The Electricity Bid Stack:

Linking coal, gas, power and emissions prices

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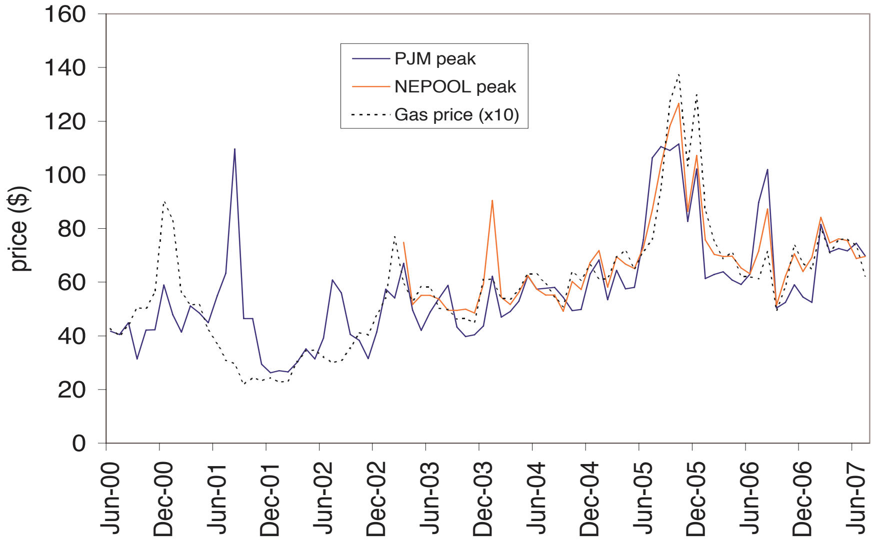
Energy Price Modelling

Main goal: Capturing high cross-commodity correlations in both spot and forward prices as well as link to demand.

Example: Spread options

$$\text{Payoff} = (P_T - HG_T - K)^+$$



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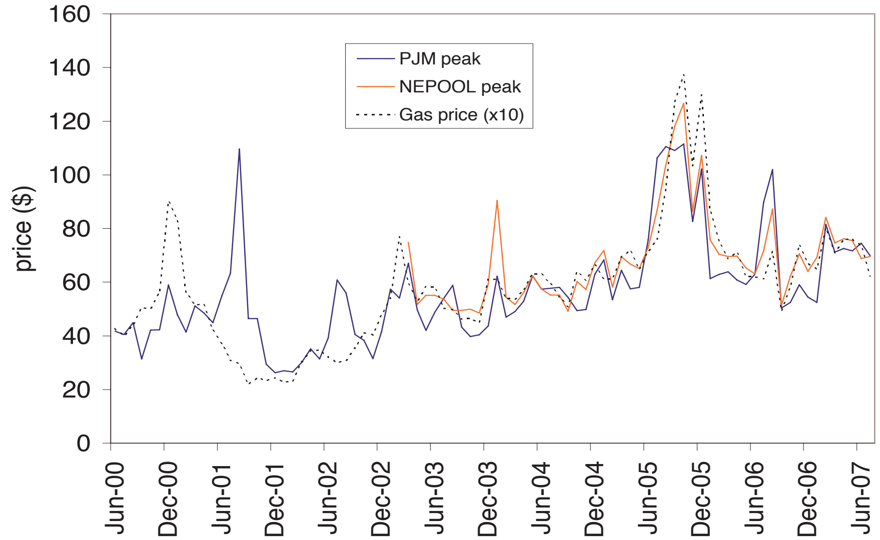


How can we attempt to model these relationships?

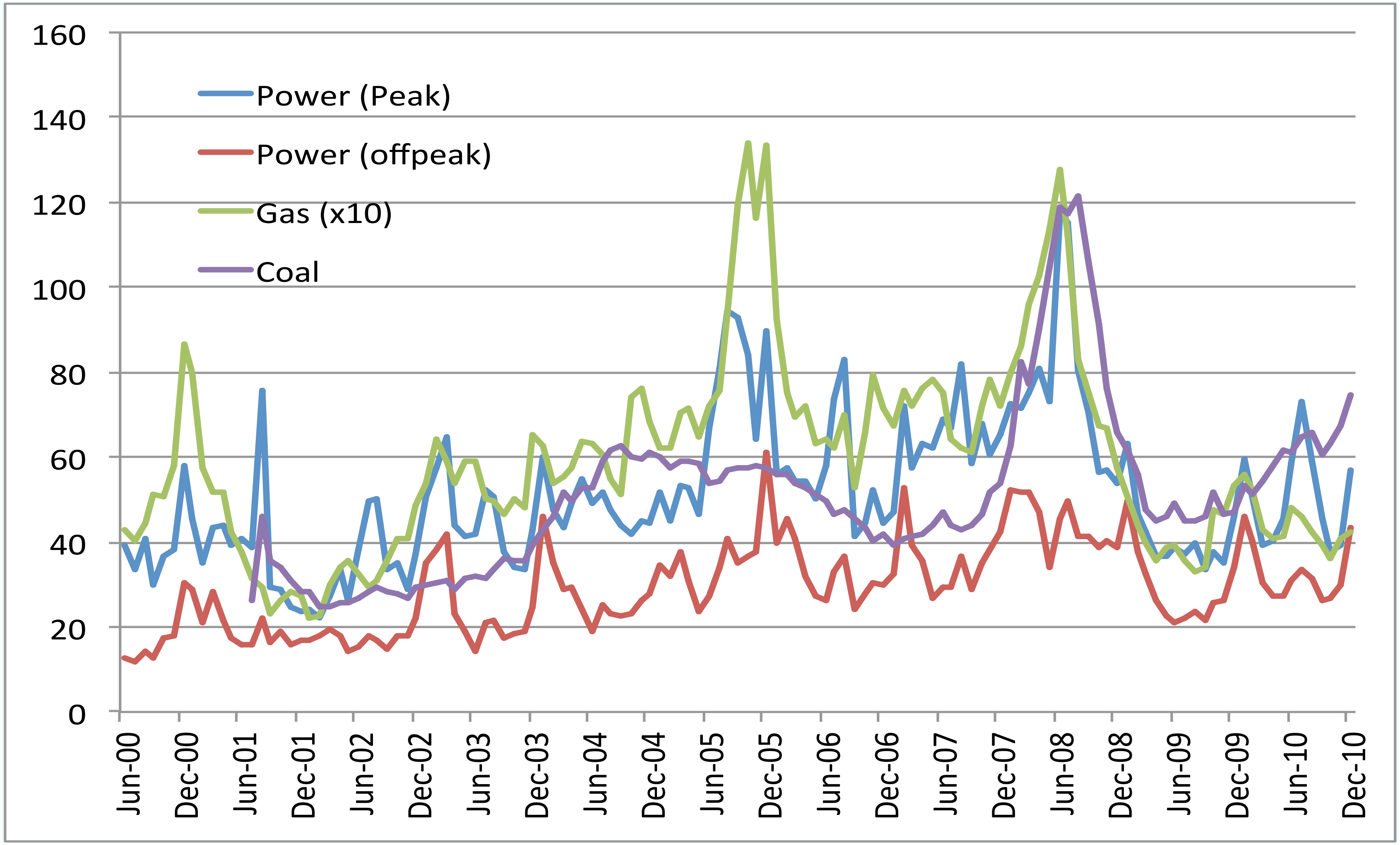
- Directly: Correlated Brownian Motions, Co-integrated processes
- Indirectly: Embedded into a model for spot power:

$$\text{Power} = f(\text{Gas}, \text{Coal}, \text{Carbon}, \ldots).$$

Hence a power market structure provides a natural starting point.

Energy Price Correlations

Recent data continues to show link between long term levels of power and gas in PJM (and to a lesser extent coal).



Structural Models for Power

Hybrid / structural models are alternatives to both reduced-form and complex optimization/equilibrium models. Key steps:

- Choice of Factors - Demand, Fuel Prices, Outages, etc.
- Choice of function $P_t = B(t, D_t, G_t, \ldots)$ to map to spot power.
- Calibration method for both components above.

