Econometrics A (Econ 210)

Problem Set 2

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Due: Oct 15, 2015; TA Session

1. Do exercises C.3 of Wooldridge (The scanned page of the book is posted on chalk).

2. Do exercise C.4 of Wooldridge (The scanned page of the book is posted on chalk).

3. We are going to estimate the true volume of a cube. To do that, we measure X_i, the length of the edge of the cube, n times, i.e. $i = 1..n$. Assume that $\{X_i\}_{i=1}^n$ are i.i.d. draws from a Normal distribution with mean μ and variance σ^2. Consider the following two estimators of the volume

$$V_n^1 = \frac{1}{n}\sum_{i=1}^{n} X_i^3$$
$$V_n^2 = \left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^3$$

   (a) Show that the odd-order moments of any symmetric distribution about zero is zero.

   (b) Use part (a) to show that $\mathbb{E}\left[(X_i - \mu)^3\right] = 0$.

   (c) use part (b) and show that $\mathbb{E}\left[X_i^3\right] = \mu^3 + 3\mu\sigma^2$.

   (d) Is V_n^1 is an unbiased estimator of the true volume? What about V_n^2? If they are biased, derive the bias of each of them. Is the bias positive or negative?

(e) Which of the estimators is more biased? The bias of which estimator gradually fades away as n gets larger?

(f) Propose an unbiased estimator for the true volume.

4. Suppose that y_i is generated according to the following model,

$$y_i = \mu + \epsilon_i$$

where μ is a constant and ϵ_i is a random variable drawn from a distribution with mean zero and variance σ^2. We do not observe μ but we do observe y_i, thus we are going to estimate μ using estimators which are a function of y_i. Two estimators are suggested as the following

$$\hat{\mu}_n = \sum_{i=1}^{n} \frac{y_i}{n}$$

$$\tilde{\mu}_n = \sum_{i=1}^{n} w_i y_i \quad \text{where} \quad w_i = \frac{i}{\sum_{i=1}^{n} i} = \frac{i}{\frac{n(n+2)}{2}}$$

(a) Show that $\hat{\mu}_n$ is a consistent estimator of μ.

(b) Derive the asymptotic distribution of $\hat{\mu}_n$ as $n \to \infty$.

(c) Show that $\tilde{\mu}_n$ is also a consistent estimator of μ.

(d) Show that the asymptotic variance of $\tilde{\mu}_n$ as $n \to \infty$ is zero.

(e) Verify your theoretical answers in the preceding parts by running the following simulation. Assume $\mu = 0$ and $\epsilon_i \sim N(0, 1)$. Play around with the number of observations, n, to resemble the theoretical results above.

5. Consider the 2000 U.S. presidential election of Bush v. Gore, the outcome of which was decided by a tiny margin in the state of Florida. Let μ denote the true fraction of votes won by Bush in Florida. In any count i of the votes, there is some error (denoted

by ϵ_i), due to human mistakes or errors in the way the machine reads the ballots. The error component for a given count of the ballots i is denoted ϵ_i, which is distributed according to $U[-.05, +.05]$. Note that this implies $E[\epsilon_i] = 0$. The error term is i.i.d. across any count of the ballots. Write the outcome of any count:

$$y_i = \mu + \epsilon_i.$$

(For this question, assume Bush and Gore are the only two candidates, and that Bush is declared the victor if he gets at least .5 of the votes.)

(a) Suppose Bush truly won 49% of the votes; that is, $\mu = .49$. What is the probability that Gore is declared the winner of the election if the votes are counted once? What is the probability that Bush is declared the winner if the votes are counted once?

(b) Florida state law mandates a recount of the votes if the first count is really close. According to law, the votes will be recounted once and the outcome of that recount will determine the election result. A Fox News commentator argues that there is no point to recounting the votes because there is just as much error in the recount as there is in the initial count. Propose an estimator $\hat{\mu}$ for μ using your data on the two counts: $\{y_i\}_{i=1}^2$.

(c) Show that your estimator is unbiased.

(d) Show that the variance of your estimator for μ is smaller than the variance of the one that follows Florida state law (i.e. Florida state law uses the recount as it's estimator: $\hat{\mu} = y_2$)