The Definite Integral

Recommended Reading: Section 5.2, Subsection "The Integral as the Limit of Riemann Sums", Section 5.3.

1. **Section 5.3, Exercise 2 (a), (b), and (f).**

2. **Section 5.3, Exercise 6.**

3. Consider the function $F(x) = \int_0^x \frac{3+t}{14+2t^2}dt$ for x in the interval $[0, 5]$.
   (a) Find $F(0)$.
   (b) Find $F'(x)$.
   (c) Find the critical point(s) of F on $[0, 5]$. Indicate whether each is a local maximum, a local minimum, or neither.
   (d) Find $F''(x)$.
   (e) Find the inflection point(s) of F on $[0, 5]$. On which subintervals is F concave up? Concave down?
   (f) Using the results of the previous questions, sketch the graph of F on $[0, 5]$.

4. **Section 5.3, Exercise 31 (a) and (b).**

5. Let f be continuous on $[a, d]$, and let b and c be points in $[a, d]$.
$$\text{Define:} \qquad F(x) = \int_b^x f(t)dt \qquad \text{and} \qquad G(x) = \int_c^x f(t)dt.$$
   (a) Show that F and G differ by a constant. *Hint:* Consider Theorems 5.3.5 and 4.2.4.
   (b) What is $F(x) - G(x)$? *Hint:* Theorem 5.3.2.

6. **Section 5.4 Exercises 5, 8, 16, 23, 24.**

Even and Odd Functions

7. Suppose that f is a real function. What does it mean for f to be *even*? What does it mean for f to be *odd*?

8. Determine whether the following functions are even, odd, or neither by evaluating them at $-x$.
   (a) $f(x) = x(x^2 + 1)$
   (b) $f(x) = \dfrac{\sin(x)\cos(x)}{x^3} + 12$
   (c) $f(x) = \left(3x\cos(2x) - \sin^3(x)\right)^9$
   (d) $f(x) = \sin(x^2) + \sin(x)$

9. State whether each of the following functions is `even`, `odd` or `neither`. No justification needed.
   (a) $f(x) = \dfrac{x^2}{|x| + 3}$
   (b) $f(x) = \tan(x)$
   (c) $f(x) = \sin(x + \pi)$
   (d) $f(x) = \sin^2(x)$
   (e) $f(x) = \cos^3(x)$
   (f) $f(x) = \sin^2(x + 1)$
   (g) $f(x) = \sin^2(1 + x) - \sin^2(1 - x)$
   (h) $f(x) = \cos(\sin(x))$
   (i) $f(x) = h(x) + h(-x)$, where $h(x)$ is a real-valued function.
   (j) $f(x) = h(x) - h(-x)$, where $h(x)$ is a real-valued function.

10. In the following questions, let $g(x)$ be an even function, and let $h(x)$ be an odd function. State whether each of the following is `even`, `odd` or `neither`. No justification needed.
    (a) $f(x) = h(x)g(x)$
    (b) $f(x) = h(x) + g(x)$
    (c) $f(x) = -g(x)$
    (d) $f(x) = h(g(x))$
    (e) $f(x) = g(h(x))$