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Examples include: Eydeland & Wolyniec (2003), Burger *et al* (2004), Cartea, Figueroa and Geman (2009), Davison *et al* (2002), Pirrong & Jermakyan (2008), Aid *et al* (2009)

Examples from literature

- Barlow (2002) - *spot price vs demand*

$$P_t = B(D_t) = (1 + \alpha D_t)^{1/\alpha}$$

- Burger et al. (2004), Cartea and Villaplana (2007) - *spot price vs demand & capacity*

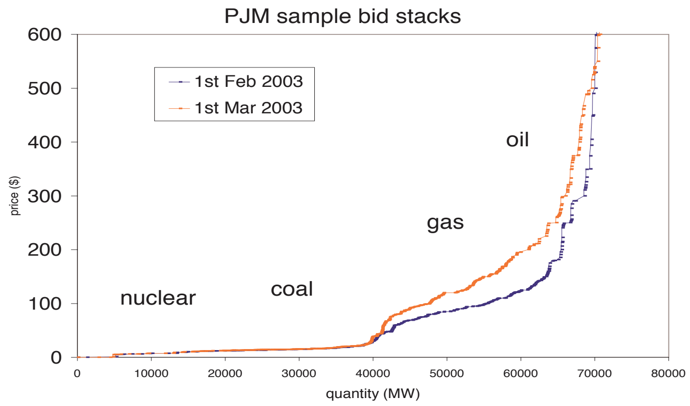
$$P_t = B(t, D_t, \xi_t)$$

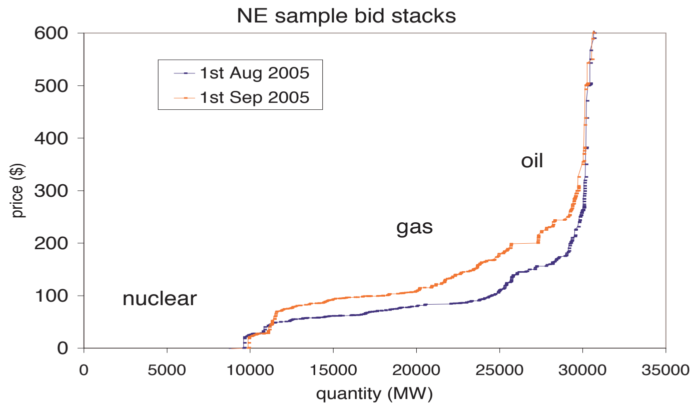
- Pirrong, Jermakyan (2005) - *spot price vs demand & gas*

$$P_t = B(D_t, G_t) = G_t f(D_t)$$

Big Challenge: Need for multiple overlapping fuels in many markets (e.g., when does coal vs gas set the power price?)

The bid stack function

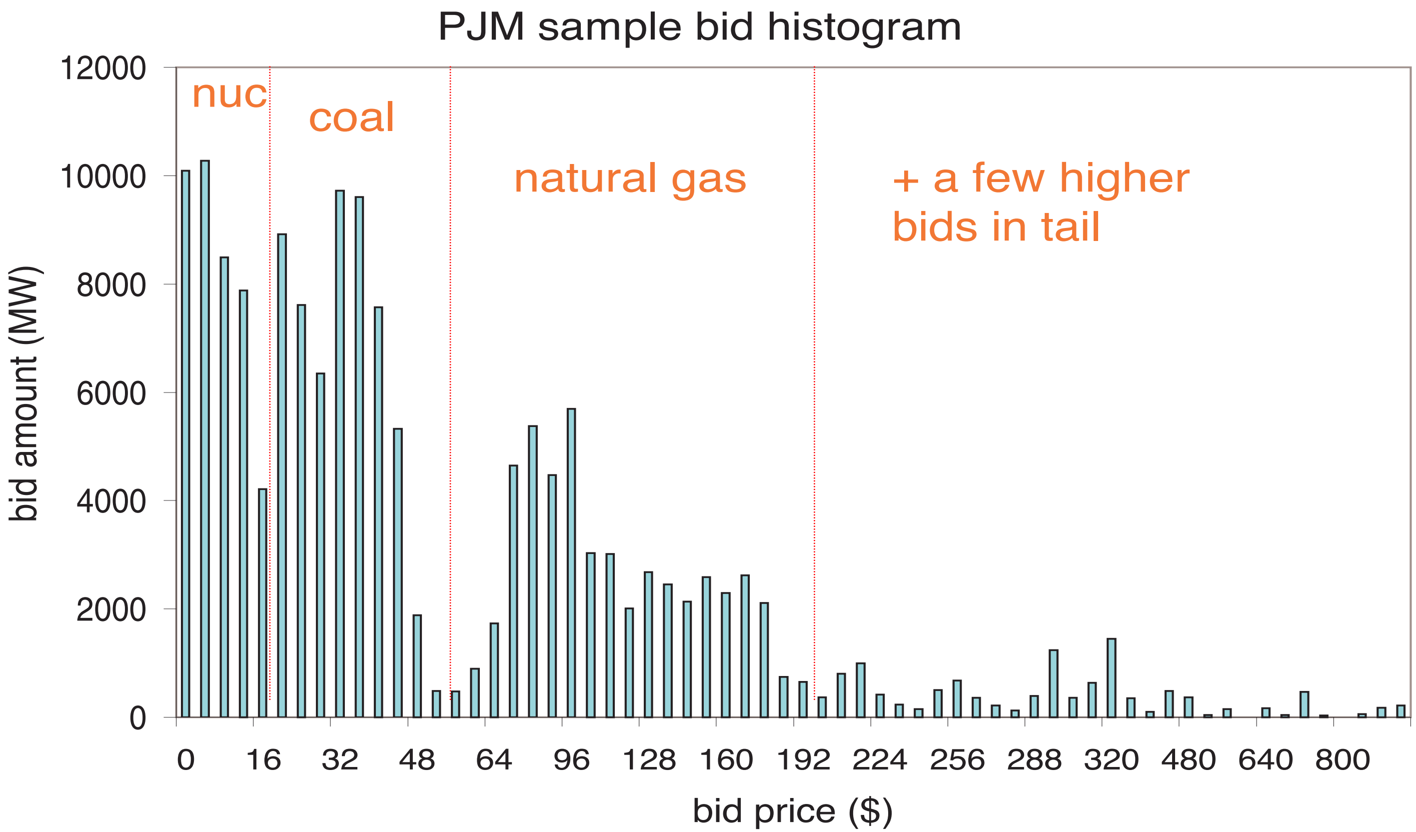




- Generators make day-ahead bids based on production costs
- Arrange by price (merit order) to form the **bid stack**
- Spot price P_t (market clearing price) is set by finding highest bid needed to match demand D_t (often assumed inelastic).
- Higher cost units are thus only needed for peak demand.
- Actual bid data available in many markets

An alternative perspective

- Can look at bid stack as a histogram of bids
- Merit order is often visible through clusters of bids



Distribution-based Bid Stack Model

Coulon, Howison (2009) - *spot price vs demand & capacity & fuel price(s)*

- Let $F_1(x), \ldots, F_N(x)$ equal the proportion of bids below x dollars for generators of fuel type $i = 1, \ldots, N$, with weights $w_1, \ldots, w_N$ (observable percentage of total capacity ξ^{max} in the market).
- We require $0 < \frac{D_t}{\xi^{max}} < 1$. (demand cannot exceed max capacity)
- Then the spot power price P_t solves:

$$\sum_{i=1}^{N} w_i F_i(P_t) = \frac{D_t}{\xi^{max}}$$

- So the bid stack function is the 'inverse cdf' of our distribution of bids.
- ξ^{max} replaced by a process ξ_t for capacity available, or alternatively $\xi_t = D_t + M_t$ where M_t is reserve margin available.
- Some bids may be removed before fitting (e.g., nuclear bidding at \$0) if never marginal.

Distribution-based Bid Stack Model

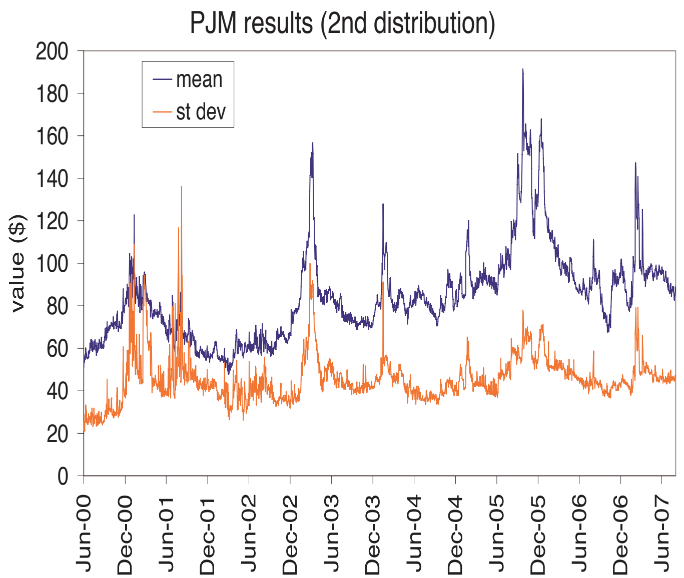
- Now choose two-parameter distributions for bids (location m_i, scale s_i) such as Gaussian, Logistic, Cauchy, Weibull.
- *One Fuel Case: (eg, New England Market):*
  - Gaussian: $P_t = m_1 + s_1 \Phi^{-1}\left(\frac{D_t}{D_t + M_t}\right)$
  - Logistic: $P_t = m_1 + s_1 \left(\log(D_t) - \log(M_t)\right)$
- *Two Fuel Case: (eg, PJM Market, with $w_1 \approx 0.5$):*
  - e.g. Gaussian: P_t solves

