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Example: f(x) = 3^{-x} on [0,1]

Closed Interval Method

1. Critical points are x = 0 & x = 1
2. f'(x) = $-3^{-x} \cdot \ln 3 \neq 0$
3. f' always exists
4. $f(0) = 1$
5. $f(1) = \frac{1}{3}$
6. $f : [0,1]- > [\frac{1}{3}, 1] \subset [0,1]$
7. =¿ There is a fixed point
8. —f'(0)— = ln3 ¿ 1
9. No guarantee of uniqueness

Interpolation and Polynomial Approximation

Interpolation refers to determining a function that exactly represents a collection of data. We need to compute values for a tabulated function at a point not in the table.

We will find a polynomial that fits a selected set of points $(x_i, f(x_i))$ & assumes the polynomial and function are exactly the same (1st degree polynomial is called linear interpolation).

Goal is to replace some times a function by a simpler one.

Applications

- Performance of a new rocket. Signals are received every 10 sec. Interpolation gives the position of the rocket as well as other information.
- Astronomy: When the motion of heavenly bodies is determined from periodic observations.

Lagrange Polynomials

x	$f(x)$
x_0	$f(x_0)$
x_1	$f(x_1)$

$P_1(x) = \frac{x-x_0}{x_0-x_1} f_0 + \frac{x-x_1}{x_1-x_0} f_1$