The nonsmooth approach for the analog simulation of switched circuits

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Abstract. The numerical integration of switching circuits is known to be a tough issue when the number of switches is high, or when sliding modes exist. Then classical analog simulators may behave poorly, or even fail. In this paper it is shown on two examples that the nonsmooth approach, which is made of 1) a specific modelling of the multivalued piecewise-linear electronic devices (ideal diodes, Zener diodes, transistors), 2) the Moreau's time-stepping scheme, and 3) specific iterative one-step solvers, supersedes simulators of the SPICE family and hybrid simulators. An academic example constructed in [Maffezzoni et al, IEEE Trans. on CADICS, Vol 25, No 11, November 2006], so that the Newton-Raphson scheme does not converge, is used to make some comparisons between the nonsmooth method and other methods of the SPICE family and a hybrid-like method. The nonsmooth approach, implemented in the SICONOS platform developed at INRIA, proves to be on this example much faster and more robust with respect to the models parameters variations.

NOTATION AND DEFINITIONS.

The following tools will be used in this paper. Let $K \subseteq I\!R^n$ be a closed non empty convex set. The normal cone to K at $x \in I\!R^n$ is $N_K(x) = \{z \in I\!R^n | \langle z, \zeta - x\rangle \leqslant 0 \text{ for all } \zeta \in K\}$. The Mixed Complementarity Problem (MCP) [11]) is defined as follows. Given a function $\mathsf{F} : I\!R^p \to I\!R^p$, lower and upper bounds $l, u \in (I\!R \cup \{+\infty, -\infty\})^p$, find $z \in I\!R^p$, $w, v \in I\!R^p_+$, such that

$$\mathsf{F}(z) = w - vl \leqslant z \leqslant u, (z-l)^T w = 0, (u-z)^T v = 0,$$

which implies $-F(z) \in N_{[l,u]}(z)$. If the $\mathsf{F}(z) = Mz + q$ for some matrix $M \in I\!R^{p\times p}$ and some vector $q \in I\!R^m$, the MCP defines a Mixed Linear Complementarity Problem (MLCP).

I. INTRODUCTION

It is well know that conventional accurate analog simulation tools, which are based on the Newton–Raphson nonlinear solver, can have serious drawbacks when they are used for the integration of switched circuits, containing switches and piecewise linear components (like ideal diodes and transistors). Then analog (SPICE-like) tools may become very time consuming, or provide very poor results with chattering [13], or even fail [7], [8], [15], [16], [19]. The same applies to "hybrid" integrators that consider an exhaustive enumeration of all the system's modes, they have a very limited scope of application because of the exponential growth of the number of modes that have to be simulated separately. Along the same lines, event-driven schemes can hardly simulate systems with large number of events, because they soon become quite time-consuming and do not allow for accumulations of events [2]. The objective of this paper is to show on an academic example taken from [15] that the nonsmooth approach allows one to simulate a nonsmooth system for which conventional analog methods fail (roughly speaking, the iterative solver for complementarity problems converges, whereas Newton-Raphson's method does not). Compared to previous works [14], [18], the ideal switches are here modelled and simulated for the first time in a completely implicit way, the advantage of which will be explained. Finally, note that more details on the modeling and simulation within the nonsmooth approach and more complex examples such as the DC/DC buck converters can be found in a longer technical report [6] with thorough comparison with the commercial package ELDO[1] and PLECS[2].

II. THE NONSMOOTH APPROACH FOR SWITCHED CIRCUITS.

A. Modelling aspects of nonsmooth components

Let us illustrate the nonsmooth modeling on two examples (ideal diode, switch).

