STAT 244
Lecture 1
1/5/16

Broadly speaking, we will cover 4 topics:

I. Probability
II. Bayesian Inference
III. Maximum Liklihood Inference
IV. Hypothesis Testing

→ see the syllabus on chalk for organizational details

I. Probability
probability Theory

Rules for calculating some probabilities from others, going from simple situations to complex

[the <u>most</u> simple situation is one where each "outcome" is equally likely]

Definitions:

Given an "experiment" (some process of observation):

Sample Space S = set of all possible outcomes

Event (e.g. E) = a set of outcomes

With events E and F:

E^c = the complement of E ("not E")

$E \cap F$ = the intersection of E and F ("both")

$E \cup F$ = the union of E and F (E or F or both)

E and F mutually exclusive if $E \cap F = \emptyset$ ("empty")

Example: $(A \cap B^c) \cup (A \cap B) = A$,

so $P((A \cap B^c) \cup (A \cap B)) = P(A)$

(whatever "P" means!)

Probability is a measure.

A measure of what?

uncertainty
belief
relative frequency in the "long run"

Properties: For any probability measure:

$$P(S) = 1; \quad 0 \le P(E) \text{ for all } E$$

$$P(E \cup F) = P(E) + P(F) \text{ if } E, F \text{ mutually exclusive}$$

Hence since $S = E \cup E^c$, $P(E) + P(E^c) = 1$

also, for any E, F:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

<u>Countable additivity</u>:

If $A_1, A_2, \ldots$ are mutually exclusive,

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$$

Permutations and Combinations

$P_{r,n}$ = number of ways to choose r objects from n distinguishable objects where order makes a difference:

$$\frac{n!}{(n-r)!} = \frac{n(n-1)(n-2)\ldots 2\cdot 1}{(n-r)(n-r-1)\ldots 1}$$

$C_{r,n}$ = number of ways of choosing r objects from n distinguishable objects where order does <u>not</u> make a difference:

$$= \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots 2\cdot 1}{r(r-1)\ldots 1\cdot(n-r)(n-r-1)\ldots 1}$$

$[0! = 1]$

Ex: If $r=2$ people are chosen from $n=5$ people to be designated president and vice president, there are $P_{2,5}=20$ ways to make the selection. If they are to make a committee of two equals, so (A,B) and (B,A) are the same committee, then there are only $\binom{5}{2}=10$ ways to select.

Conditional Probability

(or "relative probability")

Definition: If $P(F) > 0$,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Note: All probs conditional:

$$P(E) = P(E|S)$$

Note: $P(E \cap F) = P(E|F)\,P(F)$ even if $P(F) = 0$.

Any two of these determine the third.

If $P(E|F) = P(E)$ then E and F are Independent Events

Equivalently, $P(E|F) = P(E|F^c)$

or $P(E \cap F) = P(E)\,P(F)$

Ex Monte Hall Game

Three doors (A, B, C). One prize.
You pick A. Host shows B (empty)
Should you switch?

Monte Hall Game

Three doors (A, B, C). One prize.
You pick A, host shows B (empty).
Should you switch your guess to C? Assume you do.

S has 6 outcomes:

Prize in A,	you see B	Prob = 1/6
Prize in A,	you see C	Prob = 1/6
Prize in B,	you see B	Prob = 0
Prize in B,	you see C	Prob = 1/3
Prize in C,	you see C	prob = 0
Prize in C,	you see B	prob = 1/3

$$P(\text{Win} \mid \text{See } B) = \frac{P(\text{Win AND see } B)}{P(\text{See } B)}$$

$$= \frac{\left(\frac{1}{3}\right)}{\left(\frac{1}{3} + \frac{1}{6}\right)} = \frac{2}{3}$$