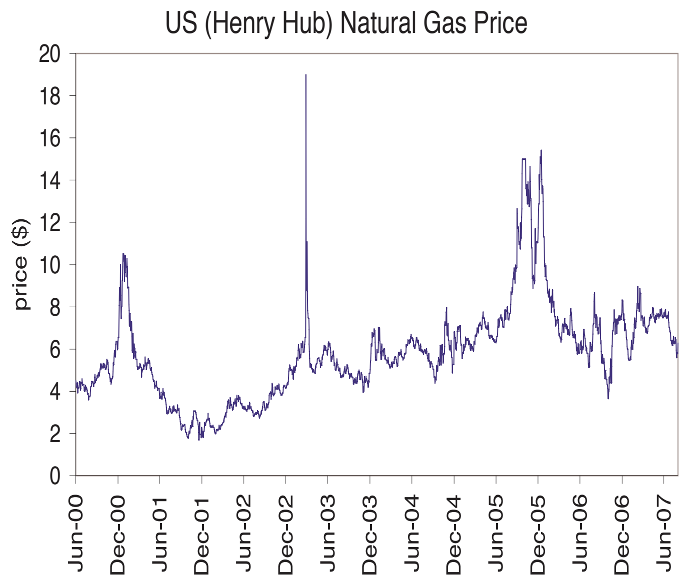
$$w_1 \Phi\left(\frac{P_t - m_1}{s_1}\right) + w_2 \Phi\left(\frac{P_t - m_2}{s_2}\right) = \frac{D_t}{D_t + M_t}$$

- We estimate m_1, s_1, m_2, s_2 by MLE independently for each day, and then observe the relationship with fuel prices.
- **Key idea:** Clusters of bids of each fuel-type move together, shifting to the right and widening as fuel price increases.

PJM Results (June 00 - Jul 07) for m_2 and s_2

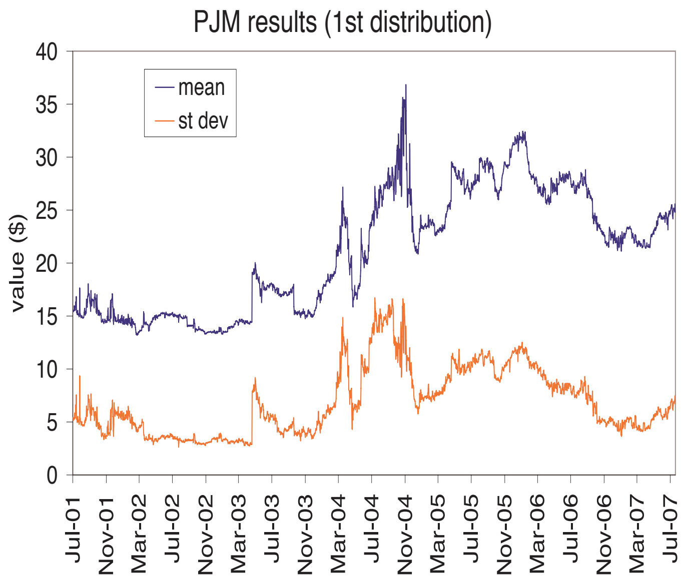
- As expected, the second distribution's parameters show very high correlation with natural gas prices (as high as 96% for m_2).

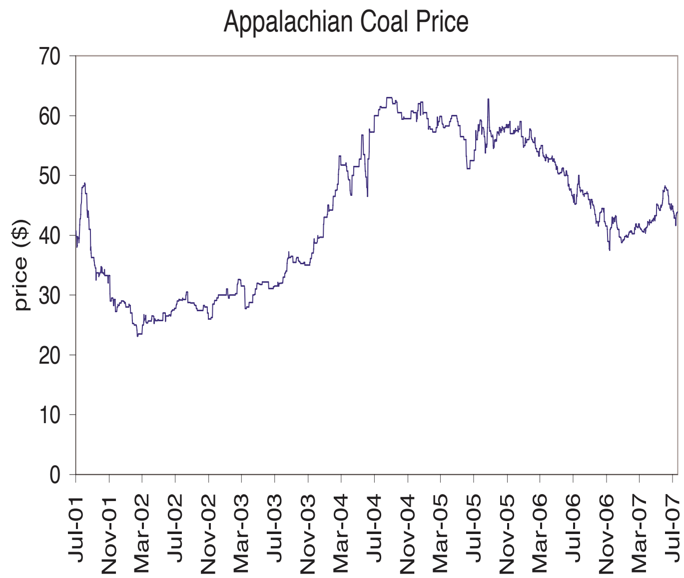




PJM Results (June 00 - Jul 07) for m_1 and s_1

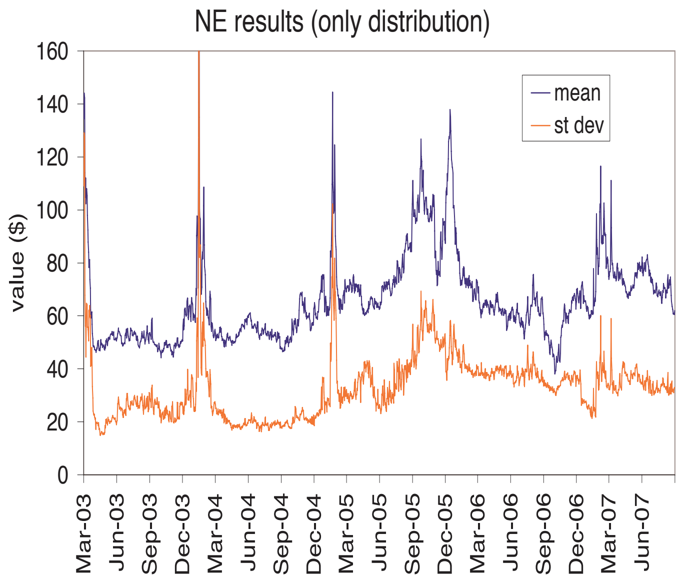
- The first distribution for PJM also shows reasonable correlation with coal prices (86% for m_1).

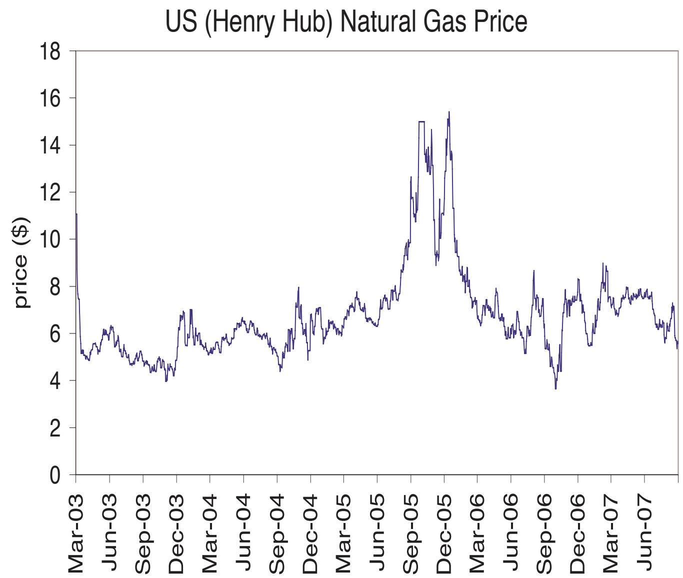




NE Results (Mar 03 - Jul 07) for m_2 and s_2

- Again, very high correlation with gas prices (as high as 95% for m_2).





Next Steps: Modelling Choices

First assume a linear dependence of parameters on fuels (coal & gas):

- For PJM, $$m_1 = \tilde{\alpha}_0 + \tilde{\alpha}_1 C_t, \quad s_1 = \tilde{\beta}_0 + \tilde{\beta}_1 C_t,$$ $$m_2 = \alpha_0 + \alpha_1 G_t, \quad s_2 = \beta_0 + \beta_1 G_t$$
- For NE, $m_1 = \alpha_0 + \alpha_1 G_t, \quad s_1 = \beta_0 + \beta_1 G_t$
- Linear regressions to find α's and β's.

