Econometrics A (Econ 210)

Problem Set 3

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Due: Oct 22, 2015; TA Session

1. Do exercise C.6 of Wooldridge.

2. Do exercise C.7 of Wooldridge.

3. In this exercise we are going to review the procedure of hypothesis testing step by step using asymptotic theory. Consider a random sample $X_1, ..., X_n$ drawn from a Bernoulli distribution with mean q. You have shown already in the previous exercises that the variance of a random variable with Bernoulli distribution is $q(1-q)$. We are going to run the following hypothesis testing

$$\begin{cases} H_0 : & q = 0 \\ H_1 : & q \neq 0 \end{cases}$$

The key idea in hypothesis testing is constructing the sample analogue of the of interest underlying parameter of hypothesis testing and finding its limiting distribution. To do this we usually appeal to the Central Limit Theorem. Then you should normalize the derived limiting distribution in order to use the well-known quantile values in your calculations to obtain confidence interval or p-value.

   (a) Construct the sample analogue of q and denote it by q_n.

(b) Use Central Limit Theorem to find the limiting distribution of q_n.

(c) Use Slutsky theorem to normalize the limiting distribution to a Standard Normal and call the final statistic which has a Standard Normal distribution T_n.

(d) Show that $Pr[T_n \geq Z_{1-\alpha}] = \alpha$.

(e) Derive the confidence interval for q, that is, derive an interval of plausible values that true q would fit in that with $(1 - \alpha)$ probability.

(f) Derive the p-value in terms of n, q_n, and q.

(g) Now consider the one-sided hypothesis testing

$$\begin{cases} H_0 : & q \leq 0 \\ H_1 : & q > 0 \end{cases}$$

In this case, part (a) through part (c) would be identical. Show that $Pr[T_n \geq Z_{1-\alpha}] \leq \alpha$. Also derive the confidence interval and p-value in this case.

4. Suppose that you know that $X \sim U(\mu - \frac{1}{2}, \mu + \frac{1}{2})$, but μ is not known.

(a) Reset the random seed to 210, and generate 10 draws from the distribution with $\mu = .2$. Estimate μ using the sample mean, $\tilde{\mu} = \bar{\mu}$.

(b) Would you be able to reject the null hypothesis that $\mu = 0$? To determine this, run a simulation where you draw 10 times from a uniform distribution with $\mu = 0$. Calculate $\hat{\mu_1^0}$. Then repeat this 500 times, obtaining $\hat{\mu_i^0}$, where i=1,...,500. Plot a histogram of all $\hat{\mu_i^0}$. Find the 95th percentile of these estimates. Is $\tilde{\mu}$ above this level? At what significance level does $\tilde{\mu}$ reject the null hypothesis that $\mu = 0$.

(c) How often would you be able to reject $H_0 : \mu = 0$ with ten draws if $\mu = .2$? To figure this out, simulate a new set of 500 iterations of ten draws using $\mu = .2$. Plot the histogram, and calculate the proportion falling above the 95th percentile from part (b). What is this proportion called in hypothesis testing?

(d) The raw second moment for a uniform distribution, $U(a,b)$, is $\frac{1}{3}(a^2+ab+b^2)$. Calculate the standard error when you have ten observations. With an asymptotic approximation, what is the critical value for rejection under the null hypothesis that $\mu = 0$ at significance level, $\alpha = 0.05$?

(e) Calculate the power of the test from (d) when $\mu = .2$.

(f) Compare the power and the significance when using the asymptotic approximation versus when using the simulated "true" distribution of your estimator.