```
In[ ]:= SetOptions[SelectedNotebook[],
         PrintingStyleEnvironment → "Printout", ShowSyntaxStyles → True]

In[ ]:= ClearAll[x, t];

        DiceFace = 6;
        HitRule = PrimeQ;

        HitRoundMax = 50;

        Print["Number of dice faces: ", DiceFace];
        Print["Number of MAXIMAL hitting rounds: ", HitRoundMax];

        Print[
          "NOTE: By setting the maximal hitting rounds to be 1000, the expection in the 6-faced
            case in the paper of Alon-Malinovsky is 2.4284..."];

        GF1 = {};
        GF2 = {};

        gf1 = 0;
        gf2 = 0;
        For[i = 1, i ≤ DiceFace, i++,
          If[HitRule[i], gf1 = gf1 + x^i, gf2 = gf2 + x^i];
         ];
        GFInitial = gf1 + gf2;
        GF1 = Append[GF1, gf1];
        GF2 = Append[GF2, gf2];

        For[HitRound = 2, HitRound ≤ HitRoundMax, HitRound++,
          gf1 = 0;
          gf2 = 0;
          gf = Expand[GFInitial * GF2[[HitRound - 1]]];
          For[i = HitRound, i ≤ HitRound * DiceFace, i++,
           If[HitRule[i],
             gf1 = gf1 + Coefficient[gf, x, i] * x^i, gf2 = gf2 + Coefficient[gf, x, i] * x^i];
          ];
          GF1 = Append[GF1, gf1];
          GF2 = Append[GF2, gf2];
         ];

        HitExpectation = 0;
        For[k = 1, k ≤ HitRoundMax, k++,
          HitExpectation = HitExpectation + k * (1/DiceFace)^k * (GF1[[k]] /. {x → 1});
         ];

        Print["Expected stopping round: ", N[HitExpectation, 50]];

        Print["log(DiceFace): ", N[Log[DiceFace], 10]];
```

```
Number of dice faces: 6

Number of MAXIMAL hitting rounds: 50

NOTE: By setting the maximal hitting rounds to be 1000, the
  expection in the 6-faced case in the paper of Alon-Malinovsky is 2.4284...

Expected stopping round: 2.428438056502905415192437877817440077716943701620 0

log(DiceFace): 1.791759469
```

In[•]:=
```
ClearAll[x, t];

DiceFace = 10;
HitRule = PrimeQ;

HitRoundMax = 50;

Print["Number of dice faces: ", DiceFace];
Print["Number of MAXIMAL hitting rounds: ", HitRoundMax];

GF1 = {};
GF2 = {};

gf1 = 0;
gf2 = 0;
For[i = 1, i ≤ DiceFace, i++,
  If[HitRule[i], gf1 = gf1 + x^i, gf2 = gf2 + x^i];
 ];
GFInitial = gf1 + gf2;
GF1 = Append[GF1, gf1];
GF2 = Append[GF2, gf2];

For[HitRound = 2, HitRound ≤ HitRoundMax, HitRound++,
  gf1 = 0;
  gf2 = 0;
  gf = Expand[GFInitial * GF2[[HitRound - 1]]];
  For[i = HitRound, i ≤ HitRound * DiceFace, i++,
   If[HitRule[i],
     gf1 = gf1 + Coefficient[gf, x, i] * x^i, gf2 = gf2 + Coefficient[gf, x, i] * x^i];
  ];
  GF1 = Append[GF1, gf1];
  GF2 = Append[GF2, gf2];
 ];

HitExpectation = 0;
For[k = 1, k ≤ HitRoundMax, k++,
  HitExpectation = HitExpectation + k * (1/DiceFace)^k * (GF1[[k]] /. {x → 1});
 ];

Print["Expected stopping round: ", N[HitExpectation, 50]];

Print["log(DiceFace): ", N[Log[DiceFace], 10]];
```

```
Number of dice faces: 10

Number of MAXIMAL hitting rounds: 50

Expected stopping round: 2.9711376392109133291448749288403453477385067339688

log(DiceFace): 2.302585093
```

