

SVD Component Mortality Model

Reproducibility Materials: Data and Code

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1 Preliminaries

This R Markdown document was created in RStudio using the Knit Directory = *Document Directory* setting. The code assumes it will execute in the document directory, and depending on how you run this, you may have to set that manually, just above.

This R Markdown document was prepared using

- R version 3.5.1 (2018-07-02) – “Feather Spray”
- RStudio version 1.1.456
- Additional R packages from CRAN
 - knitr
 - bookdown
 - formatR
 - devtools
 - reshape2
 - ggplot2
 - plyr
 - stargazer
 - xtable

- readr
- stringr
- httr

This document is distributed in R Markdown .Rmd and PDF .pdf documents. First load the necessary packages.

2 HMD Data

2.1 Download and parse HMD

First clear everything out, including directories where results will be stored.

```
rm(list = ls()) # clear R environment
system("rm -r ../data/HMD/*") # remove contents of HMD data directory
system("rm -r ../RData/*") # remove contents of Rdata directory
system("rm -r ../figures/*") # remove contents of figures directory
system("rm -r ../tables/*") # remove contents of figures directory
```

The data come from the Human Mortality Database (HMD) and are available online at www.mortality.org - [click here](#). The file *getHMD.R* contains a set of functions that automate downloading and parsing all of the HMD life tables. This code is not commented, but you can get the general gist of what's going on by reading it. In a nutshell, the HMD .zip file is downloaded and unzipped (if the HMD web site isn't working, a pre-downloaded version of the file from 2018-11-02 is used), and then all life tables of a specified type are read into a big R list that is very fast and flexible. Functions are provided for extracting a specified column from all life tables in such a list and outputting the result as a matrix. The following chunk sources the functions for automating downloading and processing of the HMD life tables.

```
# source('../R/getHMD.R') # load the 'read HMD'
# functions

# uncommented code to download the HMD Statistics file
# and organize the life tables into an
# easy-to-manipulate list
list.raw.lts <- function(hmd.dir, age, period) {

  lts <- list(female = read.raw.lt(hmd.dir, "female",
    age, period), male = read.raw.lt(hmd.dir, "male",
    age, period), both = read.raw.lt(hmd.dir, "both",
    age, period))

  return(lts)
}

read.raw.lt <- function(hmd.dir, sex, age, period) {

  # <hmd.dir> must contain a string that specifies the
  # location of the 'hmd_statistics' directory created
  # when the HMD .zip file is unzipped.

  switch <- sex.per.age.switch(sex, age, period, hmd.dir)
  data.dir <- paste(hmd.dir, switch$data.path, sep = "")
```

```

lt <- list()

files <- Sys.glob(paste(eval(data.dir), "/*.txt", sep = ""))
files.split <- strsplit(files, "\\.")

for (i in 1:length(files.split)) {
  country.name <- strsplit(basename(files[i]), "\\.")[[1]][1]
  lt[[country.name]] <- parse.lt(files[i], age)
}

return(lt)
}

sex.per.age.switch <- function(sex, age, period, root.dir) {

  subtract <- 0
  for (i in 1:3) {
    if (str_sub(root.dir, 1, 1) == "." | str_sub(root.dir,
    1, 1) == "/") {
      subtract <- subtract + 1
    }
  }
  root.length <- str_length(root.dir) - subtract

  if (sex == "female") {

    if (age == 1) {
      if (period == 1) {
        path <- "/lt_female/fltpers_1x1"
        value <- list(sap.code = 1, data.path = path)
      } else if (period == 5) {
        path <- "/lt_female/fltpers_1x5"
        value <- list(sap.code = 2, data.path = path)
      } else if (period == 10) {
        path <- "/lt_female/fltpers_1x10"
        value <- list(sap.code = 3, data.path = path)
      } else {
        value <- list(sap.code = 0, data.path = "")
      }
    } else if (age == 5) {
      if (period == 1) {
        path <- "/lt_female/fltpers_5x1"
        value <- list(sap.code = 4, data.path = path)
      } else if (period == 5) {
        path <- "/lt_female/fltpers_5x5"
        value <- list(sap.code = 5, data.path = path)
      } else if (period == 10) {
        path <- "/lt_female/fltpers_5x10"
        value <- list(sap.code = 6, data.path = path)
      } else {
        value <- list(sap.code = 0, data.path = "")
      }
    }
  }
}

```

```

        }
    } else {
        value <- list(sap.code = 0, data.path = "")
    }

} else if (sex == "male") {

    if (age == 1) {
        if (period == 1) {
            path <- "/lt_male/mltper_1x1"
            value <- list(sap.code = 7, data.path = path)
        } else if (period == 5) {
            path <- "/lt_male/mltper_1x5"
            value <- list(sap.code = 8, data.path = path)
        } else if (period == 10) {
            path <- "/lt_male/mltper_1x10"
            value <- list(sap.code = 9, data.path = path)
        } else {
            value <- list(sap.code = 0, data.path = "")
        }
    } else if (age == 5) {
        if (period == 1) {
            path <- "/lt_male/mltper_5x1"
            value <- list(sap.code = 10, data.path = path)
        } else if (period == 5) {
            path <- "/lt_male/mltper_5x5"
            value <- list(sap.code = 11, data.path = path)
        } else if (period == 10) {
            path <- "/lt_male/mltper_5x10"
            value <- list(sap.code = 12, data.path = path)
        } else {
            value <- list(sap.code = 0, data.path = "")
        }
    } else {
        value <- list(sap.code = 0, data.path = "")
    }
}

} else if (sex == "both") {

    if (age == 1) {
        if (period == 1) {
            path <- "/lt_both/bltper_1x1"
            value <- list(sap.code = 7, data.path = path)
        } else if (period == 5) {
            path <- "/lt_both/bltper_1x5"
            value <- list(sap.code = 8, data.path = path)
        } else if (period == 10) {
            path <- "/lt_both/bltper_1x10"
            value <- list(sap.code = 9, data.path = path)
        } else {
            value <- list(sap.code = 0, data.path = "")
        }
    } else if (age == 5) {

```

```

        if (period == 1) {
            path <- "/lt_both/bltper_5x1"
            value <- list(sap.code = 10, data.path = path)
        } else if (period == 5) {
            path <- "/lt_both/bltper_5x5"
            value <- list(sap.code = 11, data.path = path)
        } else if (period == 10) {
            path <- "/lt_both/bltper_5x10"
            value <- list(sap.code = 12, data.path = path)
        } else {
            value <- list(sap.code = 0, data.path = "")
        }
    } else {
        value <- list(sap.code = 0, data.path = "")
    }

    return(value)
}

parse.lt <- function(file.name, age) {

    if (age == 1) {
        w <- c(9, 11, 11, 9, 6, 8, 8, 8, 9, NA)
    } else if (age == 5) {
        w <- c(9, 11, 11, 9, 6, 8, 8, 8, 9, NA)
    } else {
        w <- NA
    }

    return(read_fwf(file = file.name, na = c("", "NA"),
        skip = 3, col_types = "ccnnnnnnnn", fwf_widths(widths = w,
        col_names = c("period", "age", "mx", "qx", "ax",
        "lx", "dx", "Lx", "Tx", "ex"))))
}

list.lts <- function(hmd.dir, age, period, download.date) {

    list.raw.lts <- list.raw.lts(hmd.dir, age, period)

    if (age == 1) {
        ages <- 111
    } else if (age == 5) {
        ages <- 24
    } else {
        ages <- NA
    }
}

```

```

}

lt.list <- list()
lt.list[["creation.date"]] <- date()
lt.list[["download.date"]] <- download.date
lt.list[["female"]] <- list()
lt.list[["male"]] <- list()
lt.list[["both"]] <- list()
lt.list[["age"]] <- age
lt.list[["age.groups"]] <- ages
lt.list[["period"]] <- period

for (sx in c("female", "male", "both")) {
  for (i in 1:length(list.raw.lts[[sx]])) {
    lt.list[[sx]][[eval(names(list.raw.lts[[sx]][i))]]] <- list()
    for (j in 1:(dim(list.raw.lts[[sx]][[i]])[1]/ages)) {
      pop <- eval(names(list.raw.lts[[sx]][[i]]))
      per <- unlist(list.raw.lts[[sx]][[i]][(ages *
        j - (ages - 1)), 1])
      per <- paste("P", str_replace_all(per, "[ - ]",
        "to"), sep = ""))
      lt.list[[sx]][[pop]][[per]] <- list.raw.lts[[sx]][[i]][((ages *
        j - (ages - 1)): (ages * j)), ]
    }
  }
}

return(lt.list)
}

count.lts <- function(lt.list, sex) {

  lt.cumsum <- 0
  for (i in 1:length(lt.list[[sex]])) {
    lt.cumsum <- lt.cumsum + length(lt.list[[sex]][[i]])
  }

  return(lt.cumsum)
}

extract.lt.col <- function(lt.list, sex, col.name) {

  if (lt.list$age == 1) {
    ages <- 111
  } else if (lt.list$age == 5) {
    ages <- 24
  } else {
    ages <- NA
  }
}

```

```

if (col.name == "period") {
  col <- 1
} else if (col.name == "age") {
  col <- 2
} else if (col.name == "mx") {
  col <- 3
} else if (col.name == "qx") {
  col <- 4
} else if (col.name == "ax") {
  col <- 5
} else if (col.name == "lx") {
  col <- 6
} else if (col.name == "dx") {
  col <- 7
} else if (col.name == "Lx") {
  col <- 8
} else if (col.name == "Tx") {
  col <- 9
} else if (col.name == "ex") {
  col <- 10
} else {
  col <- NA
}

lts.count <- count.lts(lt.list, "female")

if (col > 1) {
  lts.colmat <- matrix(data = rep(0, ages * lts.count),
                        nrow = ages, ncol = lts.count)
} else if (col == 1) {
  lts.colmat <- matrix(data = rep("", ages * lts.count),
                        nrow = ages, ncol = lts.count)
}

col.names <- rep("", lts.count)
col.index <- 1

for (i in 1:length(lt.list[[sex]])) {
  for (j in 1:length(lt.list[[sex]][[i]])) {
    if (col > 1) {
      lts.colmat[, col.index] <- as.numeric(unlist(lt.list[[sex]][[i]][[j]][,
      col]))
    } else if (col == 1) {
      lts.colmat[, col.index] <- as.character(unlist(lt.list[[sex]][[i]][[j]][,
      col]))
    }
    pop <- names(lt.list[[sex]][i])
    per <- str_sub(names(lt.list[[sex]][[i]]), j,
                  2, str_length(names(lt.list[[sex]][[i]])[j]))
    col.names[col.index] <- paste(eval(sex), ".",
                                  pop, ".", per, sep = "")
    col.index <- col.index + 1
  }
}

```

```

        }

    colnames(lts.colmat) <- col.names
    rownames(lts.colmat) <- unlist(lt.list$female[[1]][[1]][,
      2])

    return(lts.colmat)

}

download.hmd <- function(output.file, unzip.dir, hmd.user,
  hmd.pass) {

  url <- "http://www.mortality.org/hmd/zip/all_hmd/hmd_statistics.zip"
  print("Downloading HMD ...")
  hmd.zip <- try(GET(url, authenticate(hmd.user, hmd.pass),
    write_disk(output.file, overwrite = TRUE), progress(),
    show.error.messages = FALSE))
  if (class(hmd.zip) == "try-error") {
    return(hmd.zip)
    stop(attributes(hmd.zip)$condition)
  } else if (http_status(hmd.zip)$category == "Client error") {
    if (http_status(hmd.zip)$message == "Client error: (401) Unauthorized") {
      return(hmd.zip)
      stop("Check user name and password and try again.")
    } else {
      return(hmd.zip)
      stop(http_status(hmd.zip)$message)
    }
  }
  unzip(zipfile = output.file, exdir = unzip.dir)
  return(hmd.zip)
}

}

```

The following chunk uses the download.hmd() function to download the HMD life tables. To run you need to insert your own HMD user name and password. The data are unzipped into the *data/HMD/hmd_statistics* directory that needs to exist before downloading.

```

# try the download
download.result <- download.hmd(
  output.file = "../data/HMD/hmd_statistics.zip",
  unzip.dir = "../data/HMD/hmd_statistics",
#####
##### IMPORTANT #####
#####
#   with following two lines commented, this script will use      #
#     archived HMD and replicate the article                      #
#   uncomment following two lines to download and analyse       #
#     current HMD                                         #
#####
# hmd.user="sam@samclark.net",
# hmd.pass="1241662754"

```

```

)
# if download.hmd() returns an error, then use the local cached copy of the HMD
if (class(download.result)=="try-error") {
  print(paste("Something went wrong -- usually this means the HMD site is not available.",
             " Using local cached version of HMD instead of a live download.",sep=""))
  unzip(zipfile="../data/HMD Archive/hmd_statistics.zip",exdir="../data/HMD/hmd_statistics")
}

```

The `download.result` object contains a description of what happened when the download was requested. You can have a look with this code.

```

if (class(download.result) != "try-error") {
  names(download.result)
  download.result
  download.result$date
  download.result$times
  download.result$headers
} else {
  print("Download did not happen, using local cached version of HMD.")
}

```

HMD nomenclature describes $age \times period$ life tables. For example 1×1 are single calendar year by single year of age, and 5×5 are five-year age groups by five-year periods, with the first age group broken into 0 and 1–4 years. The following code creates an R list for each of various commonly used life tables. The resulting lists are saved in the `RData` directory in compressed form. Unless it has been fixed between when I write this and when you execute it, the following code will produce errors for some of the Belarus files – those files do not contain data. Finally, if you use the HMD download functions on their own, make sure not to prepend ‘/’ or ‘./’ to the path for the `hmd_statistics` directory. Several errors will appear; these are due to several HMD life tables not having any values, at this time all from Belarus.

```

# set the download date if the download was successful
if (class(download.result) != "try-error") {
  download.date <- download.result$headers$date
} else {
  download.date <- "Local cached HMD"
}

# 1-year age x 1-year period life tables
hmd.1x1.list <- list.lts("../data/HMD/hmd_statistics", 1,
                           1, download.date)
# arguments are path to 'hmd_statistics' directory age
# designator: either 1 or 5 year age groups period
# designator: either 1, 5, or 10 year age groups
save(file = "../RData/hmd-1x1.RData", compress = TRUE, list = c("hmd.1x1.list"))
# hmd.1x5.list <-
# list.lts('data/HMD/hmd_statistics', 1, 5, download.result$headers$date)
# save(file='../RData/hmd-1x5.RData',compress=TRUE,list=c('hmd.1x5.list'))
# hmd.1x10.list <-
# list.lts('data/HMD/hmd_statistics', 1, 10, download.result$headers$date)
# save(file='../RData/hmd-1x10.RData',compress=TRUE,list=c('hmd.1x10.list'))

# 5-year age x 1-year life tables
hmd.5x1.list <- list.lts("../data/HMD/hmd_statistics", 5,
                           1, download.date)
save(file = "../RData/hmd-5x1.RData", compress = TRUE, list = c("hmd.5x1.list"))

```

```

# hmd.5x5.list <-
# list.lts('data/HMD/hmd_statistics', 5, 5, download.result$headers$date)
# save(file='../../RData/hmd-5x5.RData', compress=TRUE, list=c('hmd.5x5.list'))
# hmd.5x10.list <-
# list.lts('data/HMD/hmd_statistics', 5, 10, download.result$headers$date)
# save(file='../../RData/hmd-5x10.RData', compress=TRUE, list=c('hmd.5x10.list'))

# rm(list=c('download.date', 'download.result'))

```

Have quick look at the lists saved in *Rdata*.

```
list.files("../RData/")
```

```
## [1] "hmd-1x1.RData" "hmd-5x1.RData"
```

Have a look at the top-level structure of the list.

```
str(hmd.5x1.list, max.level = 1)
```

```

## List of 8
## $ creation.date: chr "Tue Apr  9 21:18:24 2019"
## $ download.date: chr "Local cached HMD"
## $ female      :List of 49
## $ male        :List of 49
## $ both        :List of 49
## $ age         : num 5
## $ age.groups  : num 24
## $ period      : num 1

```

Have a quick look at the full structure of the lists, vastly truncated!

```
str(hmd.5x1.list, list.len = 4, vec.len = 2)
```

```

## List of 8
## $ creation.date: chr "Tue Apr  9 21:18:24 2019"
## $ download.date: chr "Local cached HMD"
## $ female      :List of 49
##   ..$ AUS    :List of 94
##     ...$ P1921:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of  10 variables:
##       ...$ period: chr [1:24] "1921" "1921" ...
##       ...$ age   : chr [1:24] "0" "1-4" ...
##       ...$ mx    : num [1:24] 0.05999 0.00602 ...
##       ...$ qx    : num [1:24] 0.0575 0.0237 0.00952 0.00637 0.0102 ...
##     ... [list output truncated]
##     ...$ P1922:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of  10 variables:
##       ...$ period: chr [1:24] "1922" "1922" ...
##       ...$ age   : chr [1:24] "0" "1-4" ...
##       ...$ mx    : num [1:24] 0.04594 0.00449 ...
##       ...$ qx    : num [1:24] 0.0444 0.0178 ...
##     ... [list output truncated]
##     ...$ P1923:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of  10 variables:
##       ...$ period: chr [1:24] "1923" "1923" ...
##       ...$ age   : chr [1:24] "0" "1-4" ...
##       ...$ mx    : num [1:24] 0.05565 0.00478 ...
##       ...$ qx    : num [1:24] 0.0535 0.0189 ...
##     ... [list output truncated]
##     ...$ P1924:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of  10 variables:

```

```

## ... .$. period: chr [1:24] "1924" "1924" ...
## ... .$. age : chr [1:24] "0" "1-4" ...
## ... .$. mx : num [1:24] 0.05379 0.00485 ...
## ... .$. qx : num [1:24] 0.0517 0.0191 ...
## ... [list output truncated]
## ... [list output truncated]
## ..$ AUT :List of 71
## ... $. P1947:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1947" "1947" ...
## ... .$. age : chr [1:24] "0" "1-4" ...
## ... .$. mx : num [1:24] 0.07981 0.00414 ...
## ... .$. qx : num [1:24] 0.0757 0.0164 ...
## ... [list output truncated]
## ... $. P1948:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1948" "1948" ...
## ... .$. age : chr [1:24] "0" "1-4" ...
## ... .$. mx : num [1:24] 0.07327 0.00316 ...
## ... .$. qx : num [1:24] 0.0698 0.0125 ...
## ... [list output truncated]
## ... $. P1949:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1949" "1949" ...
## ... .$. age : chr [1:24] "0" "1-4" ...
## ... .$. mx : num [1:24] 0.06717 0.00347 ...
## ... .$. qx : num [1:24] 0.0642 0.0138 ...
## ... [list output truncated]
## ... $. P1950:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1950" "1950" ...
## ... .$. age : chr [1:24] "0" "1-4" ...
## ... .$. mx : num [1:24] 0.05953 0.00274 ...
## ... .$. qx : num [1:24] 0.0571 0.0109 ...
## ... [list output truncated]
## ... [list output truncated]
## ..$ BEL :List of 175
## ... $. P1841:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1841" "1841" ...
## ... .$. age : chr [1:24] "0" "1-4" ...
## ... .$. mx : num [1:24] 0.1516 0.0411 ...
## ... .$. qx : num [1:24] 0.137 0.148 ...
## ... [list output truncated]
## ... -- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 960 obs. of 5 variables
## ... .$. row : int [1:960] 1753 1753 1753 1753 1753 ...
## ... .$. col : chr [1:960] "mx" "qx" ...
## ... .$. expected: chr [1:960] "a number" "a number" ...
## ... .$. actual : chr [1:960] "." "."
## ... [list output truncated]
## ... $. P1842:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1842" "1842" ...
## ... .$. age : chr [1:24] "0" "1-4" ...
## ... .$. mx : num [1:24] 0.1601 0.0463 ...
## ... .$. qx : num [1:24] 0.144 0.165 ...
## ... [list output truncated]
## ... -- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 960 obs. of 5 variables
## ... .$. row : int [1:960] 1753 1753 1753 1753 1753 ...
## ... .$. col : chr [1:960] "mx" "qx" ...

```

```

## ... . . . . $ expected: chr [1:960] "a number" "a number" ...
## ... . . . . $ actual : chr [1:960] "." "."
## ... . . . . [list output truncated]
## ... . . . $ P1843:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... . . . $ period: chr [1:24] "1843" "1843" ...
## ... . . . $ age : chr [1:24] "0" "1-4" ...
## ... . . . $ mx : num [1:24] 0.1482 0.0413 ...
## ... . . . $ qx : num [1:24] 0.135 0.149 ...
## ... . . . [list output truncated]
## ... . . . - attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 960 obs. of 5 variables
## ... . . . $ row : int [1:960] 1753 1753 1753 1753 1753 ...
## ... . . . $ col : chr [1:960] "mx" "qx" ...
## ... . . . $ expected: chr [1:960] "a number" "a number" ...
## ... . . . $ actual : chr [1:960] "." "."
## ... . . . [list output truncated]
## ... . . . $ P1844:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... . . . $ period: chr [1:24] "1844" "1844" ...
## ... . . . $ age : chr [1:24] "0" "1-4" ...
## ... . . . $ mx : num [1:24] 0.1373 0.0353 ...
## ... . . . $ qx : num [1:24] 0.125 0.129 ...
## ... . . . [list output truncated]
## ... . . . - attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 960 obs. of 5 variables
## ... . . . $ row : int [1:960] 1753 1753 1753 1753 1753 ...
## ... . . . $ col : chr [1:960] "mx" "qx" ...
## ... . . . $ expected: chr [1:960] "a number" "a number" ...
## ... . . . $ actual : chr [1:960] "." "."
## ... . . . [list output truncated]
## ... . . . [list output truncated]
## ... $ BGR :List of 64
## ... . . . $ P1947:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... . . . $ period: chr [1:24] "1947" "1947" ...
## ... . . . $ age : chr [1:24] "0" "1-4" ...
## ... . . . $ mx : num [1:24] 0.1356 0.0143 ...
## ... . . . $ qx : num [1:24] 0.1241 0.0551 ...
## ... . . . [list output truncated]
## ... . . . $ P1948:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... . . . $ period: chr [1:24] "1948" "1948" ...
## ... . . . $ age : chr [1:24] "0" "1-4" ...
## ... . . . $ mx : num [1:24] 0.1224 0.0141 ...
## ... . . . $ qx : num [1:24] 0.1129 0.0542 ...
## ... . . . [list output truncated]
## ... . . . $ P1949:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... . . . $ period: chr [1:24] "1949" "1949" ...
## ... . . . $ age : chr [1:24] "0" "1-4" ...
## ... . . . $ mx : num [1:24] 0.11473 0.00887 ...
## ... . . . $ qx : num [1:24] 0.1064 0.0347 ...
## ... . . . [list output truncated]
## ... . . . $ P1950:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... . . . $ period: chr [1:24] "1950" "1950" ...
## ... . . . $ age : chr [1:24] "0" "1-4" ...
## ... . . . $ mx : num [1:24] 0.0931 0.0072 ...
## ... . . . $ qx : num [1:24] 0.0875 0.0282 ...
## ... . . . [list output truncated]
## ... . . . [list output truncated]

```

```

## ... [list output truncated]
## $ male      :List of 49
## ..$ AUS     :List of 94
## ...$ P1921:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1921" "1921" ...
## ... .$. age   : chr [1:24] "0" "1-4" ...
## ... .$. mx    : num [1:24] 0.07653 0.00699 ...
## ... .$. qx    : num [1:24] 0.0725 0.0275 ...
## ... ... [list output truncated]
## ...$ P1922:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1922" "1922" ...
## ... .$. age   : chr [1:24] "0" "1-4" ...
## ... .$. mx    : num [1:24] 0.06353 0.00527 ...
## ... .$. qx    : num [1:24] 0.0606 0.0208 ...
## ... ... [list output truncated]
## ...$ P1923:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1923" "1923" ...
## ... .$. age   : chr [1:24] "0" "1-4" ...
## ... .$. mx    : num [1:24] 0.06939 0.00587 ...
## ... .$. qx    : num [1:24] 0.066 0.0231 ...
## ... ... [list output truncated]
## ...$ P1924:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1924" "1924" ...
## ... .$. age   : chr [1:24] "0" "1-4" ...
## ... .$. mx    : num [1:24] 0.06487 0.00561 ...
## ... .$. qx    : num [1:24] 0.0618 0.0221 ...
## ... ... [list output truncated]
## ... [list output truncated]
## ..$ AUT     :List of 71
## ...$ P1947:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1947" "1947" ...
## ... .$. age   : chr [1:24] "0" "1-4" ...
## ... .$. mx    : num [1:24] 0.0994 0.00482 0.00153 0.00131 0.00228 ...
## ... .$. qx    : num [1:24] 0.0929 0.0191 ...
## ... ... [list output truncated]
## ...$ P1948:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1948" "1948" ...
## ... .$. age   : chr [1:24] "0" "1-4" ...
## ... .$. mx    : num [1:24] 0.09421 0.00377 ...
## ... .$. qx    : num [1:24] 0.0884 0.0149 ...
## ... ... [list output truncated]
## ...$ P1949:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1949" "1949" ...
## ... .$. age   : chr [1:24] "0" "1-4" ...
## ... .$. mx    : num [1:24] 0.08592 0.00371 ...
## ... .$. qx    : num [1:24] 0.081 0.0147 ...
## ... ... [list output truncated]
## ...$ P1950:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... .$. period: chr [1:24] "1950" "1950" ...
## ... .$. age   : chr [1:24] "0" "1-4" ...
## ... .$. mx    : num [1:24] 0.07735 0.00302 ...
## ... .$. qx    : num [1:24] 0.0733 0.012 ...
## ... ... [list output truncated]
## ... [list output truncated]

```

```

## ..$ BEL      :List of 175
## ... $ P1841:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ...   ..$ period: chr [1:24] "1841" "1841" ...
## ...   ..$ age   : chr [1:24] "0" "1-4" ...
## ...   ..$ mx    : num [1:24] 0.1865 0.0395 ...
## ...   ..$ qx    : num [1:24] 0.165 0.143 ...
## ...   ... [list output truncated]
## ...   ..- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 960 obs. of 5 variables:
## ...     ..$ row   : int [1:960] 1753 1753 1753 1753 1753 ...
## ...     ..$ col   : chr [1:960] "mx" "qx" ...
## ...     ..$ expected: chr [1:960] "a number" "a number" ...
## ...     ..$ actual  : chr [1:960] "." "."
## ...     ... [list output truncated]
## ...   ..- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ...     ..$ period: chr [1:24] "1842" "1842" ...
## ...     ..$ age   : chr [1:24] "0" "1-4" ...
## ...     ..$ mx    : num [1:24] 0.1918 0.0439 ...
## ...     ..$ qx    : num [1:24] 0.169 0.157 ...
## ...     ... [list output truncated]
## ...   ..- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 960 obs. of 5 variables:
## ...     ..$ row   : int [1:960] 1753 1753 1753 1753 1753 ...
## ...     ..$ col   : chr [1:960] "mx" "qx" ...
## ...     ..$ expected: chr [1:960] "a number" "a number" ...
## ...     ..$ actual  : chr [1:960] "." "."
## ...     ... [list output truncated]
## ...   ..- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ...     ..$ period: chr [1:24] "1843" "1843" ...
## ...     ..$ age   : chr [1:24] "0" "1-4" ...
## ...     ..$ mx    : num [1:24] 0.1815 0.0408 ...
## ...     ..$ qx    : num [1:24] 0.161 0.147 ...
## ...     ... [list output truncated]
## ...   ..- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 960 obs. of 5 variables:
## ...     ..$ row   : int [1:960] 1753 1753 1753 1753 1753 ...
## ...     ..$ col   : chr [1:960] "mx" "qx" ...
## ...     ..$ expected: chr [1:960] "a number" "a number" ...
## ...     ..$ actual  : chr [1:960] "." "."
## ...     ... [list output truncated]
## ...   ..- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ...     ..$ period: chr [1:24] "1844" "1844" ...
## ...     ..$ age   : chr [1:24] "0" "1-4" ...
## ...     ..$ mx    : num [1:24] 0.1714 0.0348 ...
## ...     ..$ qx    : num [1:24] 0.153 0.127 ...
## ...     ... [list output truncated]
## ...   ..- attr(*, "problems")=Classes 'tbl_df', 'tbl' and 'data.frame': 960 obs. of 5 variables:
## ...     ..$ row   : int [1:960] 1753 1753 1753 1753 1753 ...
## ...     ..$ col   : chr [1:960] "mx" "qx" ...
## ...     ..$ expected: chr [1:960] "a number" "a number" ...
## ...     ..$ actual  : chr [1:960] "." "."
## ...     ... [list output truncated]
## ...   ... [list output truncated]
## ...   ..$ BGR      :List of 64
## ...     ..$ P1947:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ...       ..$ period: chr [1:24] "1947" "1947" ...
## ...       ..$ age   : chr [1:24] "0" "1-4" ...

```

```

## ... .$. mx : num [1:24] 0.156 0.014 ...
## ... .$. qx : num [1:24] 0.1408 0.0541 ...
## ... [list output truncated]
## ... $. P1948:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... $. period: chr [1:24] "1948" "1948" ...
## ... $. age : chr [1:24] "0" "1-4" ...
## ... $. mx : num [1:24] 0.1397 0.0131 ...
## ... $. qx : num [1:24] 0.1273 0.0505 ...
## ... [list output truncated]
## ... $. P1949:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... $. period: chr [1:24] "1949" "1949" ...
## ... $. age : chr [1:24] "0" "1-4" ...
## ... $. mx : num [1:24] 0.136 0.01 ...
## ... $. qx : num [1:24] 0.1245 0.0389 ...
## ... [list output truncated]
## ... $. P1950:Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## ... $. period: chr [1:24] "1950" "1950" ...
## ... $. age : chr [1:24] "0" "1-4" ...
## ... $. mx : num [1:24] 0.11234 0.00779 ...
## ... $. qx : num [1:24] 0.1041 0.0305 ...
## ... [list output truncated]

```

Have look at the full structure of one life table.

```
str(hmd.5x1.list[[3]][[1]][[1]])
```

```

## Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## $ period: chr "1921" "1921" "1921" "1921" ...
## $ age : chr "0" "1-4" "5-9" "10-14" ...
## $ mx : num 0.05999 0.00602 0.00192 0.00128 0.00205 ...
## $ qx : num 0.0575 0.0237 0.00952 0.00637 0.0102 ...
## $ ax : num 0.28 1.38 2.14 2.6 2.68 2.57 2.57 2.64 2.62 2.55 ...
## $ lx : num 100000 94250 92016 91140 90559 ...
## $ dx : num 5750 2234 876 580 924 ...
## $ Lx : num 95857 371152 457576 454303 450651 ...
## $ Tx : num 6317561 6221704 5850553 5392977 4938674 ...
## $ ex : num 63.2 66 63.6 59.2 54.5 ...

```

or equivalently

```
str(hmd.5x1.list$female$AUS$P1921)
```

```

## Classes 'tbl_df', 'tbl' and 'data.frame': 24 obs. of 10 variables:
## $ period: chr "1921" "1921" "1921" "1921" ...
## $ age : chr "0" "1-4" "5-9" "10-14" ...
## $ mx : num 0.05999 0.00602 0.00192 0.00128 0.00205 ...
## $ qx : num 0.0575 0.0237 0.00952 0.00637 0.0102 ...
## $ ax : num 0.28 1.38 2.14 2.6 2.68 2.57 2.57 2.64 2.62 2.55 ...
## $ lx : num 100000 94250 92016 91140 90559 ...
## $ dx : num 5750 2234 876 580 924 ...
## $ Lx : num 95857 371152 457576 454303 450651 ...
## $ Tx : num 6317561 6221704 5850553 5392977 4938674 ...
## $ ex : num 63.2 66 63.6 59.2 54.5 ...

```

Extract single calendar year 1 and 5-year age group probabilities of dying and life expectancies and save

them in $age \times lifetable$ matrices in the *RData* directory.

```
# 1x1 nqx
q1.f <- extract.lt.col(hmd.1x1.list, "female", "qx")
save(file = "../RData/q1.f.RData", compress = TRUE, list = c("q1.f"))
q1.m <- extract.lt.col(hmd.1x1.list, "male", "qx")
save(file = "../RData/q1.m.RData", compress = TRUE, list = c("q1.m"))
# remove last row where nqx = 1
q1.f <- q1.f[1:(nrow(q1.f) - 1), ]
q1.m <- q1.m[1:(nrow(q1.m) - 1), ]

# 1x1 1ax
a1.f <- extract.lt.col(hmd.1x1.list, "female", "ax")
save(file = "../RData/a1.f.RData", compress = TRUE, list = c("a1.f"))
a1.m <- extract.lt.col(hmd.1x1.list, "male", "ax")
save(file = "../RData/a1.m.RData", compress = TRUE, list = c("a1.m"))

# 1x1 lx
l1.f <- extract.lt.col(hmd.1x1.list, "female", "lx")
save(file = "../RData/l1.f.RData", compress = TRUE, list = c("l1.f"))
l1.m <- extract.lt.col(hmd.1x1.list, "male", "lx")
save(file = "../RData/l1.m.RData", compress = TRUE, list = c("l1.m"))

# 1x1 ex
e1.f <- extract.lt.col(hmd.1x1.list, "female", "ex")
save(file = "../RData/e1.f.RData", compress = TRUE, list = c("e1.f"))
e1.m <- extract.lt.col(hmd.1x1.list, "male", "ex")
save(file = "../RData/e1.m.RData", compress = TRUE, list = c("e1.m"))

# 5x1 nqx
q5.f <- extract.lt.col(hmd.5x1.list, "female", "qx")
save(file = "../RData/q5.f.RData", compress = TRUE, list = c("q5.f"))
q5.m <- extract.lt.col(hmd.5x1.list, "male", "qx")
save(file = "../RData/q5.m.RData", compress = TRUE, list = c("q5.m"))
# remove last row where nqx = 1
q5.f <- q5.f[1:(nrow(q5.f) - 1), ]
q5.m <- q5.m[1:(nrow(q5.m) - 1), ]

# 5x1 5ax
a5.f <- extract.lt.col(hmd.5x1.list, "female", "ax")
save(file = "../RData/a5.f.RData", compress = TRUE, list = c("a5.f"))
a5.m <- extract.lt.col(hmd.5x1.list, "male", "ax")
save(file = "../RData/a5.m.RData", compress = TRUE, list = c("a5.m"))

# 5x1 lx
l5.f <- extract.lt.col(hmd.5x1.list, "female", "lx")
save(file = "../RData/l5.f.RData", compress = TRUE, list = c("l5.f"))
l5.m <- extract.lt.col(hmd.5x1.list, "male", "lx")
save(file = "../RData/l5.m.RData", compress = TRUE, list = c("l5.m"))

# 5x1 ex
e5.f <- extract.lt.col(hmd.5x1.list, "female", "ex")
save(file = "../RData/e5.f.RData", compress = TRUE, list = c("e5.f"))
e5.m <- extract.lt.col(hmd.5x1.list, "male", "ex")
```

```

save(file = "../RData/e5.m.RData", compress = TRUE, list = c("e5.m"))

# rm(list=c('hmd.1x1.list', 'hmd.5x1.list'))

```

Have another quick look at the lists and now matrices saved in “Rdata”.

```

list.files("../RData/")

## [1] "a1.f.RData"      "a1.m.RData"      "a5.f.RData"
## [4] "a5.m.RData"      "e1.f.RData"      "e1.m.RData"
## [7] "e5.f.RData"      "e5.m.RData"      "hmd-1x1.RData"
## [10] "hmd-5x1.RData"   "l1.f.RData"      "l1.m.RData"
## [13] "l5.f.RData"      "l5.m.RData"      "q1.f.RData"
## [16] "q1.m.RData"      "q5.f.RData"      "q5.m.RData"

```

2.2 Clean HMD

There are two persistent problems with the HMD 1×1 life tables: 1. as mentioned just above, some of the Belarus life tables are empty, and 2. the ${}_1q_x$ values for some life tables are ‘flat’ at older ages, i.e. are constant.

