STAT 24400 Statistics Theory and Methods I

Homework 3: Due 3:00PM Thurs January 28 2016.

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1. Rice, 4.78: Show that if a density is symmetric around zero, its skewness is zero.

2. Consider the bivariate density of X and Y, $f(x, y) = 4(x + y + xy)/5$ for $0 < x < 1$, $0 < y < 1$, $f(x, y) = 0$ otherwise.

   (a) Verify that this is a bivariate density (that the total volume $\int\int f(x, y)dxdy = 1$).

   (b) Find the marginal density of Y.

   (c) Find the conditional density of X given $Y = 0.5$.

   (d) Find $E(X)$, $E(X^2)$, $Var(X)$, $E(XY)$, and $Cov(X, Y)$.

   (e) Find $P(0.2 \leq X \leq 0.5, 0.4 \leq Y \leq 0.8)$.

   (f) Find $P(X + Y \leq 1)$.

3. Rice, 4.81 and 4.82: (4.81) Find the moment-generating function of a Bernoulli random variable, and use it to find the mean, variance, and third moment. (4.82) Use the result of 4.81 to find the mgf of a binomial random variable and its mean and variance. Note that Rice has the answer to 4.81, but you must show your work.

4. Rice 5.6: Using moment-generating functions, show that as $\alpha \to \infty$, the gamma distribution with parameters α and λ, properly standardized, tends to the standard normal distribution.

5. Suppose that a Bayesian statistician has a Beta(2,1) prior distribution on the cure rate θ (= Probability of cure) for an experimental drug. The drug is tried (independently) on three subjects, and X are cured. Compute $P(\theta \leq 0.2 \mid X = k)$ and $E(\theta \mid X = k)$ for $k = 0, 1, 2, 3$.

6. Laplace's rule of succession. What is the *a posteriori* expectation of the probability that the sun will rise tomorrow given that it has risen n days in a row and that before those n days began we had an *a priori* uniform distribution for the probability the sun would rise? [This is a classical problem.]