In[•]:=
```
ClearAll[x, t];

DiceFace = 20;
HitRule = PrimeQ;

HitRoundMax = 50;

Print["Number of dice faces: ", DiceFace];
Print["Number of MAXIMAL hitting rounds: ", HitRoundMax];

GF1 = {};
GF2 = {};

gf1 = 0;
gf2 = 0;
For[i = 1, i ≤ DiceFace, i++,
  If[HitRule[i], gf1 = gf1 + x^i, gf2 = gf2 + x^i];
 ];
GFInitial = gf1 + gf2;
GF1 = Append[GF1, gf1];
GF2 = Append[GF2, gf2];

For[HitRound = 2, HitRound ≤ HitRoundMax, HitRound++,
  gf1 = 0;
  gf2 = 0;
  gf = Expand[GFInitial * GF2[[HitRound - 1]]];
  For[i = HitRound, i ≤ HitRound * DiceFace, i++,
   If[HitRule[i],
     gf1 = gf1 + Coefficient[gf, x, i] * x^i, gf2 = gf2 + Coefficient[gf, x, i] * x^i];
  ];
  GF1 = Append[GF1, gf1];
  GF2 = Append[GF2, gf2];
 ];

HitExpectation = 0;
For[k = 1, k ≤ HitRoundMax, k++,
  HitExpectation = HitExpectation + k * (1/DiceFace)^k * (GF1[[k]] /. {x → 1});
 ];

Print["Expected stopping round: ", N[HitExpectation, 50]];

Print["log(DiceFace): ", N[Log[DiceFace], 10]];
```

```
Number of dice faces: 20

Number of MAXIMAL hitting rounds: 50

Expected stopping round: 3.4707936279221485268704392916093086946271987105604

log(DiceFace): 2.995732274
```

In[•]:=
```
ClearAll[x, t];

DiceFace = 50;
HitRule = PrimeQ;

HitRoundMax = 50;

Print["Number of dice faces: ", DiceFace];
Print["Number of MAXIMAL hitting rounds: ", HitRoundMax];

GF1 = {};
GF2 = {};

gf1 = 0;
gf2 = 0;
For[i = 1, i ≤ DiceFace, i++,
  If[HitRule[i], gf1 = gf1 + x^i, gf2 = gf2 + x^i];
 ];
GFInitial = gf1 + gf2;
GF1 = Append[GF1, gf1];
GF2 = Append[GF2, gf2];

For[HitRound = 2, HitRound ≤ HitRoundMax, HitRound++,
  gf1 = 0;
  gf2 = 0;
  gf = Expand[GFInitial * GF2[[HitRound - 1]]];
  For[i = HitRound, i ≤ HitRound * DiceFace, i++,
   If[HitRule[i],
     gf1 = gf1 + Coefficient[gf, x, i] * x^i, gf2 = gf2 + Coefficient[gf, x, i] * x^i];
  ];
  GF1 = Append[GF1, gf1];
  GF2 = Append[GF2, gf2];
 ];

HitExpectation = 0;
For[k = 1, k ≤ HitRoundMax, k++,
  HitExpectation = HitExpectation + k * (1/DiceFace)^k * (GF1[[k]] /. {x → 1});
 ];

Print["Expected stopping round: ", N[HitExpectation, 50]];

Print["log(DiceFace): ", N[Log[DiceFace], 10]];
```

```
Number of dice faces: 50

Number of MAXIMAL hitting rounds: 50

Expected stopping round: 4.5129116813733563241239982554733150499064734676293

log(DiceFace): 3.912023005
```

In[•]:=
```
ClearAll[x, t];

DiceFace = 100;
HitRule = PrimeQ;

HitRoundMax = 50;

Print["Number of dice faces: ", DiceFace];
Print["Number of MAXIMAL hitting rounds: ", HitRoundMax];

GF1 = {};
GF2 = {};

gf1 = 0;
gf2 = 0;
For[i = 1, i ≤ DiceFace, i++,
  If[HitRule[i], gf1 = gf1 + x^i, gf2 = gf2 + x^i];
 ];
GFInitial = gf1 + gf2;
GF1 = Append[GF1, gf1];
GF2 = Append[GF2, gf2];

For[HitRound = 2, HitRound ≤ HitRoundMax, HitRound++,
  gf1 = 0;
  gf2 = 0;
  gf = Expand[GFInitial * GF2[[HitRound - 1]]];
  For[i = HitRound, i ≤ HitRound * DiceFace, i++,
   If[HitRule[i],
     gf1 = gf1 + Coefficient[gf, x, i] * x^i, gf2 = gf2 + Coefficient[gf, x, i] * x^i];
  ];
  GF1 = Append[GF1, gf1];
  GF2 = Append[GF2, gf2];
 ];

HitExpectation = 0;
For[k = 1, k ≤ HitRoundMax, k++,
  HitExpectation = HitExpectation + k * (1/DiceFace)^k * (GF1[[k]] /. {x → 1});
 ];

Print["Expected stopping round: ", N[HitExpectation, 50]];

Print["log(DiceFace): ", N[Log[DiceFace], 10]];
```

```
Number of dice faces: 100

Number of MAXIMAL hitting rounds: 50

Expected stopping round: 5.2725273248294885021653082354438726023966486903300

log(DiceFace): 4.605170186
```