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- For NE, $m_1 = \alpha_0 + \alpha_1 G_t, \quad s_1 = \beta_0 + \beta_1 G_t$
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Next, model underlying factors. In PJM / NEPOOL, we choose:

- Gas price G_t follows Schwartz 2-factor (exp. of OU + ABM + seas)
- Coal price C_t follows correlated GBM
- Demand D_t follows exponential OU with seasonality
- Reserve margin $M_t = D_t - \xi_t$ follows a correlated exponential OU with regime switch to spikes (ie, outages)

Parameter Estimation & Model Performance

While many parameters are needed, large quantities of historical data can be exploited *(see paper for estimation details)*:

- actual bid data (for stack parameters)
- gas and coal forward curve dynamics (eg, Kalman Filtering)
- system load / demand (or weather data ?)
- spot price and observed bids (to imply margin behaviour)
- outage / transmission data ? (direct margin fit)

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Encouraging results for PJM / New England in terms of:

- Simulated forward curves vs market data
- Behaviour / statistics of simulated price paths vs market data
- Regression coefficients in line with market heat rates
- Implied generation volumes (ie, % of power from gas vs coal)

Model Advantages & Weaknesses

- Exploits clear relationships with underlying electricity market drivers (often observable) and identify primary drivers at different time scales.
- Capture unusual features and correlations of power prices while maintaining fairly simple processes for factors.
- Balance between mathematical tractability and realism.
  e.g., In one-fuel market, power forwards:

  $$\begin{aligned} F^P(t,T) &= E_t\left[\alpha_0 + \alpha_1 G_T + (\beta_0 + \beta_1 G_T)\left(\log(D_T) - \log(M_t)\right)\right] \\ &= A(t,T,D_t,M_t) + B(t,T,D_t,M_t)F^G(t,T) \end{aligned}$$

  for quite simple functions $A(\cdot)$ and $B(\cdot)$. **But,** for multi-fuel market, no closed-form results. Calibration may be computationally demanding.
- Prices power and gas derivatives, spark spread options, even demand-dependent contracts, in a single framework.
- Adapts easily to new info, **but** many modifications for some markets.

Next challenge: closed-form!

- Multi-fuel case: no explicit expressions even for spot or forward.
- Alternative: allow slightly less flexibility in the stack but with the benefit of closed-form expressions for forwards, options and even spark or dark spread options. (e.g., payoff $V_T = (P_t - HG_T)^+$)
  (Joint work with R. Carmona and D. Schwarz.)

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  (Joint work with R. Carmona and D. Schwarz.)
- Key assumption: within each fuel type, heat rate differences lead to **exponential** bid stacks. (multiplicative in fuel price)
  - Assume coal and gas generators only, with capacity $\bar{\xi}^c$ and $\bar{\xi}^g$. Then aggregation of coal bids produces the 'sub bid stack':

$$b_c(x) = C_t e^{k_c + m_c x}, \text{ for } 0 \leq x \leq \bar{\xi}^c$$

and similarly for gas:

$$b_g(x) = G_t e^{k_g + m_g x}, \text{ for } 0 \leq x \leq \bar{\xi}^g$$

Case of exponential ‘sub bid stacks’

- The total market bid stack (as a function of demand) is given by:

$$B(x) = (b_c^{-1} + b_g^{-1})^{-1}(x), \qquad \text{for } 0 \le x \le \bar{\xi} = \bar{\xi}^c + \bar{\xi}^g$$

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- Hence, the result is **piecewise exponential**, although the precise form depends on ordering of start and endpoints of coal and gas stacks.
- For example, if $C_t e^{k_c} < G_t e^{k_g} < C_t e^{k_c + m_c \bar{\xi}_c} < G_t e^{k_g + m_g \bar{\xi}_g}$ (coal below gas but some overlap), then spot price P_t has three regions:

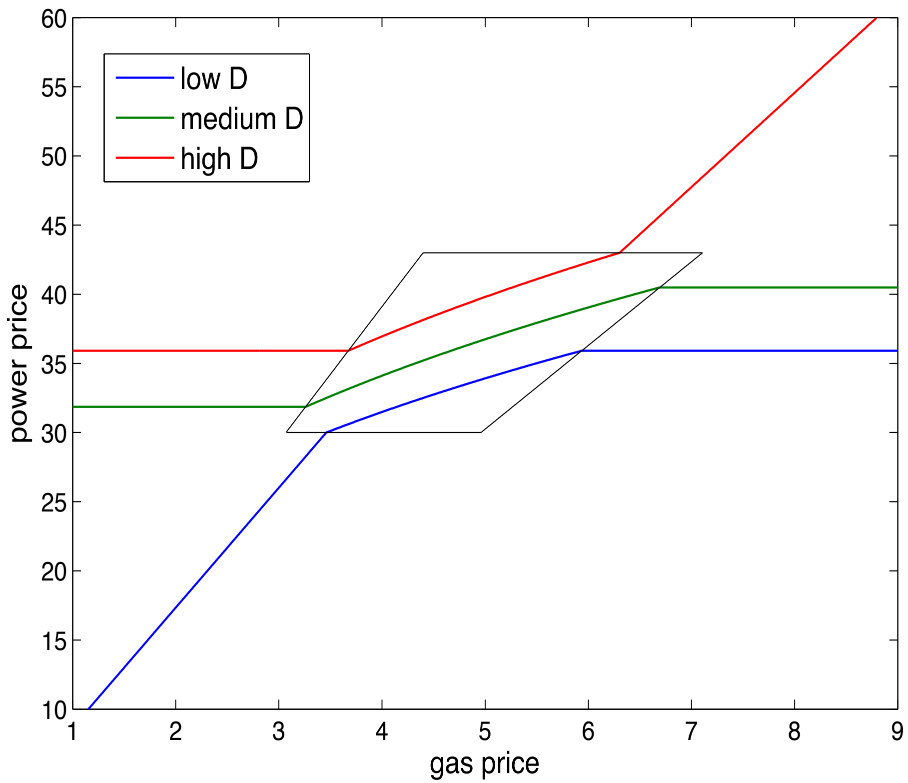
$$P_t(D, C_t, G_t) = \begin{cases} b_c(D) = C_t \mathrm{e}^{k_c + m_c D} & \text{for } 0 \le D \le D_1 \\ C_t^{\alpha} G_t^{\beta} \mathrm{e}^{\gamma + \delta D} & \text{for } D_1 \le D \le D_2 \\ b_g(D - \bar{\xi}^c) = G_t \mathrm{e}^{k_g + m_g (D - \bar{\xi}^c)} & \text{for } D_2 \le D \le \bar{\xi} \end{cases}$$

$$\alpha = \frac{m_g}{m_c + m_g}, \quad \beta = 1 - \alpha, \quad \gamma = \frac{k_c m_g + k_g m_c}{m_c + m_g}, \quad \delta = \frac{m_c m_g}{m_c + m_g},$$

and with $D_1 = \frac{1}{m_c}\left(\log(G_t / C_t) + k_g - k_c\right)$, and D_2 similar.

Case of exponential ‘sub bid stacks’

Alternatively, depicting power price P_t as a function of G_t (or similarly C_t) leads to three different demand ‘regimes’ (Case of $\bar{\xi}_c > \bar{\xi}_g$ plotted below):



High Demand: $D > \bar{\xi}_c$
(i.e., $D > \max(\bar{\xi}_c, \bar{\xi}_g)$)

Medium Demand:
$\bar{\xi}_g < D < \bar{\xi}_c$

Low Demand: $D < \bar{\xi}_g$
(i.e., $D < \min(\bar{\xi}_c, \bar{\xi}_g)$)

Quadrilateral in middle of plot represents region of coal and gas price overlap (ie, both generators at margin, setting price).

Exponential Stacks - Spot Prices

In summary, three demand regimes (low, medium, high) and three regions / cases (fuel price dependent) within each regime.

- For each regime, spot prices have a convenient form, e.g. for low D,

$$\begin{aligned} P_t^{low} &= C_t \mathrm{e}^{\lambda^c(D)} \mathbb{I}_{\{G_t > C_t \mathrm{e}^{\lambda^c(D) - \lambda^g(0)}\}} + G_t \mathrm{e}^{\lambda^g(D)} \mathbb{I}_{\{G_t < C_t \mathrm{e}^{\lambda^c(0) - \lambda^g(D)}\}} + \\ &\quad (C_t)^{\alpha} (G_t)^{\beta} \mathrm{e}^{\gamma + \delta D} \mathbb{I}_{\{C_t \mathrm{e}^{\lambda^c(0) - \lambda^g(D)} < G_t < C_t \mathrm{e}^{\lambda^c(D) - \lambda^g(0)}\}}, \end{aligned}$$

where $\lambda^c(x) = k_i + m_i x$ for $i \in \{c, g\}$ and $x \in [0, \bar{\xi}^i]$.

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  where $\lambda^c(x) = k_i + m_i x$ for $i \in \{c, g\}$ and $x \in [0, \bar{\xi}^i]$.

