STAT 24400 Statistics Theory and Methods I

Homework 2: Due 3:00PM Thurs January 21 2016 in class.

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1. Suppose that a traffic light alternates between green and red (let's ignore yellow) and stays green for 60 seconds and red for 30 seconds. You arrive at this light at a "random" time. Come up with a reasonable formula for the cdf of how long you have to wait at this light (assuming you obey traffic laws) and explain how you arrived at your answer. Sketch (accurately) your cdf.

2. Problem 12, Chapter 3: Let

$$f(x, y) = c(x^2 - y^2)e^{-x}, \qquad 0 \le x < \infty, \qquad -x \le y < x.$$

   (a) Find c.

   (b) Find the marginal densities.

   (c) Find the conditional densities.

3. Consider a two-ended laser spinner; that is a pen-like laser acting as the arrow mounted on a pin at the center of a spinner. Suppose the center of the disk is one meter away from a wall of infinite extent marked with a linear scale, with zero at the point closest to the center of the spinner and negative numbers to left, positive to the right. The laser is spun and comes to rest projecting for one of its ends at a point Y on the scale (with probability zero the laser will stop parallel to the wall and miss it; we ignore that possibility). Suppose that the angle X the laser makes to the perpendicular to the wall is uniformly distributed over $-\pi/2$ to $\pi/2$. Find the probability density of Y.

4. Suppose $X_1 \sim \text{Ber}(p_1)$, $X_2 \sim \text{Ber}(p_2)$ and X_1 and X_2 are independent.

   (a) Find the probability mass function for $X = X_1 + X_2$.

   (b) Suppose $Y \sim \text{Bin}(2, \frac{1}{2}(p_1 + p_2))$ Under what conditions is $P(X = 1) > P(Y = 1)$?

   (c) Professional basketball teams sometimes play "home-and-home" series in which two teams play consecutive games, one at the home stadium of one team and the next at the home stadium of the other team. Assuming that teams generally are more likely to win at home than when away from home, does part (b) have anything to say about the chances of each team winning one game of a "home-and-home" series?

5. Suppose U follows a uniform distribution on the interval $(0, 2\pi)$.

   (a) Find the density of $\sin U$.

   (b) Find the density of $\cos U$.

   (c) How do your answers to (a) and (b) compare? Does your result make sense? Explain.

   (d) Find the cumulative distribution function of $\sin^2 U + \cos^2 U$.

6. For X following a standard normal distribution, find $E(X^3)$ and $E(X^4)$. For $X \sim N(\mu, \sigma^2)$, find $E(X^3)$ and $E(X^4)$ using your answers for the standard normal and no further calculus.

7. Consider a Poisson process with parameter λ. Let X be the number of events in $(0,t_2]$ and Y the number of events in $(t_1,t_3]$ for $0 < t_1 < t_2 < t_3$ so that the intervals are guaranteed to overlap.

   (a) Find the mean and variance of $Y - X$.

   (b) Find $E(Y \mid X)$. Verify that $E\{E(Y \mid X)\}$ equals $E(Y)$.