In[•]:=
```
ClearAll[x, t];

DiceFace = 200;
HitRule = PrimeQ;

HitRoundMax = 50;

Print["Number of dice faces: ", DiceFace];
Print["Number of MAXIMAL hitting rounds: ", HitRoundMax];

GF1 = {};
GF2 = {};

gf1 = 0;
gf2 = 0;
For[i = 1, i ≤ DiceFace, i++,
  If[HitRule[i], gf1 = gf1 + x^i, gf2 = gf2 + x^i];
 ];
GFInitial = gf1 + gf2;
GF1 = Append[GF1, gf1];
GF2 = Append[GF2, gf2];

For[HitRound = 2, HitRound ≤ HitRoundMax, HitRound++,
  gf1 = 0;
  gf2 = 0;
  gf = Expand[GFInitial * GF2[[HitRound - 1]]];
  For[i = HitRound, i ≤ HitRound * DiceFace, i++,
   If[HitRule[i],
     gf1 = gf1 + Coefficient[gf, x, i] * x^i, gf2 = gf2 + Coefficient[gf, x, i] * x^i];
  ];
  GF1 = Append[GF1, gf1];
  GF2 = Append[GF2, gf2];
 ];

HitExpectation = 0;
For[k = 1, k ≤ HitRoundMax, k++,
  HitExpectation = HitExpectation + k * (1/DiceFace)^k * (GF1[[k]] /. {x → 1});
 ];

Print["Expected stopping round: ", N[HitExpectation, 50]];

Print["log(DiceFace): ", N[Log[DiceFace], 10]];
```

```
Number of dice faces: 200

Number of MAXIMAL hitting rounds: 50

Expected stopping round: 5.9163933689991405358167608699551054734855776200084

log(DiceFace): 5.298317367
```

In[•]:=
```
ClearAll[x, t];

DiceFace = 500;
HitRule = PrimeQ;

HitRoundMax = 50;

Print["Number of dice faces: ", DiceFace];
Print["Number of MAXIMAL hitting rounds: ", HitRoundMax];

GF1 = {};
GF2 = {};

gf1 = 0;
gf2 = 0;
For[i = 1, i ≤ DiceFace, i++,
  If[HitRule[i], gf1 = gf1 + x^i, gf2 = gf2 + x^i];
 ];
GFInitial = gf1 + gf2;
GF1 = Append[GF1, gf1];
GF2 = Append[GF2, gf2];

For[HitRound = 2, HitRound ≤ HitRoundMax, HitRound++,
  gf1 = 0;
  gf2 = 0;
  gf = Expand[GFInitial * GF2[[HitRound - 1]]];
  For[i = HitRound, i ≤ HitRound * DiceFace, i++,
   If[HitRule[i],
     gf1 = gf1 + Coefficient[gf, x, i] * x^i, gf2 = gf2 + Coefficient[gf, x, i] * x^i];
  ];
  GF1 = Append[GF1, gf1];
  GF2 = Append[GF2, gf2];
 ];

HitExpectation = 0;
For[k = 1, k ≤ HitRoundMax, k++,
  HitExpectation = HitExpectation + k * (1/DiceFace)^k * (GF1[[k]] /. {x → 1});
 ];

Print["Expected stopping round: ", N[HitExpectation, 50]];

Print["log(DiceFace): ", N[Log[DiceFace], 10]];
```

```
Number of dice faces: 500

Number of MAXIMAL hitting rounds: 50

Expected stopping round: 6.8903927514310706786602168957004662221920614412240

log(DiceFace): 6.214608098
```

In[•]:=
```
ClearAll[x, t];

DiceFace = 1000;
HitRule = PrimeQ;

HitRoundMax = 50;

Print["Number of dice faces: ", DiceFace];
Print["Number of MAXIMAL hitting rounds: ", HitRoundMax];

GF1 = {};
GF2 = {};

gf1 = 0;
gf2 = 0;
For[i = 1, i ≤ DiceFace, i++,
  If[HitRule[i], gf1 = gf1 + x^i, gf2 = gf2 + x^i];
 ];
GFInitial = gf1 + gf2;
GF1 = Append[GF1, gf1];
GF2 = Append[GF2, gf2];

For[HitRound = 2, HitRound ≤ HitRoundMax, HitRound++,
  gf1 = 0;
  gf2 = 0;
  gf = Expand[GFInitial * GF2[[HitRound - 1]]];
  For[i = HitRound, i ≤ HitRound * DiceFace, i++,
   If[HitRule[i],
     gf1 = gf1 + Coefficient[gf, x, i] * x^i, gf2 = gf2 + Coefficient[gf, x, i] * x^i];
  ];
  GF1 = Append[GF1, gf1];
  GF2 = Append[GF2, gf2];
 ];

HitExpectation = 0;
For[k = 1, k ≤ HitRoundMax, k++,
  HitExpectation = HitExpectation + k * (1/DiceFace)^k * (GF1[[k]] /. {x → 1});
 ];

Print["Expected stopping round: ", N[HitExpectation, 50]];

Print["log(DiceFace): ", N[Log[DiceFace], 10]];
```

```
Number of dice faces: 1000

Number of MAXIMAL hitting rounds: 50

Expected stopping round: 7.5925906451878283576893210809323803615661225377711

log(DiceFace): 6.907755279
```