1 Introduction

This document is a short overview about the Modified Nodal Analysis. The M.N.A. is the method used in SPICE to obtain the circuit equation formulation. For more details, read the book «Circuit Simulation Methods and Algorithms by Jan Ogrodzki».

1.1 Notations

1. U is a tension, I is a current.
2. V denotes a node's potential.
3. q denotes a capacitor's charge.
4. ψ denotes a inductor's flux.
5. Indice $_a$ denotes the current branch.
6. Indice $_b$ denotes the other branch whose voltage is a controlling variable.
7. Indice $_c$ denotes the other branch whose current is a controlling variable.

Def 1 *The branch is current-defined if its currents is a function of its own voltage, controlling variable or their derivatives:*

$$I_a = F_i(U_a, U_b, I_c, \frac{dU_a}{dt}, \frac{dU_b}{dt}, \frac{dI_c}{dt}) \tag{1}$$

Examples :
A resistor is a current-defined branch because $I_a = \frac{U_a}{R}$.
A capacitor is a current-defined branch because $I_a = C\frac{dU_a}{dt}$.

Def 2 *The branch is voltage-defined if its voltage is a function of its own current, controlling variable or their derivatives:*

$$U_a = F_i(I_a, U_b, I_c, \frac{dU_a}{dt}, \frac{dU_b}{dt}, \frac{dI_c}{dt}) \tag{2}$$

Examples :
A resistor is a voltage-defined branch because $U_a = RI_a$.
A inductor is a voltage-defined branch because $U_a = L\frac{dI_a}{dt}$.

2 Hypothesis

The M.N.A. assumes smooth branches are explicit functions of current or voltage. It means each smooth branch is Voltage Defined (V.D.) or Current Defined (C.D.)

3 Unknowns

The M.N.A. use the following unknowns:

1. Nodal voltages

2. Currents in the V.D. branches

3. Capacitor's charges and currents
4. Inductor's flux and currents
5. Currents control

These unknowns are sufficient to describe the circuit.

4 Equations

The M.N.A. use following equations:

4.1 The Kirchhoff Current Law (KCL)

Kcl 1 *At any node in an electrical circuit where charge density is not changing in time, the sum of currents flowing towards that node is equal to the sum of currents flowing away from that node.*

KCL law gives this type of equation:
$I_1 + I_2 + ... + I_n = 0$
Current from current-defined branch is replaced with relation 1. The result is a linear relation between system's unknowns.

4.2 Law in voltage-defined branches (LVD)

It consists to replace U_a by $V_i - V_j$ in the relation 2 and we obtain a linear relation between system's unknowns.

$$V_i - V_j = F_i(I_a, U_b, I_c, \frac{dU_a}{dt}, \frac{dU_b}{dt}, \frac{dI_c}{dt})$$

4.3 Capacitor laws (CAP)

A relation between capacitor charge and tension (CAP1):

$$q_a = CU_a$$

Voltage definition (CAP2):

$$U_a = V_i - V_j$$

A dynamic relation (CAP3):

$$I_a = \frac{dq_a}{dt}$$

Use a time discretisation and these equations give tow linear relations between system's unknowns.

4.4 Inductor laws (IND)

A relation between inductor flux and current (IND1):

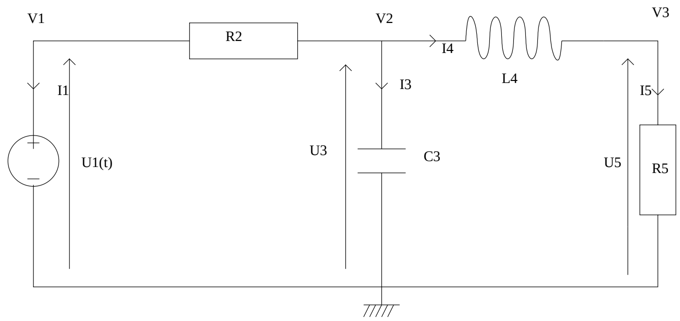
$$\psi_a = LI_a$$

A dynamic relation (IND2):

$$V_i - V_j = \frac{d\psi_a}{dt}$$

Use a time discretisation and these equations give tow linear relations between system's unknowns.

5 Example



5.1 Unknowns

1. Branch 1 is voltage defined.
2. Branch 2 is looked as current defined ($I_2 = \frac{U_2}{R}$).
3. Branch 3 is current defined.
4. Branch 4 is voltage defined.
5. Branch 5 is looked as current defined ($U_5 = RIU_5$).

Therefore the unknowns vector is:
(V1,V2,V3,I1,I3,I4,I5,U3,q3,ψ4)

5.2 Tableau Equation

$$
\left(\begin{array}{cccccccccccc}
 & V1 & V2 & V3 & I1 & I3 & I4 & I5 & U3 & q3 & \psi 4 & \\
\hline
(\ KCL1\ ) & \frac{-1}{R_2} & \frac{1}{R_2} & 0 & -1 & 0 & 0 & 0 & 0 & 0 & & \\
(\ KCL2\ ) & \frac{1}{R_2} & \frac{-1}{R_2} & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & \\
(\ KCL3\ ) & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & \\
(\ CAP1\ ) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_3 & -1 & 0 & \\
(\ IND1\ ) & 0 & 0 & 0 & 0 & 0 & L_4 & 0 & 0 & 0 & -1 & \\
(\ VDL5\ ) & 0 & 0 & \frac{-1}{R_5} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\
(\ VDL1\ ) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(\ CAP2\ ) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
(\ CAP3\ ) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{-1}{h} & 0 \\
(\ IND2\ ) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{h}
\end{array}\right)
=
\left(\begin{array}{c}
RSH \\
\hline
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ U_1(t) \\ 0 \\ \frac{-q_3(t-h)}{h} \\ \frac{-\psi_4(t-h)}{h}
\end{array}\right)
$$

6 Stamp method

The stamp method is an algorithmic method used to fill the tableau equation from the components. It consists to write a table for each type of component, this table is the contribution of the component in the tableau equation.
Following, this is stamp examples.

6.1 Resistor stamp

$$\left(\begin{array}{cccc} & V_i & V_j & RSH \\ \hline KCL(i) & \frac{-1}{R} & \frac{1}{R} & \\ KCL(j) & \frac{1}{R} & \frac{-1}{R} & \end{array}\right)$$

Where R is the branch's resistance.

6.2 Conductance stamp

$$\left(\begin{array}{ccccc} & V_i & V_j & I_a & RSH \\ \hline KCL(i) & & & 1 & \\ KCL(j) & & & -1 & \\ LVD & G & -G & 1 & \end{array}\right)$$

Where G is the branch's conductance.

6.3 Voltage source stamp

$$\left(\begin{array}{ccccc} & V_i & V_j & I_a & RSH \\ \hline KCL(i) & & & 1 & \\ KCL(j) & & & -1 & \\ LVD & 1 & -1 & & E \end{array}\right)$$

6.4 Current controlled voltage source stamp

$$\left(\begin{array}{cccccc} & V_i & V_j & I_a & I_b & RSH \\ \hline KCL(i) & & & 1 & & \\ KCL(j) & & & -1 & & \\ LVD & 1 & -1 & & \gamma & \end{array}\right)$$

With $U_a = \gamma I_b$.