1) nonsmooth diodes: The notation for the currents and the potentials at the ports of the diode is depicted in Fig. 1. Four models of diodes are depicted in Fig. 2:

a)the smooth exponential Shockley model in Fig. 2(a) defined by the smooth constitutive equation,

$$i(t) = i_s \exp(-\frac{v(t)}{\alpha} - 1), \tag{1}$$

where i_s and α are physical parameters of the diode,

b) ideal diodes with possible residual current $-a$ and voltage $-b$ in Fig. 2(b), defined by the following complementarity condition

$$0 \leqslant i(t) + a \perp v(t) + b \geqslant 0, \tag{2}$$

where the $x \perp y$ means $x^T y = 0$ and a and b are the threshold values for i and v,

c) in Fig. 2(c), the "hybrid" model which considers the two modes separately with for instance an associated Modelica [12] script

$$\begin{array}{rcl} \mathsf{off} & = & \mathsf{s} < 0 \\ v(t) & = & \textbf{if} \;\; \mathsf{off} \;\; \textbf{then} \;\; -\mathsf{s} \;\; \textbf{else} \;\; 0 \\ i(t) & = & \textbf{if} \;\; \mathsf{off} \;\; \textbf{then} \;\; 0 \;\; \textbf{else} \;\; \mathsf{s}, \end{array} \tag{3}$$

d) a piecewise-linear model in Fig. 2(d) defined by

$$v(t) = \begin{cases} -R_{\mathsf{on}}\, i(t) & \text{if } v(t) < 0 \\ -R_{\mathsf{off}}\, i(t) & \text{if } v(t) \geqslant 0 \end{cases}, \tag{4}$$

[1] http://www.mentor.com/eldo
[2] http://www.plexim.com/

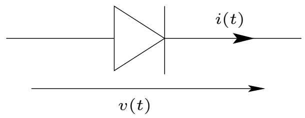


Fig. 1. Diode symbol.

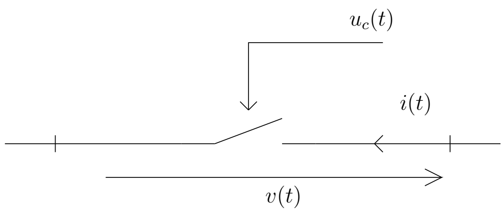


Fig. 3. Ideal switch symbol.

where $R_{\text{on}} \ll 1$ and $R_{\text{off}} \gg 1$ are the equivalent resistive values of each branches.

The set-valued ideal diode model in Fig. 2(b) is chosen in this paper. The drawbacks of the Shockley law is that it introduces high stiffness in the dynamical equations. The hybrid model becomes rapidly unusable if the number m of diodes increases, since the number of modes to be described in the associated script varies as 2^m. The model 2(d) leads to badly conditioned complementarity problems. On the contrary the ideal model of Fig. 2(b) leads, when introduced in the dynamics, to well-conditioned problems, and yields time–stepping methods for which efficient solvers exist.

2) nonsmooth switches: Depicted on Fig. 3, the switch is defined by

$$v(t) = \begin{cases} R_{\text{off}}\, i(t) & \text{if } u_c(t) < 0 \\ R_{\text{on}}\, i(t) & \text{if } u_c(t) \geqslant 0 \end{cases} \tag{5}$$

where the voltage $u_c(\cdot)$ is a state variable of the overall dynamical system, $v(\cdot)$ is the voltage of the switch and $i(\cdot)$ is the current through the switch. The resistors $R_{\text{off}} \gg 1$ and $R_{\text{on}} \ll 1$ are chosen by the designer. The switch in (5) is modeled as follows:

$$\begin{cases} v(t) = \frac{1}{2}(1+\tau(t))R_{\text{on}} i(t) + \frac{1}{2}(1-\tau(t))R_{\text{off}} i(t) \\ \\ \tau(t) \in \operatorname{sgn}(u_c(t)) \;\Leftrightarrow\; u_c(t) \in -N_{[-1,1]}(\tau(t)) \end{cases} \tag{6}$$

It is noteworthy that the voltage $v(t)$ in (5) is discontinuous at $u_c(t) = 0$ for any $i(t) \neq 0$, the jump magnitude being equal to $|(R_{\text{off}} - R_{\text{on}})i(t)|$. The choice that is made in (6) implies that the discontinuities are "filled-in" and the model is consequently multivalued at $u_c(t) = 0$, $i(t) \neq 0$. This is precisely what allows one to smoothly simulate the sliding-modes [3]. Let us conclude this section by introducing a generic form of nonsmooth components as

$$0 \in y + N_K(\lambda), \tag{7}$$

where y and λ plays the role of slackness (non necessary physical) variables for describing the complete multi–valued behavior of the component. More complex examples and more generic formulations of nonsmooth components can be found in [6].

