MTH 343 Numerical Analysis Lecture 6: Secant Method

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Suppose $\boxed{\text{f(x)} = 0}$

Technique:

1. Begin with two values x_0 & x_1 near the root r. Draw a line joining the two points. The point x_2 where this line hits the x-axis is the first approximation of the exact root r.

2. Continue this repeatedly, always choosing the last computed values for drawing this straight line.

Derivation:

$\frac{y - y_1}{x - x_1} = m$ or $y - y_1 = m(x - x_1)s$

$$\frac{y - f(P_1)}{x - P_1} = \frac{f(P_1) - f(P_0)}{P_1 - P_0}$$

$$y = f(P_1) + \frac{f(P_1) - f(P_0)}{P_1 - P_0}(x - P_1)$$

To find x-intercept, set y=0,

$$0 = f(P_1) + \frac{f(P_1) - f(P_0)}{P_1 - P_0}(x - P_1)$$

$$0 = f(P_1)(P_1 - P_0) + (f(P_1) - f(P_0))(x - P_1)$$

$$(f(P_1) - f(P_0))(x - P_1) = -f(P_1)(P_1 - P_0)$$

$$x = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)}$$

$$P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)}$$

Example:

$$f(x) = x^2 - 2$$

n	P_n	$f(P_n)$
2	1.6	0.56
3	1.47826	0.18525
4	1.418079	0.01094
5	1.414299	0.000241

Disadvantages of the Secant Method

1. If the fraction is far from linear near the root, the iterates can fly off to points far from the root. This can be fixed by plotting $f(x)$ and changing the starting points.

2. If an iterate duplicates a previous one this results in an endless loop that never reaches the value of the true solution.

3. Secant method doesn't have the bracketing property of the bisection method. Therefore, the method doesn't always converge. But if it does, it is generally faster than the bisection method.

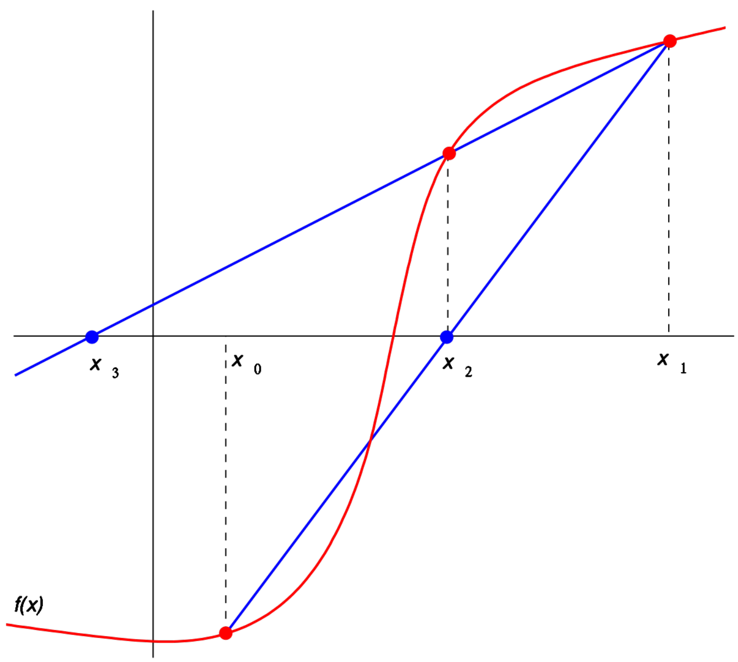

Figure 1: Visual demonstration of the secant method