STAT 24400 Statistics Theory and Methods I

Homework 4: Due 4:30PM Thurs, February 4, 2015. 107 Kent, at the Midterm

---

1. Let $Z \sim N(0,1)$, a standard normal distribution and let $X \sim N(\mu, \sigma^2)$. Let $\Phi(z)$ be the cdf of Z. Then probabilities $P(a < X < b) = P(X < b) - P(X < a)$ can be found for any a and b from any table of $\Phi(z)$ for $z > 0$ using $\Phi(z) = 1 - \Phi(-z)$ and the fact that $P(X < x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$. One such table is given in Rice, page A7. There are also many calulators and tables on the web; if you type 'normal cdf calculator' or 'normal cdf table' into google, you will have many choices. Suppose that $X \sim N(-4, 16)$; find
   (a) $P(X > 2)$
   (b) $P(0 < X < 4)$
   (c) $P(|X + 3| \geq 3)$
   (d) $P(X \leq 0 \text{ or } X \geq 3)$

2. Based on student A's performance during the first two weeks of a course, the professor has approximately a normal $N(70, 8^2)$ prior distribution about the student's true ability, on a scale of 0 to 100. Consider the midterm examination as an error-prone measure of the student's true ability, where if the true ability is x, the examination score can be modeled as approximately normally distributed, $N(x, 6^2)$. The student scores 90 on the midterm.
   (a) What are the posterior expectation and the probability that the student's true ability is above 85?
   (b) Above 90?

3. A "psychic" uses a five-card deck of cards to demonstrate psychic ability (ESP), and claims to be able to guess a card correctly with probability .5 (ordinary guessing would be right with probability $1/5 = .2$). A single experiment consists of making five guesses, reshuffling the deck after each guess. The experiment is tried and the "psychic" guesses correctly 3 times out of five. Assuming the only two possibilities are "ESP" and "ordinary guessing," how high must the a priori probability be that the "psychic" really has ESP, in order that the a posteriori probability that the "psychic" has ESP is at least .7?

4. Suppose $\hat{\theta}_1$ and $\hat{\theta}_2$ are uncorrelated and both are unbiased estimators of θ, and that $\text{Var}(\hat{\theta}_1) = 2\text{Var}(\hat{\theta}_2)$.
   (a) Show that for any constant c, the weighted average $\hat{\theta}_3 = c\hat{\theta}_1 + (1-c)\hat{\theta}_2$ is an unbiased estimator of θ.
   (b) Find the c for which $\hat{\theta}_3$ has the smallest MSE.
   (c) Are there any values of $c, 0 \leq c \leq 1$ for which $\hat{\theta}_3$ is better (in the sense of MSE) than both $\hat{\theta}_1$ and $\hat{\theta}_2$? Which?