In both cases, these life tables need to be removed. We’ll get rid of the Belarus tables first. The strategy is general: identify life tables with ‘NA’ values and remove those. They turn out to be Belarus 1914–1918.

```

## females
which(is.na(q1.f[1, ]))

## female.BEL.1914 female.BEL.1915 female.BEL.1916
##           239          240          241
## female.BEL.1917 female.BEL.1918
##           242          243

q1.f[1, which(is.na(q1.f[1, ]))]

## female.BEL.1914 female.BEL.1915 female.BEL.1916
##           NA          NA          NA
## female.BEL.1917 female.BEL.1918
##           NA          NA

which(is.na(q5.f[1, ]))

## female.BEL.1914 female.BEL.1915 female.BEL.1916
##           239          240          241
## female.BEL.1917 female.BEL.1918
##           242          243

q5.f[1, which(is.na(q5.f[1, ]))]

## female.BEL.1914 female.BEL.1915 female.BEL.1916
##           NA          NA          NA
## female.BEL.1917 female.BEL.1918
##           NA          NA

which(is.na(e1.f[1, ]))

## female.BEL.1914 female.BEL.1915 female.BEL.1916
##           239          240          241
## female.BEL.1917 female.BEL.1918
##           242          243

```

```

e1.f[1, which(is.na(e1.f[1, ]))]

## female.BEL.1914 female.BEL.1915 female.BEL.1916
##          NA          NA          NA
## female.BEL.1917 female.BEL.1918
##          NA          NA

which(is.na(e5.f[1, ]))

## female.BEL.1914 female.BEL.1915 female.BEL.1916
##          239          240          241
## female.BEL.1917 female.BEL.1918
##          242          243

e5.f[1, which(is.na(e5.f[1, ]))]

## female.BEL.1914 female.BEL.1915 female.BEL.1916
##          NA          NA          NA
## female.BEL.1917 female.BEL.1918
##          NA          NA

## males
which(is.na(q1.m[1, ]))

## male.BEL.1914 male.BEL.1915 male.BEL.1916
##          239          240          241
## male.BEL.1917 male.BEL.1918
##          242          243

q1.m[1, which(is.na(q1.m[1, ]))]

## male.BEL.1914 male.BEL.1915 male.BEL.1916
##          NA          NA          NA
## male.BEL.1917 male.BEL.1918
##          NA          NA

which(is.na(q5.m[1, ]))

## male.BEL.1914 male.BEL.1915 male.BEL.1916
##          239          240          241
## male.BEL.1917 male.BEL.1918
##          242          243

q5.m[1, which(is.na(q5.m[1, ]))]

## male.BEL.1914 male.BEL.1915 male.BEL.1916
##          NA          NA          NA
## male.BEL.1917 male.BEL.1918
##          NA          NA

which(is.na(e1.m[1, ]))

## male.BEL.1914 male.BEL.1915 male.BEL.1916
##          239          240          241
## male.BEL.1917 male.BEL.1918
##          242          243

e1.m[1, which(is.na(e1.m[1, ]))]

## male.BEL.1914 male.BEL.1915 male.BEL.1916

```

```

##          NA          NA          NA
## male.BEL.1917 male.BEL.1918
##          NA          NA
which(is.na(e5.m[1, ]))

## male.BEL.1914 male.BEL.1915 male.BEL.1916
##          239          240          241
## male.BEL.1917 male.BEL.1918
##          242          243
e5.m[1, which(is.na(e5.m[1, ]))]

## male.BEL.1914 male.BEL.1915 male.BEL.1916
##          NA          NA          NA
## male.BEL.1917 male.BEL.1918
##          NA          NA

# in all matrices, empty columns are THE SAME, numbered
# 236-340
remove <- unique(c(which(is.na(q1.f[1, ])), which(is.na(q1.m[1,
]))))

q1.f <- q1.f[, -remove]
q5.f <- q5.f[, -remove]
e1.f <- e1.f[, -remove]
e5.f <- e5.f[, -remove]
a1.f <- a1.f[, -remove]
a5.f <- a5.f[, -remove]
l1.f <- l1.f[, -remove]
l5.f <- l5.f[, -remove]

q1.m <- q1.m[, -remove]
q5.m <- q5.m[, -remove]
e1.m <- e1.m[, -remove]
e5.m <- e5.m[, -remove]
a1.m <- a1.m[, -remove]
a5.m <- a5.m[, -remove]
l1.m <- l1.m[, -remove]
l5.m <- l5.m[, -remove]

# verify all is well now
q1.f[1, which(is.na(q1.f[1, ]))]

## numeric(0)
q5.f[1, which(is.na(q5.f[1, ]))]

## numeric(0)
e1.f[1, which(is.na(q1.f[1, ]))]

## numeric(0)
e5.f[1, which(is.na(q5.f[1, ]))]

## numeric(0)

```

```

q1.m[1, which(is.na(q1.m[1, ]))]

## numeric(0)

q5.m[1, which(is.na(q5.m[1, ]))]

## numeric(0)

e1.m[1, which(is.na(e1.m[1, ]))]

## numeric(0)

e5.m[1, which(is.na(e5.m[1, ]))]

## numeric(0)

# make sure all matrices have same number of columns and
# same column names
dim(q1.f)

## [1] 110 4614

dim(q1.m)

## [1] 110 4614

dim(q5.f)

## [1] 23 4614

dim(q5.m)

## [1] 23 4614

dim(e1.f)

## [1] 111 4614

dim(e1.m)

## [1] 111 4614

dim(e5.f)

## [1] 24 4614

dim(e5.m)

## [1] 24 4614

dim(a1.f)

## [1] 111 4614

dim(a1.m)

## [1] 111 4614

dim(a5.f)

## [1] 24 4614

dim(a5.m)

## [1] 24 4614

```

```

dim(l1.f)

## [1] 111 4614
dim(l1.m)

## [1] 111 4614
dim(15.f)

## [1] 24 4614
dim(15.m)

## [1] 24 4614
# NB, last argument in str_sub intentionally empty,
# defaults to end of string
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(q1.m),
 6, )) 

## [1] TRUE
identical(str_sub(colnames(q5.f), 8, ), str_sub(colnames(q5.m),
 6, )) 

## [1] TRUE
identical(str_sub(colnames(e1.f), 8, ), str_sub(colnames(e1.m),
 6, )) 

## [1] TRUE
identical(str_sub(colnames(e5.f), 8, ), str_sub(colnames(e5.m),
 6, )) 

## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(e1.f),
 8, )) 

## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(e1.m),
 6, )) 

## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(q5.f),
 8, )) 

## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(q5.m),
 6, )) 

## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(l1.f),
 8, )) 

## [1] TRUE

```

```

identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(l1.m),
6, ))
## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(15.f),
8, ))
## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(15.m),
6, ))
## [1] TRUE
rm(list = c("remove"))

```

Now identify and remove the ‘flat’ life tables. These turn out to be Iceland 1852 and New Zealand Maori 1949, 1956, and 1959.

```

## females -- FOUR PROBLEM LIFE TABLES 1-year age
## identify flat female LTs
which(q1.f[106, ] == q1.f[110, ])

##     female.ISL.1852 female.NZL_MA.1949
##           2709          3643
## female.NZL_MA.1956 female.NZL_MA.1959
##           3650          3653

# verify that they are constant at roughly ages 80+
q1.f[75:110, which(q1.f[106, ] == q1.f[110, ])]

##     female.ISL.1852 female.NZL_MA.1949
## 74      0.06242      0.07424
## 75      0.07010      0.11634
## 76      0.08167      0.15404
## 77      0.09211      0.04880
## 78      0.09056      0.00000
## 79      0.09189      0.33434
## 80      0.10649      0.11516
## 81      0.10649      0.11516
## 82      0.10649      0.11516
## 83      0.10649      0.11516
## 84      0.10649      0.11516
## 85      0.10649      0.11516
## 86      0.10649      0.11516
## 87      0.10649      0.11516
## 88      0.10649      0.11516
## 89      0.10649      0.11516
## 90      0.10649      0.11516
## 91      0.10649      0.11516
## 92      0.10649      0.11516
## 93      0.10649      0.11516
## 94      0.10649      0.11516
## 95      0.10649      0.11516
## 96      0.10649      0.11516
## 97      0.10649      0.11516
## 98      0.10649      0.11516

```

```

## 99      0.10649      0.11516
## 100     0.10649      0.11516
## 101     0.10649      0.11516
## 102     0.10649      0.11516
## 103     0.10649      0.11516
## 104     0.10649      0.11516
## 105     0.10649      0.11516
## 106     0.10649      0.11516
## 107     0.10649      0.11516
## 108     0.10649      0.11516
## 109     0.10649      0.11516
## female.NZL_MA.1956 female.NZL_MA.1959
## 74      0.08003      0.10611
## 75      0.10351      0.23040
## 76      0.09529      0.05827
## 77      0.08515      0.07232
## 78      0.10131      0.20249
## 79      0.14296      0.08829
## 80      0.11708      0.11642
## 81      0.11708      0.11642
## 82      0.11708      0.11642
## 83      0.11708      0.11642
## 84      0.11708      0.11642
## 85      0.11708      0.11642
## 86      0.11708      0.11642
## 87      0.11708      0.11642
## 88      0.11708      0.11642
## 89      0.11708      0.11642
## 90      0.11708      0.11642
## 91      0.11708      0.11642
## 92      0.11708      0.11642
## 93      0.11708      0.11642
## 94      0.11708      0.11642
## 95      0.11708      0.11642
## 96      0.11708      0.11642
## 97      0.11708      0.11642
## 98      0.11708      0.11642
## 99      0.11708      0.11642
## 100     0.11708      0.11642
## 101     0.11708      0.11642
## 102     0.11708      0.11642
## 103     0.11708      0.11642
## 104     0.11708      0.11642
## 105     0.11708      0.11642
## 106     0.11708      0.11642
## 107     0.11708      0.11642
## 108     0.11708      0.11642
## 109     0.11708      0.11642

# 5-year age identify flat female LTs
which(q5.f[23, ] == q5.f[19, ])

##      female.ISL.1852 female.NZL_MA.1949
##      2709             3643
## female.NZL_MA.1956 female.NZL_MA.1959

```

```

##          3650          3653
# verify that they are constant at roughly ages 80+
q5.f[15:23, which(q5.f[23, ] == q5.f[19, ])]

##      female.ISL.1852 female.NZL_MA.1949
## 65-69          0.22643          0.27392
## 70-74          0.24125          0.30321
## 75-79          0.35970          0.52667
## 80-84          0.43050          0.45759
## 85-89          0.43050          0.45759
## 90-94          0.43050          0.45759
## 95-99          0.43050          0.45759
## 100-104         0.43050          0.45759
## 105-109         0.43050          0.45759
##      female.NZL_MA.1956 female.NZL_MA.1959
## 65-69          0.27451          0.22760
## 70-74          0.35737          0.32634
## 75-79          0.42851          0.51114
## 80-84          0.46347          0.46145
## 85-89          0.46347          0.46145
## 90-94          0.46347          0.46145
## 95-99          0.46347          0.46145
## 100-104         0.46347          0.46145
## 105-109         0.46347          0.46145

## males -- ALL OK 1-year age identify flat female LTs
which(q1.m[106, ] == q1.m[110, ])

## named integer(0)
# verify that they are constant at roughly ages 80+
q1.m[75:110, which(q1.m[106, ] == q1.m[110, ])]

## 
## 74
## 75
## 76
## 77
## 78
## 79
## 80
## 81
## 82
## 83
## 84
## 85
## 86
## 87
## 88
## 89
## 90
## 91
## 92
## 93
## 94

```

```

## 95
## 96
## 97
## 98
## 99
## 100
## 101
## 102
## 103
## 104
## 105
## 106
## 107
## 108
## 109

# 5-year age identify flat female LTs
which(q5.m[23, ] == q5.m[19, ])

## named integer(0)

# verify that they are constant at roughly ages 80+
q5.m[15:23, which(q5.m[23, ] == q5.m[19, ])]

## 
## 65-69
## 70-74
## 75-79
## 80-84
## 85-89
## 90-94
## 95-99
## 100-104
## 105-109

# remove all flat LTs that were flat for either females
# or males from both female and male collections.
remove.1 <- unique(c(which(q1.f[106, ] == q1.f[110, ]),
  which(q1.m[106, ] == q1.m[110, ])))
remove.5 <- unique(c(which(q5.f[23, ] == q5.f[19, ]),
  which(q5.m[23, ] == q5.m[19, ])))

# are the remove lists the same?
identical(remove.1, remove.5)

## [1] TRUE

# remove them from both sexes and verify
q1.f <- q1.f[, -remove.1]
which(q1.f[106, ] == q1.f[110, ])

## named integer(0)

q1.m <- q1.m[, -remove.1]
which(q1.m[106, ] == q1.m[110, ])

## named integer(0)

```

```

q5.f <- q5.f[, -remove.5]
which(q5.f[23, ] == q5.f[19, ])

## named integer(0)
q5.m <- q5.m[, -remove.5]
which(q5.m[23, ] == q5.m[19, ])

## named integer(0)
e1.f <- e1.f[, -remove.1]
e1.m <- e1.m[, -remove.1]

e5.f <- e5.f[, -remove.5]
e5.m <- e5.m[, -remove.5]

a1.f <- a1.f[, -remove.1]
a1.m <- a1.m[, -remove.1]

a5.f <- a5.f[, -remove.5]
a5.m <- a5.m[, -remove.5]

l1.f <- l1.f[, -remove.1]
l1.m <- l1.m[, -remove.1]

l5.f <- l5.f[, -remove.5]
l5.m <- l5.m[, -remove.5]

# make sure all matrices have same number of columns and
# same column names
dim(q1.f)

## [1] 110 4610
dim(q1.m)

## [1] 110 4610
dim(q5.f)

## [1] 23 4610
dim(q5.m)

## [1] 23 4610
dim(e1.f)

## [1] 111 4610
dim(e1.m)

## [1] 111 4610
dim(e5.f)

## [1] 24 4610
dim(e5.m)

## [1] 24 4610

```

```

dim(a1.f)

## [1] 111 4610
dim(a1.m)

## [1] 111 4610
dim(a5.f)

## [1] 24 4610
dim(a5.m)

## [1] 24 4610
dim(l1.f)

## [1] 111 4610
dim(l1.m)

## [1] 111 4610
dim(l5.f)

## [1] 24 4610
dim(l5.m)

## [1] 24 4610
# NB, last argument in str_sub intentionally empty,
# defaults to end of string
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(q1.m),
  6, )) 

## [1] TRUE
identical(str_sub(colnames(q5.f), 8, ), str_sub(colnames(q5.m),
  6, )) 

## [1] TRUE
identical(str_sub(colnames(e1.f), 8, ), str_sub(colnames(e1.m),
  6, )) 

## [1] TRUE
identical(str_sub(colnames(e5.f), 8, ), str_sub(colnames(e5.m),
  6, )) 

## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(e1.f),
  8, )) 

## [1] TRUE
identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(e1.m),
  6, )) 

## [1] TRUE

```

```

identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(q5.f),
8, ))  

## [1] TRUE  

identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(q5.m),
6, ))  

## [1] TRUE  

identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(l1.f),
8, ))  

## [1] TRUE  

identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(l1.m),
6, ))  

## [1] TRUE  

identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(15.f),
8, ))  

## [1] TRUE  

identical(str_sub(colnames(q1.f), 8, ), str_sub(colnames(15.m),
6, ))  

## [1] TRUE  

rm(list = c("remove.1", "remove.5"))

```

The last data cleaning step involves identifying $_{nq_x}$ values that are zero and replacing these with very small numbers. This is necessary so that we use the log function to transform these.

```

length(q1.f)  # values in q1.f  

## [1] 507100  

length(q1.f[q1.f == 0])  # zero cells in q1.f  

## [1] 2604  

length(q5.f)  # values in q5.f  

## [1] 106030  

length(q5.f[q5.f == 0])  # zero cells in q5.f  

## [1] 75  

length(q1.m)  # values in q1.m  

## [1] 507100  

length(q1.m[q1.m == 0])  # zero cells in q1.m  

## [1] 1380  

length(q5.m)  # values in q5.m  

## [1] 106030

```

```

length(q5.m[q5.m == 0])  # zero cells in q5.m

## [1] 24

# female
q1.f.nz <- q1.f
q1.f.nz[q1.f.nz == 0] <- 1e-06
q5.f.nz <- q5.f
q5.f.nz[q5.f.nz == 0] <- 1e-06

# male
q1.m.nz <- q1.m
q1.m.nz[q1.m.nz == 0] <- 1e-06
q5.m.nz <- q5.m
q5.m.nz[q5.m.nz == 0] <- 1e-06

cat("\n")

length(q1.f.nz)  # values in q1.f

## [1] 507100

length(q1.f.nz[q1.f.nz == 0])  # zero cells in q1.f

## [1] 0

length(q5.f.nz)  # values in q5.f

## [1] 106030

length(q5.f.nz[q5.f.nz == 0])  # zero cells in q5.f

## [1] 0

length(q1.m.nz)  # values in q1.m

## [1] 507100

length(q1.m.nz[q1.m.nz == 0])  # zero cells in q1.m

## [1] 0

length(q5.m.nz)  # values in q5.m

## [1] 106030

length(q5.m.nz[q5.m.nz == 0])  # zero cells in q5.m

## [1] 0

```

Take the log and logit transforms of the nq_x values.

```

# function for logit transformation
logit <- function(x) {
  return(log(x/(1 - x)))
}

# function for inverse logit transformation
expit <- function(x) {
  return(exp(x)/(1 + exp(x)))
}

```

```

# log and logit transform the female nqx
q11.f <- log(q1.f.nz)
q1logit.f <- logit(q1.f.nz)
q51.f <- log(q5.f.nz)
q5logit.f <- logit(q5.f.nz)

# log and logit transform the male nqx
q11.m <- log(q1.m.nz)
q1logit.m <- logit(q1.m.nz)
q51.m <- log(q5.m.nz)
q5logit.m <- logit(q5.m.nz)

```

Check how many life tables are left and be sure all the data objects have the same number of life tables and age groups.

```

dim(q11.f)

## [1] 110 4610
dim(q1logit.f)

## [1] 110 4610
dim(q51.f)

## [1] 23 4610
dim(q5logit.f)

## [1] 23 4610
dim(q11.m)

## [1] 110 4610
dim(q1logit.m)

## [1] 110 4610
dim(q51.m)

## [1] 23 4610
dim(q5logit.m)

## [1] 23 4610

```

2.3 Additional Indicator Calculation

We need child, ${}_5q_0$, and adult, ${}_{45}q_{15}$, mortality values for females and males. Calculate these from the 1×1 nq_x values and store in separate matrices, including the log and logit transformed values.

```

# function to generate 5q0 from a matrix of 1qx
convert1qxTo5q0 <- function(q1) {

  # q1 is an age by life table matrix of 1qx q5 is 1 by
  # life table matrix/vector of 5q0

  tmp.q <- rep(1, ncol(q1))
  for (i in 1:5) {

```

```

        tmp.q <- tmp.q * (1 - q1[i, ])
    }
    q5 <- as.matrix(1 - tmp.q)
    return(q5)
}

# function to generate 45q15 from a matrix of 1qx
convert1qxTo45q15 <- function(q1) {

    # q1 is an age by life table matrix of 1qx q5 is 1 by
    # life table matrix/vector of 45q15

    tmp.q <- rep(1, ncol(q1))
    for (i in 16:60) {
        tmp.q <- tmp.q * (1 - q1[i, ])
    }
    q5 <- as.matrix(1 - tmp.q)
    return(q5)
}

# now actually create the child and adult mortality
# indicators

# female

# make matrix with 5q0 in row 1 and 45q15 in row 2
Q.f <- rbind(t(convert1qxTo5q0(q1.f)), t(convert1qxTo45q15(q1.f)))
# check for zeroes
Q.f[Q.f == 0]

## numeric(0)
# log and logit
Q1.f <- log(Q.f)
Qlogit.f <- logit(Q.f)

colnames(Q.f) <- colnames(q1.f)
colnames(Q1.f) <- colnames(q1.f)
colnames(Qlogit.f) <- colnames(q1.f)

rownames(Q.f) <- c("Child Mortality", "Adult Mortality")
rownames(Q1.f) <- c("Child Mortality", "Adult Mortality")
rownames(Qlogit.f) <- c("Child Mortality", "Adult Mortality")

# male

# make matrix with 5q0 in row 1 and 45q15 in row 2
Q.m <- rbind(t(convert1qxTo5q0(q1.m)), t(convert1qxTo45q15(q1.m)))
# check for zeroes
Q.m[Q.m == 0]

## numeric(0)
# log and logit
Q1.m <- log(Q.m)

```

```

Qlogit.m <- logit(Q.m)

colnames(Q.m) <- colnames(q1.m)
colnames(Q1.m) <- colnames(q1.m)
colnames(Qlogit.m) <- colnames(q1.m)

rownames(Q.m) <- c("Child Mortality", "Adult Mortality")
rownames(Q1.m) <- c("Child Mortality", "Adult Mortality")
rownames(Qlogit.m) <- c("Child Mortality", "Adult Mortality")

```

We now have everything we need to get going. Clean up or clear stuff we don't need and save everything.

```

# rm(list=c('download.result', 'hmd.1x1.list', 'hmd.5x1.list', 'remove', 'remove.1'
# , 'remove.5', 'i', 'tmp.q', 'count.lts', 'download.hmd', 'extract.lt.col', 'list.lts'
# , 'list.raw.lts', 'parse.lt', 'read.raw.lt', 'sex.per.age.switch'))
save.image("../RData/hmd.qs.RData")
# load('../RData/hmd.qs.RData')

```

3 SVD Component Model of Mortality

3.1 *svdMod()* function

svdMod() is a function that wraps up most of the operations needed to calculate and validate SVD-Comp models. This function does a lot and can be used in a variety of ways:

- Calculate/estimate an SVD-component mortality model using a set of age-specific nq_x as inputs
- Calculate/estimate a smoothed SVD-component mortality model using a set of age-specific nq_x as inputs
- Randomly sample a set of age-specific nq_x , calculate an SVD-component model of mortality (smoothed or not), predict nq_x for the not-sampled age-specific nq_x , and summarize the prediction errors
- All of this can be repeated a specified number of times
- The return object contains very detailed results for everything that was requested

Inputs to the function:

- ‘ql’ are the logit-transformed input age-specific nq_x (life tables) arranged as age×lifetable
- ‘Ql’ are the logit-transformed summary mortality indicators: child mortality, $5q_0$, and adult mortality, $45q_{15}$, arranged in $2 \times n$ form where the first row is child mortality, the second adult mortality, and the columns correspond to life tables
- ‘N’ is the number of times to repeat sampling/validation
- ‘S’ is the fraction of life tables to include in the sample
- ‘offset’ is a number used to offset the age-specific mortality rates from the origin before calculating the SVD; this is an easy way to give each age group approximately the same weight in the SVD calculation; it is added back when predictions are made
- ‘retAll’ is a switch indicating if all results should be returned or just summaries
- ‘adult’ is a switch indicating if adult mortality, $45q_{15}$, is supplied and should be used directly as an input to the model when predictions are made; if not, then child mortality, $5q_0$, is the only direct input, and adult mortality is used *indirectly* by predicting it from child mortality and then using it together with child mortality for the predictions for other ages
- ‘q0Fix’ is a switch indicating if the $q0$ fix should be executed during prediction
- ‘smooth’ is a switch indicating if the SVD-comp model should be smoothed
- ‘C’ specifies the number of components to include in the SVD-component model

The return object is a large list that contains:

- ‘ql.samp’ - a list of the sampled age-specific nq_x , i.e. life tables, (one for each sample)

- ‘ql.nsamp’ - a list of the not sampled age-specific nq_x (one for each sample)
- ‘names’ - the names of the sampled life tables
- ‘svd’ - a list of the SVD decompositions (one for each sample) of the sampled life tables
- ‘svd.sm’ - a list of the smoothed SVD decompositions (one for each sample) of the sampled life tables
- ‘mods’ - a list of the the regression return objects (one for each sample) from the regression models for each SVD component weight in the model, the model for adult mortality, and the model for the q0 fix
- ‘recon.samp’ - a list of the predicted life tables (one for each sample) for life tables in the sample
- ‘error.samp’ - a list of the prediction errors (one for each sample) for the sampled life tables
- ‘recon.nsamp’ - a list of the predicted life tables (one for each sample) for life tables not in the sample
- ‘error.nsamp’ - a list of the prediction errors (one for each sample) for the not sampled life tables
- ‘errsum.samp’ - a list of summaries (one for each sample) of in-sample errors, uses R’s *summary()* function
- ‘errsum.nsamp’ - a list of summaries (one for each sample) of out-of-sample errors, uses R’s *summary()* function
- ‘offset’ - the *offset* value used when the function was called
- ‘retAll’ - the *retAll* value used when the function was called
- ‘adult’ - the *adult* value used when the function was called
- ‘q0fix’ - the *q0fix* value used when the function was called
- ‘smooth’ - the *smooth* value used when the function was called
- ‘C’ - the *C* value used when the function was called

A couple notes:

- The input age-specific nq_x must all be logit-transformed, the function assumes this and uses an *expit* transformation to do predictions on the natural scale
- The ‘mods’ return object is very useful for doing predictions and building additional modeling features using the return object of this function
- the ‘retAll’ option is included because full results can be *very* large, and returning the summaries is a much more compact way to do things if you need to run many times and don’t need the detailed results each time

```
svdMod <- function(ql, Ql, N, S, offset, retAll, adult,
q0Fix, smooth, C = 4, printS = FALSE) {

  # ql is input qs Ql is input summary indicators (child
  # and adult ql) N is number of samples S is sample
  # fraction offset is the SVD offset retAll is switch to
  # return 'all' or just error summaries adult is a switch
  # indicating whether to include adult mortality in the
  # model q0Fix is a switch to execute the q0 fix smooth
  # is a switch to smooth left singular vectors C is
  # number of components, default C=4 printS is a switch
  # to turn off printing the sample number at each
  # iteration

  ret.ql.samp <- list(0) # sampled qls
  ret.ql.nsamp <- list(0) # out of sample qls
  ret.samp.names <- list(0) # sampled LT names
  ret.svd <- list(0) # svd of sampled qls
  ret.svd.sm <- list(0) # smooth svd of samplped qls
  ret.mods <- list(0) # models
  ret.recon.samp <- list(0) # sample reconstructions
  ret.error.samp <- list(0) # sample errors
  ret.recon.nsamp <- list(0) # out of sample reconstructions
  ret.error.nsamp <- list(0) # out of sample errors
```

```

ret.errsum.samp <- list(0) # summary of sample errors
ret.errsum.nsamp <- list(0) # summary of out of sample errors

# Ensure C is in reasonable range: 1-4
if (C < 1 | C > 4) {
  C <- 4
  print("Setting C=4")
}

cat("\n")
print(paste("Adult mortality is direct input to predictions:",
adult))
print(paste("SVD model is smoothed:", smooth))
print(paste(N, "iterations"))
print(paste(round(S * 100, 0), "% sample fraction",
sep = ""))
print(paste(C, "components"))

if (S > 0) {

  for (i in 1:N) {

    if (printS) {
      print(paste(" Sample:", i))

    }

    # pick the sample
    if (S == 1) {
      samp <- colnames(q1) # the sample is all LTs
      nsamp <- NA # nothing in the out of sample
    } else {
      # identify sample
      samp <- sample(colnames(q1), ncol(q1) *
S)
      # identify out of sample
      nsamp <- colnames(q1)[-which(colnames(q1) %in%
samp)]
    }
    name <- paste("s", i, sep = "") # give this sample a name

    # store the sample
    ret.samp.names[[i]] <- samp # store sampled LT names to return list
    names(ret.samp.names)[i] <- name # name the sampled LT names

    # store the sampled qls
    ret ql.samp[[i]] <- ql[, samp] # store sampled qls in return list
    names(ret ql.samp)[i] <- name # name the sampled qls

    # store the out of sample qls
    if (S == 1) {
      ret ql.nsamp[[i]] <- NA # no out of sample LTs
    } else {
      ret ql.nsamp[[i]] <- ql[, nsamp] # store out of sample qls in return list
    }
  }
}

```

```

}

names(ret.ql.nsamp)[i] <- name # name out of sample qls

# calculate the svd of the sampled qls
svd <- svd(ql[, samp] - offset) # subtract offset before calculating svd

# store the SVD
ret.svd[[i]] <- svd # store svd in return list
names(ret.svd)[i] <- name # name the svd

# calculate transformations of *sampled* child
# mortality: input is logged
cm <- expit(Q1[1, samp]) # child mortality, natural scale
cml <- Q1[1, samp] # child mortality, logged
cmls <- cml^2 # square of logged child mortality
cmlc <- cml^3 # cube of logged child mortality

# calcualte transformations of *sampled* adult
# mortality: input is logged
am <- expit(Q1[2, samp]) # adult mortality, natural scale
aml <- Q1[2, samp] # adult mortality, logged
amls <- aml^2 # square of logged adult mortality
amlc <- aml^3 # cube of logged adult mortality

# calcualte one-way interaction of *sampled* child and
# adult mortality
cmlaml <- cml * aml

# model *sampled* adult mortality ~ child mortality
aml.betas <- lm(aml ~ cm + cml + cmls + cmlc)

# model *sampled* first four vs ~ child mortality and
# adult mortality
v1.betas <- lm(svd$v[, 1] ~ cm + cml + cmls +
  cmlc + am + amls + amlc + cmlaml)
v2.betas <- lm(svd$v[, 2] ~ cm + cml + cmls +
  cmlc + am + amls + amlc + cmlaml)
v3.betas <- lm(svd$v[, 3] ~ cm + cml + cmls +
  cmlc + am + amls + amlc + cmlaml)
v4.betas <- lm(svd$v[, 4] ~ cm + cml + cmls +
  cmlc + am + amls + amlc + cmlaml)

# predictions for all LTs, both sampled and out of
# sample start by transforming child mortality for all
# LTs
cml.p <- Q1[1, ]
cm.p <- expit(cml.p)
cmls.p <- cml.p^2
cmlc.p <- cml.p^3

# predict the adult mortality that goes with this child
# mortality data frame of predictors
predictors.aml <- data.frame(cbind(cm.p, cml.p,

```

```

        cmls.p, cmlc.p))
# names for predictors that match the variable in the
# original model
colnames(predictors.aml) <- c("cm", "cml", "cmls",
                               "cmlc")
# predictions for adult mortality
if (adult) {
  aml.p <- Q1[2, ]
} else {
  aml.p <- predict.lm(aml.betas, newdata = predictors.aml)
}

# predict vs using child mortality and (predicted) adult
# mortality transform predicted adult mortality
am.p <- expit(aml.p)
amls.p <- am.p^2
amlc.p <- am.p^3
cmlaml.p <- cml.p * aml.p
# data frame of predictors
predictors.vs <- data.frame(cbind(cm.p, cml.p,
                                    cmls.p, cmlc.p, am.p, amls.p, amlc.p, cmlaml.p))
# names for predictors that match the variables in the
# original regressions
colnames(predictors.vs) <- c("cm", "cml", "cmls",
                               "cmlc", "am", "amls", "amlc", "cmlaml")
# predictions for each v
v1.p <- predict.lm(v1.betas, newdata = predictors.vs)
v2.p <- predict.lm(v2.betas, newdata = predictors.vs)
v3.p <- predict.lm(v3.betas, newdata = predictors.vs)
v4.p <- predict.lm(v4.betas, newdata = predictors.vs)

# smooth left singular vectors
if (smooth) {
  for (k in 2:6) {
    t <- ksmooth(seq(1, dim(svd$u)[1], 1),
                 svd$u[, k], kernel = "normal", bandwidth = (k +
                   1))
    t$y[1:(k - 1)] <- svd$u[, k][1:(k - 1)]
    svd$u[, k] <- t$y
  }
  # store the smooth SVD
  ret.svd.sm[[i]] <- svd # store the smooth svd in return list
  names(ret.svd.sm)[i] <- name # name the smooth svd
} else {
  ret.svd.sm[[i]] <- NA
  names(ret.svd.sm)[i] <- name # name the smooth svd
}

# reconstruct the predicted LTs
v <- cbind(v1.p, v2.p, v3.p, v4.p) # matrix of new predicted vs
r.p <- ql - ql # data frame for reconstructed values with names
for (z in 1:C) {
  # loop over C components svd reconstruction; sum of

```

```

    # rank-1 matrices, one for each v
    r.p <- r.p + svd$d[z] * svd$u[, z] %*% t(v[, z])
}
r.p <- r.p + offset # add the offset back in

if (q0Fix) {
  # fix up q0 prediction child mortality for sample LTs
  cml <- Q1[1, samp] # sample child mortality
  cmls <- cml^2 # square of sample child mortality
  q0.betas <- lm(as.numeric(q1[1, samp]) ~
    cml + cmls) # q0 ~ cml + cmls
  # predictors for all LTs
  cml.p <- Q1[1, ] # child mortality for all LTs
  cmls.p <- cml.p^2 # square of child mortality for all LTs
  predictors.q0 <- data.frame(cbind(cml.p,
    cmls.p))
  colnames(predictors.q0) <- c("cml", "cmls")
  q0.p <- predict.lm(q0.betas, newdata = predictors.q0)
  # replace the predicted values for q0 with those from
  # model above
  r.p[1, ] <- q0.p
} else {
  q0.betas <- NA
}

# store the models
ret.mods[[i]] <- list(aml = aml.betas, v1 = v1.betas,
  v2 = v2.betas, v3 = v3.betas, v4 = v4.betas,
  q0 = q0.betas)
names(ret.mods)[i] <- name

# results: sampled LTs store the reconstructed sampled
# LTs
ret.recon.samp[[i]] <- r.p[, samp]
names(ret.recon.samp)[i] <- name

# store the errors in the reconstructed sampled LTs
ret.error.samp[[i]] <- expit(q1[, samp]) - expit(r.p[, samp])
names(ret.error.samp)[i] <- name

# store summaries of errors in sampled LTs
ret.errsum.samp[[i]] <- summary(as.vector(as.matrix(ret.error.samp[[i]])))
names(ret.errsum.samp)[i] <- name

if (S == 1) {
  # results: out of sample LTs store the reconstructed out
  # of sample LTs
  ret.recon.nsamp[[i]] <- NA
  names(ret.recon.nsamp)[i] <- name

  # store the errors in the reconstructed out of sample

```

```

# LTs
ret.error.nsamp[[i]] <- NA
names(ret.error.nsamp)[i] <- name

# store the summaries of errors in out of sample LTs
ret.errsum.nsamp[[i]] <- NA
names(ret.errsum.nsamp)[i] <- name
} else {
# results: out of sample LTs store the reconstructed out
# of sample LTs
ret.recon.nsamp[[i]] <- r.p[, nsamp]
names(ret.recon.nsamp)[i] <- name

# store the errors in the reconstructed out of sample
# LTs
ret.error.nsamp[[i]] <- expit(ql[, nsamp]) -
expit(r.p[, nsamp])
names(ret.error.nsamp)[i] <- name

# store the summaries of errors in out of sample LTs
ret.errsum.nsamp[[i]] <- summary(as.vector(as.matrix(ret.error.nsamp[[i]])))
names(ret.errsum.nsamp)[i] <- name
}

# put all the return lists together into one big list
if (retAll == TRUE) {
    return(list(ql.samp = ret ql.samp # sampled qls
, ql.nsamp = ret ql.nsamp # out of sample qls
, names = ret.samp.names # sampled LT names
, svd = ret.svd # svd of sampled qls
, svd.sm = ret.svd.sm # smooth svd of sampled qls
, mods = ret.mods # models
, recon.samp = ret.recon.samp # sample reconstructions
, error.samp = ret.error.samp # sample errors
, recon.nsamp = ret.recon.nsamp # out of sample reconstructions
, error.nsamp = ret.error.nsamp # out of sample errors
, errsum.samp = ret.errsum.samp # summary of sample errors
, errsum.nsamp = ret.errsum.nsamp # summary of out of sample errors
, offset = offset # the offset necessary to reconstruct
,
retAll = retAll, adult = adult, q0fix = q0Fix,
smooth = smooth, C = C))
} else {
    return(list(errsum.samp = ret.errsum.samp # summary of sample errors
, errsum.nsamp = ret.errsum.nsamp # summary of out of sample errors
))
}
} else {
print("S must be larger than 0.0")
return()
}

```

```

    print("Done")
}

```

3.2 *ltPredict()* function

ltPredict() is a function that uses a return object from *svdMod()* to predict new life tables.

