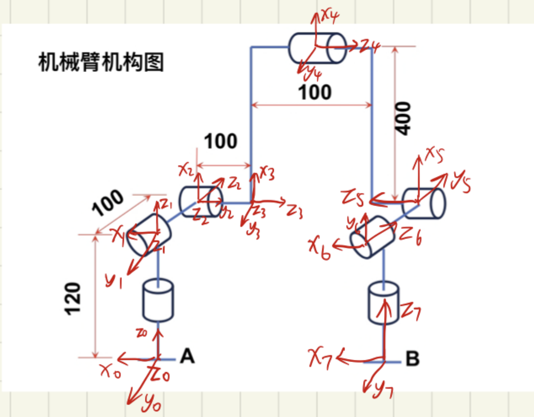
正运动学

DH参数如下：



	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	120(85)	θ_1
2	90	0	100	θ_2+90
3	-90	0	100	θ_3
4	0	400	50	θ_4
5	180	-400	-150	θ_5
6	-90	0	-100	θ_6-90
7	-90	0	-120(-85)	θ_7

仿真软件中用蓝色参数

代入以下矩阵，计算正运动学结果${}^0_7T = {}^0_1T {}^1_2T {}^2_3T {}^3_4T {}^4_5T {}^5_6T {}^6_7T$

$$
{}^{i-1}_{\ i}T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

结果如下：

$$
\begin{aligned}
t11 =& -s1(c7s(3+4-5)s6+c(3+4-5)s7)+\ldots \\
& c1(c2c6c7+s2(-c(3+4-5)c7s6+s(3+4-5)s7)) \\
t12 =& c7s(3+4)(c1c5s2-s1s5)-c(3+4)c7(c5s1+c1s2s5)+\ldots \\
& (s1s(3+4-5)s6+c1(-c2c6+c(3+4-5)s2s6))s7 \\
t13 =& -c6s1s(3+4-5)-c1(c(3+4-5)c6s2+c2s6) \\
t14 =& c1(s2(-0.4c3+0.4c(3+4)+0.0425c(3+4-5-6)+\ldots \\
& 0.0425c(3+4-5+6)+0.1s(3+4-5))+c2(-0.3+0.085s6))+\ldots \\
& s1(0.1-0.1c(3+4-5)-0.4s3+0.4s(3+4)+0.0425s(3+4-5-6)+0.0425s(3+4-5+6)) \\
t21 =& c2c6c7s1+c(3+4-5)(-c7s1s2s6+c1s7)+s(3+4-5)(c1c7s6+s1s2s7) \\
t22 =& c1(c(3+4-5)c7-s(3+4-5)s6s7)+s1(c7s2s(3+4-5)+(-c2c6+c(3+4-5)s2s6)s7) \\
t23 =& -c(3+4-5)c6s1s2+c1c6s(3+4-5)-c2s1s6 \\
t24 =& s1(s2(-0.4c3+0.4c(3+4)+0.0425c(3+4-5-6)+\ldots \\
& 0.0425c(3+4-5+6)+0.1s(3+4-5))+c2(-0.3+0.085s6))+\ldots \\
& c1(-0.1+0.1c(3+4-5)+0.4s3-0.4s(3+4)-0.0425s(3+4-5-6)-0.0425s(3+4-5+6)) \\
t31 =& c6c7s2+c2(c(3+4-5)c7s6-s(3+4-5)s7) \\
t32 =& -c6s2s7-c2(c7s(3+4-5)+c(3+4-5)s6s7) \\
t33 =& c2c(3+4-5)c6-s2s6 \\
t34 =& 0.085+c2(0.4c3-0.4c(3+4)-0.0425c(3+4-5-6)-\ldots \\
& 0.0425c(3+4-5+6)-0.1s(3+4-5))+s2(-0.3+0.085s6)
\end{aligned}
$$

其中c、s代表cos、sin函数

逆运动学

令t2=0，然后求解。

由${}_6^2T = {}_0^2T \cdot T \cdot {}_6^7T = ({}_2^0T)^{-1} \cdot T \cdot ({}_7^6T)^{-1}$得左右矩阵如下:

$$
\begin{pmatrix}
\text{Cos}[t3+t4-t5]\,\text{Sin}[t6] & \text{Cos}[t3+t4-t5]\,\text{Cos}[t6] & \text{Sin}[t3+t4-t5] & 0.4\,\text{Cos}[t3]-0.4\,\text{Cos}[t3+t4]-0.1\,\text{Sin}[t3+t4-t5] \\
-1.\,\text{Cos}[t6] & 1.\,\text{Sin}[t6] & 0. & 0.3 \\
-\text{Sin}[t3+t4-t5]\,\text{Sin}[t6] & -\text{Cos}[t6]\,\text{Sin}[t3+t4-t5] & \text{Cos}[t3+t4-t5] & -0.1\,\text{Cos}[t3+t4-t5]-0.4\,\text{Sin}[t3]+0.4\,\text{Sin}[t3+t4] \\
0. & 0. & 0. & 1.
\end{pmatrix}
$$