B. The dynamical equations and their automatic generation

There are a lot of methods to build a smooth DAE formulation of standard electrical circuits. To cite a few of them, the Sparse Tableau Analysis (STA) and the modified Nodal Analysis (MNA) are the most widespread. An automatic circuit equation generation system extending the MNA has been developed at the INRIA (see the patent [1]) yielding to the suitable following formulation

$$\left\{ \begin{array}{lll} \dot{x} & = f_1(x,z,t) + U(t) & \text{Semi-Explicit DAE} \\ 0 & = f_2(x,z,t) & \\ 0 & = h(x,z,\lambda,t) & \text{Input/output relations on} \\ y & = g(x,z,\lambda,t) & \text{nonsmooth components} \\ & 0 \in y + N_K(\lambda) & \text{"Inclusion rule"} \end{array} \right. \tag{8}$$

where $x \in \mathbb{R}^n$ corresponds to the current in the inductive branches and the voltages in the capacitive branches, $z \in \mathbb{R}^p$ collects all the node potentials, the currents in the voltage–defined and nonsmooth branches and the currents in a subset of the capacitive branches. The choice and the construction of the latter subset of branches is described in details in [1]. Starting from (8), the numerical time–discretization is an extension of the Moreau's time stepping scheme [17] which is given by,

$$\begin{cases} x_{k+1} - x_k = h f_1(x_{k+\theta}, z_{k+\theta}, t_{k+1}) + hU(t_{k+\theta}) \\ 0 = f_2(x_{k+1}, z_{k+1}, t_{k+1}) \\ 0 = h(x_{k+1}, z_{k+1}, \lambda_{k+1}, t_{k+1}), \\ y = g(x_{k+1}, z_{k+1}, \lambda_{k+1}, t_{k+1}) \\ 0 \in y_{k+1} + N_K(\lambda_{k+1}) \end{cases} . \tag{9}$$

where the notation $x_{k+\theta} = \theta x_{k+1} + (1-\theta)x_k, \theta \in [0,1]$.

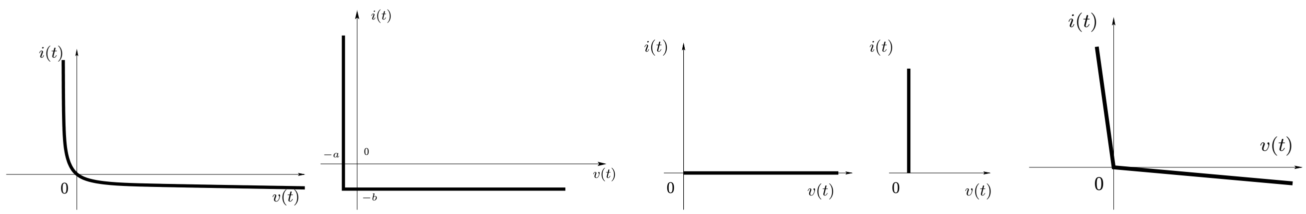
C. Software aspects

From a SPICE Netlist, the automatic generator builds all the components defined in (8). The opensource SICONOS/KERNEL library performs the time-discretization following the Moreau time–stepping scheme (9) and formulates at each time–step one instance of the associated MCP. The numerical algorithms for the latter problem are in the opensource SICONOS/NUMERICS library. The output of the simulation is a file containing the potential and current values in the SPICE format. The implementation is object-oriented and mainly in C++. The open-source platform SICONOS[3] [2], [4], [5] is under GPL license and can be freely used. The equation generator is under private license and can be obtained freely on demand for an academic use.

