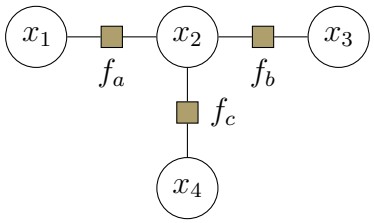
Probabilistic Machine Learning

Exercise Sheet #10

Inference in Graphical Models

1. **Exam-Type Question — The Sum-Product Algorithm.** Lecture 18 introduced the Sum-Product Algorithm for tree-structured factor graphs. Consider the graph below, which encodes the (normalized) joint distribution

$$p(x_1, \dots, x_4) = f_a(x_1, x_2) \cdot f_b(x_2, x_3) \cdot f_c(x_2, x_4).$$



   Let us designate x_3 as the root of the graph, and (x_1, x_2, x_4) as leaves. Explicitly write down all 12 messages (6 toward the root, 6 toward the leaves) constructed by the sum-product algorithm (many of them are of rather simple form).

2. **Theory Question.** Use the messages constructed for Question 1 to answer the following questions:

   (a) confirm that the product of incoming messages gives the correct marginal for x_2, i.e. that

$$\mu_{f_a \to x_2}(x_2) \cdot \mu_{f_b \to x_2}(x_2) \cdot \mu_{f_c \to x_2}(x_2) = \sum_{x_1, x_3, x_4} p(x_1, x_2, x_3, x_4) = p(x_2).$$

   (b) confirm the analogous result for the root x_3 and one of the leaves, x_1.

   (c) show that $p(x_1, x_2) = f_a(x_1, x_2) \cdot \prod_{i=1,2} \mu_{x_i \to f_a}(x_i)$.

   (d) Suppose we wish to evaluate the *joint* distribution $p(x_1, x_4)$. In contrast to the previous question, x_1 and x_4 do not belong to a common factor. Define a procedure for using the sum-product algorithm to evaluate the joint, in which one of the variables is successively clamped to each of its allowed values (slide 7 in lecture 18 explains what that means). Hint: use the product rule on $p(x_1, x_4)$.

3. **Practical Question.** The aim is to efficiently implement the Gibbs sampler for LDA inference over the State of the Union corpus. The algorithm was introduced formally in the lecture. Your task is to implement it, and do so *efficiently*. You will need to do the following:

   (a) Initialize LDA's latent variables by sampling from the prior, following the generative process.

   (b) At each iteration, sample from the full-conditional distribution of each variable group in turn.

   (c) After sampling has finished, inspect the top words of each topic. Recall that each topic is a mixture of words.

   (d) Inspect the sparsity of some of the resulting topic-word and document-topic proportions.

   (e) Visualize the change of the document-topic proportions over the year.

   Use `Exercise_10.ipynb` as your workspace.