

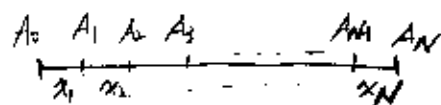
Since  $P(n) \leq P(n-1) + 2^{-101}$ ,  $n > 100$

we have  $P(n) \leq \frac{1}{2^{100}} + 2^{-101} \cdot (n-100)$

here  $n = 60 \times 60 \times 24 \times 365 \times 100 < 2^{33}$

Therefore  $P(n) \leq \frac{1}{2^{100}} + 2^{-101} \cdot 2^{33} < 0.01\%$  Yes!

7. Solution:

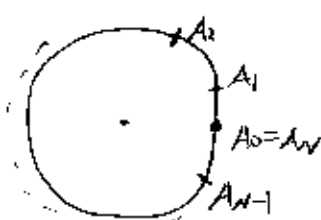


$x_1, x_2, \dots, x_N$  can form an  $N$  sided polygon

if and only if

$$\begin{aligned} 0 \leq x_1 < \frac{1}{2} \\ \textcircled{*} \quad 0 \leq x_2 < \frac{1}{2} \\ \vdots \\ 0 \leq x_N < \frac{1}{2} \end{aligned} \quad \text{and} \quad x_1 + x_2 + \dots + x_N = 1$$

If we glue  $A_0$  and  $A_N$ , then we have  $\textcircled{*}$  is false



if and only if  $A_0 = A_N, A_1, A_2, \dots, A_{N-1}$

are on a semi-circle.

Hence, we only need to compute the probability that  $N$  points are on a semi-circle.

Note that  $A_0, A_1, \dots, A_{N-1}$  are on a semi-circle if and only if starting at some  $A_i$  and look counter-clockwise,  $A_i, A_{i+1}, \dots, A_{i-1}$  are on a semi-circle.

Since  $P(\text{Start at } A_i, \text{ counter-clockwise, } A_i, A_{i+1}, \dots, A_{i-1} \text{ on a semi-circle}) = \left(\frac{1}{2}\right)^{N-1}$

and these events for different  $i$  are mutually exclusive

Hence having  $N$  points on a semi-circle is with probability  $\frac{N}{2^{N-1}}$ .

Eventually, the answer to our original problem is

$$\boxed{1 - \frac{N}{2^{N-1}}}$$