# MTH 343 Numerical Analysis Lecture 3: Errors, Solutions of Equations in one Variable

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The floating point form is obtained by terminating the mantissa using the following two ways:

1. Chopping

2. Rounding

**Definition 1 (Absolute error)** $|true\ value - approximate\ value| = |p - p^*|$

**Definition 2 (Relative error)** $|\frac{true\ value\ -\ approximate\ value}{true\ value}| = |\frac{p\ -\ p^*}{p}|$

$$(approximate\ value)\ p_1 = 1.2,\ (true\ value)\ p_1^* = 1.2$$

$$p_2 = 1000,\ p_2^* = 1000.2$$

$$
\begin{aligned}
absolute\ error\quad &=\quad |1 - 1.2| = 0.2 \\
&=\quad |1000 - 1000.2| = 0.2
\end{aligned}
$$

$$
\begin{aligned}
rel.\ error\quad &=\quad |\frac{1 - 1.2}{1}| = 0.2 \\
&=\quad |\frac{1000 - 1000.2}{1000}| = 0.0002
\end{aligned}
$$

$$find\ f(x) = x^3 = 61x^3 + 3.2x + 1.5\ at\ f(x = 4.71)$$

|                  | $x$  | $x^2$   | $x^3$     | $61x^3$   | $3.2x$ |
|------------------|------|---------|-----------|-----------|--------|
| Exact            | 4.7  | 22.1841 | 104.487111 | 135.32301 | 15.072 |
| 3-digit chopping | 4.7  | 22.1    | 104.0     | 134       | 15.0   |
| 3-digit rounding | 4.7  | 22.2    | 104       | 135       | 15.1   |

Exact: -14.263899, Chopping: -13.5. Rounding: -13.4
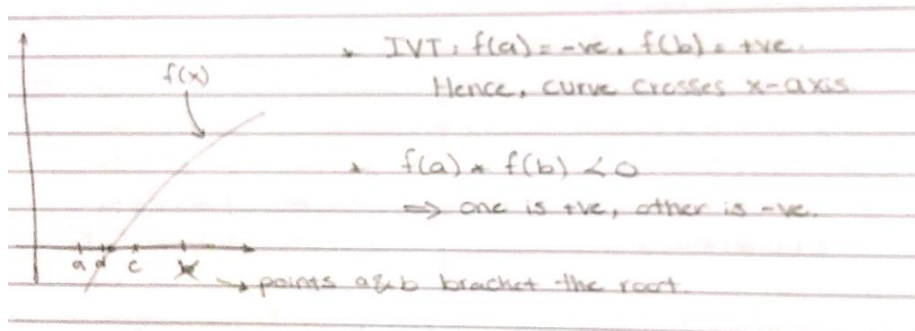Rel. error: Chopping: 0.0045, Rounding: 0.0025

Figure 1: Visual demonstration of the bisection algorithm. Credits to Luna Hatahet.

## Solutions of Equations of one variable

$$\begin{aligned}
0 &= x^3 - 16x \\
&= x(x^2 - 16) \\
&= x(x-4)(x+4)
\end{aligned}$$
$$x = 0, 4, -4$$

$$\begin{aligned}
0 &= x^2 - 4x + 3x - 12 \\
&= x(x-4) + 3(x-4) \\
&= (x-4)(x-3)
\end{aligned}$$
$$x = 4, 3$$

## Bisection Method $(\mathbf{f(x) = 0})$

Find the values of $x$ for which $f(x) = 0$, that is find the points of intersection of $f(x)$ with the x-axis.