STAT 24400 Statistics Theory and Methods I
Homework 7: Due 3:00PM Thurs, March 3, 2016.

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1. Suppose we face a pattern recognition problem, where the data consist of a single set of pixels X (where there are 16 possible pixel patterns), and there are two possible patterns θ, "0" and "6". The model is that X has the probability function $p(x \mid \theta)$ depending on θ, given by the following table. Find the best test for "0" versus "6" for which the chance of making the error of "6" when the pattern is "0" is no greater than 0.10. What is the power of this test?

	Pixel number (X_i)															
$p(x\|\theta)$:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
θ: "0"	0	0	.02	.03	.02	.03	.02	.02	.08	.12	.02	.22	.02	.23	.02	.15
θ: "6"	0	.03	.01	.13	.08	.12	0	0	.02	.20	.01	.17	.04	.11	0	.08

2. Suppose X has a $N(\mu, \sigma^2)$ distribution.
   (a) Find the Most Powerful test for testing at level $\alpha = 0.05$ the hypothesis $H_0 : \mu = 6$ and $\sigma^2 = 4$ versus $H_1 : \mu = 9$ and $\sigma^2 = 4$.
   (b) Find the power of this test.
   (c) Suppose that instead of the above H_1, we have $H_1 : \mu = \mu_1$ and $\sigma^2 = 4$, where $\mu_1 > 6$. Find and graph the power function.

3. Suppose $X_1, X_2, X_3, \ldots X_k$ are multinomial distributed, based upon n trials, with parameters $\theta_1, \theta_2, \theta_k, \ldots \theta_k$, where $\sum_k \theta_k = 1$.
   (a) For an arbitrary i, j, $i \neq j$, find $\mathrm{Cov}(X_i, X_j)$ and ρ_{X_i, X_j}. (Hint: What is the distribution of $Z = X_i + X_j$? What is $\mathrm{Var}(X_i + X_j)$? Use your knowledge of $\mathrm{Var}(X_i)$ and $\mathrm{Var}(X_j)$ to find $\mathrm{Cov}(X_i, X_j)$ from this, and then find the correlation.)
   (b) Suppose all $\theta_i = 1/k$. What is the covariance of $X_i + X_j$ and $X_i - X_j$?

4. Rice 9.12

5. Rice 9.13

6. Rice 9.24, (a)-(d)

7. Suppose X follows a geometric distribution with parameter p.
   (a) Derive the likelihood ratio for testing the hypothesis $p = p_0$ versus the alternative $p \neq p_0$.
   (b) For $p_0 = 0.01$, by some combination of numerical experimentation and mathematical analysis, find the set of possible values of x for X for which the likelihood ratio is less than 0.1.
   (c) Find the probability of a Type 1 error for the test that rejects $p_0 = 0.01$ when the likelihood ratio is less than 0.1. Find the power of this test when $p = 0.5$. Find the power of this test when $p = 0.001$.