- Next assume that coal and gas prices are each lognormally distributed, for example driven by (correlated) exponential OU processes. For a given maturity T, $\log C_T$ and $\log G_T$ are bivariate Gaussian:

$$\begin{pmatrix} \log C_T \\ \log G_T \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}, \begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix} \right)$$

Exponential Stacks - Forward Prices

To calculate forward prices (for a given D), solve expectations of the form:

$$\mathbb{E}^{\mathbb{Q}}\left[\tilde{a}_0 C_t^{\tilde{a}_1} G_t^{\tilde{a}_2} \mathbb{I}_{\{\tilde{b}_0 C_t^{\tilde{b}_1} G_t^{\tilde{b}_2} < 1\}}\right] = \mathbb{E}\left[\left(\mathrm{e}^{a_0 + a_1 X + a_2 Y}\right) \mathbb{I}_{\{b_0 + b_1 X + b_2 Y < 0\}}\right]$$

- Note the following useful result:

$$\int_{-\infty}^{\infty} e^{cx} \Phi\left(\frac{a+bx}{d}\right) \frac{\mathrm{e}^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx = \mathrm{e}^{\frac{1}{2}c^2} \Phi\left(\frac{a+bc}{\sqrt{b^2+d^2}}\right)$$

- Or more generally (and useful later),

$$\int_{-\infty}^{h} e^{cx} \Phi\left(\frac{a+bx}{d}\right) \frac{\mathrm{e}^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx = \mathrm{e}^{\frac{1}{2}c^2} \Phi_2\left(h-c, \frac{a+bc}{\sqrt{b^2+d^2}}; \frac{-b}{\sqrt{b^2+d^2}}\right)$$

where $\Phi(z)$ and $\Phi_2(z_1, z_2, \rho_{12})$ are the univariate and bivariate standard Gaussian cdf. (Used for example in the Geske closed-form expression for compound options)

Exponential Stacks - Forward Prices

Hence we have (for low demand D here, similar for others) a closed-form solution for power forwards as a function of fuel forwards.

$$\begin{aligned} F_t^{low} &= F_t^g \mathrm{e}^{k_g + m_g D} \Phi\left(\frac{R_1(0,D)}{\sigma}\right) + F_t^c \mathrm{e}^{k_c + m_c D} \Phi\left(\frac{R_3(D,0)}{\sigma}\right) + \\ & \quad (F_t^c)^\alpha (F_t^g)^\beta \mathrm{e}^{\gamma + \delta D + f(\sigma_X, \sigma_Y)} \left[\Phi\left(\frac{R_2(D,0)}{\sigma}\right) - \Phi\left(\frac{R_2(0,D)}{\sigma}\right)\right], \end{aligned}$$

where

$$\begin{aligned} \sigma^2 &= \sigma_X^2 - 2\rho\sigma_X\sigma_Y + \sigma_Y^2 \\ f(\sigma_X, \sigma_Y) &= \frac{1}{2}(\alpha^2 - \alpha)\sigma_X^2 + \frac{1}{2}(\beta^2 - \beta)\sigma_Y^2 + \rho\alpha\beta\sigma_X\sigma_Y \\ R_1(x,y) &= \lambda^c(x) - \lambda^g(y) + \log(F_t^c) - \log(F_t^g) - \frac{1}{2}\sigma_X^2 - \frac{1}{2}\sigma_Y^2 + \rho\sigma_X\sigma_Y \\ R_3(x,y) &= -\lambda^c(x) + \lambda^g(y) - \log(F_t^c) + \log(F_t^g) - \frac{1}{2}\sigma_X^2 - \frac{1}{2}\sigma_Y^2 + \rho\sigma_X\sigma_Y \end{aligned}$$

Exponential Stacks - Forward Prices

More realistically, suppose that demand D is not known in advance. Then integrate (or sum) over demand distribution $f_D(x)$:

$$F_t^T = \int_0^{\bar{\xi}^g} F_t^{low}(x) f_D(x) dx + \int_{\bar{\xi}^g}^{\bar{\xi}^c} F_t^{med}(x) f_D(x) dx + \int_{\bar{\xi}^c}^{\bar{\xi}} F_t^{high}(x) f_D(x) dx$$

Consider Gaussian demand $D \sim N(\mu_z, \sigma_z^2)$ along with max spot price p^{max} and min spot price p^{min} (occurring if $D > \bar{\xi}$ and $D < 0$ respectively). Then forward price can be written explicitly (though not easily on slides!)...

- Let $\sigma_{cz}^2 = m_c^2 \sigma_z^2 + \sigma^2$, and $\sigma_{gz}^2 = m_g^2 \sigma_z^2 + \sigma^2$, and

$$\Phi_2^{2\times 1}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, y; \rho\right) = \Phi_2(x_1, y; \rho) - \Phi_2(x_2, y; \rho)$$

$$\Phi_2^{2\times 2}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}; \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}\right) = \Phi_2(x_1, y_1; \rho_1) - \Phi_2(x_2, y_1; \rho_1) - \Phi_2(x_1, y_2; \rho_2) + \Phi_2(x_2, y_2; \rho_2)$$

A nice closed-form result for forwards...

Forward power price (for some T) is given by

$$F_t = p^{min}\Phi\left(\frac{-\mu_z}{\sigma_z}\right) + p^{max}\Phi\left(\frac{\mu_z - \bar{\xi}}{\sigma_z}\right) +$$

$$F_t^g \mathrm{e}^{k_g + m_g\mu_z \frac{1}{2} m_g^2 \sigma_z^2} \Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - m_g\sigma_z \\ \frac{-\mu_z}{\sigma_z} - m_g\sigma_z \end{bmatrix}, \frac{R_1(0,\mu_z) - m_g^2\sigma_z^2}{\sigma_{gz}}; \frac{m_g\sigma_z}{\sigma_{gz}}\right) +$$

$$F_t^c \mathrm{e}^{k_c + m_c\mu_z + \frac{1}{2} m_c^2 \sigma_z^2} \Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - m_c\sigma_z \\ \frac{-\mu_z}{\sigma_z} - m_c\sigma_z \end{bmatrix}, \frac{R_3(\mu_z,0) - m_c^2\sigma_z^2}{\sigma_{cz}}; \frac{m_c\sigma_z}{\sigma_{cz}}\right) +$$

$$(F_t^c)^\alpha (F_t^g)^\beta \mathrm{e}^{\gamma + \delta\mu_z + \frac{1}{2}\delta^2\sigma_z^2 + f(\sigma_X,\sigma_Y)} \Phi_2^{2\times 2}\left(\begin{bmatrix} \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - \delta\sigma_z \\ \frac{-\mu_z}{\sigma_z} - \delta\sigma_z \end{bmatrix}, \begin{bmatrix} (R_2(\mu_z,0) + \delta m_c\sigma_z)/ \\ (R_2(0,\mu_z) - \delta m_g\sigma_z)/ \end{bmatrix}\right.$$

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$$F_t^g e^{k_g+m_g\mu_z\frac{1}{2}m_g^2\sigma_z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix}\frac{\bar{\xi}^g-\mu_z}{\sigma_z}-m_g\sigma_z\\ \frac{-\mu_z}{\sigma_z}-m_g\sigma_z\end{bmatrix}, \frac{R_1(0,\mu_z)-m_g^2\sigma_z^2}{\sigma_{gz}}; \frac{m_g\sigma_z}{\sigma_{gz}}\right) +$$

$$F_t^c e^{k_c+m_c\mu_z+\frac{1}{2}m_c^2\sigma_z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix}\frac{\bar{\xi}^g-\mu_z}{\sigma_z}-m_c\sigma_z\\ \frac{-\mu_z}{\sigma_z}-m_c\sigma_z\end{bmatrix}, \frac{R_3(\mu_z,0)-m_c^2\sigma_z^2}{\sigma_{cz}}; \frac{m_c\sigma_z}{\sigma_{cz}}\right) +$$

$$(F_t^c)^\alpha(F_t^g)^\beta e^{\gamma+\delta\mu_z+\frac{1}{2}\delta^2\sigma_z^2+f(\sigma_X,\sigma_Y)}\Phi_2^{2\times 2}\left(\begin{bmatrix}\frac{\bar{\xi}^g-\mu_z}{\sigma_z}-\delta\sigma_z\\ \frac{-\mu_z}{\sigma_z}-\delta\sigma_z\end{bmatrix}, \begin{bmatrix}(R_2(\mu_z,0)+\delta m_c\sigma_z)/\\ (R_2(0,\mu_z)-\delta m_g\sigma_z)/\end{bmatrix}\right.$$

$$F_t^c e^{k_c+m_c(\mu_z-\bar{\xi}^g)+\frac{1}{2}m_c^2\sigma_z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix}\frac{\bar{\xi}^c-\mu_z}{\sigma_z}-m_c\sigma_z\\ \frac{\bar{\xi}^g-\mu_z}{\sigma_z}-m_c\sigma_z\end{bmatrix}, \frac{-R_3(\mu_z-\bar{\xi}^g,\bar{\xi}^g)+m_c^2\sigma_z^2}{\sigma_{cz}}; \frac{-m_c}{\sigma_{cz}}\right.$$

$$F_t^c e^{k_c+m_c\mu_z+\frac{1}{2}m_c^2\sigma_z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix}\frac{\bar{\xi}^c-\mu_z}{\sigma_z}-m_c\sigma_z\\ \frac{\bar{\xi}^g-\mu_z}{\sigma_z}-m_c\sigma_z\end{bmatrix}, \frac{R_3(\mu_z,0)-m_c^2\sigma_z^2}{\sigma_{cz}}; \frac{m_c\sigma_z}{\sigma_{cz}}\right) +$$

$$(F_t^c)^\alpha(F_t^g)^\beta e^{\gamma+\delta\mu_z+\frac{1}{2}\delta^2\sigma_z^2+f(\sigma_X,\sigma_Y)}\Phi_2^{2\times 2}\left(\begin{bmatrix}\frac{\bar{\xi}^c-\mu_z}{\sigma_z}-\delta\sigma_z\\ \frac{\bar{\xi}^g-\mu_z}{\sigma_z}-\delta\sigma_z\end{bmatrix}, \begin{bmatrix}(R_2(\mu_z,0)+\delta m_c\sigma\\ (R_2(\ldots\bar{\xi}^c\ldots\bar{\xi}^g)+\delta m\end{bmatrix}\right.$$

A nice closed-form result for forwards...

