually Gaussian but not jointly Gaussian.
Solution:
X and Z are independent, X is Gaussian and p(z=1)=p(z=-1)=0.5. Let Y=XZ, Y=-XZ. (X,Y) satisfy the requirement since conditional probability of Y given X is not Gaussian.

6. How to generate bivariate Gaussian if we only have only have one generator?
Solution:

How do you generate two $N(0,1)$ (standard normal distribution) random variables with correlation ρ if you have a random number generator for standard normal distribution?

Solution: Two $N(0,1)$ random variables x_1, x_2 with a correlation ρ can be generated from independent $N(0,1)$ random variables z_1, z_2 using the following equations:

$$x_1 = z_1$$

$$x_2 = \rho z_1 + \sqrt{1-\rho^2}\, z_2$$

It is easy to confirm that $\text{var}(x_1) = \text{var}(z_1) = 1$, $\text{var}(x_2) = \rho^2\, \text{var}(z_1) + (1-\rho^2)\, \text{var}(z_2) = 1$, and $\text{cov}(x_1, x_2) = \text{cov}(z_1, \rho z_1 + \sqrt{1-\rho^2}\, z_2) = \text{cov}(z_1, \rho z_1) = \rho$.

This approach is a basic example using Cholesky decomposition to generate correlated random numbers. To generate correlated random variables that follow a n-dimensional multivariate normal distribution $X = [X_1, X_2, \cdots, X_n]^T \sim N(\mu, \Sigma)$ with mean $\mu = [\mu_1, \mu_2, \cdots, \mu_n]^T$ and covariance matrix Σ (a $n \times n$ positive definite matrix)[18], we can decompose the covariance matrix Σ into $R^T R$ and generate n independent $N(0, 1)$ random variables $z_1, z_2, \cdots, z_n$. Let vector $Z = [z_1, z_2, \cdots, z_n]^T$, then X can be generated as $X = \mu + R^T Z$.[19]

Alternatively, X can also be generated using another important matrix decomposition called **singular value decomposition (SVD)**: For any $n \times p$ matrix X, there exists a factorization of the form $X = UDV^T$, where U and V are $n \times p$ and $p \times p$ orthogonal matrices, with columns of U spanning the column space of X, and the columns of V spanning the row space; D is a $p \times p$ diagonal matrix called the singular values of X. For a positive definite covariance matrix, we have $V = U$ and $\Sigma = UDU^T$. Furthermore, D is the diagonal matrix of eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ and U is the matrix of n corresponding eigenvectors. Let $D^{1/2}$ be a diagonal matrix with diagonal elements $\sqrt{\lambda_1}, \sqrt{\lambda_2}, \cdots, \sqrt{\lambda_n}$, then it is clear that $D = (D^{1/2})^2 = (D^{1/2})(D^{1/2})^T$ and $\Sigma = UD^{1/2}(UD^{1/2})^T$. Again, if we generate a vector of n independent $N(0, 1)$ random variables $Z = [z_1, z_2, \cdots, z_n]^T$, X can be generated as $X = \mu + (UD^{1/2})Z$.

7.Can you explain the relationship between Interest rate and inflation rate.
In general, as interest rates are lowered, more people are able to borrow more money. The result is that consumers have more money to spend, causing the economy to grow and inflation to increase.

8. Do you know the recent Fed decision on interest rate? And how Fed works to determine the interest rate?
Using economic indicators such as the CPI, the Fed will establish interest rate targets intended to keep the economy in balance. By moving interest rate targets up or down, the Fed attempts to achieve maximum employment, stable prices and stable economic growth. The Fed will tighten interest rates (or increase rates) to stave off inflation. Conversely, the Fed will ease (or decrease rates) to spur economic growth.

9. Effect of raising interest rate for the commodity price, bond price and stock price? (This is a common question.)
Solution:
http://investorplace.com/247trader/markets-101-stocks-bonds-currencies-and-commodities/#.VgbNYstVhBc
http://www.marketwatch.com/story/the-decision-for-fed-on-interest-rates-could-rock-these-6-markets-2015-09-15