Inputs to the function:

- ‘mods’ is an output object from *svdMod()*
- ‘smooth’ is a switch indicating if the smoothed left singular vectors should be used for the prediction
- ‘cml’ is a value/vector for the input level(s) of logit-scale child mortality, ${}_5q_0$
- ‘aml’ is a value/vector for the input level(s) of logit-scale adult mortality, ${}_{45}q_{15}$

The return object is:

- ‘r.p’ – a data frame containing the predicted life table(s)

```

ltPredict <- function(mods, smooth, cml, aml) {

  # mods is an output object from svdMod() cml is a vector
  # of logit child mortality rates aml is a vector of
  # logit adult mortality rates if aml not supplied, then
  # aml predicted from cml smooth is a switch to use
  # smoothed SVD if it's available

  cm <- expit(cml)
  cmls <- cml^2
  cmlc <- cml^3
  preds.aml <- data.frame(cm = as.numeric(cm), cml = as.numeric(cml),
    cmls = as.numeric(cmls), cmlc = as.numeric(cmlc))
  if (missing(aml)) {
    # predict adult mortality
    aml <- predict(mods$mods$s1$aml, newdata = preds.aml)
  }

  # predict vs
  am <- expit(aml)
  amls <- aml^2
  amlc <- aml^3
  cmlaml <- cml * aml
  preds.vs <- data.frame(cm = as.numeric(cm), cml = as.numeric(cml),
    cmls = as.numeric(cmls), cmlc = as.numeric(cmlc),
    am = as.numeric(am), amls = as.numeric(amls), amlc = as.numeric(amlc),
    cmlaml = as.numeric(cmlaml))
  v1 <- predict(mods$mods$s1$v1, newdata = preds.vs)
  v2 <- predict(mods$mods$s1$v2, newdata = preds.vs)
  v3 <- predict(mods$mods$s1$v3, newdata = preds.vs)
  v4 <- predict(mods$mods$s1$v4, newdata = preds.vs)

  # if smoothed SVD available
  if (smooth & mods$smooth) {
    svd <- mods$svd.sm$s1
  } else {

```

```

        svd <- mods$svd$s1
    }

    # construct LTs
    v <- cbind(v1, v2, v3, v4)
    r.p <- matrix(data = 0, ncol = length(cml), nrow = length(svd$u[, 1]))
    for (z in 1:4) {
        r.p <- r.p + svd$d[z] * svd$u[, z] %*% t(v[, z])
    }
    r.p <- r.p + mods$offset

    # fix q0
    if (mods$q0fix) {
        cmls <- cml^2
        preds.q0 <- data.frame(cml = as.numeric(cml), cmls = as.numeric(cmls))
        r.p[1, ] <- predict(mods$mods$s1$q0, newdata = preds.q0)
    }

    # fix up r.p
    r.p <- data.frame(r.p)
    colnames(r.p) <- paste("cml.", cml, sep = "")
    rownames(r.p) <- rownames(mods$ql.samp$s1)

    # returns the matrix of predicted values
    return(r.p)
}

}

```

4 Validation

First, run the *svdComp()* function with one iteration and a 100% sample in both ‘base’ and ‘smoothed’ form using only child mortality as a direct input, i.e. ‘adult’ is set to FALSE, and specifying two components. This will yield SVD-Comp models calibrated on the entire HMD data set. The *svdComp()* function provides a little feedback, here indicating that two-component models were run on one sample of 100% with the child mortality-only model, either base or smoothed.

```

# specify some key parameters, just to be a bit more
# readable!
adult <- FALSE
smooth <- FALSE
N <- 1
S <- 1
C <- 4
offset <- 10
# base model
mod.1_0.m <- svdMod(qlogit.m, Qlogit.m, N, S, offset, TRUE,
                      adult, TRUE, smooth, C)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"

```

```

## [1] "4 components"
mod.1_0.f <- svdMod(q1logit.f, Qlogit.f, N, S, offset, TRUE,
                      adult, TRUE, smooth, C)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

# smooth now
smooth <- TRUE
mod.1_0.sm.m <- svdMod(q1logit.m, Qlogit.m, N, S, offset,
                        TRUE, adult, TRUE, smooth, C)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: TRUE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

mod.1_0.sm.f <- svdMod(q1logit.f, Qlogit.f, N, S, offset,
                        TRUE, adult, TRUE, smooth, C)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: TRUE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

```

To compare the predicted results from the SVD-comp model calibrated with the entire HMD to results produced by Wilmoth et al.'s Log Quad model, we must calculate five-year age group probabilities of dying, tq_x , because Log Quad operates only with five-year age groups. The following code uses the single-year age group predictions from SVD-comp to calculate five-year age group probabilities of dying.

```

# fast functions to calculate a vector of 5qx for any
# five-year age group indexed from 0 by 1

# ages 1-4
oneToFourYear <- function(oneYear) {
  return(1 - prod(sapply(2:5, function(x, y) (1 - y[x]),
                        y = oneYear)))
}

# five-year age groups
fiveYear <- function(start, oneYear) {
  return(1 - prod(sapply((start * 5 + 1):(start * 5 +
                                         5), function(x, y) (1 - y[x]), y = oneYear)))
}

# fast function to convert full schedule of 1qx into
# full schedule of 5qx
fiveYearQ <- function(oneYear) {

```

```

    sapply(0:(trunc(length(oneYear)/5 - 1)), function(x,
      y) fiveYear(x, y), y = oneYear)
}

# fast function to calculate 45q15 from 1qx
adultQ <- function(oneYear) {
  return(1 - prod(sapply(16:60, function(x, y) (1 - y[x]),
    y = oneYear)))
}

# fast function to calculate 5q0 from standard 5qx
childQ5 <- function(fiveYear) {
  return(1 - prod(sapply(1:2, function(x, y) (1 - y[x]),
    y = fiveYear)))
}

# fast function to calculate 45q15 from 5qx
adultQ5 <- function(fiveYear) {
  return(1 - prod(sapply(5:13, function(x, y) (1 - y[x]),
    y = fiveYear)))
}

# fast function to calculate a full standard 5-year age
# schedule
standardFiveYear <- function(oneYear) {
  l <- trunc(length(oneYear)/5)
  c(oneYear[1], oneToFourYear(oneYear), fiveYearQ(oneYear)[2:l])
}

# examples using single age schedule of 1qx
# adultQ(<data>) fiveYearQ(<data>)
# standardFiveYear(<data>) using a matrix of age
# schedules of 1qx apply(data,2,<function:adultQ or
# fiveYearQ or standardFiveYear>

# function to calculate a matrix of 5qx from a matrix of
# 1qx
convert1qxTo5qxApply <- function(q1) {

  # q1 contains values of 1qx and is an age by life table
  # matrix with at least two columns

  # q5 is returned: an age by life table matrix with 5qx

  q5 <- apply(q1, 2, standardFiveYear)
  colnames(q5) <- colnames(q1)
  rNames <- c("0", "1-4")
  for (i in seq(1, (trunc(nrow(q1)/5) - 1), 1)) {
    rNames <- c(rNames, paste(i * 5, (i * 5 + 4), sep = "-"))
  }
  rownames(q5) <- rNames
  return(q5)
}

```

```

# simpler and more readable approach which turns out to
# be roughly as fast or faster in this markdown
# document!

# function to convert 1qx to standard age group 5qx
convert1qxTo5qx <- function(q1) {

  # q1 contains values of 1qx and is an age by life table
  # matrix q5 is returned: an age by life table matrix with
  # 5qx

  q5 <- matrix(data = 0, ncol = ncol(q1), nrow = 23)
  rNames <- rep("", 23)
  # age 0
  q5[1, ] <- as.matrix(q1[1, ])
  rNames[1] <- "0"
  # ages 1-4
  tmp.q <- rep(1, ncol(q1))
  for (i in 2:5) {
    tmp.q <- tmp.q * (1 - q1[i, ])
  }
  q5[2, ] <- as.matrix(1 - tmp.q)
  rNames[2] <- "1-4"
  # five-year age groups for ages 5-105 (ending 110)
  for (j in 1:21) {
    tmp.q <- rep(1, ncol(q1))
    for (i in (j * 5 + 1):(j * 5 + 5)) {
      tmp.q <- tmp.q * (1 - q1[i, ])
    }
    q5[(j + 2), ] <- as.matrix(1 - tmp.q)
    rNames[(j + 2)] <- paste((j * 5), "-", (j * 5 +
      4), sep = "")
  }
  rownames(q5) <- rNames
  colnames(q5) <- colnames(q1)
  return(q5)
}

# compare the speed of the two approaches
start.time.apply <- Sys.time()
tmp.apply <- convert1qxTo5qxApply(expit(mod.1_0.f$recon.samp$s1))
stop.time.apply <- Sys.time()
start.time.loop <- Sys.time()
tmp.loop <- convert1qxTo5qxApply(expit(mod.1_0.f$recon.samp$s1))
stop.time.loop <- Sys.time()
# results!
print(paste("Loop way:", stop.time.loop - start.time.loop))

## [1] "Loop way: 1.88644886016846"
print(paste("Apply way:", stop.time.apply - start.time.apply))

## [1] "Apply way: 1.80632400512695"

```

```

# apply way usually a tiny bit faster

# check to be sure they produce same answer
all.equal(tmp.loop, tmp.apply)

## [1] TRUE

# looks like it!
rm(list = c("tmp.loop", "tmp.apply"))

# Now actually calculate the 5qx schedules from the
# predicted 1qx

# female
q5p.f <- convert1qxTo5qxApply(expit(mod.1_0.f$recon.samp$s1))

# male
q5p.m <- convert1qxTo5qxApply(expit(mod.1_0.m$recon.samp$s1))

```

R code supplied by Wilmoth et al. is used to calculate the predicted five-year age group probabilities of dying using the Log Quad model using the same inputs as those used by SVD-Comp: the ${}_5q_0$ and ${}_45q_{15}$ values stored in the ‘Q.f’ and ‘Q.m’ matrices – logit-transformed (‘Qlogit.f’ and ‘Qlogit.m’) for SVD-Comp. For more information on the Log Quad model code download here (www.demog.berkeley.edu/~jrw/LogQuad). First create a function to do the comparisons.

```

# function to conduct comparison of predicted 5qx from SVD-Comp
# and Log-Quad
doComparison <- function(q1logit.f,Qlogit.f,q1logit.m,Qlogit.m
                          ,q5.f,q5.m,N,S,offset,adult,smooth,C) {

  # q1logit.f - female logit 1qx
  # Qlogit.f - female child and adult mortality levels
  # q1logit.m - male logit 1qx
  # Qlogit.m - male child and adult mortality levels
  # q5.f - female 5qx values from HMD site
  # q5.m - male 5qx values from HMD site
  # N - number of samples taken
  # S - sample fraction
  # offset - size of offset
  # adult - include adult mortality directly
  # smooth - use smoothing
  # C - number of components to use

  # rerun models setting using adult mortality directly
  mod.1_0.m <- svdMod(q1logit.m,Qlogit.m,N,S,10,TRUE,adult,TRUE,smooth,C)
  mod.1_0.f <- svdMod(q1logit.f,Qlogit.f,N,S,10,TRUE,adult,TRUE,smooth,C)

  # store the predicted values from the model in five-year age groups
  q5p.f <- convert1qxTo5qxApply(expit(mod.1_0.f$recon.samp$s1))
  q5p.m <- convert1qxTo5qxApply(expit(mod.1_0.m$recon.samp$s1))

  # create Log-Quad predictions
  # fit the log-quad using child mortality only

  # Source functions file

```

```

source("../R/logQuad/DataProgramsExamples/R/functions.R")

# Create labels for age vectors
ages.5x1 <- c("0","1-4",paste(seq(5,105,5),seq(9,109,5),sep="-"),"110+")
sexes <- c("Female","Male","Total")

# Import matrix of model coefficients
tmp1 <- read.csv("../R/logQuad/DataProgramsExamples/Data/coefs.logquad.HMD719.csv")
tmp2 <- array(c(as.matrix(tmp1[, 3:6]))
              , dim=c(24, 3, 4)
              , dimnames=list(ages.5x1, sexes, c("ax", "bx", "cx", "vx")))
coefs <- aperm(tmp2, c(1,3,2))

# female
q5.lq.f <- q5.f - q5.f
e5.lq.f <- rbind(q5.f - q5.f,rep(0,ncol(q5.f)))
a5.lq.f <- rbind(q5.f - q5.f,rep(0,ncol(q5.f)))
l5.lq.f <- rbind(q5.f - q5.f,rep(0,ncol(q5.f)))
for (i in 1:ncol(q5.f)) {
  if (adult) {
    lqfit <- lthat.any2.logquad(coefs,"Female",Q5=Q.f[1,i],QQa=Q.f[2,i]) # with adult
  } else {
    lqfit <- lthat.any2.logquad(coefs,"Female",Q5=Q.f[1,i],k=0) # without adult
  }
  q5.lq.f[,i] <- lqfit$lt[1:23,2]
  a5.lq.f[,i] <- lqfit$lt[1:24,3]
  e5.lq.f[,i] <- lqfit$lt[1:24,8]
  l5.lq.f[,i] <- lqfit$lt[1:24,4]
}
q5.lq.f <- log(q5.lq.f)
q5logit.lq.f <- logit(q5.lq.f)

# male
q5.lq.m <- q5.m - q5.m
e5.lq.m <- rbind(q5.m - q5.m,rep(0,ncol(q5.m)))
a5.lq.m <- rbind(q5.m - q5.m,rep(0,ncol(q5.m)))
l5.lq.m <- rbind(q5.m - q5.m,rep(0,ncol(q5.m)))
for (i in 1:ncol(q5.m)) {
  if (adult) {
    lqfit <- lthat.any2.logquad(coefs,"Male",Q5=Q.m[1,i],QQa=Q.m[2,i]) # with adult
  } else {
    lqfit <- lthat.any2.logquad(coefs,"Male",Q5=Q.m[1,i],k=0) # without adult
  }
  q5.lq.m[,i] <- lqfit$lt[1:23,2]
  a5.lq.m[,i] <- lqfit$lt[1:24,3]
  e5.lq.m[,i] <- lqfit$lt[1:24,8]
  l5.lq.m[,i] <- lqfit$lt[1:24,4]
}
q5.lq.m <- log(q5.lq.m)
q5logit.lq.m <- logit(q5.lq.m)

# compare the fits using the 5qx values obtained
# directly from the HMD web site, q5.f and q5.m

```

```

# construct a vector of comparison descriptors

# females
comps.f <- matrix(data = 0, nrow = 2, ncol = 3)
colnames(comps.f) <- c("total.abs.error","mean.abs.error","max.error")
rownames(comps.f) <- c("comp","lq")
comps.f[1,1] <- sum(abs(q5.f - q5p.f))
comps.f[2,1] <- sum(abs(q5.f - q5.lq.f))
comps.f[,2] <- comps.f[,1]/(ncol(q5p.f)*nrow(q5p.f))
comps.f[1,3] <- max(q5.f - q5p.f)
comps.f[2,3] <- max(q5.f - q5.lq.f)

# males
comps.m <- matrix(data = 0, nrow = 2, ncol = 3)
colnames(comps.m) <- c("total.abs.error","mean.abs.error","max.error")
rownames(comps.m) <- c("comp","lq")
comps.m[1,1] <- sum(abs(q5.m - q5p.m))
comps.m[2,1] <- sum(abs(q5.m - q5.lq.m))
comps.m[,2] <- comps.m[,1]/(ncol(q5p.m)*nrow(q5p.m))
comps.m[1,3] <- max(q5.m - q5p.m)
comps.m[2,3] <- max(q5.m - q5.lq.m)

comps <- list(
  female = comps.f
  ,male = comps.m

  ,q5p.f = q5p.f
  ,q5p.m = q5p.m

  ,q5.lq.f = q5.lq.f
  ,e5.lq.f = e5.lq.f
  ,a5.lq.f = a5.lq.f
  ,15.lq.f = 15.lq.f
  ,q51.lq.f = q51.lq.f
  ,q5logit.lq.f = q5logit.lq.f

  ,q5.lq.m = q5.lq.m
  ,e5.lq.m = e5.lq.m
  ,a5.lq.m = a5.lq.m
  ,15.lq.m = 15.lq.m
  ,q51.lq.m = q51.lq.m
  ,q5logit.lq.m = q5logit.lq.m
)

return(comps)
}

Now compare the models first using only child mortality  ${}_5q_0$  to predict and then using both child  ${}_5q_0$  and adult  ${}_{15}q_{15}$  mortality to predict.

```

```

# set basic model parameters
smooth <- FALSE
N <- 1

```

```

S <- 1
C <- 4
offset <- 10

# do comparison between SVD-Comp and Log-Quad using only
# child mortality as an input
comps.child <- doComparison(q1logit.f, Qlogit.f, q1logit.m,
    Qlogit.m, q5.f, q5.m, N, S, offset, adult = FALSE, smooth,
    C)

## 
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

# do comparison between SVD-Comp and Log-Quad using both
# child and adult mortality as inputs
comps.adult <- doComparison(q1logit.f, Qlogit.f, q1logit.m,
    Qlogit.m, q5.f, q5.m, N, S, offset, adult = TRUE, smooth,
    C)

## 
## [1] "Adult mortality is direct input to predictions: TRUE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: TRUE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

# have a look
cat("\n")

comps.child$female

##      total.abs.error mean.abs.error max.error
## comp        1445.741     0.01363521 0.3306283
## lq         1501.606     0.01416209 0.3968440

comps.child$male

##      total.abs.error mean.abs.error max.error
## comp        1674.491     0.01579262 0.3773859
## lq         1777.257     0.01676183 0.3792040

```

```

cat("\n")
comps.adult$female

##      total.abs.error mean.abs.error max.error
## comp        1297.786     0.01223980 0.2204735
## lq         1399.336     0.01319754 0.3865020

comps.adult$male

##      total.abs.error mean.abs.error max.error
## comp        1378.421     0.01300029 0.3866729
## lq         1472.245     0.01388518 0.3532550

Run 50 50% samples to summarize one-year age group prediction errors.

# 50 runs with 50% sampling proportion
adult <- FALSE
smooth <- FALSE
N <- 50
S <- 0.5
mod.0_5.50.m <- svdMod(q1logit.m, Qlogit.m, N, S, 10, TRUE,
                        adult, TRUE, smooth, C)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "50% sample fraction"
## [1] "4 components"

mod.0_5.50.f <- svdMod(q1logit.f, Qlogit.f, N, S, 10, TRUE,
                        adult, TRUE, smooth, C)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "50% sample fraction"
## [1] "4 components"

# female

# sampled errors aggregate errors by age
error.age.f <- matrix(data = 0, ncol = 0, nrow = length(mod.0_5.50.f$error.samp$s1[, 1]))
for (i in 1:N) {
  # print(i)
  error.age.f <- cbind(error.age.f, as.matrix(mod.0_5.50.f$error.samp[[i]]))
}

# out of sample errors aggregate errors by age
error.age.nsamp.f <- matrix(data = 0, ncol = 0, nrow = length(mod.0_5.50.f$error.nsamp$s1[, 1]))
for (i in 1:N) {
  # print(i)
  error.age.nsamp.f <- cbind(error.age.nsamp.f, as.matrix(mod.0_5.50.f$error.nsamp[[i]]))
}

```

```

}

# male

# sampled errors aggregate errors by age
error.age.m <- matrix(data = 0, ncol = 0, nrow = length(mod.0_5.50.m$error.samp$s1[, 1]))
for (i in 1:N) {
  # print(i)
  error.age.m <- cbind(error.age.m, as.matrix(mod.0_5.50.m$error.samp[[i]]))
}

# out of sample errors aggregate errors by age
error.age.nsamp.m <- matrix(data = 0, ncol = 0, nrow = length(mod.0_5.50.m$error.nsamp$s1[, 1]))
for (i in 1:N) {
  # print(i)
  error.age.nsamp.m <- cbind(error.age.nsamp.m, as.matrix(mod.0_5.50.m$error.nsamp[[i]]))
}

```

Run 50 samples with sampling fractions 10%, 30%, 50%, 70%, and 90% to summarize and characterize prediction errors as the sample fraction varies.

```

# female
adult <- FALSE
smooth <- FALSE
N <- 50
for (S in seq(0.1, 0.9, 0.2)) {
  assign(paste("qlPred_", S, ".f", sep = ""), svdMod(q1logit.f,
    Q1.f, N, S, 10, FALSE, adult, TRUE, smooth, C))
}

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "10% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "30% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "50% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "70% sample fraction"

```

```

## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "90% sample fraction"
## [1] "4 components"

# ... this could be more elegant as a list or by usign
# 'assign' like above, but ... summary errors
errsum.meds.1.f <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.1.f <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.1.f[i, 1] <- qlPred_0.1.f$errsum.samp[[i]][3]
  errsum.meds.1.f[i, 2] <- qlPred_0.1.f$errsum.nsamp[[i]][3]
  errsum.iqrs.1.f[i, 1] <- qlPred_0.1.f$errsum.samp[[i]][5] -
    qlPred_0.1.f$errsum.samp[[i]][2]
  errsum.iqrs.1.f[i, 2] <- qlPred_0.1.f$errsum.nsamp[[i]][5] -
    qlPred_0.1.f$errsum.nsamp[[i]][2]
}
# summary errors
errsum.meds.3.f <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.3.f <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.3.f[i, 1] <- qlPred_0.3.f$errsum.samp[[i]][3]
  errsum.meds.3.f[i, 2] <- qlPred_0.3.f$errsum.nsamp[[i]][3]
  errsum.iqrs.3.f[i, 1] <- qlPred_0.3.f$errsum.samp[[i]][5] -
    qlPred_0.3.f$errsum.samp[[i]][2]
  errsum.iqrs.3.f[i, 2] <- qlPred_0.3.f$errsum.nsamp[[i]][5] -
    qlPred_0.3.f$errsum.nsamp[[i]][2]
}
# summary errors
errsum.meds.5.f <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.5.f <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.5.f[i, 1] <- qlPred_0.5.f$errsum.samp[[i]][3]
  errsum.meds.5.f[i, 2] <- qlPred_0.5.f$errsum.nsamp[[i]][3]
  errsum.iqrs.5.f[i, 1] <- qlPred_0.5.f$errsum.samp[[i]][5] -
    qlPred_0.5.f$errsum.samp[[i]][2]
  errsum.iqrs.5.f[i, 2] <- qlPred_0.5.f$errsum.nsamp[[i]][5] -
    qlPred_0.5.f$errsum.nsamp[[i]][2]
}
# summary errors
errsum.meds.7.f <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.7.f <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.7.f[i, 1] <- qlPred_0.7.f$errsum.samp[[i]][3]
  errsum.meds.7.f[i, 2] <- qlPred_0.7.f$errsum.nsamp[[i]][3]
  errsum.iqrs.7.f[i, 1] <- qlPred_0.7.f$errsum.samp[[i]][5] -
    qlPred_0.7.f$errsum.samp[[i]][2]
  errsum.iqrs.7.f[i, 2] <- qlPred_0.7.f$errsum.nsamp[[i]][5] -
    qlPred_0.7.f$errsum.nsamp[[i]][2]
}

```

```

}

# summary errors
errsum.meds.9.f <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqr.9.f <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.9.f[i, 1] <- qlPred_0.9.f$errsum.samp[[i]][3]
  errsum.meds.9.f[i, 2] <- qlPred_0.9.f$errsum.nsamp[[i]][3]
  errsum.iqr.9.f[i, 1] <- qlPred_0.9.f$errsum.samp[[i]][5] -
    qlPred_0.9.f$errsum.samp[[i]][2]
  errsum.iqr.9.f[i, 2] <- qlPred_0.9.f$errsum.nsamp[[i]][5] -
    qlPred_0.9.f$errsum.nsamp[[i]][2]
}

# male
adult <- FALSE
smooth <- FALSE
N <- 50
for (S in seq(0.1, 0.9, 0.2)) {
  assign(paste("qlPred_", S, ".m", sep = ""), svdMod(q1logit.m,
    Q1.m, N, S, 10, FALSE, adult, TRUE, smooth, C))
}

## 
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "10% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "30% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "50% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "70% sample fraction"
## [1] "4 components"
##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "50 iterations"
## [1] "90% sample fraction"
## [1] "4 components"

```

```

# ... this could be more elegant as a list or by usign
# 'assign' like above, but ... summary errors
errsum.meds.1.m <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.1.m <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.1.m[i, 1] <- qlPred_0.1.m$errsum.samp[[i]][3]
  errsum.meds.1.m[i, 2] <- qlPred_0.1.m$errsum.nsamp[[i]][3]
  errsum.iqrs.1.m[i, 1] <- qlPred_0.1.m$errsum.samp[[i]][5] -
    qlPred_0.1.m$errsum.samp[[i]][2]
  errsum.iqrs.1.m[i, 2] <- qlPred_0.1.m$errsum.nsamp[[i]][5] -
    qlPred_0.1.m$errsum.nsamp[[i]][2]
}
# summary errors
errsum.meds.3.m <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.3.m <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.3.m[i, 1] <- qlPred_0.3.m$errsum.samp[[i]][3]
  errsum.meds.3.m[i, 2] <- qlPred_0.3.m$errsum.nsamp[[i]][3]
  errsum.iqrs.3.m[i, 1] <- qlPred_0.3.m$errsum.samp[[i]][5] -
    qlPred_0.3.m$errsum.samp[[i]][2]
  errsum.iqrs.3.m[i, 2] <- qlPred_0.3.m$errsum.nsamp[[i]][5] -
    qlPred_0.3.m$errsum.nsamp[[i]][2]
}
# summary errors
errsum.meds.5.m <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.5.m <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.5.m[i, 1] <- qlPred_0.5.m$errsum.samp[[i]][3]
  errsum.meds.5.m[i, 2] <- qlPred_0.5.m$errsum.nsamp[[i]][3]
  errsum.iqrs.5.m[i, 1] <- qlPred_0.5.m$errsum.samp[[i]][5] -
    qlPred_0.5.m$errsum.samp[[i]][2]
  errsum.iqrs.5.m[i, 2] <- qlPred_0.5.m$errsum.nsamp[[i]][5] -
    qlPred_0.5.m$errsum.nsamp[[i]][2]
}
# summary errors
errsum.meds.7.m <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.7.m <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.7.m[i, 1] <- qlPred_0.7.m$errsum.samp[[i]][3]
  errsum.meds.7.m[i, 2] <- qlPred_0.7.m$errsum.nsamp[[i]][3]
  errsum.iqrs.7.m[i, 1] <- qlPred_0.7.m$errsum.samp[[i]][5] -
    qlPred_0.7.m$errsum.samp[[i]][2]
  errsum.iqrs.7.m[i, 2] <- qlPred_0.7.m$errsum.nsamp[[i]][5] -
    qlPred_0.7.m$errsum.nsamp[[i]][2]
}
# summary errors
errsum.meds.9.m <- matrix(data = 0, ncol = 2, nrow = N)
errsum.iqrs.9.m <- matrix(data = 0, ncol = 2, nrow = N)
for (i in 1:N) {
  errsum.meds.9.m[i, 1] <- qlPred_0.9.m$errsum.samp[[i]][3]
}

```

```

errsum.meds.9.m[i, 2] <- qlPred_0.9.m$errsum.nsamp[[i]][3]
errsum.iqrs.9.m[i, 1] <- qlPred_0.9.m$errsum.samp[[i]][5] -
  qlPred_0.9.m$errsum.samp[[i]][2]
errsum.iqrs.9.m[i, 2] <- qlPred_0.9.m$errsum.nsamp[[i]][5] -
  qlPred_0.9.m$errsum.nsamp[[i]][2]
}

```

5 Plotting

Most plotting is done using *ggplot*. First load the necessary packages. The plots are not generated in the same order of appearance as the paper.

Plot the basic age \times age relationships among $5q_x$ for all ages and both sexes and save as (very large) PDF files.

```

# age relationships

# female
pdf(file = "../figures/femaleLogit(q)AgeScatterplots.pdf")
qln.f <- as.matrix(q1logit.f)
nr <- nrow(qln.f)
for (i in seq(0, 100, 5)) {
  for (j in seq(i, 100, 5)) {
    plot(qln.f[(j + 1), ] ~ qln.f[(i + 1), ], cex = 0.1,
        xlab = paste("Age ", i, sep = ""), ylab = paste("Age ",
                  j, sep = ""), xlim = c(-12, 0), ylim = c(-12,
                  0), main = "Logit(q) by Logit(q) in 5-year Age Groups")
  }
}
dev.off()

## pdf
## 2

# male
pdf(file = "../figures/maleLogit(q)AgeScatterplots.pdf")
qln.m <- as.matrix(q1logit.m)
nr <- nrow(qln.m)
for (i in seq(0, 100, 5)) {
  for (j in seq(i, 100, 5)) {
    plot(qln.m[(j + 1), ] ~ qln.m[(i + 1), ], cex = 0.1,
        xlab = paste("Age ", i, sep = ""), ylab = paste("Age ",
                  j, sep = ""), xlim = c(-12, 0), ylim = c(-12,
                  0), main = "Logit(q) by Logit(q) in 5-year Age Groups")
  }
}
dev.off()

## pdf
## 2
rm(list = c("nr", "i", "j"))

```

Plot the SVD-comp and Log Quad error distributions by sex and age using 25%, 50%, and 75% quantiles and whiskers to 10% and 90%.

```

# the predicted values from both models are stored in
# comps.child and comps.adult; we are making comparisons
# using the child-only predictions the q5.x values are
# straight from HMD errors on natural scale female
errors.comp.f <- q5.f - comps.child$q5p.f
errors.lq.f <- q5.f - comps.child$q5.lq.f
# male
errors.comp.m <- q5.m - comps.child$q5p.m
errors.lq.m <- q5.m - comps.child$q5.lq.m

# reshape the data

# female
ecf <- melt(as.matrix(errors.comp.f))
elf <- melt(as.matrix(errors.lq.f))
ecf <- cbind(ecf[, c(1, 3)], rep("SVD-Comp", nrow(ecf)))
colnames(ecf) <- c("Age (years)", "Error", "Model")
elf <- cbind(elf[, c(1, 3)], rep("Log-Quad", nrow(elf)))
colnames(elf) <- c("Age (years)", "Error", "Model")
ef <- rbind(ecf, elf)

# male
ecm <- melt(as.matrix(errors.comp.m))
elm <- melt(as.matrix(errors.lq.m))
ecm <- cbind(ecm[, c(1, 3)], rep("SVD-Comp", nrow(ecm)))
colnames(ecm) <- c("Age (years)", "Error", "Model")
elm <- cbind(elm[, c(1, 3)], rep("Log-Quad", nrow(elm)))
colnames(elm) <- c("Age (years)", "Error", "Model")
em <- rbind(ecm, elm)

efn <- cbind(em, "Female")
colnames_efn <- c("Age (years)", "Error", "Model", "Sex")
emn <- cbind(em, "Male")
colnames_emn <- c("Age (years)", "Error", "Model", "Sex")

e <- rbind(efn, emn)

e.sum <- ddply(e, .(Sex, `Age (years)`), summarize,
  ymin = quantile(Error, 0.1), ymax = quantile(Error,
  0.9), middle = median>Error, lower = quantile>Error,
  0.25), upper = quantile>Error, 0.75))

s.names <- list(`$`1` = expression(bold("Female")), `$`2` = expression(bold("Male")))
# s.names

s.labeller <- function(variable, value) {
  return(s.names[value])
}

ggplot(data = e.sum, aes(x = `Age (years)`)) + geom_boxplot(aes(fill = Model,
  ymin = ymin, ymax = ymax, middle = middle, upper = upper,
  lower = lower), stat = "identity", size = 0.2) + scale_y_continuous(limits = c(-0.08,
  0.08)) + # theme(legend.justification=c(1,0),

