COP 5536 Advanced Data Structures

University of Florida

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Exam 1 Solution

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Question 1

a) Implement a QUEUE with two STACKs having constant amortized cost for each QUEUE operation (6 points).

Name the two STACKs as $Stack_1$ and $Stack_2$, we can implement the QUEUE as follows:

- ENQUEUE(x): PUSH x into $Stack_1$
- DEQUEUE(x): If $Stack_2$ is not empty, then simply POP from $Stack_2$ and return the element. If $Stack_2$ is empty, POP all the elements of $Stack_1$, PUSH them into $Stack_2$, then POP from $Stack_2$ and return the result.

b) Choose any two from the three methods to prove the amortized cost for each QUEUE operation is O(1) (4 points each).

- Aggregate method

Consider a sequence of n operations. The sequence of operations will involve at most n elements. The cost associated with each element will be at most 4 i.e. (pushed into $Stack_1$, popped from $Stack_1$, pushed to $Stack_2$, and popped from $Stack_2$). Hence, the actual cost of n operations will be upper bounded by T(n)= 4 n. Hence, the amortized cost of each operation can be T(n)/n = 4n / n = 4 = O(1).

- Accounting method

We guess that the amortized costs for ENQUEUE and DEQUEUE are 3 and 1. We show that the potential function P(n) satisfies P(n) - P(0) >= 0 for all n.

We have P(0) = 0. If an element is not popped, then it's only pushed twice and popped once. Thus, the cost of 3 is paid for by ENQUEUE operation. The cost for last pop operation is paid for by the DEQUEUE.

Note: Alternatively, we can set the costs for ENQUEUE and DEQUEUE as 4 and 0 respectively.