Forward power price (for some T) is given by

$$F_t = p^{min}\Phi\left(\frac{-\mu_z}{\sigma_z}\right) + p^{max}\Phi\left(\frac{\mu_z - \bar{\xi}}{\sigma_z}\right) +$$

$$F_t^g \mathrm{e}^{k_g + m_g\mu_z \frac{1}{2} m_g^2\sigma_z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - m_g\sigma_z \\ \frac{-\mu_z}{\sigma_z} - m_g\sigma_z \end{bmatrix}, \frac{R_1(0,\mu_z) - m_g^2\sigma_z^2}{\sigma_{gz}}; \frac{m_g\sigma_z}{\sigma_{gz}}\right) +$$

$$F_t^c \mathrm{e}^{k_c + m_c\mu_z + \frac{1}{2} m_c^2\sigma_z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - m_c\sigma_z \\ \frac{-\mu_z}{\sigma_z} - m_c\sigma_z \end{bmatrix}, \frac{R_3(\mu_z,0) - m_c^2\sigma_z^2}{\sigma_{cz}}; \frac{m_c\sigma_z}{\sigma_{cz}}\right) +$$

$$(F_t^c)^\alpha (F_t^g)^\beta \mathrm{e}^{\gamma + \delta\mu_z + \frac{1}{2}\delta^2\sigma_z^2 + f(\sigma_X,\sigma_Y)}\Phi_2^{2\times 2}\left(\begin{bmatrix} \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - \delta\sigma_z \\ \frac{-\mu_z}{\sigma_z} - \delta\sigma_z \end{bmatrix}, \begin{bmatrix} (R_2(\mu_z,0) + \delta m_c\sigma_z)/ \\ (R_2(0,\mu_z) - \delta m_g\sigma_z)/ \end{bmatrix}\right.$$

$$F_t^c \mathrm{e}^{k_c + m_c(\mu_z - \bar{\xi}^g) + \frac{1}{2} m_c^2\sigma_z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}^c - \mu_z}{\sigma_z} - m_c\sigma_z \\ \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - m_c\sigma_z \end{bmatrix}, \frac{-R_3(\mu_z - \bar{\xi}^g, \bar{\xi}^g) + m_c^2\sigma_z^2}{\sigma_{cz}}; \frac{-m_c}{\sigma_{cz}}\right.$$

$$F_t^c \mathrm{e}^{k_c + m_c\mu_z + \frac{1}{2} m_c^2\sigma_z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}^c - \mu_z}{\sigma_z} - m_c\sigma_z \\ \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - m_c\sigma_z \end{bmatrix}, \frac{R_3(\mu_z,0) - m_c^2\sigma_z^2}{\sigma_{cz}}; \frac{m_c\sigma_z}{\sigma_{cz}}\right) +$$

$$(F_t^c)^\alpha (F_t^g)^\beta \mathrm{e}^{\gamma + \delta\mu_z + \frac{1}{2}\delta^2\sigma_z^2 + f(\sigma_X,\sigma_Y)}\Phi_2^{2\times 2}\left(\begin{bmatrix} \frac{\bar{\xi}^c - \mu_z}{\sigma_z} - \delta\sigma_z \\ \frac{\bar{\xi}^g - \mu_z}{\sigma_z} - \delta\sigma_z \end{bmatrix}, \begin{bmatrix} (R_2(\mu_z,0) + \delta m_c\sigma \\ (R_2(\ldots \bar{\xi}^g, \bar{\xi}^g) + \delta m \end{bmatrix}\right.$$

Or to put it a little more clearly...

Here the contribution from the five possible expressions for P_t is visible:

$$
\begin{aligned}
F_t = p^{min}\Phi\left(\frac{-\mu_z}{\sigma_z}\right) + p^{max}\Phi\left(\frac{\mu_z-\bar{\xi}}{\sigma_z}\right) + \\
(F_t^g)\mathrm{e}^{k_g+m_g\mu_z+\frac{1}{2}m_g^2\sigma_z^2}\sum_j \Phi_2\left(A_j^g(F_t^c,F_t^g), B_j^g(F_t^c,F_t^g); \rho_j^g\right) + \\
(F_t^c)\mathrm{e}^{k_c+m_c\mu_z+\frac{1}{2}m_c^2\sigma_z^2}\sum_j \Phi_2\left(A_j^c(F_t^c,F_t^g), B_j^c(F_t^c,F_t^g); \rho_j^c\right) + \\
(F_t^c)^\alpha(F_t^g)^\beta\mathrm{e}^{\gamma+\delta\mu_z+\frac{1}{2}\delta^2\sigma_z^2+f(\sigma_X,\sigma_Y)}\sum_j \Phi_2\left(A_j^{cg}(F_t^c,F_t^g), B_j^{cg}(F_t^c,F_t^g); \rho_j^{cg}\right) + \\
(F_t^g)\mathrm{e}^{k_g+m_g(\mu_z-\bar{\xi}^c)+\frac{1}{2}m_g^2\sigma_z^2}\sum_j \Phi_2\left(A_j^g(F_t^c,F_t^g), B_j^g(F_t^c,F_t^g); \rho_j^g\right) + \\
(F_t^c)\mathrm{e}^{k_c+m_c(\mu_z-\bar{\xi}^g)+\frac{1}{2}m_c^2\sigma_z^2}\sum_j \Phi_2\left(A_j^c(F_t^c,F_t^g), B_j^c(F_t^c,F_t^g); \rho_j^c\right)
\end{aligned}
$$

where $A_j^g, B_j^g, A_j^c, B_j^c, A_j^{cg}, B_j^{cg}$ are all linear functions of $\log(F_t^c), \log(F_t^g)$, and also of $\bar{\xi}^c$ and $\bar{\xi}^c$ (and μ_z).

Other Applications: Spread Options

For options on power spot or spark / dark spreads, similar calculations.

$$\text{Payoff } = V_T = (P_T - K)^+, \quad \text{or } (P_T - HG_T)^+ \quad \text{or } (P_T - HC_T)^+$$

As for forwards, first solve for a given demand D and consider regimes:

$$V_t = \mathbb{E}_t^{\mathbb{Q}}[(P_T - HC_T)^+] = \begin{cases} V_t^{low}, & \text{for } 0 \le D \le \min(\bar{\xi}^c, \bar{\xi}^g) \\ V_t^{med}, & \text{for } \min(\bar{\xi}^c, \bar{\xi}^g) < D \le \max(\bar{\xi}^c, \bar{\xi}^g) \\ V_t^{high}, & \text{for } \max(\bar{\xi}^c, \bar{\xi}^g) < D \le \bar{\xi}, \end{cases}$$

Next consider cases corresponding to value of heat rate H, e.g. for low D,

$$V_t^{low} = \begin{cases} V_t^{low,1}, & \text{for } 0 \le H \le \mathrm{e}^{k_c} \\ V_t^{low,2}, & \text{for } \mathrm{e}^{k_c} \le H \le \mathrm{e}^{k_c + m_c D} \\ V_t^{low,3} (= 0), & \text{for } H > \mathrm{e}^{k_c + m_c D} \end{cases}$$

Required expectations all have the same convenient form as before.

Other Challenges: Spikes, More Fuels

Basic model above may prove useful for certain applications, but many additional considerations.