$$
\begin{pmatrix}
t31\,\text{Cos}[t7]-t32\,\text{Sin}[t7] & 0.+t33 & -t32\,\text{Cos}[t7]-t31\,\text{Sin}[t7] & -0.085+0.085\,t33+t34 \\
\text{Cos}[t1]\,(-t11\,\text{Cos}[t7]+t12\,\text{Sin}[t7])+\text{Sin}[t1]\,(-t21\,\text{Cos}[t7]+t22\,\text{Sin}[t7]) & -t13\,\text{Cos}[t1]-t23\,\text{Sin}[t1] & \text{Cos}[t1]\,(t12\,\text{Cos}[t7]+t11\,\text{Sin}[t7])+\text{Sin}[t1]\,(t22\,\text{Cos}[t7]+t21\,\text{Sin}[t7]) & (-0.085\,t13-1.\,t14)\,\text{Cos}[t1]+(-0.085\,t23-1.\,t24)\,\text{Sin}[t1] \\
\text{Sin}[t1]\,(t11\,\text{Cos}[t7]-t12\,\text{Sin}[t7])+\text{Cos}[t1]\,(-t21\,\text{Cos}[t7]+t22\,\text{Sin}[t7]) & -t23\,\text{Cos}[t1]+t13\,\text{Sin}[t1] & -\text{Sin}[t1]\,(t12\,\text{Cos}[t7]+t11\,\text{Sin}[t7])+\text{Cos}[t1]\,(t22\,\text{Cos}[t7]+t21\,\text{Sin}[t7]) & -0.1+(-0.085\,t23-1.\,t24)\,\text{Cos}[t1]+(0.085\,t13+t14)\,\text{Sin}[t1] \\
0. & 0. & 0. & 1.
\end{pmatrix}
$$

第二行第四列相等得

$$(-0.085t13-t14)cos(t1)+(-0.085t23-t24)sin(t1)=0.3$$

求解上式得

$$
\begin{gathered}
t1=atan2(m,-n)-atan2(0.3,\pm\sqrt{m^2+n^2-0.3^2}) \\
m=-0.085t13-t14 \\
n=-0.085t23-t24
\end{gathered}
$$

第二行第二列相等得

$$
\begin{gathered}
-t13cos(t1)-t23sin(t1)=sin(t6) \\
t6=asin(-t13cos(t1)-t23sin(t1))
\end{gathered}
$$

第二行第三列相等:

$$(t12cos(t1)+t22sin(t1))cos(t7)+(t11cos(t1)+t21sin(t1))sin(t7)=0$$

得到

$$
\begin{gathered}
t7=atan2(m,-n)-\pi, atan2(m,-n) \\
m=t12cos(t1)+t22sin(t1) \\
n=t11cos(t1)+t21sin(t1)
\end{gathered}
$$

由${}^2_5T = {}^2_0T \cdot T \cdot {}^7_5T = ({}^0_2T)^{-1} \cdot T \cdot ({}^5_7T)^{-1}$得如下矩阵：

$$\begin{pmatrix} \mathrm{Cos}[t3+t4-t5] & \mathrm{Sin}[t3+t4-t5] & 0 & \mathrm{Cos}[t3]\,(0.4-0.4\,\mathrm{Cos}[t4])+0.4\,\mathrm{Sin}[t3]\,\mathrm{Sin}[t4] \\ 0. & 0. & -1. & 0.3 \\ -\mathrm{Sin}[t3+t4-t5] & \mathrm{Cos}[t3+t4-t5] & 0. & (-0.4+0.4\,\mathrm{Cos}[t4])\,\mathrm{Sin}[t3]+0.4\,\mathrm{Cos}[t3]\,\mathrm{Sin}[t4] \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