```

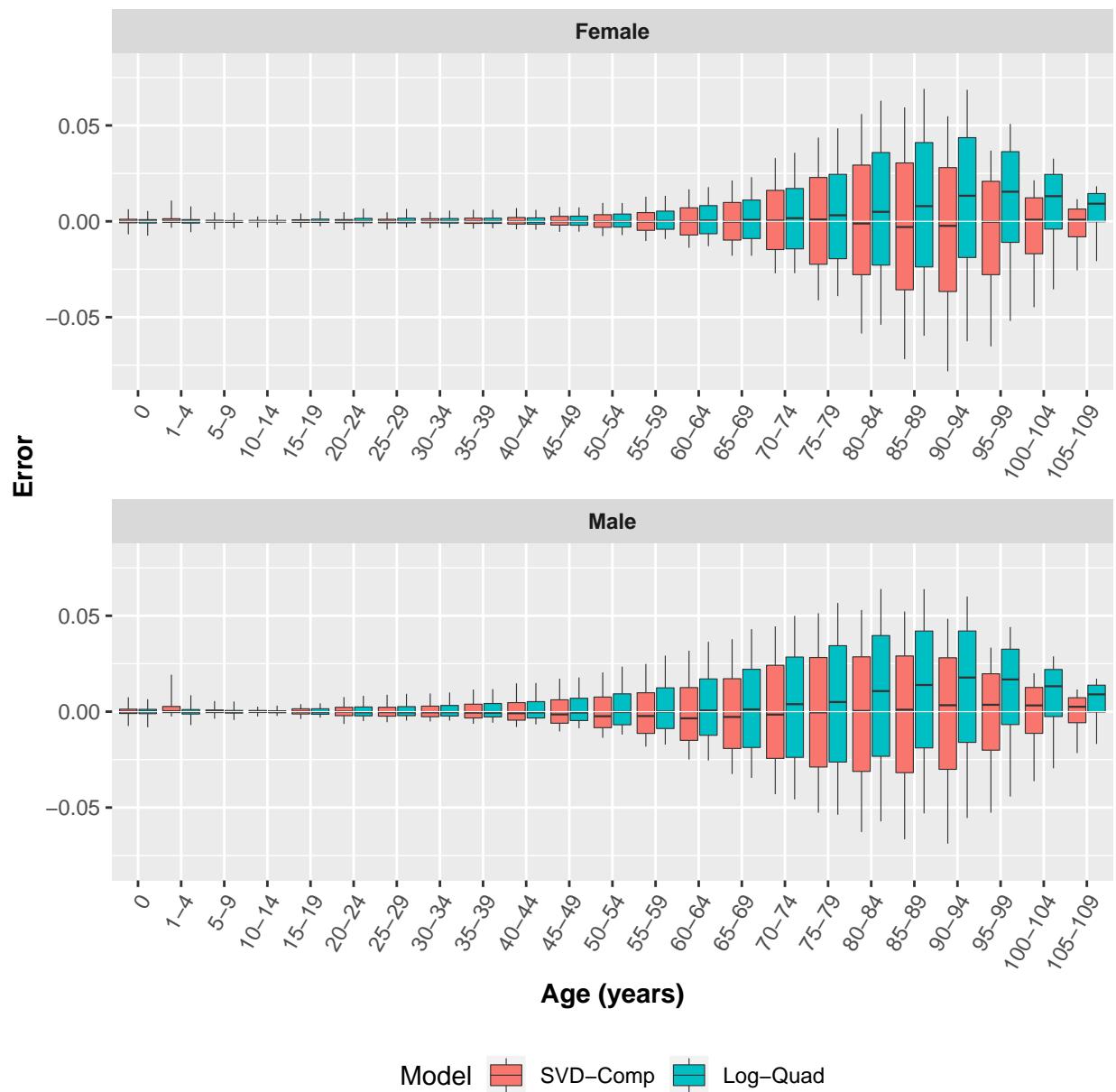
```

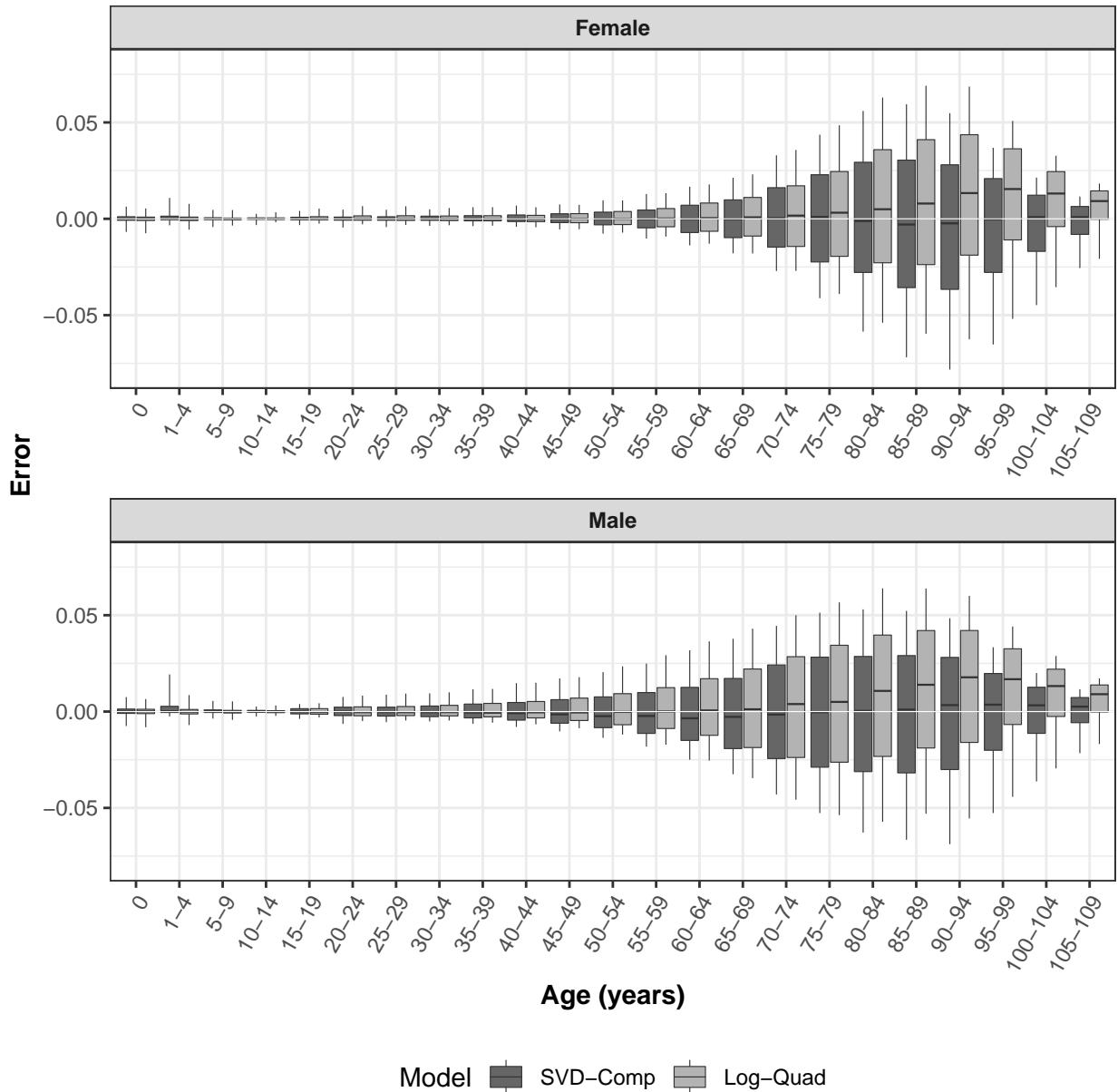
# legend.position=c(.22,0.02)) +
theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(axis.text.x = element_text(angle = 60, hjust = 1)) +
  geom_hline(yintercept = 0, colour = "white", lwd = 0.2) +
  labs(y = expression(bold("Error")), x = expression(bold("Age (years)"))) +
  facet_wrap(~Sex, ncol = 1, scales = "free", labeller = s.labeller) +
  ggsave("../figures/fig3.pdf", width = 6.5, height = 6.5,
         units = c("in"))

# grayscale
ggplot(data = e.sum, aes(x = `Age (years)`)) + geom_boxplot(aes(fill = Model,
  ymin = ymin, ymax = ymax, middle = middle, upper = upper,
  lower = lower), stat = "identity", size = 0.2) + scale_y_continuous(limits = c(-0.08,
  0.08)) + # theme(legend.justification=c(1,0),
# legend.position=c(.22,0.02)) +
theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(axis.text.x = element_text(angle = 60, hjust = 1),
        ) + geom_hline(yintercept = 0, colour = "white",
        lwd = 0.2) + labs(y = expression(bold("Error")), x = expression(bold("Age (years)"))) +
  facet_wrap(~Sex, ncol = 1, scales = "free", labeller = s.labeller) +
  scale_fill_grey(start = 0.4, end = 0.7) + ggsave("../figures/fig3-BW.pdf",
  width = 6.5, height = 6.5, units = c("in"))

# clean up
rm(list = c("e.sum", "e", "efn", "emn", "em", "elm", "ecm",
  "ef", "elf", "ecf"))

```





Plot the example mortality schedules with data and predictions from SVD-Comp. Use France 1978 as a low mortality example and Sweden 1751 as a high mortality example.

```
# calculate the total absolute error per life table
tot.abs.err.f <- colSums(abs(errors.comp.f))
tot.abs.err.m <- colSums(abs(errors.comp.m))
# using that metric, look for best fitting female and
# male LTs
best.f <- which(tot.abs.err.f == min(tot.abs.err.f))
best.f

## female.DEUTE.2006
##                 824

best.m <- which(tot.abs.err.m == min(tot.abs.err.m))
best.m
```

```

## male.DNK.2010
##          1095
# looks like East Germany 2006 for females and Denmark
# 2010 for males

# find the earliest Swedish LT
# cat(colnames(q1.f),sep='\n') # lists all the female
# LTs; use with caution - very long
swe.f.1751 <- which(colnames(q1.f) == "female.SWE.1751")
swe.f.1751

## [1] 4161
swe.m.1751 <- which(colnames(q1.m) == "male.SWE.1751")
swe.m.1751

## [1] 4161
# looks like Sweden 1751 is number 4,140; double check
cat(colnames(q1.f)[(swe.f.1751 - 3):(swe.f.1751 + 3)], sep = "\n")

## female.SVN.2012
## female.SVN.2013
## female.SVN.2014
## female.SWE.1751
## female.SWE.1752
## female.SWE.1753
## female.SWE.1754

# have a quick look at both the 'best' fitting LTs and
# choose one
i <- best.f
plot(q1logit.f[, i], )
points(q1logit.m[, i], col = "red")
points(mod.1_0.f$recon.samp$s1[, i], type = "l")
points(mod.1_0.m$recon.samp$s1[, i], type = "l", col = "red")
i <- best.m
plot(q1logit.f[, i], )
points(q1logit.m[, i], col = "red")
points(mod.1_0.f$recon.samp$s1[, i], type = "l")
points(mod.1_0.m$recon.samp$s1[, i], type = "l", col = "red")
# don't like either of them much as pretty examples

# Austria 1990 is nice example of low mortality - will
# use that for low mortality example
aut.f.1990 <- which(colnames(q1.f) == "female.AUT.1990")
aut.f.1990

## [1] 138
aut.m.1990 <- which(colnames(q1.m) == "male.AUT.1990")
aut.m.1990

## [1] 138
i <- aut.f.1990
plot(q1logit.f[, i], )
points(q1logit.m[, i], col = "red")

```

```

points(mod.1_0.f$recon.samp$s1[, i], type = "l")
points(mod.1_0.m$recon.samp$s1[, i], type = "l", col = "red")

# data - use Sweden 1751 and Austria 1990:
i.low <- aut.f.1990
i.high <- swe.f.1751
tmp.1.m <- cbind(rep("Male", 110), rownames(q1logit.m),
  rep("Sweden, 1751", 110), "Data", q1logit.m[, i.high])
tmp.1.f <- cbind(rep("Female", 110), rownames(q1logit.m),
  rep("Sweden, 1751", 110), "Data", q1logit.f[, i.high])
tmp.2.m <- cbind(rep("Male", 110), rownames(q1logit.m),
  rep("Austria, 1990", 110), "Data", q1logit.m[, i.low])
tmp.2.f <- cbind(rep("Female", 110), rownames(q1logit.m),
  rep("Austria, 1990", 110), "Data", q1logit.f[, i.low])
data.fig1 <- rbind(tmp.1.m, tmp.1.f, tmp.2.m, tmp.2.f)

data.fig1.df <- data.frame(Sex = as.character(data.fig1[, 1]),
  Age = as.numeric(data.fig1[, 2]), LT = as.character(data.fig1[, 3]),
  Type = as.character(data.fig1[, 4]), Value = as.numeric(data.fig1[, 5]))

# predictions
m.1.p <- mod.1_0.m$recon.samp$s1[, i.high]
f.1.p <- mod.1_0.f$recon.samp$s1[, i.high]
m.2.p <- mod.1_0.m$recon.samp$s1[, i.low]
f.2.p <- mod.1_0.f$recon.samp$s1[, i.low]
tmp.1.m <- cbind(rep("Male", 110), rownames(q1logit.m),
  rep("Sweden, 1751", 110), "Predicted", m.1.p)
tmp.1.f <- cbind(rep("Female", 110), rownames(q1logit.m),
  rep("Sweden, 1751", 110), "Predicted", f.1.p)
tmp.2.m <- cbind(rep("Male", 110), rownames(q1logit.m),
  rep("Austria, 1990", 110), "Predicted", m.2.p)
tmp.2.f <- cbind(rep("Female", 110), rownames(q1logit.m),
  rep("Austria, 1990", 110), "Predicted", f.2.p)
pred.fig1 <- rbind(tmp.1.m, tmp.1.f, tmp.2.m, tmp.2.f)

pred.fig1.df <- data.frame(Sex = as.character(pred.fig1[, 1]),
  Age = as.numeric(pred.fig1[, 2]), LT = as.character(pred.fig1[, 3]),
  Type = as.character(pred.fig1[, 4]), Value = as.numeric(pred.fig1[, 5]))

# Plot
ggplot(data = data.fig1.df, aes(x = Age, y = Value, group = interaction(Sex,
  LT), colour = Sex, shape = LT)) + geom_point(size = 1.5) +
  geom_line(data = pred.fig1.df, aes(x = Age, y = Value,
    group = interaction(Sex, LT), colour = Sex), size = 1) +
  scale_x_continuous(breaks = seq(0, 110, 5)) + labs(y = expression(""["bold(1)"] *
  bolditalic("q") [bolditalic(x)] * bold(" (logit scale)")),
  x = expression(bold("Age (years)"))) + # theme(legend.justification=c(1,0),
# legend.position=c(0.98,0.02)) +
  theme(legend.position = "bottom", legend.box = "horizontal") +
  scale_shape_discrete(name = "Life Table") + ggsave("../figures/fig1.pdf",
  width = 6.5, height = 6.5, units = c("in"))

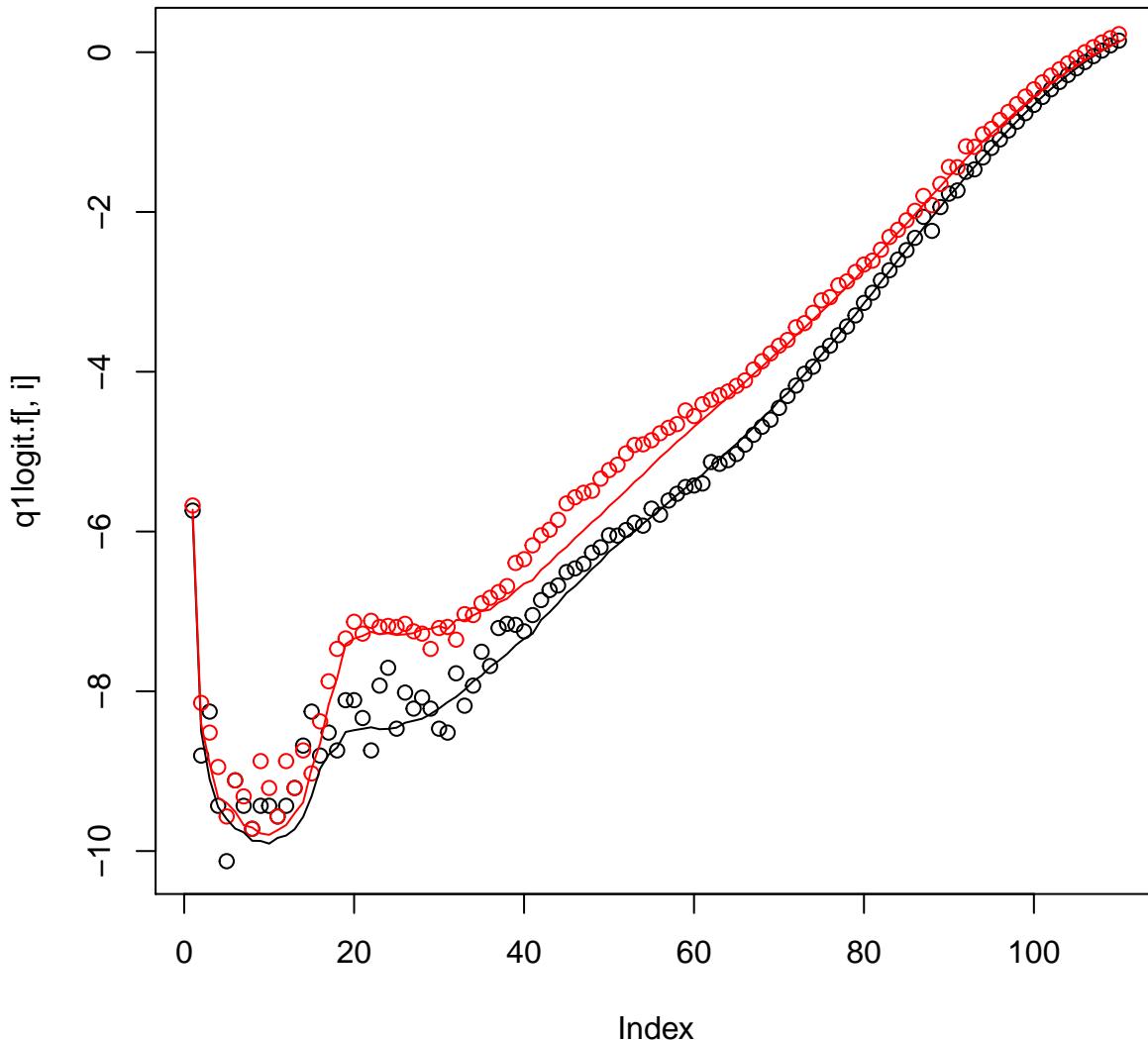
```

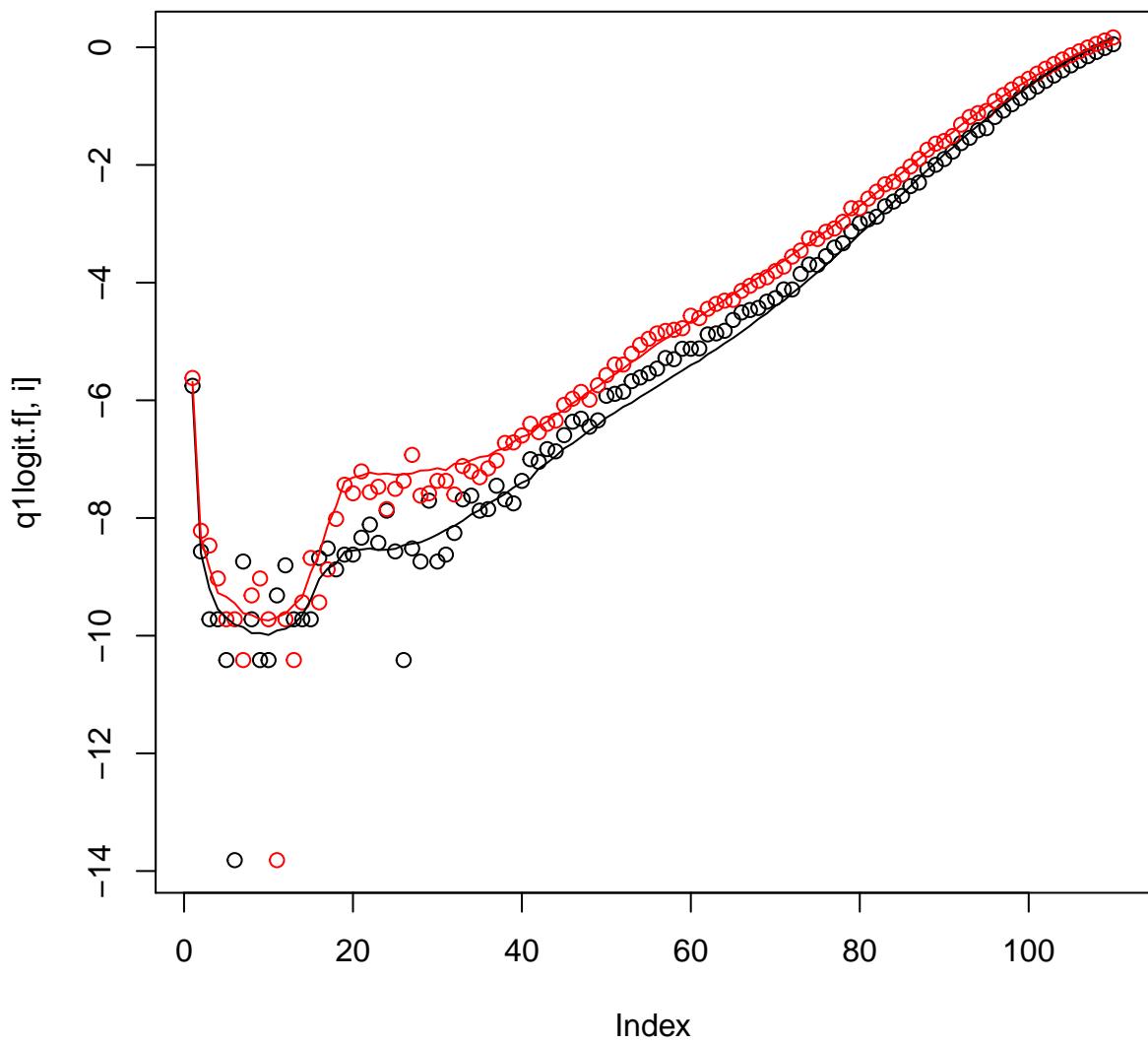
```

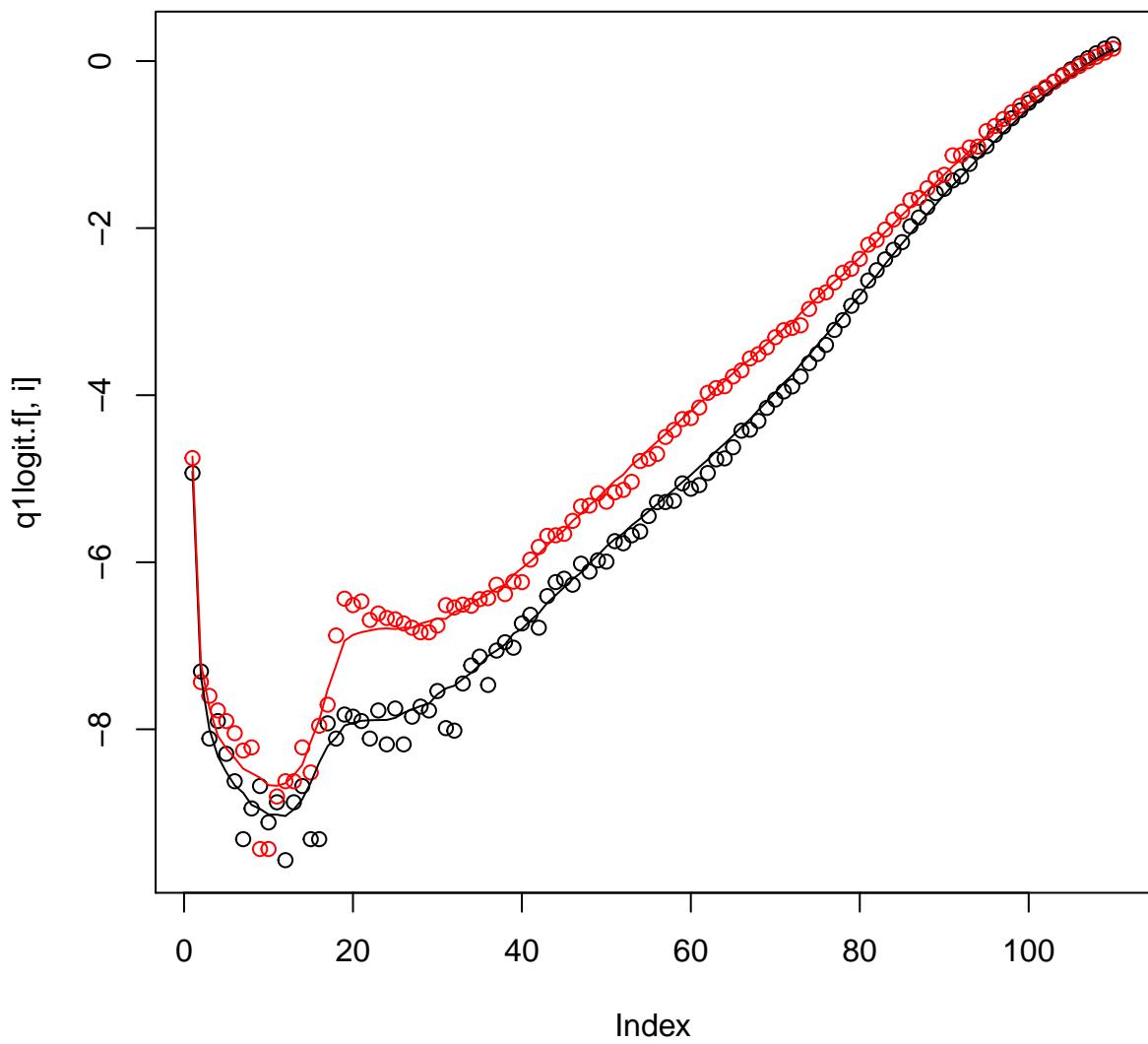
# grayscale
ggplot(data = data.fig1.df, aes(x = Age, y = Value, group = interaction(Sex,
LT), colour = Sex, shape = LT)) + geom_point(size = 1.5) +
geom_line(data = pred.fig1.df, aes(x = Age, y = Value,
group = interaction(Sex, LT), colour = Sex), size = 1) +
scale_x_continuous(breaks = seq(0, 110, 5)) + labs(y = expression(""\textbf{[bold(1)]} *
bolditalic("q")\textbf{[bolditalic(x)]} * bold(" (logit scale)")),
x = expression(bold("Age (years)")))) + # theme(legend.justification=c(1,0),
# legend.position=c(0.98,0.02)) +
theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
scale_shape_discrete(name = "Life Table") + scale_colour_grey(start = 0,
end = 0.7) + ggsave("../figures/fig1-BW.pdf", width = 6.5,
height = 6.5, units = c("in"))

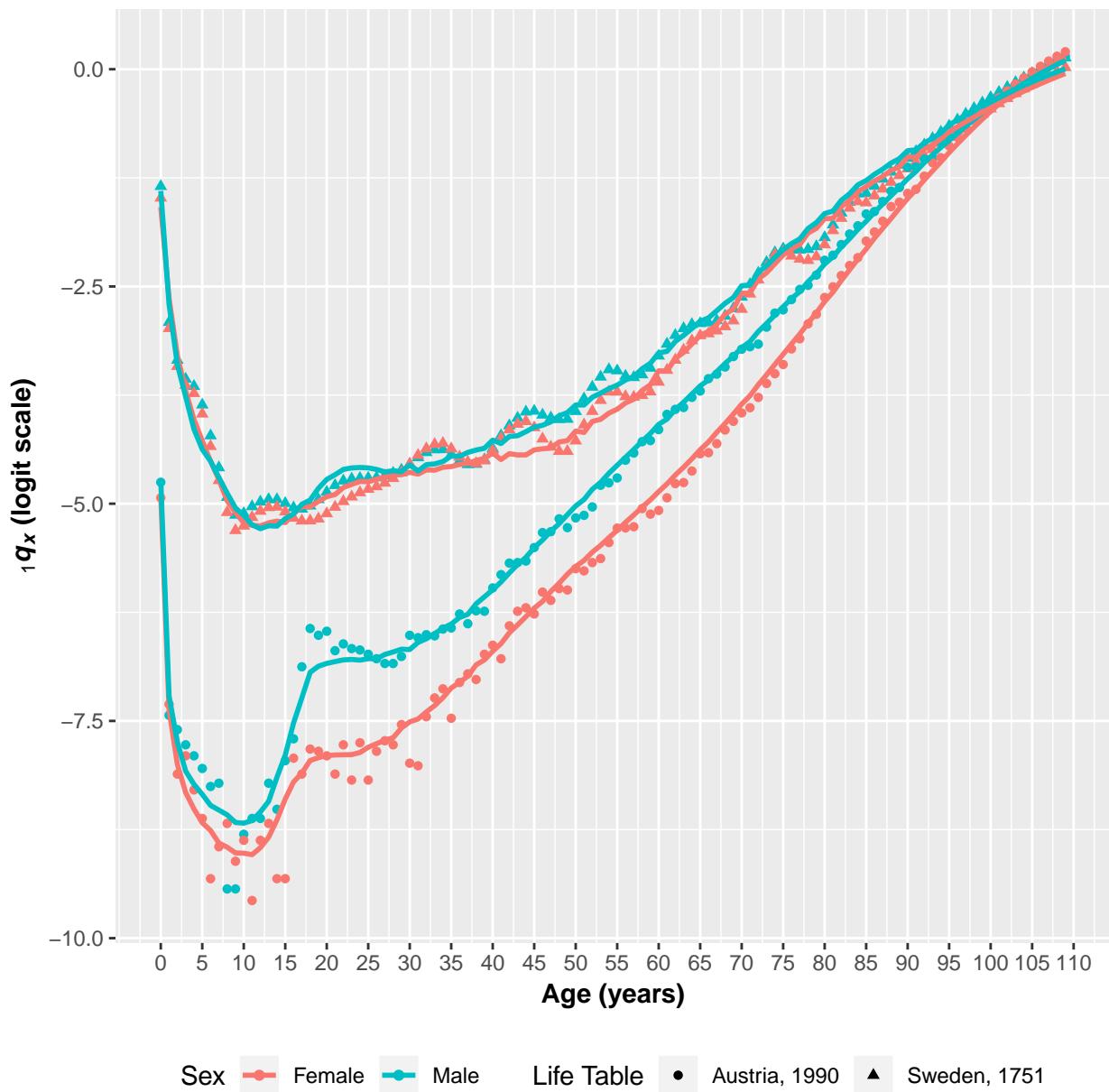
# clean up
rm(list = c("pred.fig1.df", "pred.fig1", "tmp.2.f", "tmp.2.m",
"tmp.1.f", "tmp.1.m", "f.2.p", "m.2.p", "f.1.p", "m.1.p",
"data.fig1.df", "data.fig1", "i.low", "i.high", "tot.abs.err.f",
"tot.abs.err.m"))

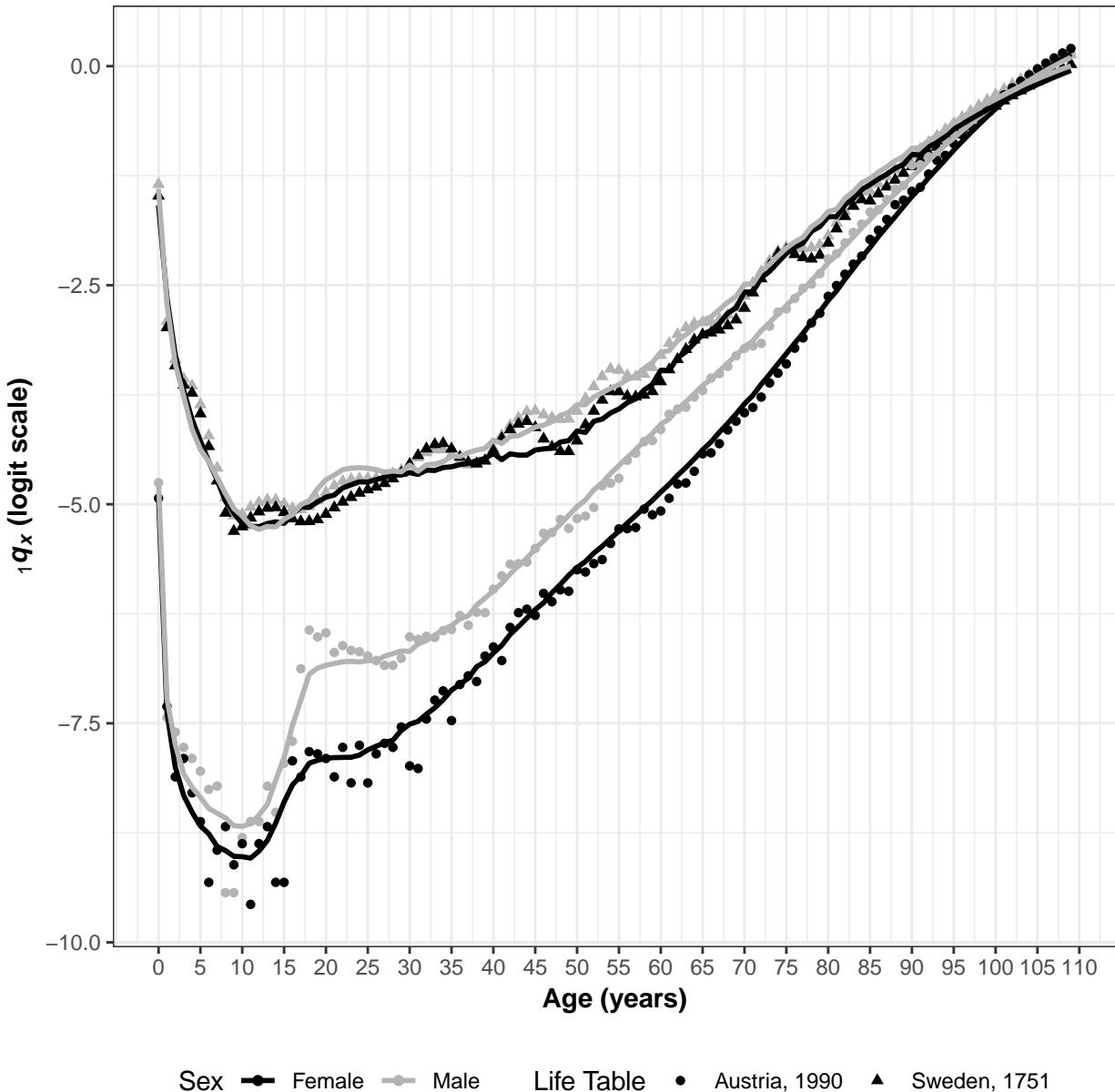
```











Plot the scaled left singular vectors of the SVD decompositions of logit-transformed $1q_x$.

```
# svds
svd.m <- mod.1_0.m$svd$s1
svd.f <- mod.1_0.f$svd$s1

# scaled us
u1.m <- cbind(rep("Male", 110), rownames(q1logit.m), rep("u1",
  110), svd.m$d[1] * svd.m$u[, 1])
u2.m <- cbind(rep("Male", 110), rownames(q1logit.m), rep("u2",
  110), svd.m$d[2] * svd.m$u[, 2])
u3.m <- cbind(rep("Male", 110), rownames(q1logit.m), rep("u3",
  110), -1 * svd.m$d[3] * svd.m$u[, 3])
u4.m <- cbind(rep("Male", 110), rownames(q1logit.m), rep("u4",
  110), svd.m$d[4] * svd.m$u[, 4])
```

```

u1.f <- cbind(rep("Female", 110), rownames(q1logit.m), rep("u1",
  110), svd.f$d[1] * svd.f$u[, 1])
u2.f <- cbind(rep("Female", 110), rownames(q1logit.m), rep("u2",
  110), svd.f$d[2] * svd.f$u[, 2])
u3.f <- cbind(rep("Female", 110), rownames(q1logit.m), rep("u3",
  110), svd.f$d[3] * svd.f$u[, 3])
u4.f <- cbind(rep("Female", 110), rownames(q1logit.m), rep("u4",
  110), svd.f$d[4] * svd.f$u[, 4])

us <- rbind(u1.m, u2.m, u3.m, u4.m, u1.f, u2.f, u3.f, u4.f)

us.df <- data.frame(Sex = as.character(us[, 1]), Age = as.numeric(us[, 2]), U = as.character(us[, 3]), Value = as.numeric(us[, 4]))
save(file = "../RData/us.RData", compress = TRUE, list = c("us.df"))
write.csv(us.df, file = "../tables/us.csv")
# str(us.df)

# smooth svds svds
svd.sm.m <- mod.1_0.sm.m$svd.sm$s1
svd.sm.f <- mod.1_0.sm.f$svd.sm$s1

# us
u1.sm.m <- cbind(rep("Male", 110), rownames(q1logit.m),
  rep("u1", 110), svd.sm.m$d[1] * svd.sm.m$u[, 1])
u2.sm.m <- cbind(rep("Male", 110), rownames(q1logit.m),
  rep("u2", 110), svd.sm.m$d[2] * svd.sm.m$u[, 2])
u3.sm.m <- cbind(rep("Male", 110), rownames(q1logit.m),
  rep("u3", 110), -1 * svd.sm.m$d[3] * svd.sm.m$u[, 3])
u4.sm.m <- cbind(rep("Male", 110), rownames(q1logit.m),
  rep("u4", 110), svd.sm.m$d[4] * svd.sm.m$u[, 4])

u1.sm.f <- cbind(rep("Female", 110), rownames(q1logit.m),
  rep("u1", 110), svd.sm.f$d[1] * svd.sm.f$u[, 1])
u2.sm.f <- cbind(rep("Female", 110), rownames(q1logit.m),
  rep("u2", 110), svd.sm.f$d[2] * svd.sm.f$u[, 2])
u3.sm.f <- cbind(rep("Female", 110), rownames(q1logit.m),
  rep("u3", 110), svd.sm.f$d[3] * svd.sm.f$u[, 3])
u4.sm.f <- cbind(rep("Female", 110), rownames(q1logit.m),
  rep("u4", 110), svd.sm.f$d[4] * svd.sm.f$u[, 4])

us.sm <- rbind(u1.sm.m, u2.sm.m, u3.sm.m, u4.sm.m, u1.sm.f,
  u2.sm.f, u3.sm.f, u4.sm.f)

us.sm.df <- data.frame(Sex = as.character(us.sm[, 1]), Age = as.numeric(us.sm[, 2]), U = as.character(us.sm[, 3]), Value = as.numeric(us.sm[, 4]))
save(file = "../RData/us-smooth.RData", compress = TRUE,
  list = c("us.sm.df"))
write.csv(us.sm.df, file = "../tables/us-smooth.csv")
# str(us.sm.df)

# Plot

