The Mean Value Theorem, Extreme Values, and Curve Sketching

Recommended Reading: Chapter 4, Section 4.1 – 4.8.

1. Suppose that f is differentiable on the interval $(0, 2)$, and that $-1 \leq f'(x) \leq 2$ for all x in $(0, 2)$. If $f(0) = 0$, then what are the possible values of $f(2)$? *Hint:* Use the Mean Value Theorem.

2. Prove Theorem 4.2.2(iii), that is, prove that if f is a differentiable function on an open interval I, and that $f'(x) = 0$ for every x in I, then f is constant on I. *Hint:* Mean Value Theorem.

3. (a) Give the definition of a *critical point* of a function f.

   (b) Give the definition of a *local maximum* and *local minimum* of a function f.

   (c) Given a function f, summarize the procedure for finding local and absolute extreme values of f.

4. **Section 4.3, Exercise 10.**

5. **Section 4.4, Exercise 3, 12, 45.**

6. **Section 4.4, Exercise 39.** *Hint:* Mean Value Theorem.

7. **Section 4.5, Exercise 6, 11.**

8. For the following question, consult Section 4.6.

   (a) Let f be a differentiable function on an open interval I. Define what it means for f to be *concave up* or *concave down* on I. Illustrate the definition with the graph of an appropriate function.

   (b) Let f be continuous at a point c and differentiable near c. Define what it means for $(c, f(c))$ to be a *point of inflection.* If $(c, f(c))$ is an inflection point, then what must be true of $f''(c)$? Illustrate the definition with the graph of an appropriate function.

9. For the following question, consult Section 4.7.

   (a) Let f be a real valued function. What does it mean for f to have a *vertical asymptote*? Give an example of a function with a vertical asymptote and sketch its graph.

   (b) What does it mean for f to have a *horizontal asymptote*? Give an example of a function with a horizontal asymptote and sketch its graph.

10. (a) Read the seven step procedure for sketching curves given in Section 4.8.

    (b) **Section 4.8, Exercise 5, 21, 22.** Sketch the graphs of these curves. Label any endpoints, vertical or horizontal asymptotes, x or y intercepts, critical points, extreme values, or inflection points.