$$\begin{pmatrix} t33\,\mathrm{Cos}[t6]+\mathrm{Sin}[t6]\,(t31\,\mathrm{Cos}[t7]-t32\,\mathrm{Sin}[t7]) & -t32\,\mathrm{Cos}[t7]-t31\,\mathrm{Sin}[t7] & -t33\,\mathrm{Sin}[t6]+\mathrm{Cos}[t6]\,(t31\,\mathrm{Cos}[t7]-t32\,\mathrm{Sin}[t7]) & -0.085+0.085\,t33+t34-0.1\,t32\,\mathrm{Cos}[t7]-0.1\,t31\,\mathrm{Sin}[t7] \\ -\mathrm{Cos}[t1]\,(t13\,\mathrm{Cos}[t6]+\mathrm{Sin}[t6]\,(t11\,\mathrm{Cos}[t7]-t12\,\mathrm{Sin}[t7]))-\mathrm{Sin}[t1]\,(t23\,\mathrm{Cos}[t6]+\mathrm{Sin}[t6]\,(t21\,\mathrm{Cos}[t7]-t22\,\mathrm{Sin}[t7])) & \mathrm{Cos}[t1]\,(t12\,\mathrm{Cos}[t7]+t11\,\mathrm{Sin}[t7])+\mathrm{Sin}[t1]\,(t22\,\mathrm{Cos}[t7]+t21\,\mathrm{Sin}[t7]) & \mathrm{Cos}[t1]\,(t13\,\mathrm{Sin}[t6]+\mathrm{Cos}[t6]\,(-t11\,\mathrm{Cos}[t7]+t12\,\mathrm{Sin}[t7]))+\mathrm{Sin}[t1]\,(t23\,\mathrm{Sin}[t6]+\mathrm{Cos}[t6]\,(-t21\,\mathrm{Cos}[t7]+t22\,\mathrm{Sin}[t7])) & \mathrm{Cos}[t1]\,(-0.085\,t13-1.\,t14+0.1\,t12\,\mathrm{Cos}[t7]+0.1\,t11\,\mathrm{Sin}[t7])+\mathrm{Sin}[t1]\,(-0.085\,t23-1.\,t24+0.1\,t22\,\mathrm{Cos}[t7]+0.1\,t21\,\mathrm{Sin}[t7]) \\ \mathrm{Sin}[t1]\,(t13\,\mathrm{Cos}[t6]+\mathrm{Sin}[t6]\,(t11\,\mathrm{Cos}[t7]-t12\,\mathrm{Sin}[t7]))-\mathrm{Cos}[t1]\,(t23\,\mathrm{Cos}[t6]+\mathrm{Sin}[t6]\,(t21\,\mathrm{Cos}[t7]-t22\,\mathrm{Sin}[t7])) & -\mathrm{Sin}[t1]\,(t12\,\mathrm{Cos}[t7]+t11\,\mathrm{Sin}[t7])+\mathrm{Cos}[t1]\,(t22\,\mathrm{Cos}[t7]+t21\,\mathrm{Sin}[t7]) & -\mathrm{Sin}[t1]\,(t13\,\mathrm{Sin}[t6]+\mathrm{Cos}[t6]\,(-t11\,\mathrm{Cos}[t7]+t12\,\mathrm{Sin}[t7]))+\mathrm{Cos}[t1]\,(t23\,\mathrm{Sin}[t6]+\mathrm{Cos}[t6]\,(-t21\,\mathrm{Cos}[t7]+t22\,\mathrm{Sin}[t7])) & -0.1+\mathrm{Sin}[t1]\,(0.085\,t13+t14-0.1\,t12\,\mathrm{Cos}[t7]-0.1\,t11\,\mathrm{Sin}[t7])+\mathrm{Cos}[t1]\,(-0.085\,t23-1.\,t24+0.1\,t22\,\mathrm{Cos}[t7]+0.1\,t21\,\mathrm{Sin}[t7]) \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

第一行第四列相等和第三行第四列相等得：

$$\begin{aligned} cos(t3)(0.4-0.4cos(t4))+0.4sin(t3)sin(t4) &= -0.085+0.085t33+t34-0.1t32cos(t7)-0.1t31sin(t7) \\ (-0.4+0.4cos(t4))sin(t3)+0.4cos(t3)sin(t4) &= -0.1+sin(t1)(0.085t13+t14-0.1t12cos(t7)-0.1t11sin(t7)+\ldots \\ & cos(t1)(-0.085t23-t24+0.1t22cos(t7)+0.1t21sin(t7) \end{aligned}$$

即

$$\begin{gathered} 0.4(c3-c34)=m \\ 0.4(-s3+s34)=n \\ m=-0.085+0.085t33+t34-0.1t32cos(t7)-0.1t31sin(t7) \\ n=-0.1+sin(t1)(0.085t13+t14-0.1t12cos(t7)-0.1t11sin(t7)+\ldots \\ cos(t1)(-0.085t23-t24+0.1t22cos(t7)+0.1t21sin(t7) \end{gathered}$$

得到：

$$t4=acos(\frac{0.32-m^2-n^2}{0.32})$$

反代入原式，得

$$\begin{gathered} sin(t3)=\frac{1.25(-1+cos(t4))*n+1.25sin(t4)*m}{1-cos(t4)} \\ cos(t3)=\frac{1.25(1-cos(t4))*m+1.25sin(t4)*n}{1-cos(t4)} \\ t3=atan2((-1+cos(t4))*n+sin(t4)*m,(1-cos(t4))*m+sin(t4)*n) \end{gathered}$$

又第一行第一列相等和第一行第二列相等得：

$$\begin{gathered} cos(t3+t4-t5)=t33cos(t6)+sin(t6)(t31cos(t7)-t32sin(t7))=n \\ sin(t3+t4-t5)=-t32cos(t7)-t31sin(t7)=m \\ t5=t3+t4-atan2(m,n) \end{gathered}$$