```

```

# data for horizontal lines at 0
hlines <- data.frame(U = as.character(c("u1", "u2", "u3",
                                         "u4")), y = as.numeric(c(NA, 0, 0, 0)))

u.names <- list(`U#1` = expression(italic("s")[1] * bold("u")[1]),
                 `U#2` = expression(italic("s")[2] * bold("u")[2]), `U#3` = expression(italic("s")[3] *
                     bold("u")[3]), `U#4` = expression(italic("s")[4] *
                     bold("u")[4]))
# u.names

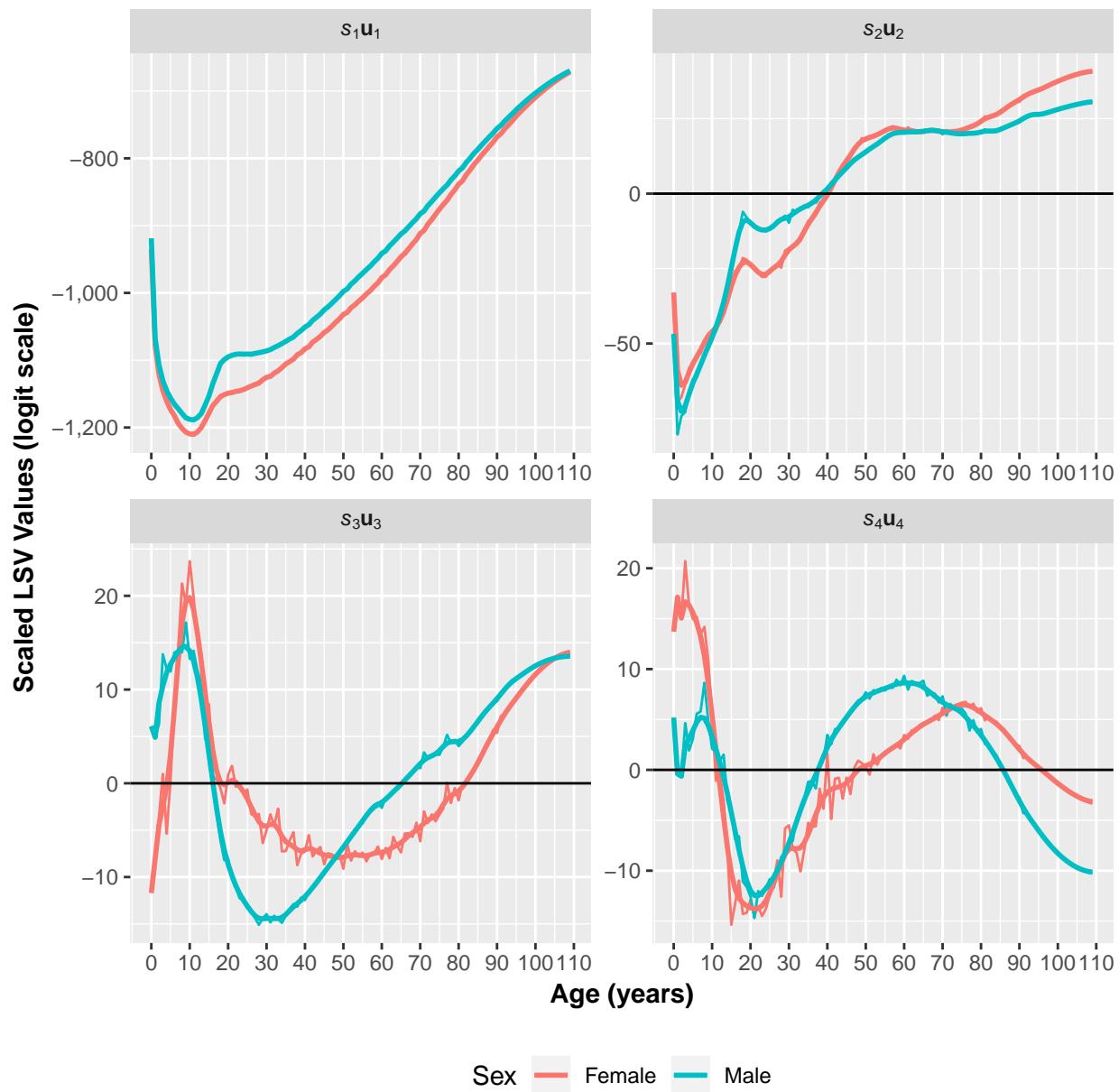
u.labeller <- function(variable, value) {
  return(u.names[value])
}

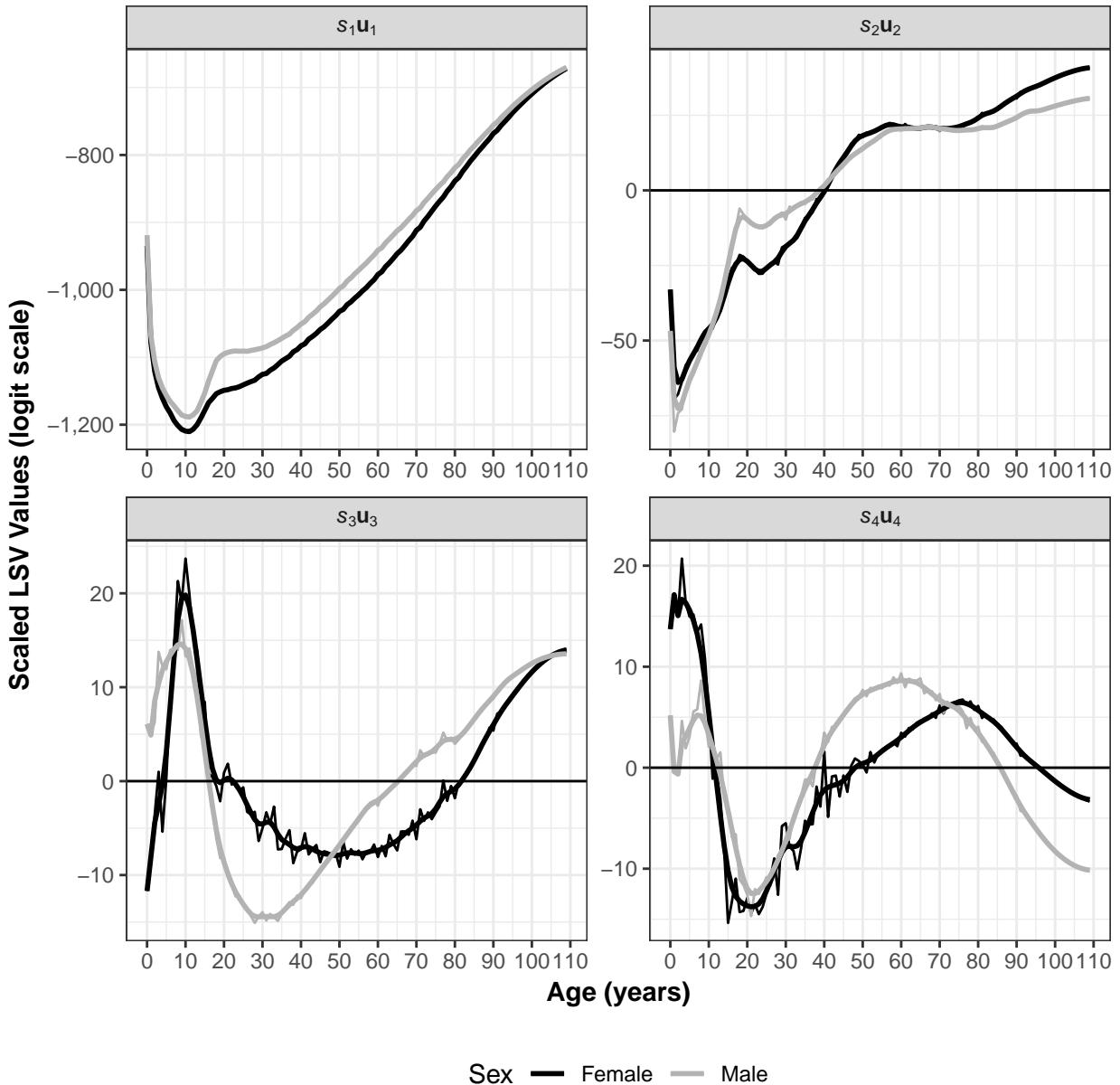
# Plot
ggplot(data = us.df, aes(x = Age, y = Value, group = Sex,
                           colour = Sex)) + geom_line() + geom_line(data = us.sm.df,
                           aes(x = Age, y = Value), size = 1) + geom_hline(data = hlines,
                           aes(yintercept = y)) + scale_x_continuous(breaks = seq(0,
                           110, 10)) + scale_y_continuous(labels = function(x) format(x,
                           big.mark = ",", decimal.mark = ".", scientific = FALSE)) +
  facet_wrap(~U, scales = "free", labeller = u.labeller) +
  labs(y = expression(bold("Scaled LSV Values (logit scale)")),
       x = expression(bold("Age (years)"))) + # theme(legend.justification=c(1,0),
# legend.position=c(0.99,0.56))
theme(legend.position = "bottom", legend.box = "horizontal") +
  ggsave("../figures/fig2.pdf", width = 6.5, height = 6.5,
         units = c("in"))

# grayscale
ggplot(data = us.df, aes(x = Age, y = Value, group = Sex,
                           colour = Sex)) + geom_line() + geom_line(data = us.sm.df,
                           aes(x = Age, y = Value), size = 1) + geom_hline(data = hlines,
                           aes(yintercept = y)) + scale_x_continuous(breaks = seq(0,
                           110, 10)) + scale_y_continuous(labels = function(x) format(x,
                           big.mark = ",", decimal.mark = ".", scientific = FALSE)) +
  facet_wrap(~U, scales = "free", labeller = u.labeller) +
  labs(y = expression(bold("Scaled LSV Values (logit scale)")),
       x = expression(bold("Age (years)"))) + # theme(legend.justification=c(1,0),
# legend.position=c(0.99,0.56))
theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
  scale_colour_grey(start = 0, end = 0.7) + ggsave("../figures/fig2-BW.pdf",
  width = 6.5, height = 6.5, units = c("in"))

# clean up
rm(list = c("u.labeller", "u.names", "hlines", "us.sm.df",
            "us.sm", "u4.sm.f", "u3.sm.f", "u2.sm.f", "u1.sm.f",
            "u4.sm.m", "u3.sm.m", "u2.sm.m", "u1.sm.m", "svd.sm.f",
            "svd.sm.m", "us.df", "us", "u4.f", "u3.f", "u2.f", "u1.f",
            "u4.m", "u3.m", "u2.m", "u1.m", "svd.f", "svd.m"))

```





Sex — Female — Male

Plot the single-year prediction error distributions from 50 50% samples.

```
# female
esf <- melt(error.age.f)
esf <- cbind(esf[, c(1, 3)], "In", "Female")
colnames(esf) <- c("Age", "Error", "Sample", "Sex")

ensf <- melt(error.age.nsamp.f)
ensf <- cbind(ensf[, c(1, 3)], "Out", "Female")
colnames(ensf) <- c("Age", "Error", "Sample", "Sex")
# rm(list=c('error.age.f', 'error.age.nsamp.f'))

ef <- rbind(esf, ensf)
ef$Age <- factor(ef$Age)
rm(list = c("esf", "ensf"))
```

```

# male
esm <- melt(error.age.m)
esm <- cbind(esm[, c(1, 3)], "In", "Male")
colnames(esm) <- c("Age", "Error", "Sample", "Sex")

ensm <- melt(error.age.nsamp.m)
ensm <- cbind(ensm[, c(1, 3)], "Out", "Male")
colnames(ensm) <- c("Age", "Error", "Sample", "Sex")
# rm(list=c('error.age.m', 'error.age.nsamp.m'))

em <- rbind(esm, ensm)
em$Age <- factor(em$Age)
rm(list = c("esm", "ensm"))

# combine male and female
eb <- rbind(em, ef)
rm(list = c("em", "ef"))

# order female first
eb[, 4] <- factor(eb[, 4], levels = c("Female", "Male"))
# str(eb)

e.sum <- ddply(eb, .(Sex, Age, Sample), summarize, ymin = quantile(Error,
  0.1), ymax = quantile(Error, 0.9), middle = median>Error),
  lower = quantile>Error, 0.25), upper = quantile>Error,
  0.75))

s.names <- list(`S#1` = expression(bold("Female")), `S#2` = expression(bold("Male")))
# s.names

s.labeller <- function(variable, value) {
  return(s.names[value])
}

# Plot
ggplot(data = e.sum, aes(x = Age)) + geom_boxplot(aes(fill = Sample,
  ymin = ymin, ymax = ymax, middle = middle, upper = upper,
  lower = lower), stat = "identity", size = 0.2) + # scale_y_continuous(limits = c(-0.03,0.03)) +
  scale_x_discrete(breaks = seq(0, 110, 10)) + # theme(legend.justification=c(1,0),
  # legend.position=c(.15,0.02)) +
  theme(legend.position = "bottom", legend.box = "horizontal") +
  # facet_wrap(~Sex, ncol=1) +
  facet_wrap(~Sex, ncol = 1, scales = "free", labeller = s.labeller) +
  labs(x = expression(bold("Age (years)")), y = expression(bold("Error"))) +
  ggsave("../figures/fig4.pdf", width = 6.5, height = 6.5,
  units = c("in"))

# grayscale
ggplot(data = e.sum, aes(x = Age)) + geom_boxplot(aes(fill = Sample,
  ymin = ymin, ymax = ymax, middle = middle, upper = upper,
  lower = lower), stat = "identity", size = 0.2) + # scale_y_continuous(limits = c(-0.03,0.03)) +
  scale_x_discrete(breaks = seq(0, 110, 10)) + # theme(legend.justification=c(1,0),
  # legend.position=c(.15,0.02)) +

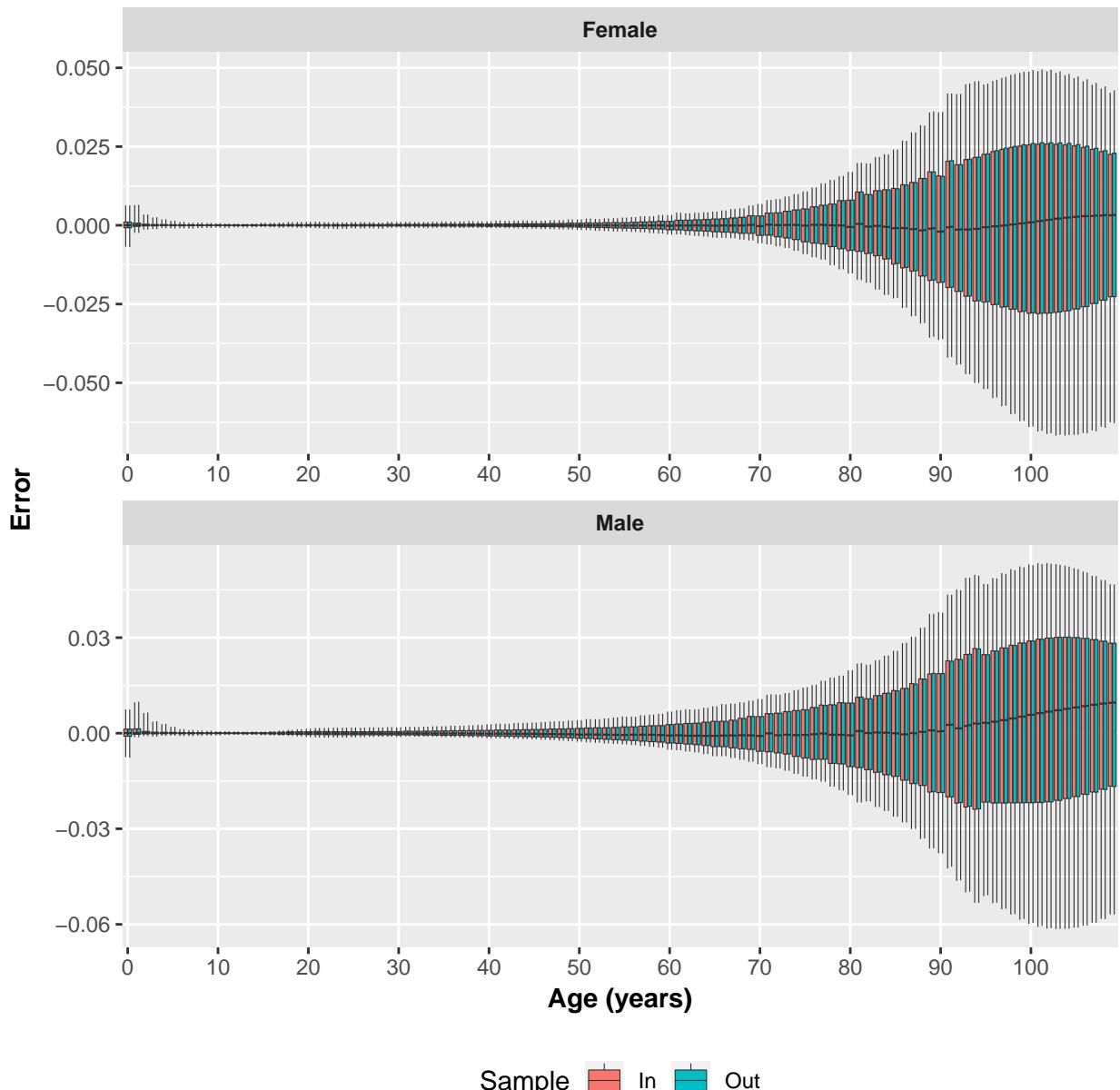
```

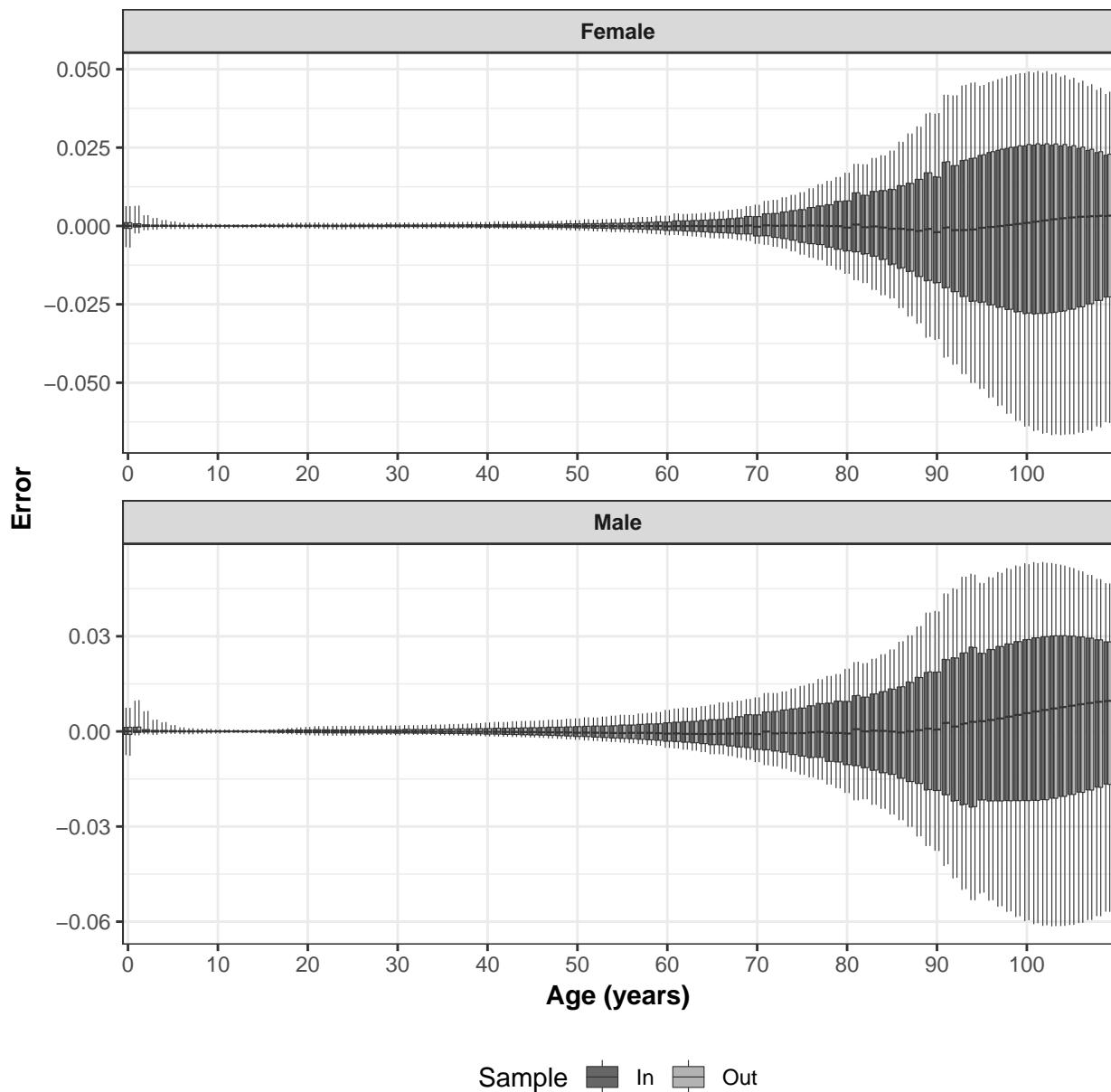
```

theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
  # facet_wrap(~Sex, ncol=1) +
  facet_wrap(~Sex, ncol = 1, scales = "free", labeller = s.labeller) +
  labs(x = expression(bold("Age (years)")), y = expression(bold("Error"))) +
  scale_fill_grey(start = 0.4, end = 0.7) + ggsave("../figures/fig4-BW.pdf",
  width = 6.5, height = 6.5, units = c("in"))

# clean up
rm(list = c("e.sum", "eb", "mod.0_5.50.f", "mod.0_5.50.m"))

```





Plot the prediction error distributions by sample fraction for 50 50% samples

```
# medians female
es.s.f <- rbind(cbind("Female", "In", "10%", errsum.meds.1.f[, 1]),
                 cbind("Female", "In", "30%", errsum.meds.3.f[, 1]),
                 cbind("Female", "In", "50%", errsum.meds.5.f[, 1]),
                 cbind("Female", "In", "70%", errsum.meds.7.f[, 1]),
                 cbind("Female", "In", "90%", errsum.meds.9.f[, 1]))

es.ns.f <- rbind(cbind("Female", "Out", "10%", errsum.meds.1.f[, 2]),
                  cbind("Female", "Out", "30%", errsum.meds.3.f[, 2]),
                  cbind("Female", "Out", "50%", errsum.meds.5.f[, 2]),
                  cbind("Female", "Out", "70%", errsum.meds.7.f[, 2]),
                  cbind("Female", "Out", "90%", errsum.meds.9.f[, 2]))
```

```

es.f <- rbind(es.s.f, es.ns.f)

es.f.df <- data.frame(Sex = as.character(es.f[, 1]), Sample = as.character(es.f[, 2]), Fraction = as.character(es.f[, 3]), Median = as.numeric(es.f[, 4]))
# str(es.f.df)

# male
es.s.m <- rbind(cbind("Male", "In", "10%", errsum.meds.1.m[, 1]), cbind("Male", "In", "30%", errsum.meds.3.m[, 1]), cbind("Male", "In", "50%", errsum.meds.5.m[, 1]), cbind("Male", "In", "70%", errsum.meds.7.m[, 1]), cbind("Male", "In", "90%", errsum.meds.9.m[, 1]))

es.ns.m <- rbind(cbind("Male", "Out", "10%", errsum.meds.1.m[, 2]), cbind("Male", "Out", "30%", errsum.meds.3.m[, 2]), cbind("Male", "Out", "50%", errsum.meds.5.m[, 2]), cbind("Male", "Out", "70%", errsum.meds.7.m[, 2]), cbind("Male", "Out", "90%", errsum.meds.9.m[, 2]))

es.m <- rbind(es.s.m, es.ns.m)

es.m.df <- data.frame(Sex = as.character(es.m[, 1]), Sample = as.character(es.m[, 2]), Fraction = as.character(es.m[, 3]), Median = as.numeric(es.m[, 4]))
# str(es.m.df)

es.df <- rbind(es.f.df, es.m.df)
# str(es.df)

# order female first
es.df[, 1] <- factor(es.df[, 1], levels = c("Female", "Male"))
# str(es.df)

e.sum <- ddply(es.df, .(Sex, Sample, Fraction), summarize,
  ymin = quantile(Median, 0.1), ymax = quantile(Median, 0.9), middle = median(Median), lower = quantile(Median, 0.25), upper = quantile(Median, 0.75))

s.names <- list(`S#1` = expression(bold("Female")), `S#2` = expression(bold("Male")))
# s.names

s.labeller <- function(variable, value) {
  return(s.names[value])
}

# Plot
ggplot(data = e.sum, aes(x = Fraction)) + geom_boxplot(aes(fill = Sample, ymin = ymin, ymax = ymax, middle = middle, upper = upper, lower = lower), stat = "identity", size = 0.2) + geom_hline(yintercept = 0) +
  # scale_y_continuous(limits = c(-2e-05, 4.5e-05)) +
  # theme(legend.justification=c(1,0),
  # legend.position=c(0.85,.56)) +

```

```

theme(legend.position = "bottom", legend.box = "horizontal") +
  labs(y = expression(bold("Median Error")), x = expression(bold("Sample Percentage")))) +
  facet_wrap(~Sex, ncol = 1, scales = "free", labeller = s.labeller) +
  ggsave("../figures/fig5a.pdf", width = 6.5, height = 6.5,
         units = c("in"))

# grayscale
ggplot(data = e.sum, aes(x = Fraction)) + geom_boxplot(aes(fill = Sample,
  ymin = ymin, ymax = ymax, middle = middle, upper = upper,
  lower = lower), stat = "identity", size = 0.2) + geom_hline(yintercept = 0) +
  # scale_y_continuous(limits = c(-2e-05,4.5e-05)) +
# theme(legend.justification=c(1,0),
# legend.position=c(0.85,.56)) +
theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
  labs(y = expression(bold("Median Error")), x = expression(bold("Sample Percentage")))) +
  facet_wrap(~Sex, ncol = 1, scales = "free", labeller = s.labeller) +
  scale_fill_grey(start = 0.4, end = 0.7) + ggsave("../figures/fig5a-BW.pdf",
width = 6.5, height = 6.5, units = c("in"))

# iqrs female
es.s.f <- rbind(cbind("Female", "In", "10%", errsum.iqrs.1.f[, 1]),
  cbind("Female", "In", "30%", errsum.iqrs.3.f[, 1]),
  cbind("Female", "In", "50%", errsum.iqrs.5.f[, 1]),
  cbind("Female", "In", "70%", errsum.iqrs.7.f[, 1]),
  cbind("Female", "In", "90%", errsum.iqrs.9.f[, 1])))

es.ns.f <- rbind(cbind("Female", "Out", "10%", errsum.iqrs.1.f[, 2]),
  cbind("Female", "Out", "30%", errsum.iqrs.3.f[, 2]),
  cbind("Female", "Out", "50%", errsum.iqrs.5.f[, 2]),
  cbind("Female", "Out", "70%", errsum.iqrs.7.f[, 2]),
  cbind("Female", "Out", "90%", errsum.iqrs.9.f[, 2])))

es.f <- rbind(es.s.f, es.ns.f)

es.f.df <- data.frame(Sex = as.character(es.f[, 1]), Sample = as.character(es.f[, 2]),
  Fraction = as.character(es.f[, 3]), Median = as.numeric(es.f[, 4]))
# str(es.f.df)

# male
es.s.m <- rbind(cbind("Male", "In", "10%", errsum.iqrs.1.m[, 1]),
  cbind("Male", "In", "30%", errsum.iqrs.3.m[, 1]),
  cbind("Male", "In", "50%", errsum.iqrs.5.m[, 1]), cbind("Male",
    "In", "70%", errsum.iqrs.7.m[, 1]), cbind("Male",
    "In", "90%", errsum.iqrs.9.m[, 1]))

es.ns.m <- rbind(cbind("Male", "Out", "10%", errsum.iqrs.1.m[, 2]),
  cbind("Male", "Out", "30%", errsum.iqrs.3.m[, 2]),
  cbind("Male", "Out", "50%", errsum.iqrs.5.m[, 2]), cbind("Male",
    "Out", "70%", errsum.iqrs.7.m[, 2]), cbind("Male",
    "Out", "90%", errsum.iqrs.9.m[, 2]))