III. AN ELEMENTARY SWITCHING CIRCUIT

This section is devoted to the modelling and the simulation of the circuit in Fig. 4. In [15] it is shown that Newton-Raphson based methods fail to converge on such a circuit, with

[3] http://siconos.gforge.inria.fr/



(a) smooth modeling (b) nonsmooth modeling (c) hybrid modelling (d) equivalent resistor model

Fig. 2. Four models of diodes.

the switch model as in (5). The diode model is the equivalent resistor model of Fig. 2 (d). On the contrary the one–step nonsmooth solver correctly behaves on the same model.

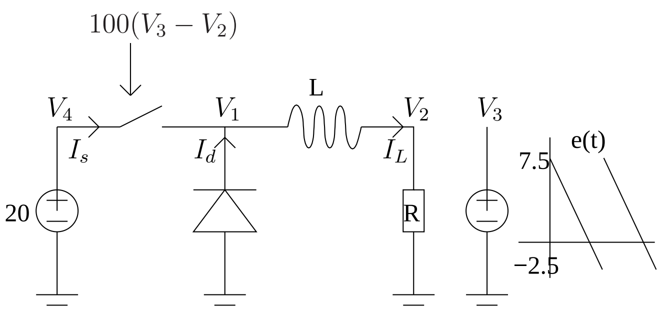


Fig. 4. A simple switched circuit.

A. The dynamical system

Applying the automatic equations generation algorithm to the circuit in Fig. 4 leads to the following 9-dimensional state vector: $X = (V_1\ V_2\ V_3\ V_4\ I_L\ I_{03}\ I_{04}\ I_s\ I_d)^T$, where the node voltages and the currents are depicted on Fig. 4. The two nonsmooth devices are the diode and the switch. It yields the following system, that fits within the general framework in (8): for the semi–explicit DAE, we obtain

$$\left\{\begin{array}{ll} L\frac{dI_L}{dt}(t) = V_1(t) - V_2(t) & I_d(t) + I_s(t) - I_L(t) = 0 \\ I_L(t) - \frac{V_2(t)}{R} = 0 & I_{03}(t) = 0 \\ I_{04}(t) - I_s(t) = 0 & V_4(t) = 20, V_3 = e(t) \end{array}\right. . \tag{10}$$

For the input/output relations on nonsmooth components, we get

$$\left\{\begin{array}{l} V_1(t) = \frac{1}{2}(\tau_1(t) - 1)R_{\text{off}}I_d(t) - \frac{1}{2}(\tau_1(t) + 1)R_{\text{on}}I_d(t) \\ V_1(t) - V_4(t) = \dfrac{1}{2}(1 + \tau_2(t))R_{\text{off}}I_s(t) \\ \qquad\qquad\qquad + \dfrac{1}{2}(1 - \tau_2(t))R_{\text{on}}I_s(t) \end{array}\right. . \tag{11}$$

Finally, the inclusion rule is written as

$$\left\{\begin{array}{l} V_1(t) \in -N_{[-1,1]}(\tau_1(t)) \\ 100(V_3(t) - V_2(t)) \in -N_{[-1,1]}(\tau_2(t)) \end{array}\right. . \tag{12}$$

B. Numerical results with SICONOS

The time step has been fixed to $0.1\mu s$. Fig. 5(a) depicts the current evolution through the inductor L. In [15], it has been shown that the Newton-Raphson algorithm fails when the state of the diode and of the switch changes at $t = t_s$ (see Fig. 5(a)). Indeed, the linearization performed at each Newton-Raphson iteration leads to an oscillation between two incorrect states and never converges to the correct one. The Newton-Raphson iterations enter into a infinite loop without converging. In the nonsmooth framework, The PATH solver [11] for MCP or a semi-smooth Newton method [9] converges and computes the correct state. In [15] an event-driven numerical method is proposed to solve the non convergence issue. However it is reliable only if the switching times can be precisely estimated, a shortcoming not encountered with the Moreau's time-stepping method.

