MTH 343 Numerical Analysis Chapter 1: Mathematical Preliminaries & Error Analysis

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Abstract

Numerical Analysis is a way to do math problems on a computer.

Two types of solutions: Analytical (Exact) and Approximate (Numerical).

$$Example: \int_0^1 2x(1+x)^{-1/2}\ dx$$

We may solve this **Analytically** using u-substitution:

$$u = 1 + x^2,\ du = 2x$$

$$\int 2x(1+x)^{-1/2}\ dx = \int u^{-1/2} du = \frac{u^{1/2}}{\frac{1}{2}}$$

$$= 2\sqrt{1+x^2}\Big|_0^1 = 2\sqrt{2} - 2$$

Numerical Solution:

Advantages:

1. Results approach arbitrary precision with the help of a computer.
2. An answer can be obtained even when a problem has no exact solution.

Disadvantages:

1. It is only an approximate solution.
2. The solution's behavior is not known.

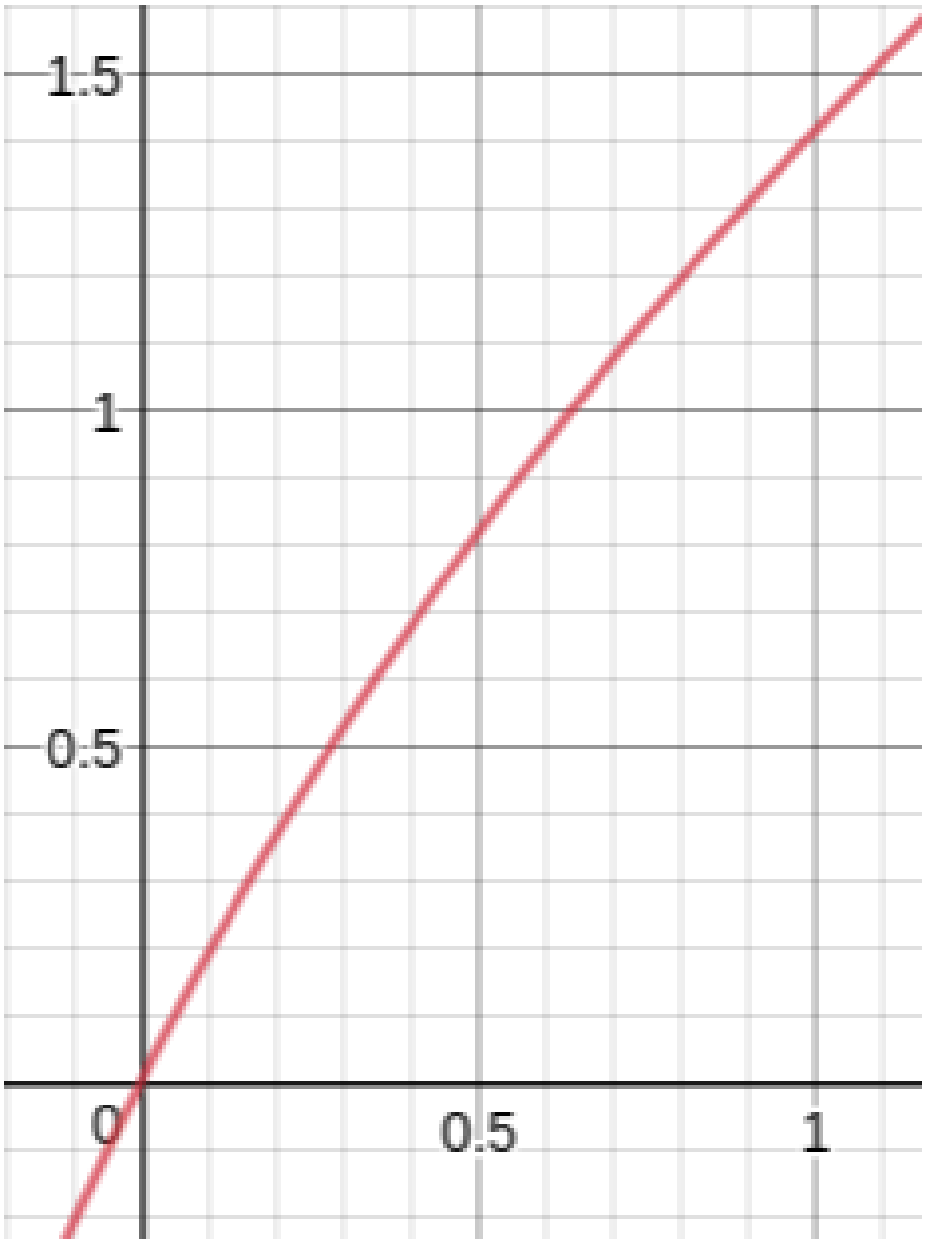

Figure 1: Graph of $2x(1+x)^{-1/2}$ from 0 to 1. The numerical solution would be to graph the function and estimate the area using slices, AKA a Riemann sum.

Analytical Solution:

Advantages:

1. It is exact.
2. The solution's properties (e.g behavior at infinity, where it is continuous, maxima-minima) are known.

Disadvantages:

1. Is difficult to determine.
2. Often does not exist (Example $\int_0^{\pi} \sqrt{1+\cos^2(x)}dx$ or $\int e^{x^3}dx$) so numerical is the best we can do.
3. Even if a closed form solution is known, most of the time you have to approximate it in order to interpret it.

One integral that we got after 140 years: $\int e^{-x^2}\,dx$ (using polar-co-ordinates).