Econometrics A (Econ 210)

Virtual Final Exam

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PRINTED NAME: _____

- **Note that the parts in this exam are independent. If you could not solve earlier parts you can still use their result to solve later parts.**
- The exam is closed book and closed notes. However, you can have one summary sheet (both sides are allowed).
- No calculators are allowed.
- There are a total of 100 possible points. Each part in this exam worth 5 points.
- Please write your answers in the space provided in the exam. There is additional scratch paper at the end of the exam.
- You have 120 minutes to complete the exam.
- Good luck!

1. (45 points) True/False/Uncertain. You should justify your short answer, otherwise you would not get any point.

   (a) In multi-variable linear regression model no perfect collinearity necessarily implies that matrix $\mathbb{E}[XX']$ where $X = (1, X^1, X^2, ..., X^k)'$ is invertible.

   (b) Imagine an i.i.d. sample of size n, $X_1, .., X_n$. The estimator $\frac{1}{n-3}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is not a consistent estimator for $\text{Var}[X]$, where $\bar{X}$ is the sample mean.

(c) Consider a random variable X. Then we can conclude that $\text{Var}[\mathbb{E}[X]] = 0$.

(d) If X and Y are two independent random variables, then $\mathbb{E}\left[\frac{X}{Y}\right] = \frac{\mathbb{E}[X]}{\mathbb{E}[Y]}$.

(e) If X and Y are two independent random variables, then $\text{Var}[X^3 + \log(X)Y|X] = X^6 + (\log(X))^2 \text{Var}[Y|X]$.

(f) Adjusted R^2 could be negative.

(g) In linear regression model $Y = \beta_0 + \beta_1 X + U$ a necessary condition for OLS estimation is "no perfect collinearity".

(h) The OLS estimates in linear regression model would not be necessarily consistent if homoscedasticity assumption is violated.

(i) Consider $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$ model. If the null hypothesis of $\beta_1 = \beta_2 = 0$ is rejected then one can also reject the null hypothesis of $\beta_1 = 0$.

2. (25 points) Suppose

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U \ .$$

You, the researcher, wish to interpret this regression as a model of the determinants of Y.

(a) Assume that X_1 is endogenous, while X_2 is exogenous. Write down the conditions that pin down endogeneity or exogeneity. Regarding the fact that X_1 is endogenous, is it reasonable to assume that $\mathbb{E}[U] = 0$? Explain briefly.

(b) Suppose for the remainder of this question that you have access to two instruments for X_1, Z_1 and Z_2. Define instrument relevance and instrument validity conditions.

(c) Write down moment conditions that yield to consistent estimates for β_1 and β_2.

(d) Describe how you would test the null hypothesis $H_0 : \dfrac{\beta_1}{\beta_2} = 2$ versus the alternative that $H_1 : \dfrac{\beta_1}{\beta_2} \neq 2$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

(e) Describe how you would test the null hypothesis $H_0 : \beta_1 = 1 \ \textit{and} \ \beta_2 = 0$ versus the alternative $H_1 : \beta_1 \neq 1 \ \textit{or} \ \beta_2 \neq 0$ at the 5% significance level. In particular, describe your test statistic, your critical value, and the rule you would use to determine whether or not to reject the null hypothesis.

3. (20 points) You as a researcher have a dataset including wages, Y, experience in years, exp, years of schooling, edu, and a dummy variable, M, which takes 1 for individuals who work in manufacturing sector and 0 for individuals who work in other sectors of then economy. In the following parts, you are asked to propose a linear regression model besides a null hypothesis to formulate the hypothesis posited by the question. You do not need to describe how to run the test. (Hint: you may consider adding quadratic and/or interactive term into your regression)

   (a) Hypothesis 1: Workers in the manufacturing sector earn less (conditional on having the same level of education and experience).

(b) Hypothesis 2: The profile of experience in time is hump-shaped. That is, in early years of a career the wages grow as experience accumulates but it hits a maximum at the middle of the career and after that experience has a negative effect on wages.

(c) Hypothesis 3: The return of education (that is positive effect of education on wages) is lower in manufacturing sector relative to the other sectors holding experience fixed.

(d) Hypothesis 4: The positive effect of education on wages gradually disappears as workers get more experienced.

4. (10 points) Suppose that in the model,

$$Y = \beta_0 + \beta_1 X + u,$$

where the variable X is subject to measurement error, being underestimated by a fixed amount α in all observations.

(a) Discuss whether it is true that the OLS estimator of β_1 will be downward biased by an amount proportional to both α and β_1.

(Scratch work)

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