```

```

    "Out", "90%", errsum.iqrs.9.m[, 2])))

es.m <- rbind(es.s.m, es.ns.m)

es.m.df <- data.frame(Sex = as.character(es.m[, 1]), Sample = as.character(es.m[, 2]), Fraction = as.character(es.m[, 3]), Median = as.numeric(es.m[, 4]))
# str(es.m.df)

es.df <- rbind(es.m.df, es.f.df)

# order female first
es.df[, 1] <- factor(es.df[, 1], levels = c("Female", "Male"))
# str(es.df)

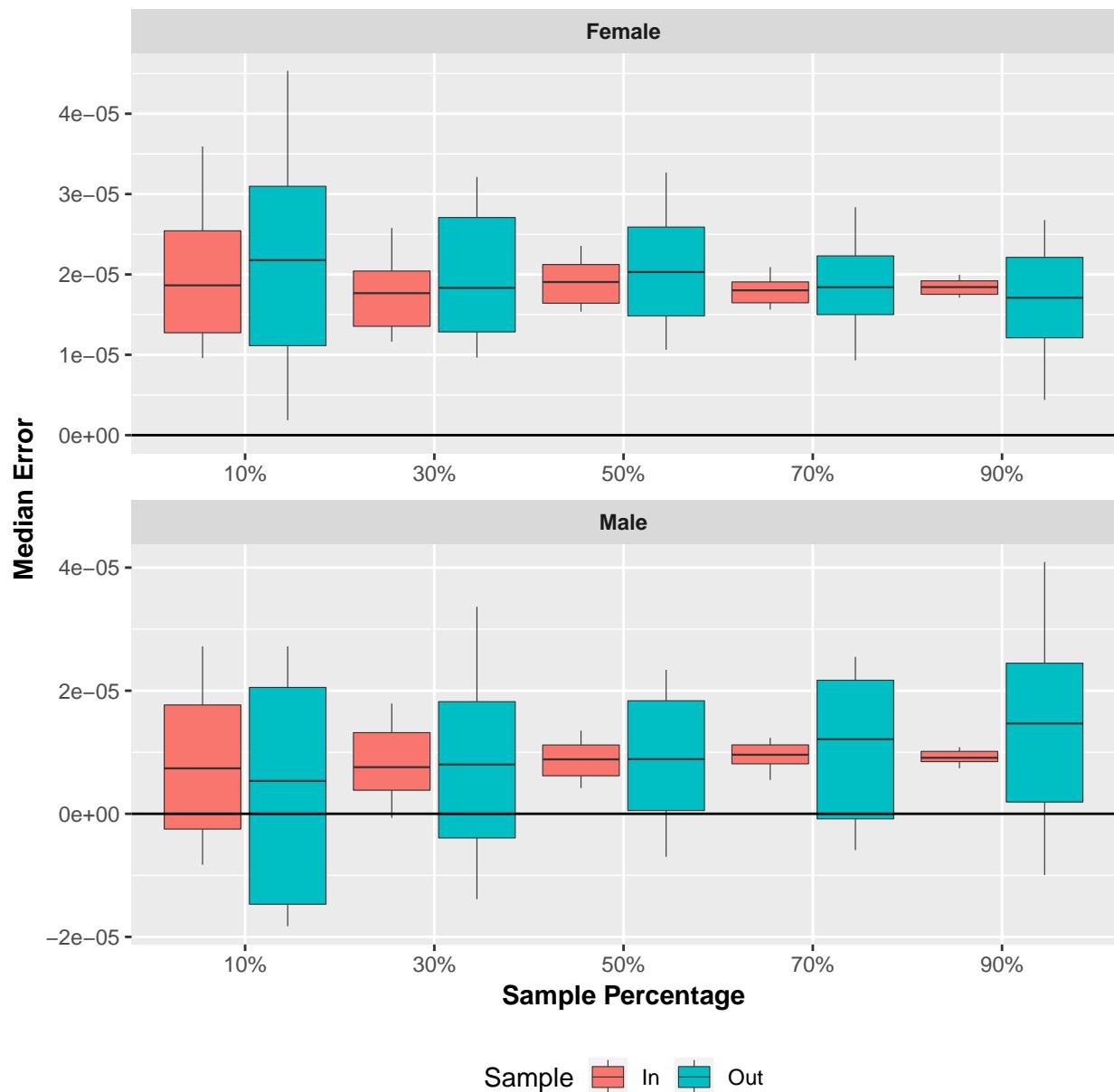
e.sum <- ddply(es.df, .(Sex, Sample, Fraction), summarize,
  ymin = quantile(Median, 0.1), ymax = quantile(Median,
  0.9), middle = median(Median), lower = quantile(Median,
  0.25), upper = quantile(Median, 0.75))

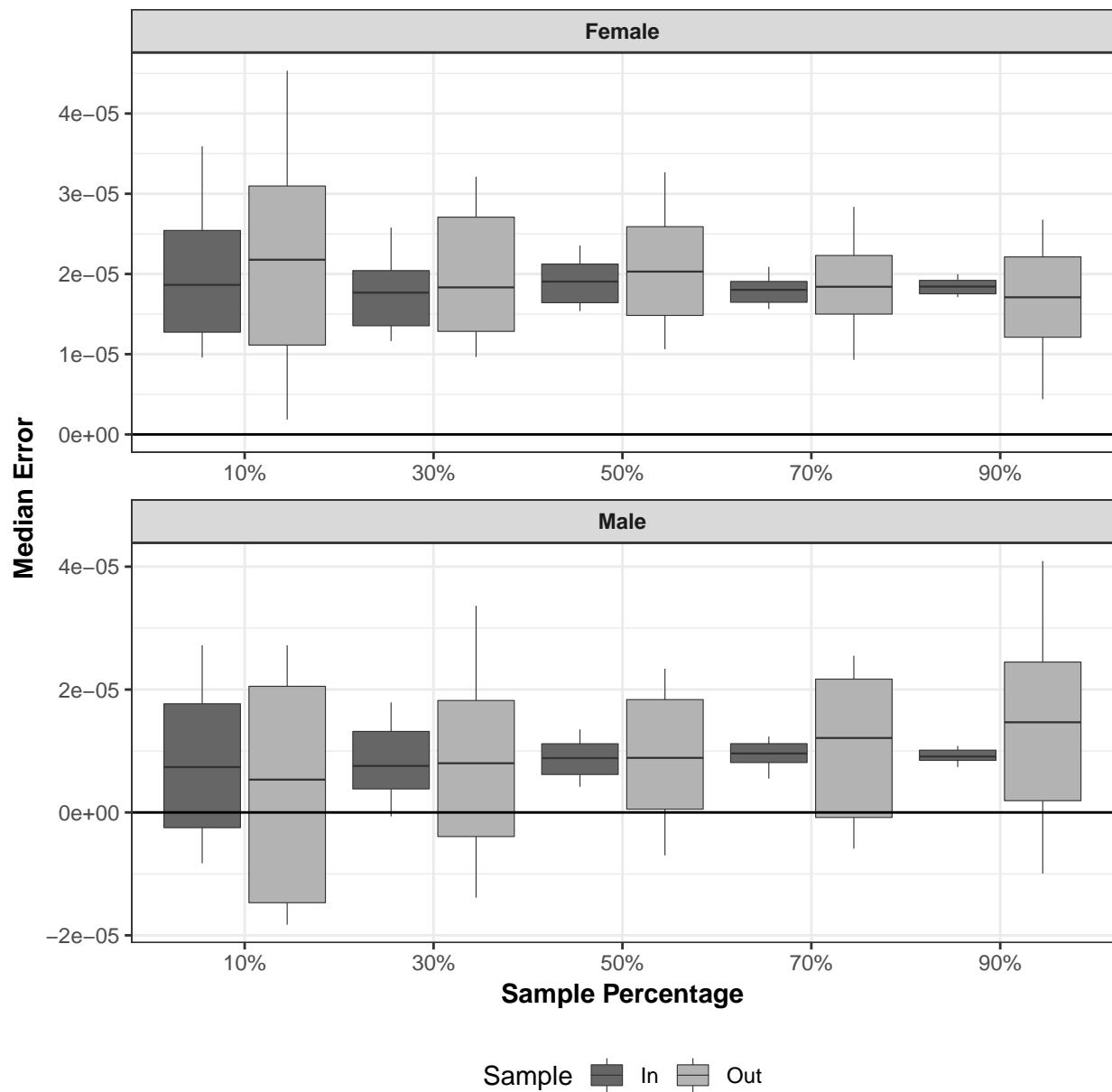
# Plot
ggplot(data = e.sum, aes(x = Fraction)) + geom_boxplot(aes(fill = Sample,
  ymin = ymin, ymax = ymax, middle = middle, upper = upper,
  lower = lower), stat = "identity", size = 0.2) + # geom_hline(yintercept=0) + scale_y_continuous(lin
# c(0.0025,0.005)) + theme(legend.justification=c(1,0),
# legend.position=c(0.85,.84)) +
theme(legend.position = "bottom", legend.box = "horizontal") +
  labs(y = expression(bold("Error Interquartile Range")),
  x = expression(bold("Sample Percentage")))) + facet_wrap(~Sex,
  ncol = 1, scales = "free", labeller = s.labeller) +
  ggsave("../figures/fig5b.pdf", width = 6.5, height = 6.5,
  units = c("in"))

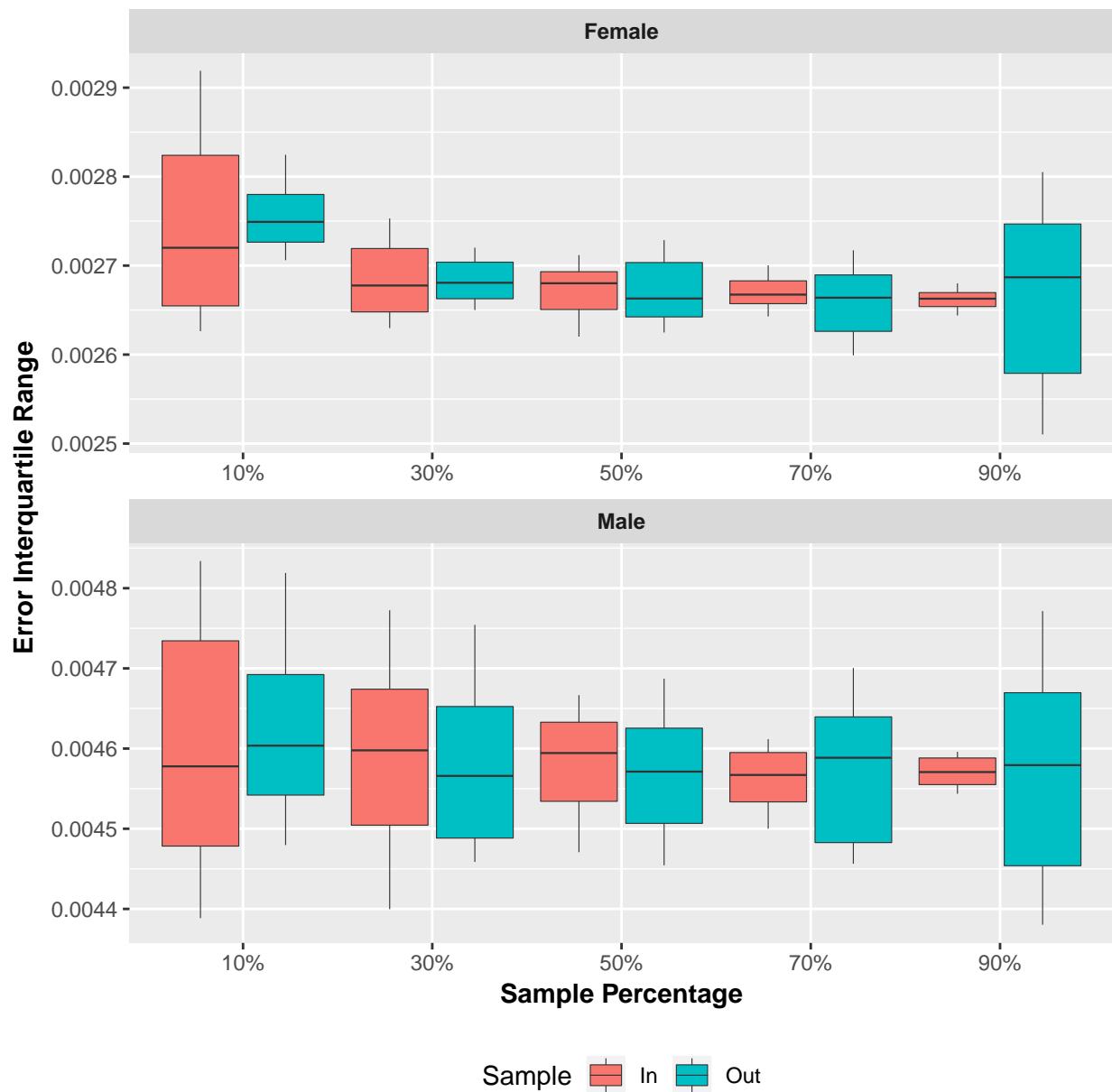
# grayscale
ggplot(data = e.sum, aes(x = Fraction)) + geom_boxplot(aes(fill = Sample,
  ymin = ymin, ymax = ymax, middle = middle, upper = upper,
  lower = lower), stat = "identity", size = 0.2) + # geom_hline(yintercept=0) + scale_y_continuous(lin
# c(0.0025,0.005)) + theme(legend.justification=c(1,0),
# legend.position=c(0.85,.84)) +
theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
  labs(y = expression(bold("Error Interquartile Range")),
  x = expression(bold("Sample Percentage")))) + facet_wrap(~Sex,
  ncol = 1, scales = "free", labeller = s.labeller) +
  scale_fill_grey(start = 0.4, end = 0.7) + ggsave("../figures/fig5b-BW.pdf",
  width = 6.5, height = 6.5, units = c("in"))

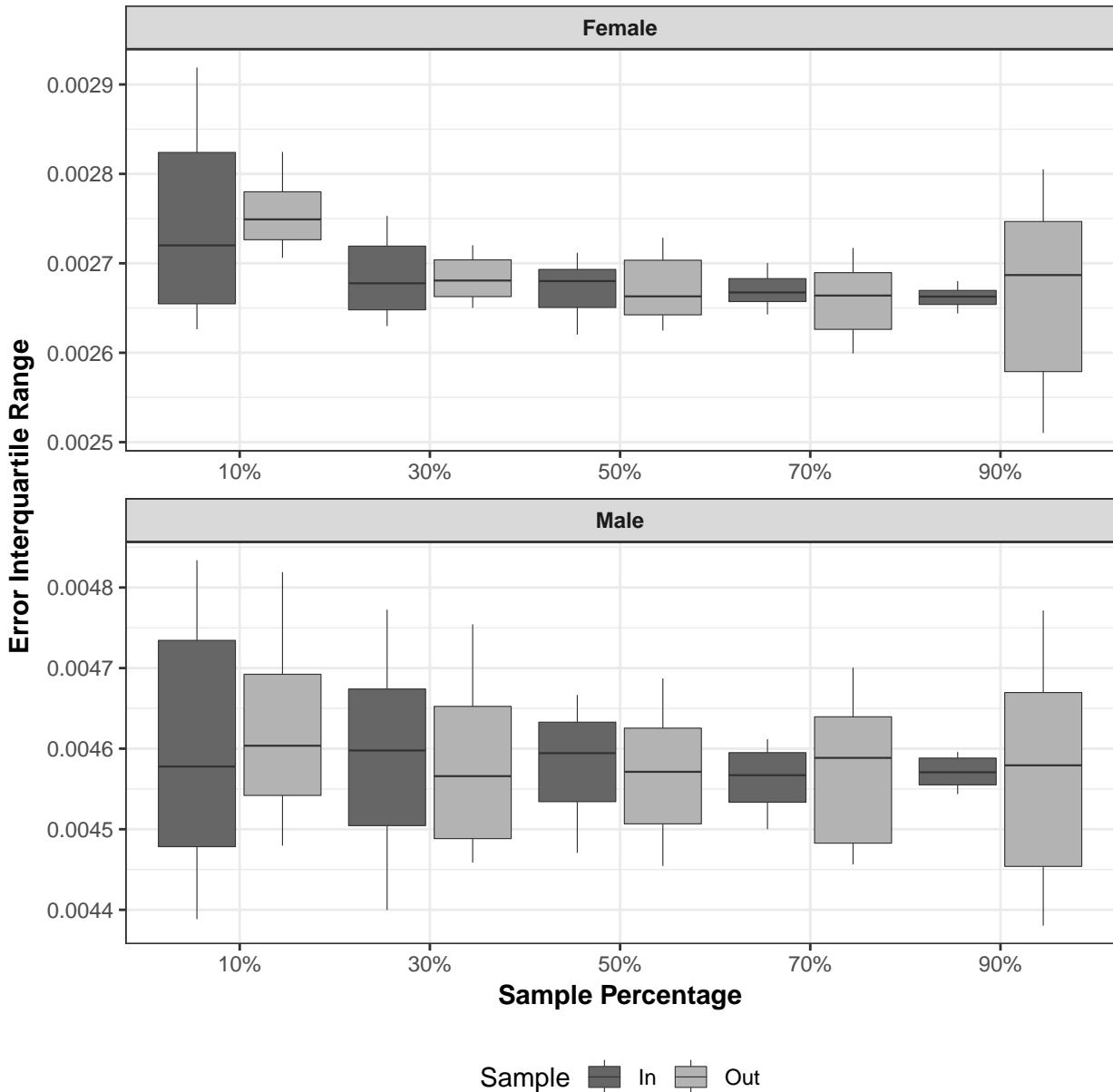
# clean up
rm(list = c("e.sum", "es.df", "es.m.df", "es.m", "es.ns.m",
  "es.s.m", "es.f.df", "es.f", "es.ns.f", "es.s.f"))

```









Plot right singular vectors versus child mortality, $5q_0$.

```
# svds
svd.m <- mod.1_0.m$svd$s1
svd.f <- mod.1_0.f$svd$s1

# right singular vectors
vs.cm <- rbind(cbind("Female", "v1", svd.f$v[, 1], Qlogit.f[1,
  ]), cbind("Female", "v2", svd.f$v[, 2], Qlogit.f[1,
  ]), cbind("Female", "v3", svd.f$v[, 3], Qlogit.f[1,
  ]), cbind("Female", "v4", svd.f$v[, 4], Qlogit.f[1,
  ]), cbind("Male", "v1", svd.m$v[, 1], Qlogit.m[1, ]),
  cbind("Male", "v2", svd.m$v[, 2], Qlogit.m[1, ]), cbind("Male",
  "v3", svd.m$v[, 3], Qlogit.m[1, ]), cbind("Male",
  "v4", svd.m$v[, 4], Qlogit.m[1, ]))
```

```

vs.cm.df <- data.frame(Sex = as.character(vs.cm[, 1]), V = as.character(vs.cm[, 2]), Value = as.numeric(vs.cm[, 3]), CM = as.numeric(vs.cm[, 4]))
# str(vs.cm.df)

# predicted right singular vectors
vs.cm.p <- rbind(cbind("Female", "v1", predict(mod.1_0.f$mods$s1$v1),
  Qlogit.f[1, ]), cbind("Female", "v2", predict(mod.1_0.f$mods$s1$v2),
  Qlogit.f[1, ]), cbind("Female", "v3", predict(mod.1_0.f$mods$s1$v3),
  Qlogit.f[1, ]), cbind("Female", "v4", predict(mod.1_0.f$mods$s1$v4),
  Qlogit.f[1, ]), cbind("Male", "v1", predict(mod.1_0.m$mods$s1$v1),
  Qlogit.m[1, ]), cbind("Male", "v2", predict(mod.1_0.m$mods$s1$v2),
  Qlogit.m[1, ]), cbind("Male", "v3", predict(mod.1_0.m$mods$s1$v3),
  Qlogit.m[1, ]), cbind("Male", "v4", predict(mod.1_0.m$mods$s1$v4),
  Qlogit.m[1, ]))

vs.cm.p.df <- data.frame(Sex = as.character(vs.cm.p[, 1]),
  V = as.character(vs.cm.p[, 2]), Value = as.numeric(vs.cm.p[, 3]),
  CM = as.numeric(vs.cm.p[, 4]))
# str(vs.cm.p.df)

v.names <- list(`V#1` = expression(bold("v")[1]), `V#2` = expression(bold("v")[2]),
  `V#3` = expression(bold("v")[3]), `V#4` = expression(bold("v")[4]))

v.labeller <- function(variable, value) {
  return(v.names[value])
}

vs.cm <- rbind(cbind(vs.cm.df, Type = "Data"), cbind(vs.cm.p.df,
  Type = "Predicted"))
# str(vs.cm)

# Plot female
ggplot(data = vs.cm[which(vs.cm[, 1] == "Female"), ], aes(x = CM,
  y = Value, group = Type, colour = Type)) + geom_point(size = 0.2) +
  labs(y = expression(bold("RSV Element Values")), x = expression(bold("Child Mortality ")[bold(5)] *
    bolditalic("q")*[bold(0)] * bold(" (logit scale)"))) +
  # theme(legend.justification=c(1,0),
  # legend.position=c(0.99,.88)) +
  theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(legend.title = element_blank()) + facet_wrap(~V,
  scale = "free", labeller = v.labeller) + ggsave("../figures/fig2-1f.pdf",
  width = 6.5, height = 6.5, units = c("in"))

# grayscale
ggplot(data = vs.cm[which(vs.cm[, 1] == "Female"), ], aes(x = CM,
  y = Value, group = Type, colour = Type)) + geom_point(size = 0.2) +
  labs(y = expression(bold("RSV Element Values")), x = expression(bold("Child Mortality ")[bold(5)] *
    bolditalic("q")*[bold(0)] * bold(" (logit scale)"))) +
  # theme(legend.justification=c(1,0),
  # legend.position=c(0.99,.88)) +
  theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(legend.title = element_blank()) + facet_wrap(~V,

```

```

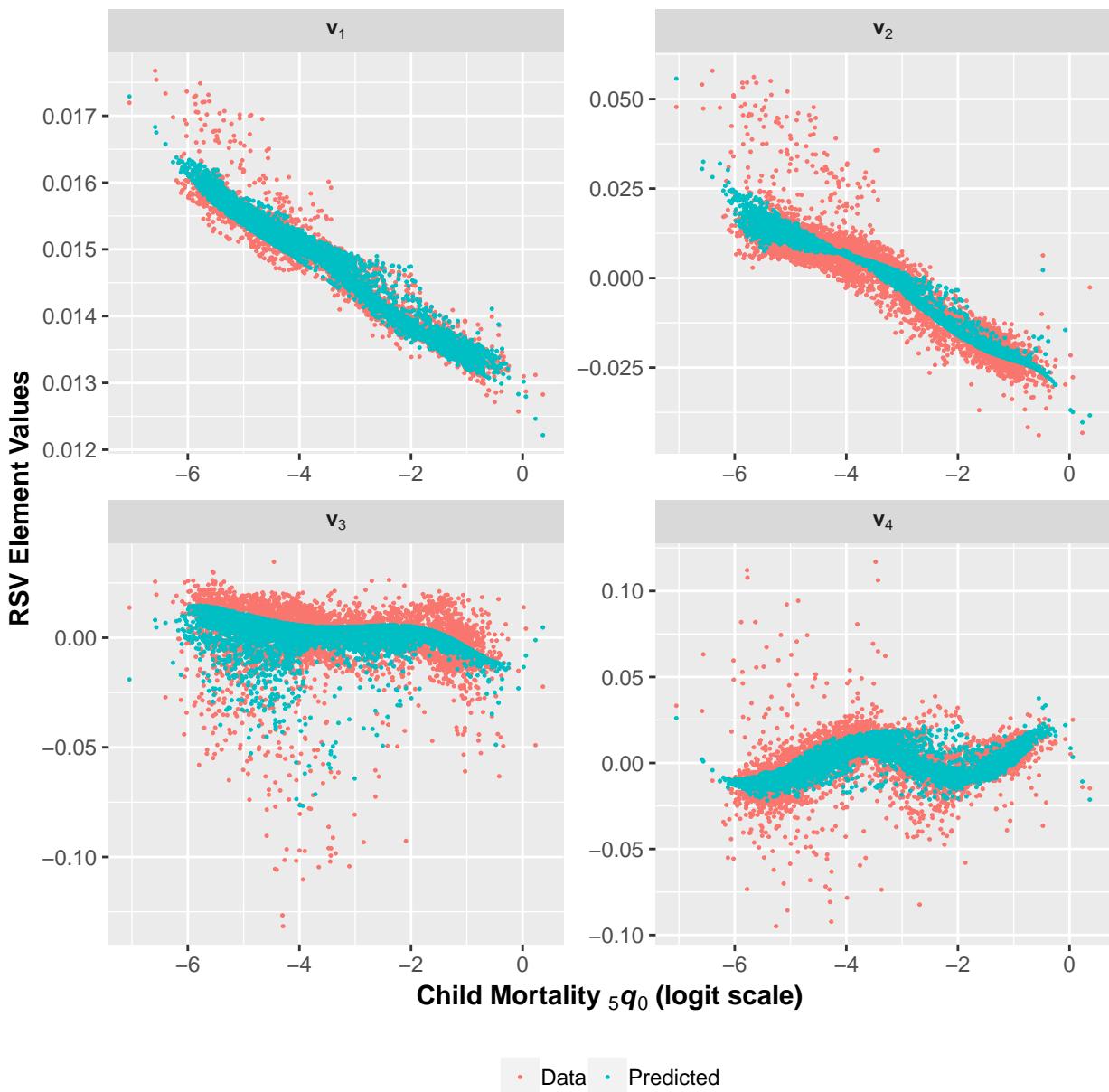
scale = "free", labeller = v.labeller) + scale_colour_grey(start = 0,
end = 0.7) + ggsave("../figures/fig2-1f-BW.pdf", width = 6.5,
height = 6.5, units = c("in"))

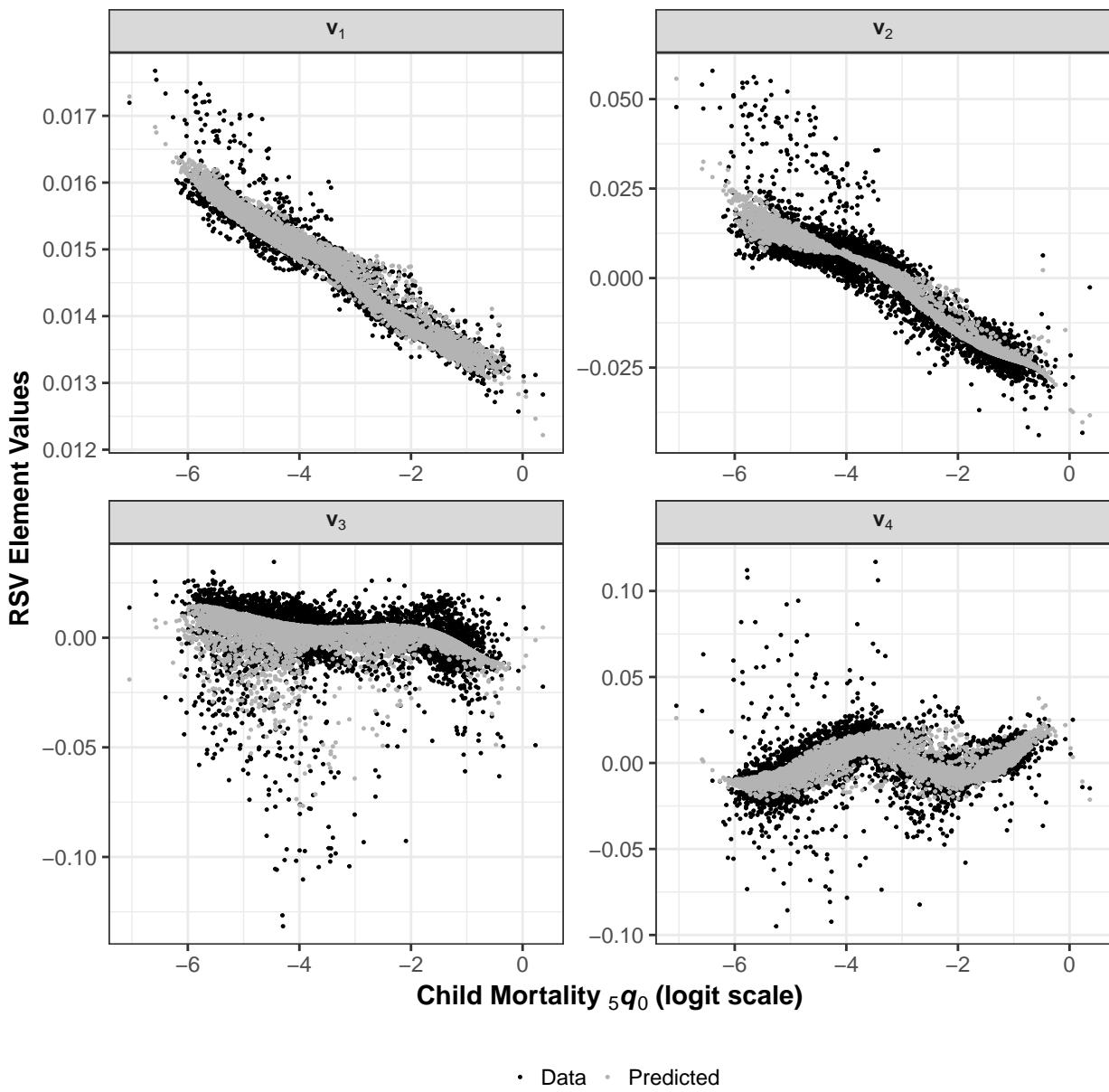
# Plot male
ggplot(data = vs.cm[which(vs.cm[, 1] == "Male"), ], aes(x = CM,
y = Value, group = Type, colour = Type)) + geom_point(size = 0.2) +
labs(y = expression(bold("RSV Element Values")), x = expression(bold("Child Mortality ")[bold(5)] * bolditalic("q")*[bold(0)] * bold(" (logit scale)")))) +
# theme(legend.justification=c(1,0),
# legend.position=c(0.99,.88)) +
theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
theme(legend.title = element_blank()) + facet_wrap(~V,
scale = "free", labeller = v.labeller) + ggsave("../figures/fig2-1m.pdf",
width = 6.5, height = 6.5, units = c("in"))

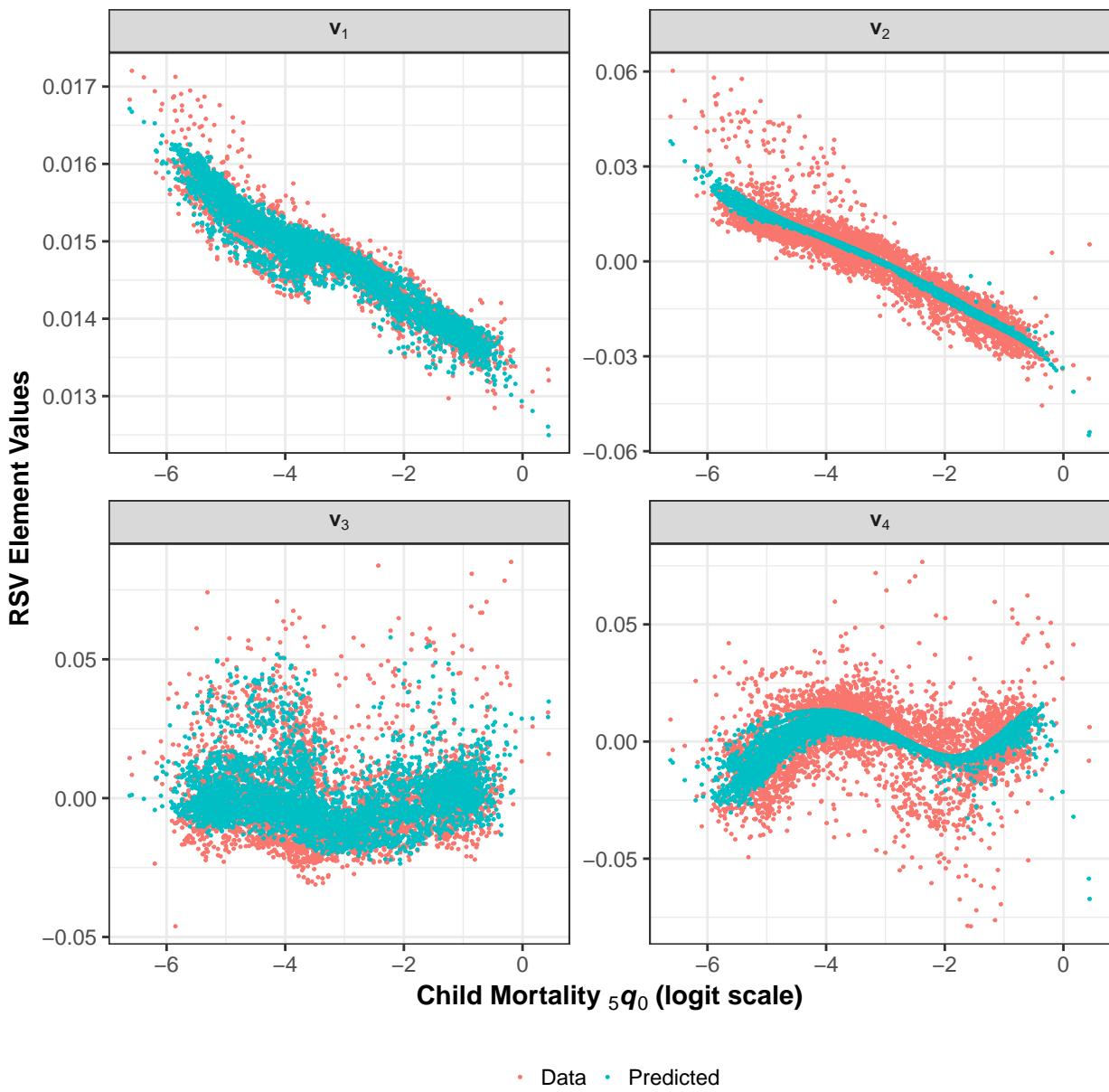
# grayscale
ggplot(data = vs.cm[which(vs.cm[, 1] == "Male"), ], aes(x = CM,
y = Value, group = Type, colour = Type)) + geom_point(size = 0.2) +
labs(y = expression(bold("RSV Element Values")), x = expression(bold("Child Mortality ")[bold(5)] * bolditalic("q")*[bold(0)] * bold(" (logit scale)")))) +
# theme(legend.justification=c(1,0),
# legend.position=c(0.99,.88)) +
theme(legend.position = "bottom", legend.box = "horizontal") +
theme(legend.title = element_blank()) + facet_wrap(~V,
scale = "free", labeller = v.labeller) + scale_colour_grey(start = 0,
end = 0.7) + ggsave("../figures/fig2-1m-BW.pdf", width = 6.5,
height = 6.5, units = c("in"))

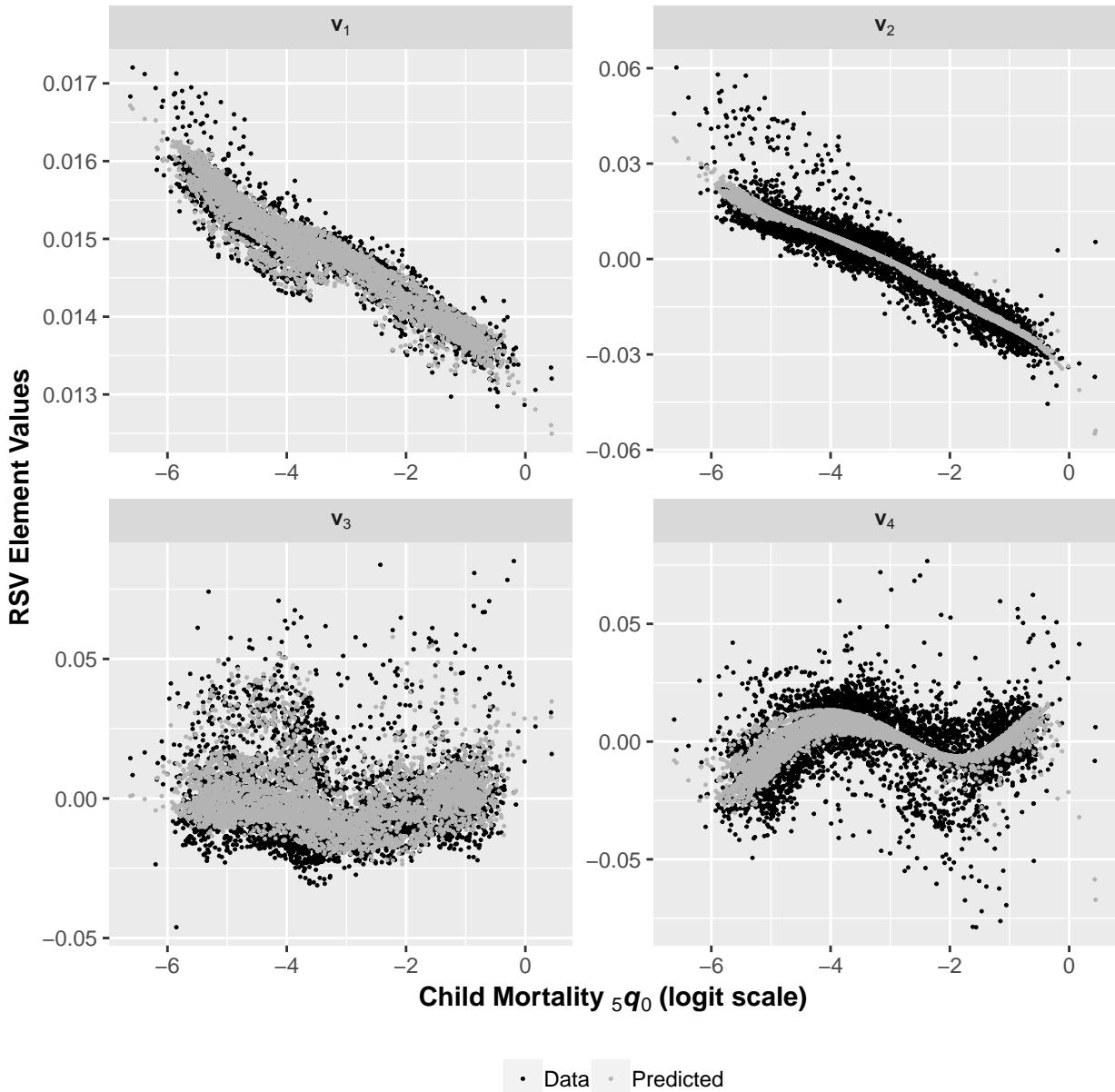
# clean up
rm(list = c("vs.cm", "v.labeller", "v.names", "vs.cm.p.df",
"vs.cm.p", "vs.cm.df", "vs.cm", "svd.m", "svd.f"))