C. Numerical results with ELDO

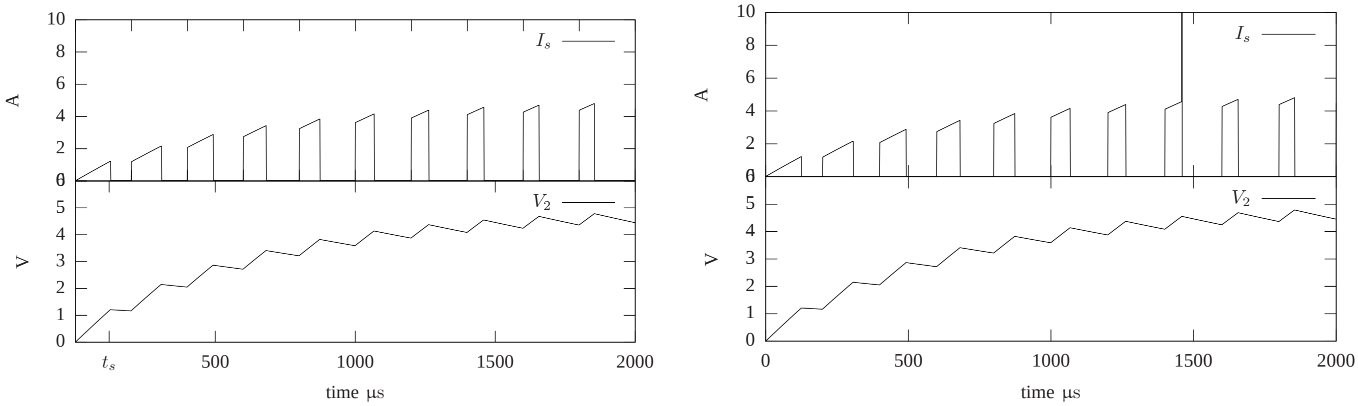
ELDO does not provide any nonsmooth switch model. But it furnishes the 'VSWITCH' one described in (13), where R_S is the controlled resistor value of the switch, and V_C the voltage control. Setting V_{off} to 0, and choosing a small value for V_{on} lead to: $R_S(t) =$

$$\left\{\begin{array}{l} R_{\text{on}} \text{ if } V_C(t) \geqslant V_{\text{on}}, \quad R_{\text{off}} \text{ if } V_C(t) \leqslant V_{off} \\ (V_C(t)(R_{\text{off}} - R_{\text{on}}) + R_{\text{on}} V_{\text{off}} - R_{\text{off}} V_{\text{on}})/(V_{\text{off}} - V_{\text{on}}) \quad \text{otherwise} \end{array}\right. \tag{13}$$

which is close to (5) for the chosen parameters. Simulations have been done using different sets of parameters. It is noteworthy that the behavior of ELDO depends on these values. For example, using a Backward Euler with the time step fixed to $0.1\mu s$ and $V_{on} = 1e - 4$V, $V_{off} = 0$V, $R_{\text{off}} = 1000\Omega$, $R_{\text{on}} = 0.001\Omega$ cause troubles during the ELDO simulation: some messages like 'Newton no-convergence' appear. Fig. 5(b) shows the ELDO simulation. The values are very close to the SICONOS simulation, except for the steps corresponding to the no-convergence messages. In this case, the resulting current value is absurd.

IV. CONCLUSIONS

In this paper it is shown that Moreau's time-stepping scheme allows one to integrate an academic example on which Newton-Raphson based methods fail. Comparisons with other analog simulators are presented (see [6] for further comparisons with PLECS). This academic example demonstrates that



(a) SICONOS simulation (b) ELDO simulation

Fig. 5. Switched circuit simulations.

analog tools can fail to simulate a switched circuit. Due to length restrictions, only few of the many advantages of the nonsmooth approach have been illustrated. More generally (see [6]), it allows one to:

- avoid computing the dynamics changes due to topology variations, since the circuits are treated as a global system with a fixed state dimension; modes transitions are taken care of by the complementarity problem solvers, which usually are polynomial in time;
- simulate circuits with very large number of events without slowing down too much the simulation;
- avoid regularization and consequently stiff systems of ODEs;
- accurately calculate the initial steady-state of the system;
- accurately simulate sliding mode trajectories without spurious oscillations around the switching surface;
- compute state jumps (initial jumps due to inconsistent states, or in the course of the integration).

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