MTH 343 Numerical Analysis Lecture 7:

Sheikh Abdul Raheem Ali

February 12, 2019

1 Method of False Position

Bisection/Secant Method that constructs approximating lines similar to those of secant method but always brackets the root in the manner of the bisection method.

$$P_{n+1} = b_n - \frac{f(b_n)(b_n - a_n)}{f(b_n) - f(a_n)}$$

Example: Solve $x^2 - 2 = 0$ for x. $f(x) = x^2 - 2, a_1 = 1, b_1 = 2$.

n	a_n	b_n	P_{n+1}	$f(P_{n+1})$
1	1	2	1.3333	-0.22222
2	1.3333	2	1.4	-0.04
3	1.4	2	1.4117	-0.00692

Although the method of False Position may appear superior to the Secant Method, it generally converges more slowly.

2 Newton's Method

One of the most widely used methods.

Technique

1. Start from a single initial point (estimate) x_0 that is close to the root, then move along the tangent line to its intersection with the x-axis & take that point as the first approximation.

2. The procedure is continued until either the successive x-values are sufficiently close or the values of the function is sufficiently near zero.

$$
\begin{aligned}
\frac{f(x_0) - 0}{x_0 - x_1} &= f'(x_0) \\
\frac{f(x_0)}{f'(x_0)} &= x_0 - x_1 \\
x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\
x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)}
\end{aligned}
$$

This method is of order two, which means that on average it converges to a solution at roughly two additional decimal points of precision per iteration.

It fails when it encounters a maximum or minimum (as $f'(x_n)$ would be 0 and the tangent would be parallel to the x-axis), which means it's not as suitable for functions that oscillate.