```









Plot adult mortality, $45q_{15}$, by child mortality, $5q_0$.

```
# data
am.cm <- rbind(cbind("Male", Qlogit.m[1, ], Qlogit.m[2,
]), cbind("Female", Qlogit.f[1, ], Qlogit.f[2, ]))

am.cm.df <- data.frame(Sex = as.character(am.cm[, 1]), CM = as.numeric(am.cm[, 2]),
AM = as.numeric(am.cm[, 3]))
# str(am.cm.df)

# predicted
am.cm.p <- rbind(cbind("Male", Qlogit.m[1, ], predict(mod.1_0.m$mods$s1$aml)),
cbind("Female", Qlogit.f[1, ], predict(mod.1_0.f$mods$s1$aml)))

am.cm.p.df <- data.frame(Sex = as.character(am.cm.p[, 1]),
CM = as.numeric(am.cm.p[, 2]), AM = as.numeric(am.cm.p[, 3]))
```

```

  3]))
# str(am.cm.p.df)

am.cm <- rbind(cbind(am.cm.df, Type = "Data"), cbind(am.cm.p.df,
  Type = "Predicted"))
# str(am.cm)

s.names <- list(`S#1` = expression(bold("Female")), `S#2` = expression(bold("Male")))
# s.names

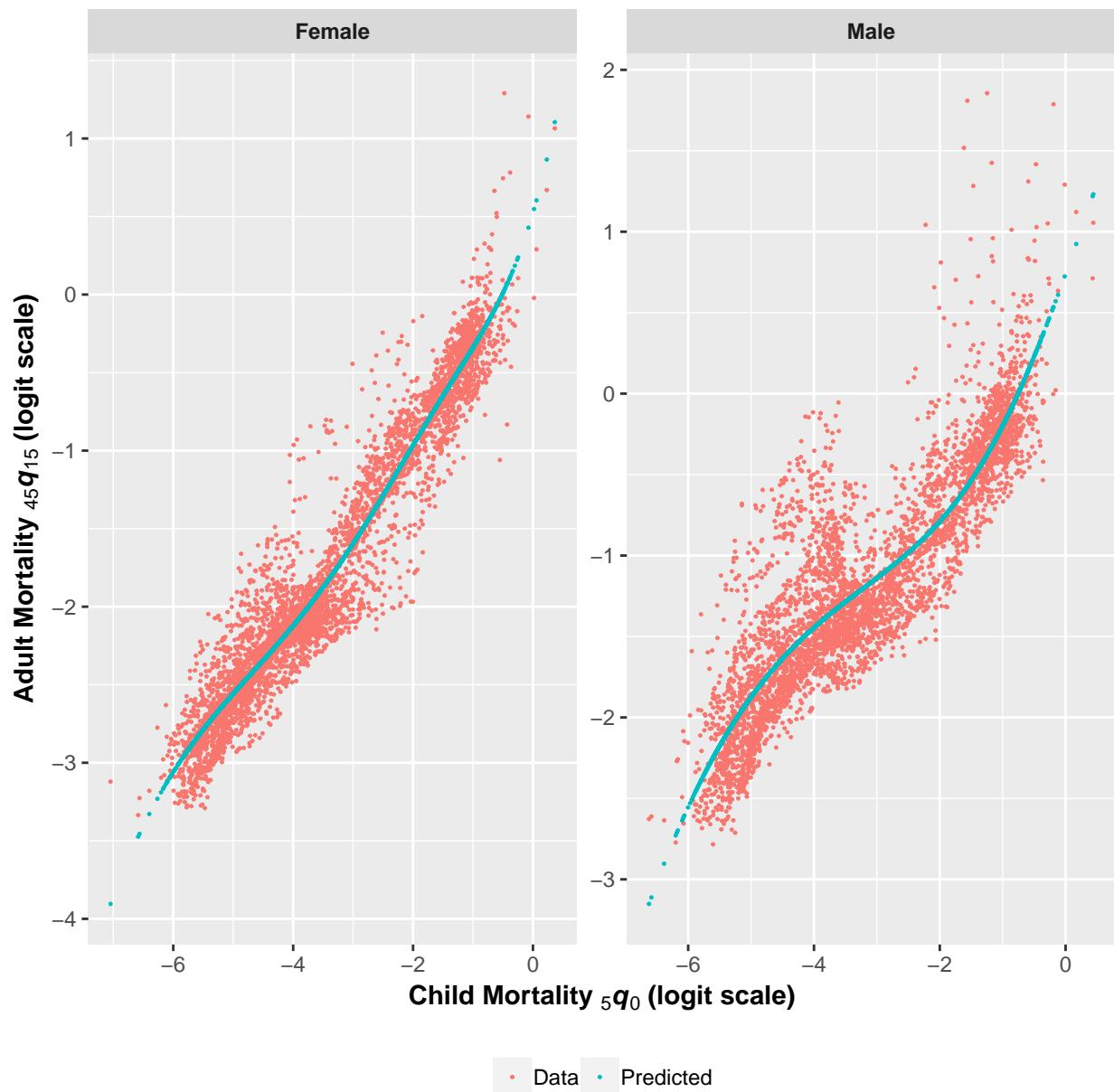
s.labeller <- function(variable, value) {
  return(s.names[value])
}

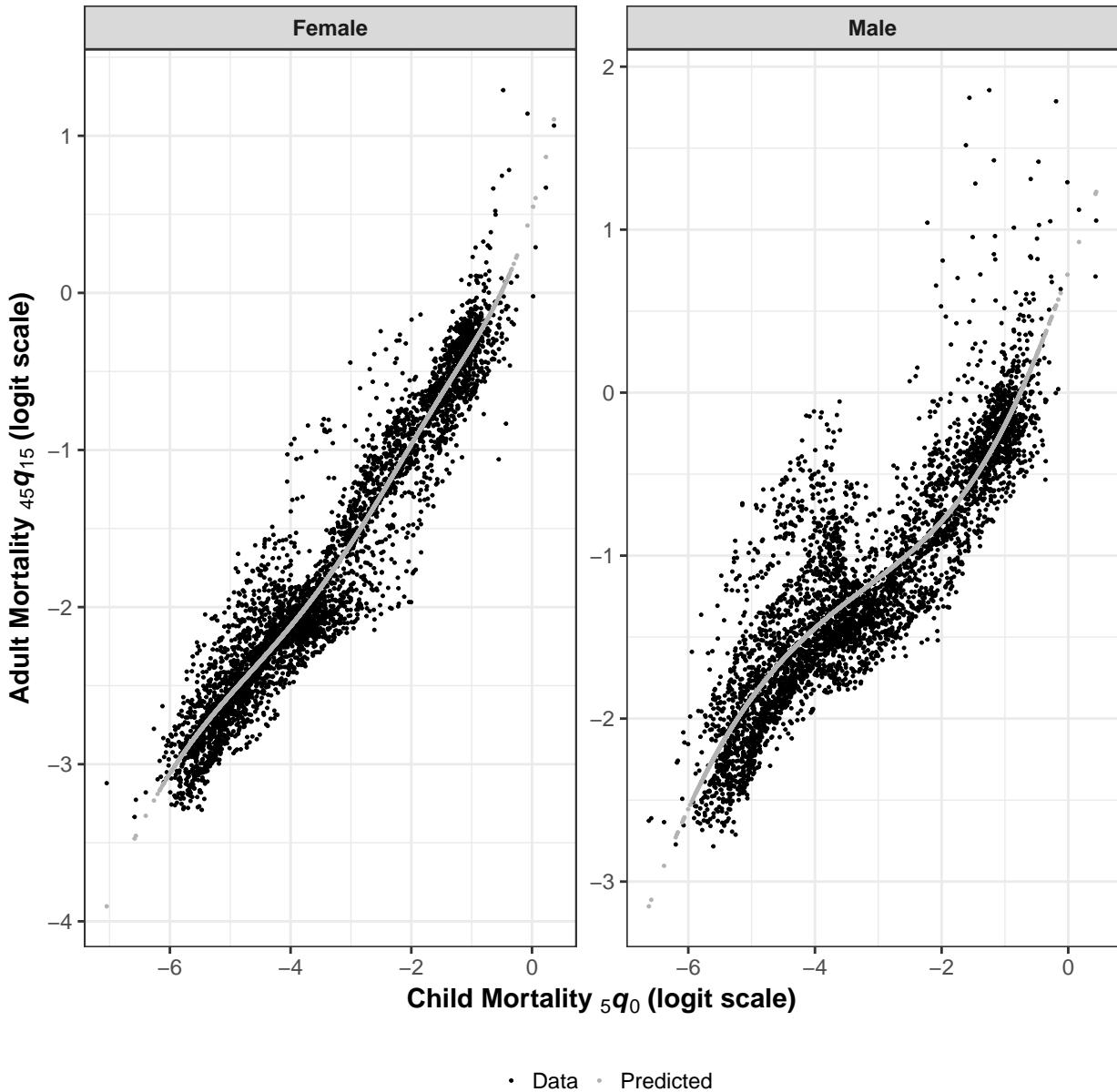
# Plot
ggplot(data = am.cm, aes(x = CM, y = AM, group = Type, colour = Type)) +
  geom_point(size = 0.2) + labs(y = expression(bold("Adult Mortality ")[bold(45)] *
  bolditalic("q")[bold(15)] * bold(" (logit scale)")),
  x = expression(bold("Child Mortality ")[bold(5)] * bolditalic("q")[bold(0)] *
  bold(" (logit scale)")))) + # theme(legend.justification=c(1,0),
# legend.position=c(0.99,0.02)) +
theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(legend.title = element_blank()) + facet_wrap(~Sex,
  scale = "free", labeller = s.labeller) + ggsave("../figures/fig2-2.pdf",
  width = 6.5, height = 6.5, units = c("in"))

# grayscale
ggplot(data = am.cm, aes(x = CM, y = AM, group = Type, colour = Type)) +
  geom_point(size = 0.2) + labs(y = expression(bold("Adult Mortality ")[bold(45)] *
  bolditalic("q")[bold(15)] * bold(" (logit scale)")),
  x = expression(bold("Child Mortality ")[bold(5)] * bolditalic("q")[bold(0)] *
  bold(" (logit scale)")))) + # theme(legend.justification=c(1,0),
# legend.position=c(0.99,0.02)) +
theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(legend.title = element_blank()) + facet_wrap(~Sex,
  scale = "free", labeller = s.labeller) + scale_colour_grey(start = 0,
  end = 0.7) + ggsave("../figures/fig2-2-BW.pdf", width = 6.5,
  height = 6.5, units = c("in"))

# clean up
rm(list = c("am.cm", "am.cm.p.df", "am.cm.p", "am.cm.df"))

```





Plot probability of dying in the first year of life, ${}_1q_0$, versus child mortality, ${}_5q_0$.

```
# data
q0.cm.m <- data.frame(Sex = as.character("Male"), CM = as.numeric(Qlogit.m[1,
  ]), q0 = as.numeric(q1logit.m[1, ]))
# str(q0.cm.m)

q0.cm.f <- data.frame(Sex = as.character("Female"), CM = as.numeric(Qlogit.f[1,
  ]), q0 = as.numeric(q1logit.f[1, ]))
# str(q0.cm.f)

q0.cm.df <- rbind(q0.cm.m, q0.cm.f)

# predicted
q0.cm.m.p <- data.frame(Sex = as.character("Male"), CM = as.numeric(Qlogit.m[1,
  ]), q0 = as.numeric(predict(mod.1_0.m$mods$s1$q0)))
```

```

# str(q0.cm.m.p)

q0.cm.f.p <- data.frame(Sex = as.character("Female"), CM = as.numeric(Qlogit.f[1,
  ]), q0 = as.numeric(predict(mod.1_0.f$mods$s1$q0)))
# str(q0.cm.f.p)

q0.cm.p.df <- rbind(q0.cm.m.p, q0.cm.f.p)

q0 <- rbind(cbind(q0.cm.df, Type = "Data"), cbind(q0.cm.p.df,
  Type = "Predicted"))

# order female first
q0[, 1] <- factor(q0[, 1], levels = c("Female", "Male"))
# str(q0)

s.names <- list(`S#1` = expression(bold("Female")), `S#2` = expression(bold("Male")))
# s.names

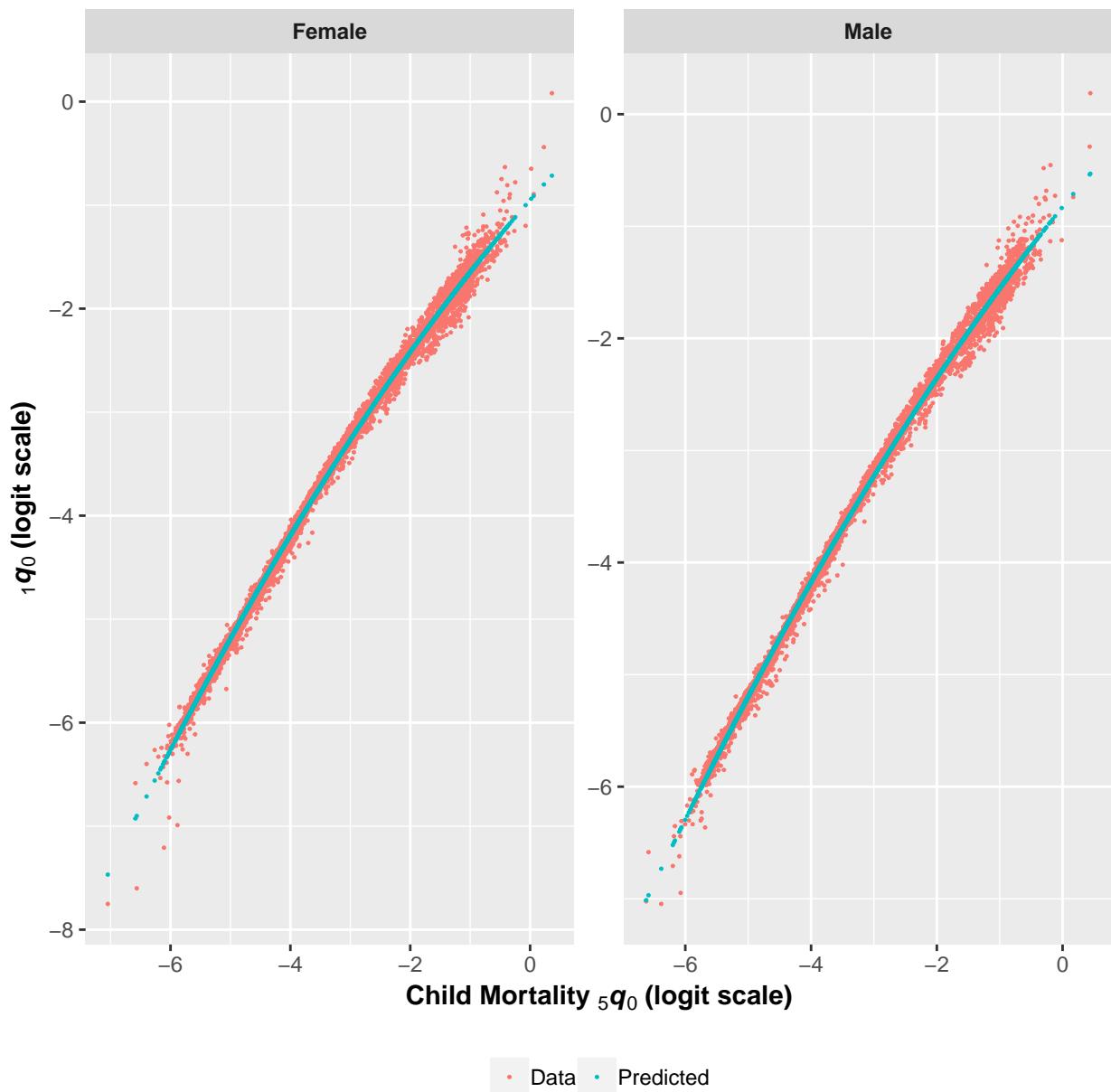
s.labeller <- function(variable, value) {
  return(s.names[value])
}

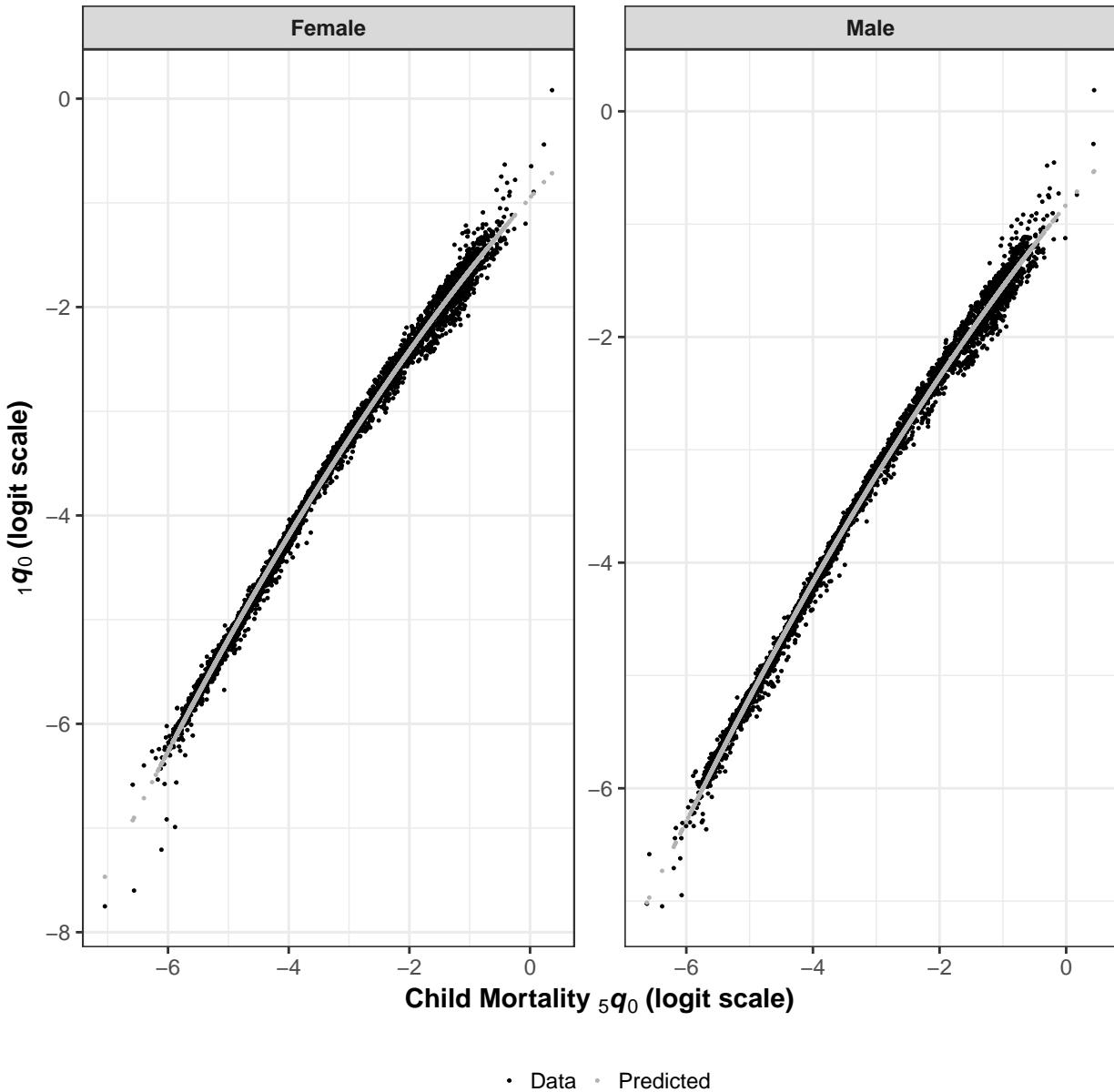
# Plot
ggplot(data = q0, aes(x = CM, y = q0, group = Type, colour = Type)) +
  geom_point(size = 0.2) + labs(y = expression("[bold(1)] *",
    bolditalic("q")[0] * bold(" (logit scale)")), x = expression(bold("Child Mortality ")[bold(5)] *,
    bolditalic("q")[[bold(0)]] * bold(" (logit scale)"))) +
  # theme(legend.justification=c(1,0),
  # legend.position=c(0.99,0.02)) +
  theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(legend.title = element_blank()) + facet_wrap(~Sex,
  scale = "free", labeller = s.labeller) + ggsave("../figures/fig2-3.pdf",
  width = 6.5, height = 6.5, units = c("in"))

# grayscale
ggplot(data = q0, aes(x = CM, y = q0, group = Type, colour = Type)) +
  geom_point(size = 0.2) + labs(y = expression("[bold(1)] *",
    bolditalic("q")[0] * bold(" (logit scale)")), x = expression(bold("Child Mortality ")[bold(5)] *,
    bolditalic("q")[[bold(0)]] * bold(" (logit scale)"))) +
  # theme(legend.justification=c(1,0),
  # legend.position=c(0.99,0.02)) +
  theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(legend.title = element_blank()) + facet_wrap(~Sex,
  scale = "free", labeller = s.labeller) + scale_colour_grey(start = 0,
  end = 0.7) + ggsave("../figures/fig2-3-BW.pdf", width = 6.5,
  height = 6.5, units = c("in"))

# clean up
rm(list = c("q0", "q0.cm.p.df", "q0.cm.f.p", "q0.cm.m.p",
  "q0.cm.df", "q0.cm.f", "q0.cm.m"))

```





Plot heuristic predictions or life tables at three levels of child mortality from very low to very high.

```
# some values for logit-scale 5q0
cml.input <- c(-5.5, -3.2, -1.5)

# predict life tables using the basic models we fit
# earlier
lt.f <- ltPredict(mod.1_0.sm.f, smooth = TRUE, cml.input)
# str(lt.f)
lt.m <- ltPredict(mod.1_0.sm.m, smooth = TRUE, cml.input)
# str(lt.m)

lt.p <- rbind(cbind("Female", as.numeric(rownames(lt.f)),
  cml.input[1], lt.f[, 1]), cbind("Female", as.numeric(rownames(lt.f)),
  cml.input[2], lt.f[, 2]), cbind("Female", as.numeric(rownames(lt.f)),
  cml.input[3], lt.f[, 3]), cbind("Male", as.numeric(rownames(lt.m)),
```

```

cml.input[1], lt.m[, 1]), cbind("Male", as.numeric(rownames(lt.m)),
cml.input[2], lt.m[, 2]), cbind("Male", as.numeric(rownames(lt.m)),
cml.input[3], lt.m[, 3]))
```

```

lt.p.df <- data.frame(Sex = as.character(lt.p[, 1]), Age = as.numeric(lt.p[, 2]),
cml = as.character(lt.p[, 3]), ql = as.numeric(lt.p[, 4]))
# str(lt.p.df)
```

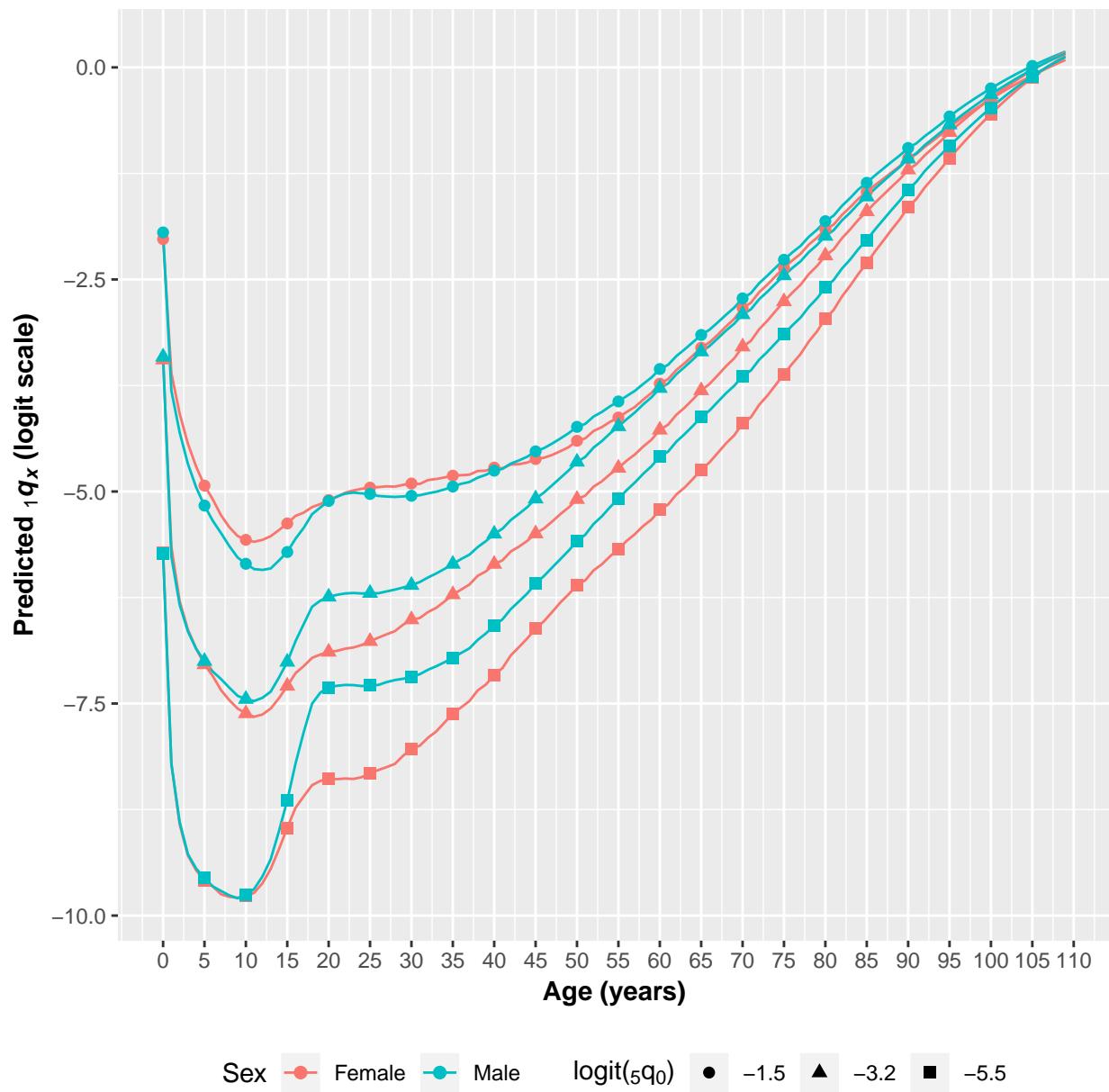
```

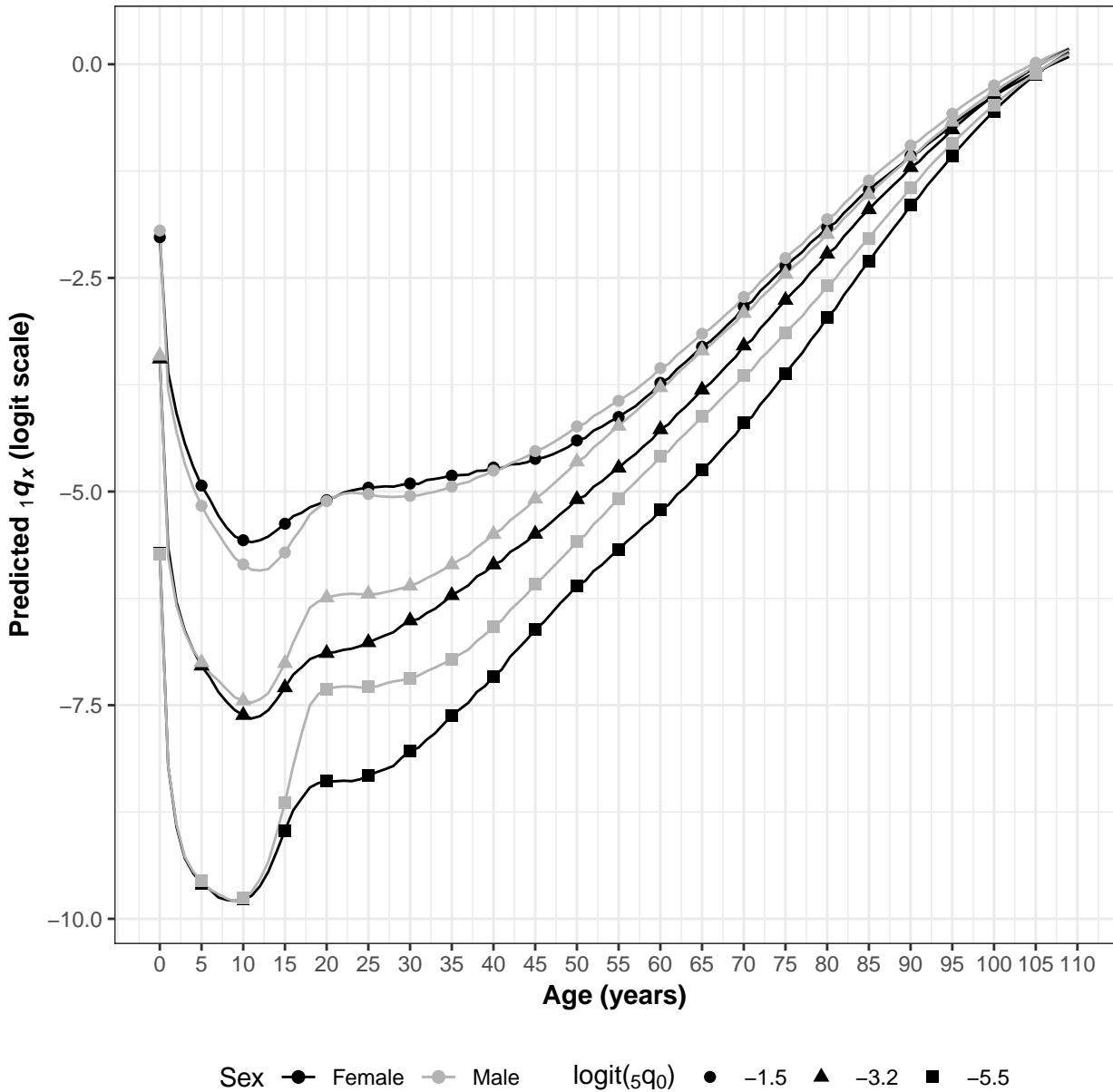
# Plot
ggplot(data = lt.p.df, aes(x = Age, y = ql, group = interaction(Sex,
cml), colour = Sex, shape = cml)) + geom_line() + geom_point(data = lt.p.df[seq(1,
nrow(lt.p.df), 5), ], size = 2) + scale_x_continuous(breaks = c(seq(0,
110, 5))) + labs(y = expression(bold("Predicted ")[bold(1)] *
bolditalic("q") [bolditalic(x)] * bold(" (logit scale)")),
x = expression(bold("Age (years)")))) + # theme(legend.justification=c(1,0),
# legend.position=c(0.95,0.05)) +
theme(legend.position = "bottom", legend.box = "horizontal") +
scale_shape_discrete(name = expression("logit("[5] *
"q"[0] * ")")) + ggsave("../figures/fig6.pdf", width = 6.5,
height = 6.5, units = c("in"))

# Plot
ggplot(data = lt.p.df, aes(x = Age, y = ql, group = interaction(Sex,
cml), colour = Sex, shape = cml)) + geom_line() + geom_point(data = lt.p.df[seq(1,
nrow(lt.p.df), 5), ], size = 2) + scale_x_continuous(breaks = c(seq(0,
110, 5))) + labs(y = expression(bold("Predicted ")[bold(1)] *
bolditalic("q") [bolditalic(x)] * bold(" (logit scale)")),
x = expression(bold("Age (years)")))) + # theme(legend.justification=c(1,0),
# legend.position=c(0.95,0.05)) +
theme_bw() + theme(legend.position = "bottom", legend.box = "horizontal") +
scale_shape_discrete(name = expression("logit("[5] *
"q"[0] * ")")) + scale_colour_grey(start = 0, end = 0.7) +
ggsave("../figures/fig6-BW.pdf", width = 6.5, height = 6.5,
units = c("in"))

# clean up
rm(list = c("lt.p.df", "lt.p", "lt.m", "lt.f", "cml.input"))

```





6 Make Tables

Load the Stargazer and xtable packages for making nice LaTeX tables from regression output. The code uses `capture.output()` to redirect output to text files where either complete LaTeX tables are stored, or the rows of LaTeX tables are stored. The rows-only tables slot nicely into headers and footers that are nicely formatted in the manuscript, and the whole tables (Stargazer output) are completely ready to go and slot straight into the LaTeX.

Create table describing which HMD life tables are included in the analysis.

```
# recall that the same life tables are used for females
# and males
all.equal(colnames(q1.f), paste("fe", colnames(q1.m), sep = ""))
## [1] TRUE
```

```

# data frame with the life table names with years
allLifetables <- read.table(text = colnames(q1.f), sep = ".",
  colClasses = "character")
colnames(allLifetables) <- c("sex", "country", "year")
allLifetables$year <- as.numeric(allLifetables$year)

# summarize life tables
country <- allLifetables[1, 2]
year <- allLifetables[1, 3]
ltList <- list()
ltList[[1]] <- c(country, year, "")
index <- 1
for (i in 2:nrow(allLifetables)) {
  if ((allLifetables[i, 2] != country) | (allLifetables[i,
    3] != (allLifetables[(i - 1), 3] + 1))) {
    ltList[[index]][3] <- allLifetables[(i - 1), 3]
    country <- allLifetables[i, 2]
    year <- allLifetables[i, 3]
    index <- index + 1
    ltList[[index]] <- c(country, year, "")
  }
  if (i == nrow(allLifetables)) {
    ltList[[index]][3] <- allLifetables[i, 3]
  }
}
# have a look at the list of life countries with their
# life tables ltList

# create LaTeX for life table summaries

# function to parse out full country names from first
# line of HMD life table text file
parse.countries <- function(file.name) {

  con <- file(file.name, "r")
  first.line <- readLines(con, n = 1)
  close(con)

  return(str_split(first.line, ",")[[1]][1])
}

# read and split the list of file names from HMD
files <- Sys.glob("../data/HMD/hmd_statistics/lt_female/fltpcr_5x1/*")
# files

# # make a list of country abbreviations and their full
# names country.names <- list() for (i in
# 1:length(files)) { country.names[[i]] <-
# c(strsplit(basename(files[i]), '\.')[[1]][1]
# , parse.countries(files[[i]])) } # have a look at the
# list of full country names # country.names

```

```

# make a list of country abbreviations and their full
# names
country.names <- list()
for (i in 1:length(files)) {
  country.names[[i]] <- c(strsplit(basename(files[i]),
    "\\.")[[1]][1], parse.countries(files[[i]]))
  # 'by hand' correction to full country names
  if (country.names[[i]][1] == "GBRTENW") {
    country.names[[i]][2] <- "England and Wales -- Total Population"
  }
  if (country.names[[i]][1] == "GBRCENW") {
    country.names[[i]][2] <- "England and Wales -- Civilian National Population"
  }
  if (country.names[[i]][1] == "FRACNP") {
    country.names[[i]][2] <- "France -- Civilian Population"
  }
  if (country.names[[i]][1] == "FRATNP") {
    country.names[[i]][2] <- "France -- Total Population"
  }
}
# have a look at the list of full country names
# country.names

# print life table summaries to local output
ltGrandTot <- 0
for (i in 1:length(ltList)) {
  for (j in 1:length(country.names)) {
    if (ltList[[i]][1] == str_to_upper(country.names[[j]][1])) {
      ltTot <- as.numeric(ltList[[i]][3]) - as.numeric(ltList[[i]][2]) +
        1
      ltGrandTot <- ltGrandTot + ltTot
      cat(country.names[[j]][1], "&", country.names[[j]][2],
        "&", ltList[[i]][2], "--", ltList[[i]][3],
        "&", ltTot, " \\\\", "
      )
    }
  }
}

## AUS & Australia & 1921 -- 2014 & 94 \\
## AUT & Austria & 1947 -- 2017 & 71 \\
## BEL & Belgium & 1841 -- 1913 & 73 \\
## BEL & Belgium & 1919 -- 2015 & 97 \\
## BGR & Bulgaria & 1947 -- 2010 & 64 \\
## BLR & Belarus & 1959 -- 2016 & 58 \\
## CAN & Canada & 1921 -- 2011 & 91 \\
## CHE & Switzerland & 1876 -- 2016 & 141 \\
## CHL & Chile & 1992 -- 2008 & 17 \\
## CZE & Czechia & 1950 -- 2016 & 67 \\
## DEUTE & East Germany & 1956 -- 2015 & 60 \\
## DEUTNP & Germany & 1990 -- 2015 & 26 \\
## DEUTW & West Germany & 1956 -- 2015 & 60 \\
## DNK & Denmark & 1835 -- 2016 & 182 \\
## ESP & Spain & 1908 -- 2016 & 109 \\
## EST & Estonia & 1959 -- 2017 & 59 \\

```

```

## FIN & Finland & 1878 -- 2015 & 138 \\
## FRACNP & France -- Civilian Population & 1816 -- 2016 & 201 \\
## FRATNP & France -- Total Population & 1816 -- 2016 & 201 \\
## GBRCENW & England and Wales -- Civilian National Population & 1841 -- 2016 & 176 \\
## GBRTENW & England and Wales -- Total Population & 1841 -- 2016 & 176 \\
## GBR_NIR & Northern Ireland & 1922 -- 2016 & 95 \\
## GBR_NP & United Kingdom & 1922 -- 2016 & 95 \\
## GBR_SCO & Scotland & 1855 -- 2016 & 162 \\
## GRC & Greece & 1981 -- 2013 & 33 \\
## HRV & Croatia & 2002 -- 2016 & 15 \\
## HUN & Hungary & 1950 -- 2017 & 68 \\
## IRL & Ireland & 1950 -- 2014 & 65 \\
## ISL & Iceland & 1838 -- 1851 & 14 \\
## ISL & Iceland & 1853 -- 2016 & 164 \\
## ISR & Israel & 1983 -- 2014 & 32 \\
## ITA & Italy & 1872 -- 2014 & 143 \\
## JPN & Japan & 1947 -- 2016 & 70 \\
## KOR & Korea & 2003 -- 2016 & 14 \\
## LTU & Lithuania & 1959 -- 2017 & 59 \\
## LUX & Luxembourg & 1960 -- 2014 & 55 \\
## LVA & Latvia & 1959 -- 2017 & 59 \\
## NLD & Netherlands & 1850 -- 2016 & 167 \\
## NOR & Norway & 1846 -- 2014 & 169 \\
## NZL_MA & New Zealand -- Maori & 1948 -- 1948 & 1 \\
## NZL_MA & New Zealand -- Maori & 1950 -- 1955 & 6 \\
## NZL_MA & New Zealand -- Maori & 1957 -- 1958 & 2 \\
## NZL_MA & New Zealand -- Maori & 1960 -- 2008 & 49 \\
## NZL_NM & New Zealand -- Non-Maori & 1901 -- 2008 & 108 \\
## NZL_NP & New Zealand & 1948 -- 2013 & 66 \\
## POL & Poland & 1958 -- 2016 & 59 \\
## PRT & Portugal & 1940 -- 2015 & 76 \\
## RUS & Russia & 1959 -- 2014 & 56 \\
## SVK & Slovakia & 1950 -- 2014 & 65 \\
## SVN & Slovenia & 1983 -- 2014 & 32 \\
## SWE & Sweden & 1751 -- 2016 & 266 \\
## TWN & Taiwan & 1970 -- 2014 & 45 \\
## UKR & Ukraine & 1959 -- 2013 & 55 \\
## USA & The United States of America & 1933 -- 2016 & 84 \\

# create a dataframe with the country names,
# abbreviations, and number of consecutive life tables
ltGrandTot <- 0
life.tables <- data.frame(country = character(length(ltList)),
                           abbreviation = character(length(ltList)), startYear = numeric(length(ltList)),
                           stopYear = numeric(length(ltList)), tables = numeric(length(ltList)),
                           stringsAsFactors = FALSE)
for (i in 1:length(ltList)) {
  for (j in 1:length(country.names)) {
    if (ltList[[i]][1] == str_to_upper(country.names[[j]][1])) {
      ltTot <- as.numeric(ltList[[i]][3]) - as.numeric(ltList[[i]][2]) +
        1
      ltGrandTot <- ltGrandTot + ltTot
      life.tables$abbreviation[i] <- as.character(country.names[[j]][1])
      life.tables$country[i] <- as.character(country.names[[j]][2])
    }
  }
}

