Econometrics A (Econ 210)

Multivariate Regression and IV Review Practice

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Consider the following model,

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + u_i$$

More compactly, we may write this as

$$y = x'\beta + u \ ,$$

where $\beta = (\beta_0, \beta_1, \beta_2)'$ and $y = (1, x_1, x_2)'$. Assume that homoscedasticity holds. We observe a sample of size n. From the sample we can calculate the sample analogue of moments, that is

$$\sum x_{i1}, \sum x_{i1} y_i, \sum x_{i1}^2, \sum x_{i2} y_i, \sum x_{i2}^2, \sum y_i^2$$

Using these sample moments answer the following questions.

1. In the following parts assume that both x_1 and x_2 are exogenous.

   (a) Discuss about three different interpretations of the model. In what interpretation $\mathbb{E}[u] = 0$ is given? In what interpretation $\mathbb{E}[u|x_1, x_2] = 0$ is given?

(b) Write down moment conditions to derive OLS estimates.

(c) Calculate $\hat{\beta}^{OLS}$.

(d) Calculate $\hat{\text{Var}}[u] = \dfrac{1}{n-3}\sum_{i=1}^{n}\hat{u}_i^2$.

(e) calculate the estimate of variance-covariance matrix, $\hat{\Sigma}^{OLS}$.

(f) Describe how you would test the null hypothesis $H_0 : \beta_1 = 0$ versus the alternative that $H_1 : \beta_1 \neq 0$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

(g) Construct a confidence interval for β_1.

(h) Describe how you would test the null hypothesis $H_0 : a\beta_1 + b\beta_2 = c$ versus the alternative that $H_1 : a\beta_1 + b\beta_2 \neq c$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

(i) Describe how you would test the null hypothesis $H_0 : \beta_1 = 0 \ \textit{and} \ \beta_2 = 0$ versus the alternative $H_1 : \beta_1 \neq 0 \ \textit{or} \ \beta_2 \neq 0$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

(j) Describe how you would test the null hypothesis $H_0 : \beta_1 = c_1 \ \textit{and} \ \beta_2 = c_2$ versus the alternative $H_1 : \beta_1 \neq c_1 \ \textit{or} \ \beta_2 \neq c_2$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

(k) Describe how you would test the null hypothesis $H_0 : a_1\beta_1 + b_1\beta_2 = c_1 \ \ a_2\beta_1 + b_2\beta_2 = c_2$ versus the alternative that $H_1 : \textit{otherwise}$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

2. Now, assume that x_2 is still exogenous but x_1 is endogenous. You have find instrument z_1 for x_1.

   (a) Discuss about three different interpretations of the model. In what interpretation $\mathbb{E}[u] = 0$ is given? In what interpretation $\mathbb{E}[u|x_1, x_2] = 0$ is given? Discuss why OLS estimates are not consistent in this case.

   (b) Write down moment conditions to derive IV estimates.

   (c) Calculate $\hat{\beta}^{IV}$.

   (d) Calculate $\hat{\text{Var}}[u] = \dfrac{1}{n-3}\sum_{i=1}^{n}\hat{u}_i^2$.

   (e) calculate the estimate of variance-covariance matrix, $\hat{\Sigma}^{IV}$.

   (f) Describe how you would test the null hypothesis $H_0 : \beta_1 = 0$ versus the alternative that $H_1 : \beta_1 \neq 0$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

   (g) Construct a confidence interval for β_1.

   (h) Describe how you would test the null hypothesis $H_0 : a\beta_1 + b\beta_2 = c$ versus the alternative that $H_1 : a\beta_1 + b\beta_2 \neq c$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

   (i) Describe how you would test the null hypothesis $H_0 : \beta_1 = 0 \ \textit{and} \ \beta_2 = 0$ versus the alternative $H_1 : \beta_1 \neq 0 \ \textit{or} \ \beta_2 \neq 0$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

   (j) Describe how you would test the null hypothesis $H_0 : \beta_1 = c_1 \ \textit{and} \ \beta_2 = c_2$ versus the alternative $H_1 : \beta_1 \neq c_1 \ \textit{or} \ \beta_2 \neq c_2$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

(k) Describe how you would test the null hypothesis $H_0 : a_1\beta_1 + b_1\beta_2 = c_1 \;\; a_2\beta_1 + b_2\beta_2 = c_2$ versus the alternative that H_1 : *otherwise* at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

3. Next, still assume that x_2 is exogenous and x_1 is endogenous. But, now you have find two instruments z_1 and z_2 for x_1.

   (a) Discuss about three different interpretations of the model. In what interpretation $\mathbb{E}[u] = 0$ is given? In what interpretation $\mathbb{E}[u|x_1, x_2] = 0$ is given? Discuss why OLS estimates are not consistent in this case.

   (b) Write down moment conditions to derive TSLS estimates.

   (c) Calculate $\hat{\beta}^{TSLS}$.

   (d) Calculate $\hat{\text{Var}}[u] = \dfrac{1}{n-3}\sum_{i=1}^{n} \hat{u}_i^2$.

   (e) calculate the estimate of variance-covariance matrix, $\hat{\Sigma}^{TSLS}$.

   (f) Describe how you would test the null hypothesis $H_0 : \beta_1 = 0$ versus the alternative that $H_1 : \beta_1 \neq 0$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

   (g) Construct a confidence interval for β_1.

   (h) Describe how you would test the null hypothesis $H_0 : a\beta_1 + b\beta_2 = c$ versus the alternative that $H_1 : a\beta_1 + b\beta_2 \neq c$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

   (i) Describe how you would test the null hypothesis $H_0 : \beta_1 = 0 \;\; and \;\; \beta_2 = 0$ versus the alternative $H_1 : \beta_1 \neq 0 \;\; or \;\; \beta_2 \neq 0$ at the 5% significance level. In particular,

describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

(j) Describe how you would test the null hypothesis $H_0 : \beta_1 = c_1 \ \textit{and} \ \beta_2 = c_2$ versus the alternative $H_1 : \beta_1 \neq c_1 \ \textit{or} \ \beta_2 \neq c_2$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

(k) Describe how you would test the null hypothesis $H_0 : a_1\beta_1 + b_1\beta_2 = c_1 \ \ a_2\beta_1 + b_2\beta_2 = c_2$ versus the alternative that $H_1 : \textit{otherwise}$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

4. If we drop the intercept from the model, that is,

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + u_i$$

How your answer to the preceding parts would change?