Review: Trigonometry

1. Read Section 1.6, "The Trigonometric Functions" up to the end of page 37.
   (a) In your own words, write definitions of the functions sine and cosine in terms of the unit circle. Illustrate the definitions with an appropriately labelled diagram of the unit circle. Draw the graphs of these functions on the interval $[-2\pi, 2\pi]$.
   (b) State the definition of tangent, cotangent, secant, and cosecant in terms of sine and cosine. You should remember these definitions!
   (c) Explain why $\sin^2(\theta) + \cos^2(\theta) = 1$. Illustrate your explanation with a diagram of the unit circle. Using the formulas for the tangent and secant functions, deduce that $\tan^2(\theta) + 1 = \sec^2(\theta)$.
   (d) Using diagrams in the style of Figures 1.6.9 and 1.6.10, explain why $\sin(\frac{1}{2}\pi - \theta) = \cos(\theta)$ and $\cos(\frac{1}{2}\pi - \theta) = \sin(\theta)$.
2. **Section 1.6, Exercise 60, 63, 64, 66, 68.**
3. Read Section 3.6, up through to the end of Example 3 on page 143.
   (a) Derive $\frac{d}{dx}(\cot(x))$ using the derivative of sine and cosine, and the quotient rule.
   (b) Referring to the diagram of the unit circle, and the graphs of $\sin(\theta)$ and $\cos(\theta)$ from Question 1 (a), observe that cosine is positive and decreasing on $[0, \pi]$, and that sine is positive and increasing on $[0, \pi]$. Explain how these observations can help us remember that the derivative of $\sin(x)$ is **positive** $\cos(x)$, whereas the derivative of $\cos(x)$ is **negative** $\sin(x)$.
4. **Section 3.6, Exercise 4, 6, 10, 11, 16.**
5. **Section 3.6, Exercise 48.**

Curve Sketching

Recommended Reading: Section 4.7, 4.8.

6. **Section 4.7, Exercises 3, 5, 7, 8, 14, 15.**
7. **Section 4.8, Exercise 13, 29, 34.** Sketch the graphs of these curves. Label any endpoints, vertical or horizontal asymptotes, x or y intercepts, critical points, extreme values, or inflection points.

The Definite Integral

Recommended Reading: Section 5.1, Section 5.2, up to (and including) Example 5.

8. Let f be a function continuous on an interval $[a, b]$.
   (a) Give the definition of a *partition* P of $[a, b]$.

(b) Given a partition $P = \{x_0, x_1, \ldots, x_n\}$ of $[a, b]$, define the upper sum $U_f(P)$ and lower sum $L_f(P)$ of f on $[a, b]$.

(c) Give the formal definition of the *definite integal* of f on $[a, b]$. (It is not enough to say "area under the curve"!).

9. Taking $f(x) = 1 - x^2$ on the interval $[-2, 2]$, compute $U_f(P)$ and $L_f(P)$ for the partition $P = \{-2, -1, \frac{1}{2}, 2\}$. Draw a graph of $f(x)$ and show how the upper sum can be interpreted as a measure of signed area by shading in the corresponding area on the graph (as in Figure 5.2.1). Do the same for the lower sum.

10. **Section 5.2, Exercise 12.**