- *Spikes:* Consider adding a third 'sub bid stack' with low capacity $\bar{\xi}^s$ and steep exponential shape (large m_s):

$$b_s(x) = e^{k_s + m_s x}, \text{ for } 0 \le x \le \bar{\xi}^s$$

- Then slight changes in formulas required, but no extra difficulty, e.g. in region of coal and gas overlap,

$$P_t = C_t^{\tilde{\alpha}} G_t^{\tilde{\beta}} \mathrm{e}^{\tilde{\gamma} + \tilde{\delta} D}$$

where $\tilde{\alpha} = \frac{m_g m_s}{m_c m_g + m_g m_s + m_c m_s}, \quad \tilde{\beta} = \frac{m_c m_s}{m_c m_g + m_g m_s + m_c m_s}, \ldots$

- *Multiple Fuels:* Similar extension possible but trivariate integrals and increasing number of merit order permutations add complications!

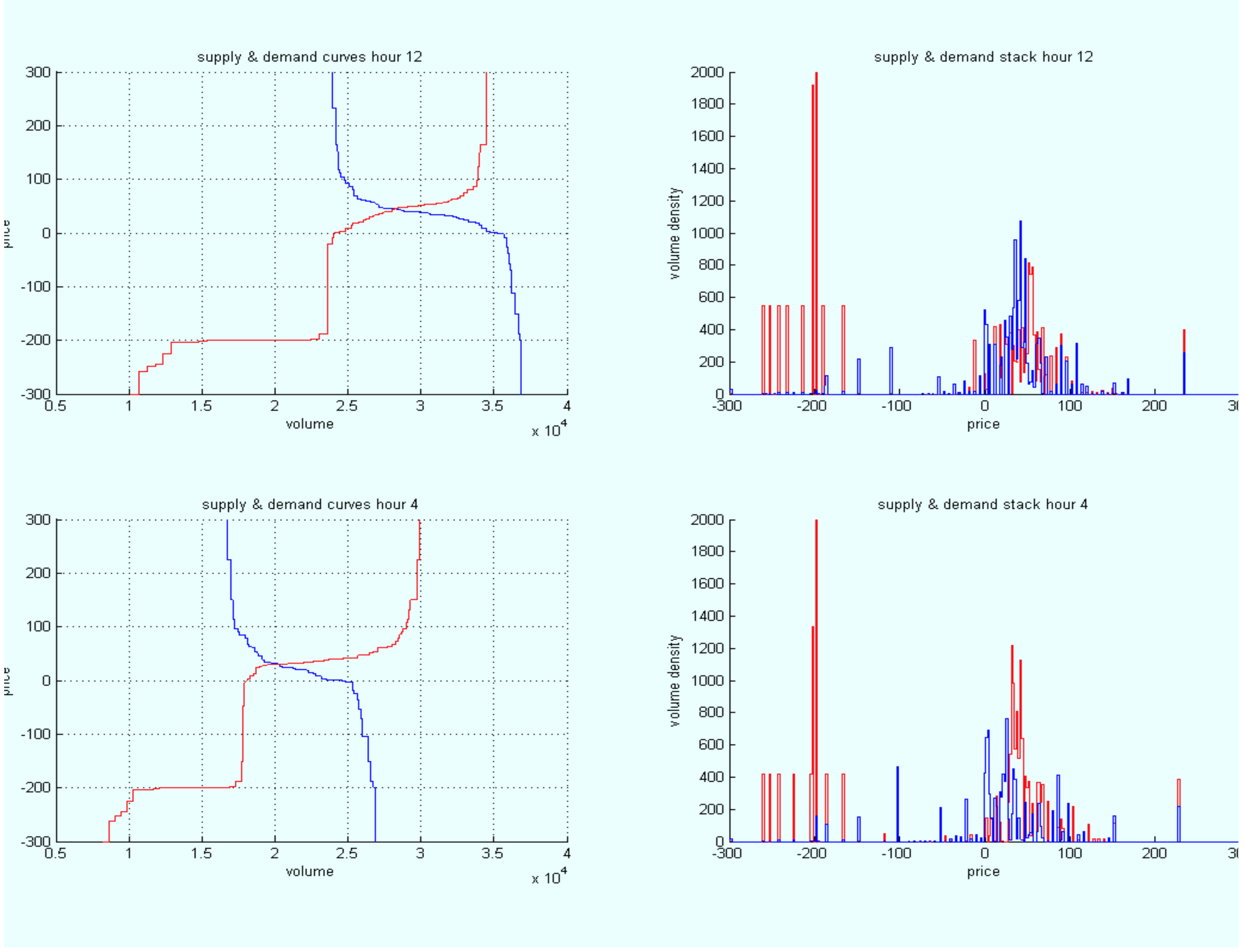
A bigger challenge: EEX!

In the German market (EEX), there are many more challenges to tackle *(work in progress - with C Jacobsson and J Strojby)*. These include:

- Non-mandatory bidding means that total capacity and weights w_i in the bid stack vary significantly. (eg, strategic bidding ?)
- Demand elasticity to price is significant $\implies$ combined bid and offer curve behaviour replaces bid stack.
- EEX represents $< 25\%$ of national demand. Hence "make or buy" bidding behaviour is common, increasing demand elasticity.
- Big variety of power sources including: nuclear, lignite, coal, gas, oil, hydro and wind (with particularly volatile availability),
- EU ETS carbon market: a key additional underlying factor!
- Market coupling: a key new (and ongoing) structural change!

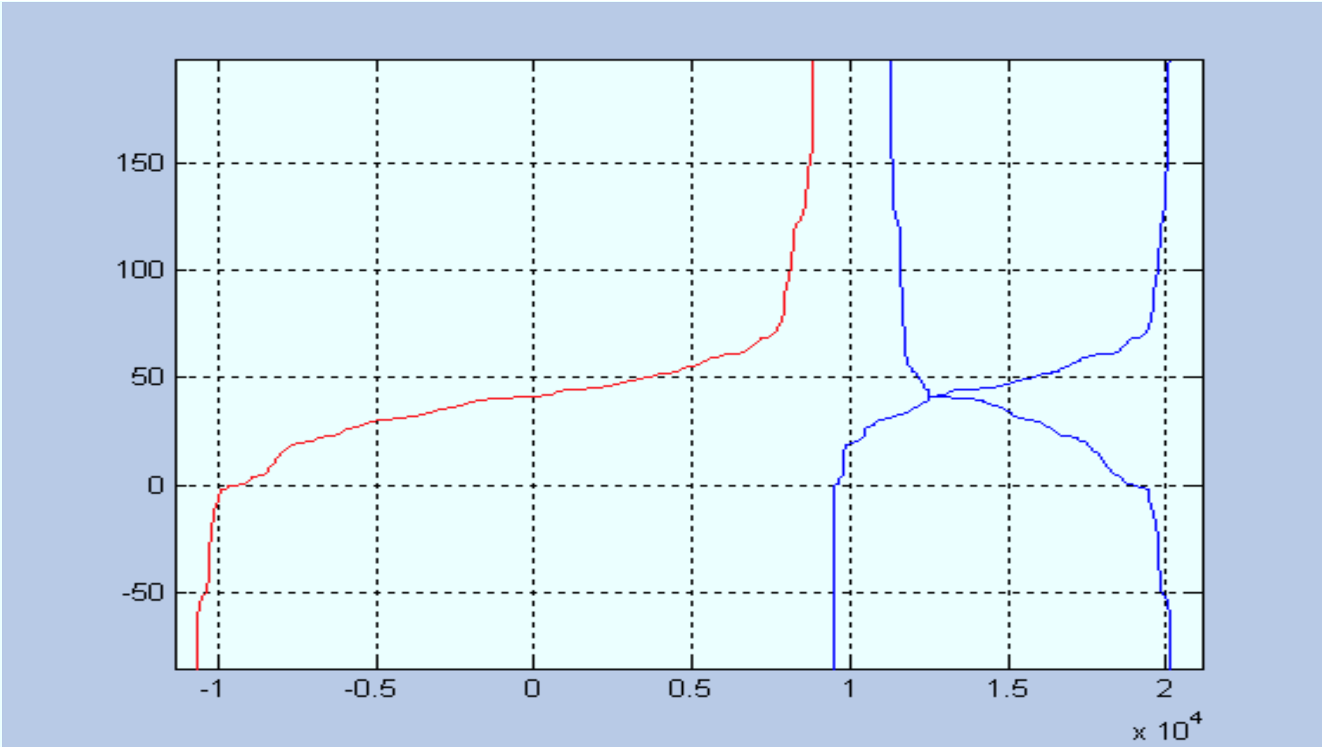
EEX study: Suppy & Demand Stacks

'Demand stack' often mirrors 'supply stack', but with clusters of bids shifted slightly downwards. Why? 'Make or buy' bidding!



