

Plot	Corn Yield	Corn Pixels
10	149.94	316
11	64.75	145
12	127.07	355
13	133.55	295
14	77.70	223
15	206.39	459
16	108.33	290
17	118.17	307

C.5 Let Y denote a Bernoulli(θ) random variable with $0 < \theta < 1$. Suppose we are interested in estimating the *odds ratio*, $\gamma = \theta/(1 - \theta)$, which is the probability of success over the probability of failure. Given a random sample $\{Y_1, \dots, Y_n\}$, we know that an unbiased and consistent estimator of θ is \bar{Y} , the proportion of successes in n trials. A natural estimator of γ is $G = \{\bar{Y}/(1 - \bar{Y})\}$, the proportion of successes over the proportion of failures in the sample.

- (i) Why is G not an unbiased estimator of γ ?
- (ii) Use PLIM.2(iii) to show that G is a consistent estimator of γ .

C.6 You are hired by the governor to study whether a tax on liquor has decreased average liquor consumption in your state. You are able to obtain, for a sample of individuals selected at random, the difference in liquor consumption (in ounces) for the years before and after the tax. For person i who is sampled randomly from the population, Y_i denotes the change in liquor consumption. Treat these as a random sample from a Normal(μ, σ^2) distribution.

- (i) The null hypothesis is that there was no change in average liquor consumption. State this formally in terms of μ .
- (ii) The alternative is that there was a decline in liquor consumption; state the alternative in terms of μ .
- (iii) Now, suppose your sample size is $n = 900$ and you obtain the estimates $\bar{y} = -32.8$ and $s = 466.4$. Calculate the t statistic for testing H_0 against H_1 ; obtain the p -value for the test. (Because of the large sample size, just use the standard normal distribution tabulated in Table G.1.) Do you reject H_0 at the 5% level? at the 1% level?
- (iv) Would you say that the estimated fall in consumption is large in magnitude? Comment on the practical versus statistical significance of this estimate.

- (v) What has been implicitly assumed in your analysis about other determinants of liquor consumption over the two-year period in order to infer causality from the tax change to liquor consumption?

C.7 The new management at a bakery claims that workers are now more productive than they were under old management, which is why wages have “generally increased.” Let W_i^b be Worker i ’s wage under the old management and let W_i^a be Worker i ’s wage after the change. The difference is $D_i \equiv W_i^a - W_i^b$. Assume that the D_i are a random sample from a $\text{Normal}(\mu, \sigma^2)$ distribution.

- (i) Using the following data on 15 workers, construct an exact 95% confidence interval for μ .
- (ii) Formally state the null hypothesis that there has been no change in average wages. In particular, what is $E(D_i)$ under H_0 ? If you are hired to examine the validity of the new management’s claim, what is the relevant alternative hypothesis in terms of $\mu = E(D_i)$?
- (iii) Test the null hypothesis from part (ii) against the stated alternative at the 5% and 1% levels.
- (iv) Obtain the p -value for the test in part (iii).

Worker	Wage Before	Wage After
1	8.30	9.25
2	9.40	9.00
3	9.00	9.25
4	10.50	10.00
5	11.40	12.00
6	8.75	9.50
7	10.00	10.25
8	9.50	9.50
9	10.80	11.50
10	12.55	13.10
11	12.00	11.50
12	8.65	9.00

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