The Definition of the Limit

Recommended Reading: Section 2.2.

1. Let a, b, c be any real numbers. Write a formal proof that

$$\lim_{x \to c} ax + b = ac + b.$$

Hint: Remember that, for each $\epsilon > 0$, the corresponding value of δ must be positive.

2. Write a formal proof that

$$\lim_{x \to 1} (x^2 - 1) = 0$$

Continuity

Recommended Reading: Section 2.4, "Continuity at a Point" up to Example 3.

3. Let f be a real function. State what it means for f to be continuous at a point c, and what it means for f to be continuous on an open interval (a, b).

4. In Question (1), you proved that the function $f(x) = ax + b$ is continuous on the real line. On Question (11) of the previous homework, you proved that the function $g(x) = |x|$ is continuous on the real line. Show that the functions

$$F(x) = |ax + b| \qquad \text{and} \qquad G(x) = a|x| + b$$

are continuous on the real line. *Hint: Theorem 2.4.4.*

The Derivative

Recommended Reading: Chapter 3.

5. Using the definition of the derivative, compute the derivative of the function $f(x) = \dfrac{1}{x}$ at the point $x = c$.

6. Write a proof by induction of the following statement:

For any integer $n \geq 1$, the derivative of the function $f_n(x) = x^n$ is $f_n'(x) = n(x^{n-1})$.

You may assume that you know the derivative of $f(x) = x$ and the product rule, but *do not* assume you already know how to differentiate x^n.

Hint: For the inductive step, notice that $x^{k+1} = (x)(x^k)$, and use the product rule.

7. Let f and g be differentiable functions, and g never equal to 0. Give expressions for the following derivatives in terms of f, g, f', and g'.
   (a) $\left(\frac{1}{g}\right)'(x)$
   (b) $\left(\frac{1}{g^2}\right)'(x)$
   (c) $\left(\frac{1}{g \circ f}\right)'(x)$
   (d) $\left(\frac{1}{g} \circ f\right)'(x)$
8. **Section 3.3, Exercise 3, 7, 8, 19.**
9. **Section 3.5, Exercise 12, 15, 19.**
10. **Section 3.5, Exercise 34, 35, 36.**