EEX study: ‘Slide Stack’ Idea

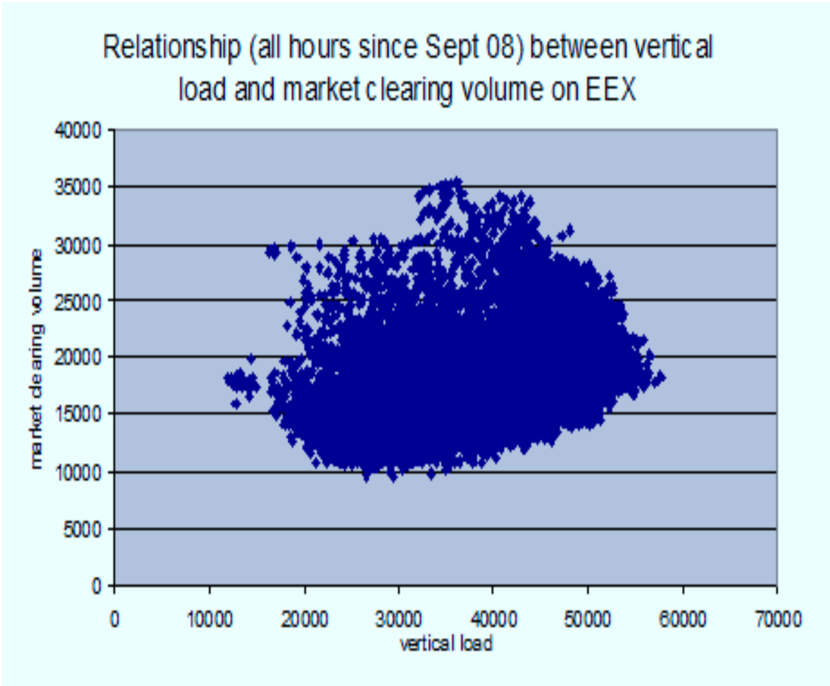
The sum of supply and demand ‘clusters’ can be interpreted as a ‘slide stack’:

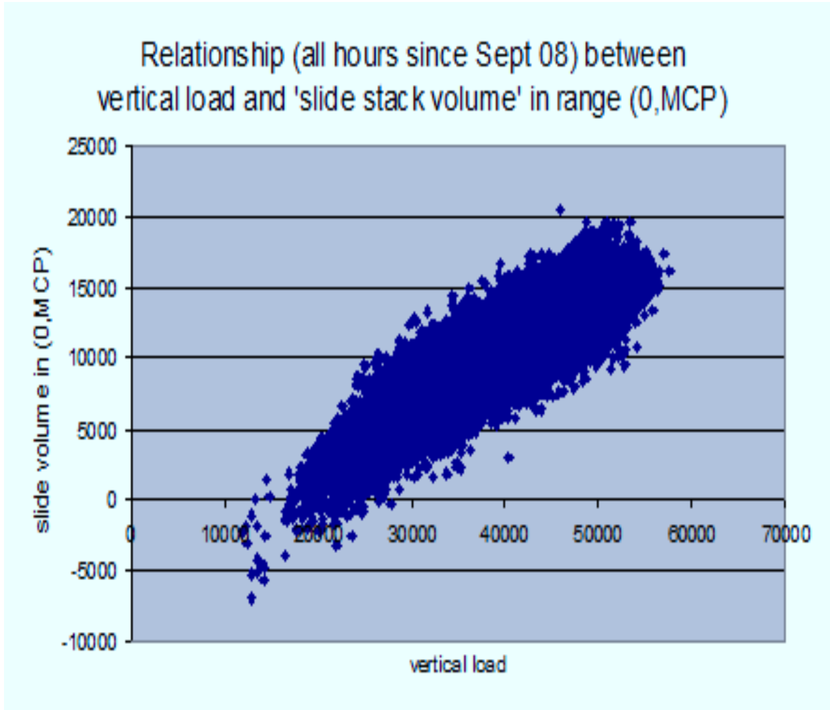


EEX study: ‘Slide Stack’ Idea

- This substantially resolves the lack of correlation between EEX volume and national load (which is easier to model):
- Also allows a return to typical inelastic demand modelling.

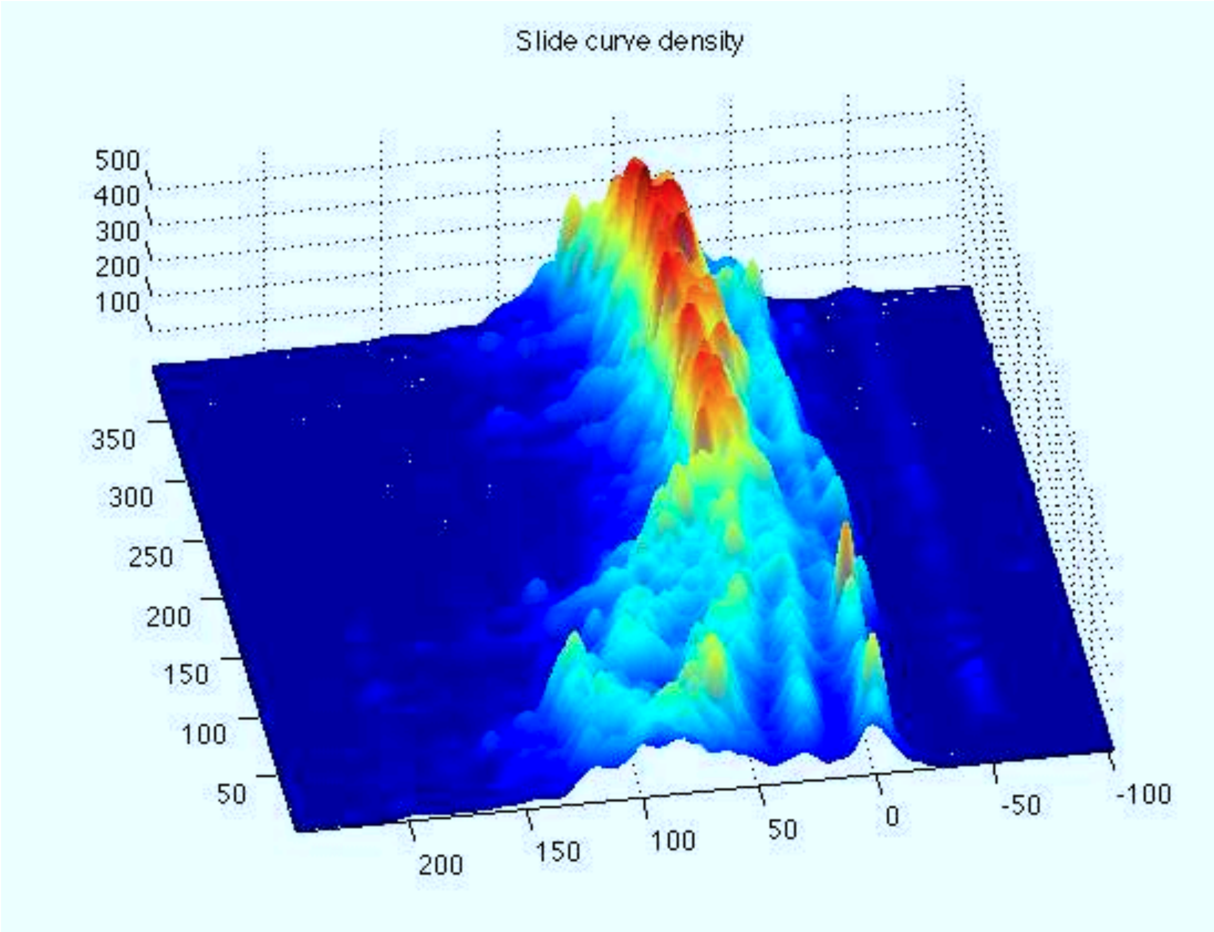
All hours since Sept 08: National load vs EEX volume (left) or slide stack volume (right):





EEX study: 'Slide Stack' Modeling

- Original bid stack approach shows promising results for 'slide stack'.
- Plot below shows drop and convergence of bids between mid 2008 and mid 2009. This matches drop in fuel prices and convergence of production costs.



Carbon Market Modeling - Bid Stack

How do we adapt the bid stack models to carbon markets?

- 'Bid cluster' approach: m_i (and s_i) are linear in allowance price A_t, eg:

$$m_1 = \tilde{\alpha}_0 + \tilde{\alpha}_1(C_t + \tilde{\alpha}_2 A_t), \qquad m_2 = \alpha_0 + \alpha_1(G_t + \alpha_2 A_t)$$

- 'Exponential stack' approach: again C_t replaced by $C_t + \tilde{\alpha}_2 A_t$, etc.
- Expect $\tilde{\alpha}_2 = e_C$, and $\alpha_2 = e_G$, avg. emission rates of coal and gas.
- Note that $e_C >> e_G \implies$ merit order changes for high A_t.
- 'Fuel switching' (changes in w_i) amplifies these of merit order changes.

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But how to model A_t? Exogenously? Lognormally?

- Closed-form approximations (forwards, options) plausible if lognormal.
- **But** A_t is a financial contract dependent on probability of excess emissions each period $\implies$ Higher A_t means more gas than coal burned and hence lower probability, lower A_t. Equilibrium price exists!