MTH 343 Numerical Analysis Lecture 3: Errors, Solutions of Equations in one Variable

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The floating point form is obtained by terminating the mantissa using the following two ways:

1. Chopping
2. Rounding

Definition 1 (Absolute error) $|true\ value - approximate\ value| = |p - p^*|$

Definition 2 (Relative error) $|\frac{true\ value\ -\ approximate\ value}{true\ value}| = |\frac{p\ -\ p^*}{p}|$

$$(approximate\ value)\ p_1 = 1.2,\ (true\ value)\ p_1^* = 1.2$$

$$p_2 = 1000,\ p_2^* = 1000.2$$

$$\begin{aligned} absolute\ error &= |1 - 1.2| = 0.2 \\ &= |1000 - 1000.2| = 0.2 \end{aligned}$$

$$\begin{aligned} rel.\ error &= |\frac{1 - 1.2}{1}| = 0.2 \\ &= |\frac{1000 - 1000.2}{1000}| = 0.0002 \end{aligned}$$

$$find\ f(x) = x^3 = 61x^3 + 3.2x + 1.5\ at\ f(x = 4.71)$$

	x	x^2	x^3	$61x^3$	$3.2x$
Exact	4.7	22.1841	104.487111	135.32301	15.072
3-digit chopping	4.7	22.1	104.0	134	15.0
3-digit rounding	4.7	22.2	104	135	15.1

Exact: -14.263899, Chopping: -13.5. Rounding: -13.4

Rel. error: Chopping: 0.0045, Rounding: 0.0025

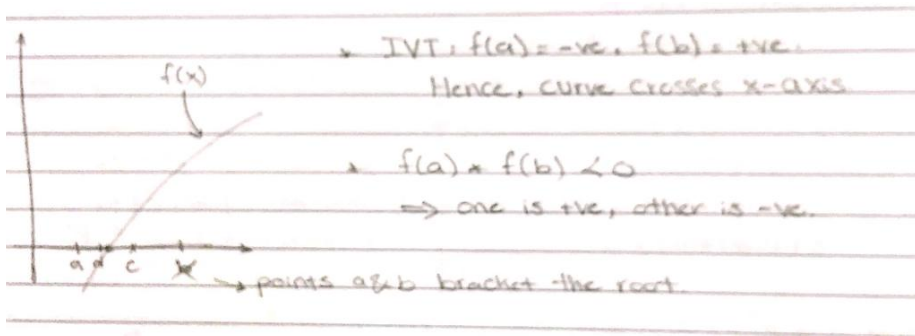

Figure 1: Visual demonstration of the bisection algorithm. Credits to Luna Hatahet.

Solutions of Equations of one variable

$$\begin{aligned} 0 &= x^3 - 16x \\ &= x(x^2 - 16) \\ &= x(x-4)(x+4) \end{aligned}$$
$$x = 0, 4, -4$$

$$\begin{aligned} 0 &= x^2 - 4x + 3x - 12 \\ &= x(x-4) + 3(x-4) \\ &= (x-4)(x-3) \end{aligned}$$
$$x = 4, 3$$

Next time: Bisection Method (f(x) = 0)

Find the values of x for which $f(x) = 0$, that is find the points of intersection of $f(x)$ with the x-axis.