```

```

        life.tables$startYear[i] <- as.numeric(ltList[[i]][2])
        life.tables$stopYear[i] <- as.numeric(ltList[[i]][3])
        life.tables$tables[i] <- as.numeric(ltTot)
    }
}
}

lts.print <- cbind(life.tables[, c(1, 2)], paste(life.tables[, 3], "--", life.tables[, 4], sep = ""), life.tables[, 5])

# save rows of table summarizing life tables in a text file
capture.output(file = "../tables/LTSummaries.txt", print.xtable(xtable(lts.print,
  booktabs = TRUE, digits = 0), only.contents = TRUE,
  include.rownames = FALSE, include.colnames = FALSE,
  hline.after = NULL))

# save the number of life tables for each sex
capture.output(file = "../tables/LTtot.txt", cat(format(ltGrandTot,
  nsmall = 0, big.mark = ",")))

# save the number of life tables for both sexes
capture.output(file = "../tables/LTtotBoth.txt", cat(format(2 *
  ltGrandTot, nsmall = 0, big.mark = ",")))

# save the download date
capture.output(file = "../tables/HMDdate.txt", cat(download.date))

print("")

## [1] ""

print(paste("Total number of life tables in raw data (q1.f):",
  length(colnames(q1.f)))))

## [1] "Total number of life tables in raw data (q1.f): 4610"
print(paste("Check, totalling up country counts of life tables:",
  ltGrandTot))

## [1] "Check, totalling up country counts of life tables: 4610"
print("")

## [1] ""

print("Four 'by-hand' corrections")

## [1] "Four 'by-hand' corrections"
print("GBRTENW: England and Wales - Total Population")

## [1] "GBRTENW: England and Wales - Total Population"
print("GBRCENW: England and Wales - Civilian National Population")

## [1] "GBRCENW: England and Wales - Civilian National Population"

```

```

print("FRACNP: France - Civilian Population")

## [1] "FRACNP: France - Civilian Population"
print("FRATNP: France - Total Population")

## [1] "FRATNP: France - Total Population"
rm(list = c("country", "year", "files", "i", "j", "index"))

```

Make lines that describe the sum of squares explained by each SVD component.

```

# sum of squares explained by each component is the
# square of the corresponding singular value

# calculate the ss for females
ss.1.f <- (mod.1_0.f$svd$s1$d^2)[1]/sum(mod.1_0.f$svd$s1$d^2)
ss.2.f <- (mod.1_0.f$svd$s1$d^2)[2]/sum(mod.1_0.f$svd$s1$d^2)
ss.3.f <- (mod.1_0.f$svd$s1$d^2)[3]/sum(mod.1_0.f$svd$s1$d^2)
ss.4.f <- (mod.1_0.f$svd$s1$d^2)[4]/sum(mod.1_0.f$svd$s1$d^2)
ss.1to4.f <- format(round(sum((mod.1_0.f$svd$s1$d^2)[1:4])/sum(mod.1_0.f$svd$s1$d^2),
  6), format = "d")
ss.f <- format(round(c(ss.1.f, ss.2.f, ss.3.f, ss.4.f),
  6), format = "d")
# write the line for males
capture.output(file = "../tables/ssFullFemale.txt", cat(paste(paste(ss.f[1:3],
  collapse = ", "), sep = ""), ", and ", ss.f[4], sep = "")))
capture.output(file = "../tables/ssFullFemaleTotal.txt",
  cat(ss.1to4.f))

# calculate the ss for males
ss.1.m <- (mod.1_0.m$svd$s1$d^2)[1]/sum(mod.1_0.m$svd$s1$d^2)
ss.2.m <- (mod.1_0.m$svd$s1$d^2)[2]/sum(mod.1_0.m$svd$s1$d^2)
ss.3.m <- (mod.1_0.m$svd$s1$d^2)[3]/sum(mod.1_0.m$svd$s1$d^2)
ss.4.m <- (mod.1_0.m$svd$s1$d^2)[4]/sum(mod.1_0.m$svd$s1$d^2)
ss.1to4.m <- format(round(sum((mod.1_0.m$svd$s1$d^2)[1:4])/sum(mod.1_0.m$svd$s1$d^2),
  6), format = "d")
ss.m <- format(round(c(ss.1.m, ss.2.m, ss.3.m, ss.4.m),
  6), format = "d")
# write the line for males
capture.output(file = "../tables/ssFullMale.txt", cat(paste(paste(ss.m[1:3],
  collapse = ", "), sep = ""), ", and ", ss.m[4], sep = "")))
capture.output(file = "../tables/ssFullMaleTotal.txt", cat(ss.1to4.m))

# calculate fractions of 2+ component ss explained by
# components 2-4

# female
ss.2plus.tot.f <- sum(mod.1_0.f$svd$s1$d[2:length(mod.1_0.f$svd$s1$d)]^2)
ss.2plus.2.f <- (mod.1_0.f$svd$s1$d^2)[2]/ss.2plus.tot.f
ss.2plus.3.f <- (mod.1_0.f$svd$s1$d^2)[3]/ss.2plus.tot.f
ss.2plus.4.f <- (mod.1_0.f$svd$s1$d^2)[4]/ss.2plus.tot.f
ss.2plus.f <- c(ss.2plus.2.f, ss.2plus.3.f, ss.2plus.4.f)
ss.2plus.format.f <- format(round(ss.2plus.f, 6), format = "d")
ss.2plus.tot.f <- format(round(sum(ss.2plus.f), 6), format = "d")
capture.output(file = "../tables/ssFemale.txt", cat(paste(paste(ss.2plus.format.f[1:2],

```

```

collapse = " ", sep = ""), ", and ", ss.2plus.format.f[3],
sep = ""))
capture.output(file = "../tables/ssFemaleTotal.txt", cat(ss.2plus.tot.f))
# male
ss.2plus.tot.m <- sum(mod.1_0.m$svd$s1$d[2:length(mod.1_0.m$svd$s1$d)]^2)
ss.2plus.2.m <- (mod.1_0.m$svd$s1$d^2)[2]/ss.2plus.tot.m
ss.2plus.3.m <- (mod.1_0.m$svd$s1$d^2)[3]/ss.2plus.tot.m
ss.2plus.4.m <- (mod.1_0.m$svd$s1$d^2)[4]/ss.2plus.tot.m
ss.2plus.m <- c(ss.2plus.2.m, ss.2plus.3.m, ss.2plus.4.m)
ss.2plus.format.m <- format(round(ss.2plus.m, 6), format = "d")
ss.2plus.tot.m <- format(round(sum(ss.2plus.m), 6), format = "d")
# write the line for males
capture.output(file = "../tables/ssMale.txt", cat(paste(paste(ss.2plus.format.m[1:2],
collapse = " "), sep = ""), ", and ", ss.2plus.format.m[3],
sep = )))
capture.output(file = "../tables/ssMaleTotal.txt", cat(ss.2plus.tot.m))

```

Make table of summary comparison results.

```

# the summary comparisons are stored in comps. ...
# objects
comps.child$female

##      total.abs.error mean.abs.error max.error
## comp      1445.741     0.01363521 0.3306283
## lq       1501.606     0.01416209 0.3968440

comps.child$male

##      total.abs.error mean.abs.error max.error
## comp      1674.491     0.01579262 0.3773859
## lq       1777.257     0.01676183 0.3792040
cat("\n")

comps.adult$female

##      total.abs.error mean.abs.error max.error
## comp      1297.786     0.01223980 0.2204735
## lq       1399.336     0.01319754 0.3865020

comps.adult$male

##      total.abs.error mean.abs.error max.error
## comp      1378.421     0.01300029 0.3866729
## lq       1472.245     0.01388518 0.3532550

# make lines for the table

# female make code more readable ... a, c b, d
a.f <- comps.child$female[1, 1] # child-only SVD-Comp
b.f <- comps.child$female[2, 1] # child-only Log-Quad
c.f <- comps.adult$female[1, 1] # child/adult SVD-Comp
d.f <- comps.adult$female[2, 1] # child/adult Log-Quad
# write the few lines
capture.output(file = "../tables/compsFemale.txt", cat(paste("R1 & SVD-Comp &",
format(a.f, big.mark = ",", digits = 0), "&", format(c.f,
big.mark = ",", digits = 0), "&", format(c.f - a.f,

```

```

    big.mark = "", digits = 0), " \\\\", sep = " "),
"\n"), cat(paste("R2 & Log-Quad &", format(b.f, big.mark = "", digits = 0), "&", format(d.f, big.mark = "", digits = 0),
"&", format(d.f - b.f, big.mark = "", digits = 0),
" \\\\", sep = " "), "\n"), cat(paste("R3 & R2-R1 &",
format(round(b.f - a.f, 0), nsmall = 0), "&", format(round(d.f -
c.f, 0), nsmall = 0), "&", format(round((d.f - b.f) -
(c.f - a.f), 0), nsmall = 0), " \\\\", sep = " "),
"\n"), cat(paste("R4 & R3/R1 (\%) &", format(round(100 *
(b.f - a.f)/a.f, 1), nsmall = 1), "&", format(round(100 *
(d.f - c.f)/c.f, 1), nsmall = 1), "&", format(round(100 *
((d.f - b.f) - (c.f - a.f))/(c.f - a.f), 1), nsmall = 1),
" \\\\", sep = " "), "\n"))

# male make code more readable ... a, c b, d
a.m <- comps.child$male[1, 1] # child-only SVD-Comp
b.m <- comps.child$male[2, 1] # child-only Log-Quad
c.m <- comps.adult$male[1, 1] # child/adult SVD-Comp
d.m <- comps.adult$male[2, 1] # child/adult Log-Quad
# write the few lines
capture.output(file = "../tables/compsMale.txt", cat(paste("R1 & SVD-Comp &",
format(a.m, big.mark = "", digits = 0), "&", format(c.m,
big.mark = "", digits = 0), "&", format(c.m - a.m,
big.mark = "", digits = 0), " \\\\", sep = " "),
"\n"), cat(paste("R2 & Log-Quad &", format(b.m, big.mark = "", digits = 0), "&", format(d.m, big.mark = "", digits = 0),
"&", format(d.m - b.m, big.mark = "", digits = 0),
" \\\\", sep = " ), "\n"), cat(paste("R3 & R2-R1 &",
format(round(b.m - a.m, 0), nsmall = 0), "&", format(round(d.m -
c.m, 0), nsmall = 0), "&", format(round((d.m - b.m) -
(c.m - a.m), 0), nsmall = 0), " \\\\", sep = " ),
"\n"), cat(paste("R4 & R3/R1 (%) &", format(round(100 *
(b.m - a.m)/a.m, 1), nsmall = 1), "&", format(round(100 *
(d.m - c.m)/c.m, 1), nsmall = 1), "&", format(round(100 *
((d.m - b.m) - (c.m - a.m))/(c.m - a.m), 1), nsmall = 1),
" \\\\", sep = " ), "\n"))

```

Make nice LaTeX tables from the regression models that are part of SVD-Comp. Don't worry about the warnings *Stargazer* raises.

```

# adult mortality model
capture.output(file = "../tables/adultMortality.txt", stargazer(mod.1_0.f$mods$s1$aml,
mod.1_0.m$mods$s1$aml, title = "Adult Mortality Models: $\logit(\qff)_z = f(qf_z, z \\\\"
label = "tab:appA:adultMxMod", dep.var.labels.include = FALSE,
dep.var.caption = "$\logit(\qff)$", model.numbers = FALSE,
column.labels = c("Female", "Male"), covariate.labels = c("$qf$",
"$\mbox{logit}(\qff)$", "$\mbox{logit}(\qff)^2$",
"$\mbox{logit}(\qff)^3$"), omit.stat = c("LL",
"ser"), single.row = TRUE))

# infant mortality
capture.output(file = "../tables/infantMortality.txt", stargazer(mod.1_0.f$mods$s1$q0,
mod.1_0.m$mods$s1$q0, title = "Infant Mortality Models: $\logit(qoz)_z = f(qf_z, z \\\\"
label = "tab:appA:infantMxMod", dep.var.labels.include = FALSE,

```

```

dep.var.caption = "$\\logit(\\qoz)$", model.numbers = FALSE,
column.labels = c("Female", "Male"), covariate.labels = c("$\\mbox{logit}(\\qf)$",
"$\\mbox{logit}(\\qf^2)$"), omit.stat = c("LL",
"ser"), single.row = TRUE)

# vs - female
capture.output(file = "../tables/vsFemale.txt", stargazer(mod.1_0.f$mods$s1$v1,
mod.1_0.f$mods$s1$v2, mod.1_0.f$mods$s1$v3, mod.1_0.f$mods$s1$v4,
title = "Female RSV Models: $v_{\\ell i} = f_i(\\qf_{\\ell}, \\qff_{\\ell})$",
label = "tab:appA:femaleRSVMod", dep.var.labels.include = FALSE,
dep.var.caption = "Right Singular Vector Elements",
model.numbers = FALSE, column.labels = c("$\\mbf{v}_1$",
"$\\mbf{v}_2$", "$\\mbf{v}_3$", "$\\mbf{v}_4$"),
covariate.labels = c("$\\qf$", "$\\mbox{logit}(\\qf)$",
"$\\mbox{logit}(\\qf^2)$", "$\\mbox{logit}(\\qf)^3$",
"$\\qff$", "$\\mbox{logit}(\\qff)^2$", "$\\mbox{logit}(\\qff)^3$",
"$\\qf \\times \\qff$"), omit.stat = c("LL", "ser")))

# vs - male
capture.output(file = "../tables/vsMale.txt", stargazer(mod.1_0.m$mods$s1$v1,
mod.1_0.m$mods$s1$v2, mod.1_0.m$mods$s1$v3, mod.1_0.m$mods$s1$v4,
title = "Male RSV Models: $v_{\\ell i} = f_i(\\qf_{\\ell}, \\qff_{\\ell})$",
label = "tab:appA:maleRSVMod", dep.var.labels.include = FALSE,
dep.var.caption = "Right Singular Vector Elements",
model.numbers = FALSE, column.labels = c("$\\mbf{v}_1$",
"$\\mbf{v}_2$", "$\\mbf{v}_3$", "$\\mbf{v}_4$"),
covariate.labels = c("$\\qf$", "$\\mbox{logit}(\\qf)$",
"$\\mbox{logit}(\\qf^2)$", "$\\mbox{logit}(\\qf)^3$",
"$\\qff$", "$\\mbox{logit}(\\qff)^2$", "$\\mbox{logit}(\\qff)^3$",
"$\\qf \\times \\qff$"), omit.stat = c("LL", "ser")))

```

Create lines for age-specific error tables. These compare the l_x -weighted age-specific total absolute error (tae) in prediction between SVD-Comp and Log Quad. The coding strategy is to write a couple functions to automate this set of calculations so that it can be repeated several times later using different numbers of components in the SVD-Comp predictions. The first function *lthat()* creates full life tables from the SVD-Comp predictions – so that we can get age-specific expectations of life, e_x . The second function *ageSpecificErrorComparisons()* used the first and actually calculates the age-weighted prediction errors and their differences and organizes them into a nice return object.

```

# function to calculate life table from matrix of 1qx
lthat <- function(q, sex, a1.f, a1.m) {
  # calculate lx
  zeroes <- matrix(0, nrow = (nrow(q) + 1), ncol = ncol(q))
  l <- zeroes
  l[1, ] <- 1e+05 # l0 = 100000
  # loop through ages and calculate lx
  for (i in 2:nrow(l)) {
    l[i, ] <- l[(i - 1), ] * (1 - q[(i - 1), ])
  }
  # calculate Lx
  L <- zeroes
  # loop through ages and calculate Lx
  for (i in 1:(nrow(l) - 1)) {
    L[i, ] <- l[(i + 1), ] + ifelse(str_to_lower(sex) ==

```

```

    "female", a1.f[i, ], a1.m[i, ]) * (l[i, ] -
    l[(i + 1), ])
}
L[nrow(L), ] <- ifelse(str_to_lower(sex) == "female",
    a1.f[nrow(L), ], a1.m[nrow(L), ]) * l[nrow(L), ]
# calculate Tx
T <- zeroes
for (i in 1:(nrow(l) - 1)) {
    T[i, ] <- colSums(L[(i:nrow(T)), ])
}
T[nrow(T), ] <- L[nrow(T), ]
# calculate ex
e <- T/l
lt <- list(qx = q, lx = l, Lx = L, Tx = T, ex = e)
return(lt)
}

# function to conduct age-specific comparisons of
# prediction errors between SVD-Comp and Log Quad
ageSpecificErrorComparisons <- function(mod.f, mod.m, lt.lq,
    q, l, e, a1.f, a1.m) {

    # mod.f is svdMod() return object for females mod.m is
    # svdMod() return object for males lt.q is object with
    # q, e, l columns from Log Quad predictions, five-year
    # age groups q is input HMD life table 5qx columns l is
    # input HMD life table lx columns, five-year age groups
    # e is input HMD life table e columns, five-year age
    # groups

    # create five-year age groups of predicted values female

    # female
    qp.f <- expit(mod.f$recon.samp$s1)
    q5p.f <- convert1qxTo5qxApply(qp.f)
    # male
    qp.m <- expit(mod.m$recon.samp$s1)
    q5p.m <- convert1qxTo5qxApply(qp.m)

    # predicted life tables at five-year age group start
    # ages female
    lt.comp.f <- lthat(qp.f, "Female", a1.f, a1.m) # female life tables
    lt.comp.f.qx <- q5p.f
    lt.comp.f.lx <- lt.comp.f$lx[c(1, 2, seq(6, 111, 5)),
        ]
    lt.comp.f.ex <- lt.comp.f$ex[c(1, 2, seq(6, 111, 5)),
        ]
    # male
    lt.comp.m <- lthat(qp.m, "Male", a1.f, a1.m) # male life tables
    lt.comp.m.qx <- q5p.m
    lt.comp.m.lx <- lt.comp.m$lx[c(1, 2, seq(6, 111, 5)),
        ]
    lt.comp.m.ex <- lt.comp.m$ex[c(1, 2, seq(6, 111, 5)),
        ]
}

```

```

]

# log quad predicted life tables female
lt.lq.f.qx <- lt.lq$q5.lq.f
lt.lq.f.lx <- lt.lq$15.lq.f
lt.lq.f.ex <- lt.lq$e5.lq.f
# male
lt.lq.m.qx <- lt.lq$q5.lq.m
lt.lq.m.lx <- lt.lq$15.lq.m
lt.lq.m.ex <- lt.lq$e5.lq.m

# age-schedule of weights based on HMD lx values female
weights.f <- rowSums(l1$15.f)/sum(rowSums(l1$15.f))
# sum(weight.f) male
weights.m <- rowSums(l1$15.m)/sum(rowSums(l1$15.m))
# sum(weight.m)

# sum age-specific absolute errors in 5qx

# female
tae.comp.q.f <- rowSums(abs(lt.comp.f.qx - q$q5.f)) *
  weights.f[1:23]
tae.lq.q.f <- rowSums(abs(lt.lq.f.qx - q$q5.f)) * weights.f[1:23]
tae.diff.q.f <- tae.comp.q.f - tae.lq.q.f
# male
tae.comp.q.m <- rowSums(abs(lt.comp.m.qx - q$q5.m)) *
  weights.m[1:23]
tae.lq.q.m <- rowSums(abs(lt.lq.m.qx - q$q5.m)) * weights.m[1:23]
tae.diff.q.m <- tae.comp.q.m - tae.lq.q.m

# store it all
tae.q <- cbind(tae.comp.q.f, tae.lq.q.f, tae.diff.q.f,
  tae.comp.q.m, tae.lq.q.m, tae.diff.q.m)
tae.q <- rbind(tae.q, colSums(tae.q))

# sum age-specific absolute errors in ex

# female
tae.comp.e.f <- rowSums(abs(lt.comp.f.ex - e$e5.f)) *
  weights.f
tae.lq.e.f <- rowSums(abs(lt.lq.f.ex - e$e5.f)) * weights.f
tae.diff.e.f <- tae.comp.e.f - tae.lq.e.f
# male
tae.comp.e.m <- rowSums(abs(lt.comp.m.ex - e$e5.m)) *
  weights.m
tae.lq.e.m <- rowSums(abs(lt.lq.m.ex - e$e5.m)) * weights.m
tae.diff.e.m <- tae.comp.e.m - tae.lq.e.m

# store it all
tae.e <- cbind(tae.comp.e.f, tae.lq.e.f, tae.diff.e.f,
  tae.comp.e.m, tae.lq.e.m, tae.diff.e.m)
tae.e <- rbind(tae.e, colSums(tae.e))

```

```

# total absolute error in e0 female
tot.tae.comp.e0.f <- sum(abs(lt.comp.f.ex[1, ] - e$e5.f[1,
    ]))
tot.tae.lq.e0.f <- sum(abs(lt.lq.f.ex[1, ] - e$e5.f[1,
    ]))
tot.tae.diff.e0.f <- tot.tae.comp.e0.f - tot.tae.lq.e0.f
# male
tot.tae.comp.e0.m <- sum(abs(lt.comp.m.ex[1, ] - e$e5.m[1,
    ]))
tot.tae.lq.e0.m <- sum(abs(lt.lq.m.ex[1, ] - e$e5.m[1,
    ]))
tot.tae.diff.e0.m <- tot.tae.comp.e0.m - tot.tae.lq.e0.m

# have a look at it all
tot.tae.e0 <- rbind(c(tot.tae.comp.e0.f, tot.tae.lq.e0.f,
    tot.tae.diff.e0.f), c(tot.tae.comp.e0.m, tot.tae.lq.e0.m,
    tot.tae.diff.e0.m))
rownames(tot.tae.e0) <- c("Female", "Male")

return(list(tae.q = tae.q, tae.e = tae.e, tot.tae.e0 = tot.tae.e0))

}

# create list of five-year age group q, e, and l columns
# from Log Quad predictions conducted earlier
lt.lq <- list(q5.lq.f = comps.child$q5.lq.f, e5.lq.f = comps.child$e5.lq.f,
    15.lq.f = comps.child$15.lq.f, q5.lq.m = comps.child$q5.lq.m,
    e5.lq.m = comps.child$e5.lq.m, 15.lq.m = comps.child$15.lq.m)

# create list of 5qx (five-year age groups) from HMD
# life tables
q <- list(q5.f = q5.f, q5.m = q5.m)

# create list of lx (five-year age groups) from HMD life
# tables
l <- list(15.f = 15.f, 15.m = 15.m)

# create list of ex (five-year age groups) from HMD life
# tables
e <- list(e5.f = e5.f, e5.m = e5.m)

# calculate age-specific comparisons in prediction
# errors
age.comps <- ageSpecificErrorComparisons(mod.1_0.f, mod.1_0.m,
    lt.lq, q, l, e, a1.f, a1.m)
# have a look
age.comps

## $tae.q
##          tae.comp.q.f   tae.lq.q.f   tae.diff.q.f
## 0      1.285808191  1.309623169 -0.0238149773
## 1-4    1.405605466  1.297656974  0.1079484913
## 5-9    0.774404676  0.739540137  0.0348645388
## 10-14  0.509533972  0.488058117  0.0214758555

```

```

## 15-19  0.631768929  0.704215885 -0.0724469560
## 20-24  0.776761164  0.856790241 -0.0800290770
## 25-29  0.767838972  0.847677326 -0.0798383535
## 30-34  0.759797072  0.801390372 -0.0415933000
## 35-39  0.835094853  0.835957359 -0.0008625060
## 40-44  0.949378962  0.918270079  0.0311088829
## 45-49  1.104900947  1.088777855  0.0161230918
## 50-54  1.446209914  1.442450719  0.0037591953
## 55-59  1.890110030  1.908891692 -0.0187816618
## 60-64  2.426462671  2.509440007 -0.0829773354
## 65-69  2.975484183  3.094838412 -0.1193542288
## 70-74  3.859023820  4.004770837 -0.1457470169
## 75-79  4.452347552  4.511095621 -0.0587480699
## 80-84  4.256483071  4.364023689 -0.1075406180
## 85-89  3.032195850  3.032863378 -0.0006675275
## 90-94  1.466339018  1.497975829 -0.0316368113
## 95-99  0.373584075  0.398302792 -0.0247187170
## 100-104 0.045990476  0.051727293 -0.0057368173
## 105-109 0.002843693  0.003311803 -0.0004681095
##           36.027967560 36.707649587 -0.6796820277
##           tae.comp.q.m   tae.lq.q.m   tae.diff.q.m
## 0          1.522463e+00  1.541778e+00 -0.0193146311
## 1-4         2.006411e+00  1.546716e+00  0.4596952560
## 5-9         8.794502e-01  8.656519e-01  0.0137982698
## 10-14        5.178406e-01  4.864013e-01  0.0314393559
## 15-19        8.854434e-01  8.426238e-01  0.0428195914
## 20-24        1.691406e+00  1.619755e+00  0.0716517282
## 25-29        1.567782e+00  1.504629e+00  0.0631531799
## 30-34        1.502594e+00  1.457767e+00  0.0448277045
## 35-39        1.674806e+00  1.621177e+00  0.0536296841
## 40-44        1.951835e+00  1.895655e+00  0.0561803576
## 45-49        2.351121e+00  2.316191e+00  0.0349304345
## 50-54        2.893663e+00  2.923593e+00 -0.0299296940
## 55-59        3.446941e+00  3.593629e+00 -0.1466873652
## 60-64        4.120316e+00  4.439985e+00 -0.3196690361
## 65-69        4.537801e+00  4.918807e+00 -0.3810053609
## 70-74        4.686842e+00  5.135102e+00 -0.4482603087
## 75-79        4.272008e+00  4.563438e+00 -0.2914298592
## 80-84        3.078585e+00  3.282934e+00 -0.2043487757
## 85-89        1.700815e+00  1.766515e+00 -0.0657000210
## 90-94        6.657873e-01  7.064783e-01 -0.0406910103
## 95-99        1.285520e-01  1.440494e-01 -0.0154973660
## 100-104      1.334220e-02  1.559056e-02 -0.0022483617
## 105-109      8.105134e-04  9.597548e-04 -0.0001492414
##           4.609662e+01  4.718942e+01 -1.0928054694
##
## $tae.e
##           tae.comp.e.f   tae.lq.e.f   tae.diff.e.f
## 0          4.108094e+02  4.147746e+02 -3.965212e+00
## 1-4         4.502406e+02  4.594468e+02 -9.206167e+00
## 5-9         4.164232e+02  4.257798e+02 -9.356524e+00
## 10-14        3.954913e+02  4.044524e+02 -8.961155e+00
## 15-19        3.815325e+02  3.889475e+02 -7.414935e+00
## 20-24        3.618813e+02  3.668358e+02 -4.954584e+00

```

```

## 25-29  3.401536e+02 3.433747e+02 -3.221157e+00
## 30-34  3.209064e+02 3.236770e+02 -2.770591e+00
## 35-39  3.042722e+02 3.084361e+02 -4.163924e+00
## 40-44  2.879651e+02 2.936671e+02 -5.702007e+00
## 45-49  2.700467e+02 2.762443e+02 -6.197647e+00
## 50-54  2.487578e+02 2.548594e+02 -6.101545e+00
## 55-59  2.240441e+02 2.294080e+02 -5.363838e+00
## 60-64  1.956326e+02 2.005822e+02 -4.949684e+00
## 65-69  1.648870e+02 1.678420e+02 -2.955035e+00
## 70-74  1.308905e+02 1.327151e+02 -1.824629e+00
## 75-79  9.421929e+01 9.527098e+01 -1.051684e+00
## 80-84  5.958329e+01 6.014278e+01 -5.594867e-01
## 85-89  3.100424e+01 3.091626e+01  8.798583e-02
## 90-94  1.240199e+01 1.235616e+01  4.582874e-02
## 95-99  3.209910e+00 3.197597e+00  1.231316e-02
## 100-104 4.565116e-01 4.550265e-01  1.485138e-03
## 105-109 3.451248e-02 3.425943e-02  2.530459e-04
## 110+    5.822578e-03 2.470463e-03  3.352115e-03
##           5.104850e+03 5.193418e+03 -8.856859e+01
##           tae.comp.e.m   tae.lq.e.m   tae.diff.e.m
## 0        6.189620e+02 6.306460e+02 -1.168393e+01
## 1-4      6.577537e+02 6.605873e+02 -2.833600e+00
## 5-9      6.085742e+02 6.336571e+02 -2.508288e+01
## 10-14     5.910655e+02 6.149495e+02 -2.388406e+01
## 15-19     5.808574e+02 6.045474e+02 -2.368999e+01
## 20-24     5.590702e+02 5.820842e+02 -2.301396e+01
## 25-29     5.200264e+02 5.445539e+02 -2.452747e+01
## 30-34     4.849374e+02 5.120736e+02 -2.713621e+01
## 35-39     4.506845e+02 4.799306e+02 -2.924610e+01
## 40-44     4.143479e+02 4.441654e+02 -2.981748e+01
## 45-49     3.740160e+02 4.020256e+02 -2.800958e+01
## 50-54     3.277359e+02 3.532945e+02 -2.555868e+01
## 55-59     2.759549e+02 2.979205e+02 -2.196555e+01
## 60-64     2.203616e+02 2.383392e+02 -1.797765e+01
## 65-69     1.637741e+02 1.764317e+02 -1.265764e+01
## 70-74     1.119345e+02 1.193255e+02 -7.391022e+00
## 75-79     6.791521e+01 7.092950e+01 -3.014291e+00
## 80-84     3.590373e+01 3.653155e+01 -6.278165e-01
## 85-89     1.588130e+01 1.578827e+01  9.303009e-02
## 90-94     5.518531e+00 5.459889e+00  5.864214e-02
## 95-99     1.154423e+00 1.150808e+00  3.615189e-03
## 100-104   1.399408e-01 1.404794e-01 -5.385160e-04
## 105-109   1.030230e-02 1.022130e-02  8.100027e-05
## 110+      2.868997e-03 1.001968e-03  1.867030e-03
##           7.086583e+03 7.424544e+03 -3.379612e+02
##
## $tot.tae.e0
##           [,1]      [,2]      [,3]
## Female  6134.350 6193.56 -59.20994
## Male    8595.722 8757.98 -162.25837
# create lines for tables
capture.output(file = "../tables/ageCompQ-1.txt", print.xtable(xtable(age.comps$tae.q[(1:nrow(age.comps)
1), ], booktabs = TRUE, digits = 4), only.contents = TRUE,

```

```

    include.rownames = TRUE, include.colnames = FALSE, hline.after = NULL,
    format.args = list(big.mark = ",")))
capture.output(file = "../tables/ageCompQ-2.txt", cat(paste("0-109 & ",
  paste(format(round(age.comps$tae.q[nrow(age.comps$tae.q),
    ], 4), format = "d", big.mark = ","), collapse = " & "),
  " \\\\", sep = ""))
capture.output(file = "../tables/ageCompE-1.txt", print.xtable(xtable(age.comps$tae.e[(1:nrow(age.comps
  1), ], booktabs = TRUE, digits = 2), only.contents = TRUE,
  include.rownames = TRUE, include.colnames = FALSE, hline.after = NULL,
  format.args = list(big.mark = ",")))
capture.output(file = "../tables/ageCompE-2.txt", cat(paste("0+ & ",
  paste(format(round(age.comps$tae.e[nrow(age.comps$tae.e),
    ], 2), format = "d", big.mark = ","), collapse = " & "),
  " \\\\", sep = ""))
capture.output(file = "../tables/ageCompTot.txt", print.xtable(xtable(age.comps$tot.tae.e0,
  booktabs = TRUE, digits = 2), only.contents = TRUE,
  include.rownames = TRUE, include.colnames = FALSE, hline.after = NULL,
  format.args = list(big.mark = ",")))

```

Create lines for tables with scaled component values.

```

# calculate the scaled components
su.f <- mod.1_0.f$svd$s1$u %*% diag(mod.1_0.f$svd$s1$d)
su.m <- mod.1_0.m$svd$s1$u %*% diag(mod.1_0.m$svd$s1$d)
# first 4 components of both
su <- cbind(seq(0, 109, 1), su.f[, 1:4], su.m[, 1:4])
# make the table rows
capture.output(file = "../tables/us.txt", print.xtable(xtable(su,
  booktabs = TRUE, digits = c(0, 0, 2, 2, 2, 2, 2, 2,
  2, 2)), only.contents = TRUE, include.rownames = FALSE,
  include.colnames = FALSE, hline.after = NULL))

```

Conduct age-specific error comparison using 1–4 components. To do this, rerun the models with *svdMod()* asking for 1–4 components, and then recalculate the error comparisons for each of those models. These results are for discussion in text only, no tables produced.

```

# 1 component re-run models with 1 component
adult.1 <- FALSE
smooth.1 <- FALSE
N.1 <- 1
S.1 <- 1
C.1 <- 1
# base model
mod.1_0.m.1 <- svdMod(q1logit.m, Qlogit.m, N.1, S.1, 10,
  TRUE, adult.1, TRUE, smooth.1, C.1)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "1 components"

```

```

mod.1_0.f.1 <- svdMod(q1logit.f, Qlogit.f, N.1, S.1, 10,
  TRUE, adult.1, TRUE, smooth.1, C.1)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "1 components"

# calculate age-specific comparisons in prediction
# errors use the lt.q, q, l, and e from above
age.comps.1 <- ageSpecificErrorComparisons(mod.1_0.f.1,
  mod.1_0.m.1, lt.lq, q, l, e, a1.f, a1.m)
# have a look
cat("\n\n")

age.comps.1

## $tae.q
##          tae.comp.q.f   tae.lq.q.f tae.diff.q.f
## 0        1.28580819  1.309623169 -0.023814977
## 1-4      6.50578917  1.297656974  5.208132196
## 5-9      1.94209962  0.739540137  1.202559488
## 10-14     1.04369833  0.488058117  0.555640218
## 15-19     1.09731300  0.704215885  0.393097114
## 20-24     1.36293794  0.856790241  0.506147700
## 25-29     1.41027018  0.847677326  0.562592856
## 30-34     1.26783812  0.801390372  0.466447749
## 35-39     1.00444260  0.835957359  0.168485238
## 40-44     0.97341055  0.918270079  0.055140469
## 45-49     1.88886195  1.088777855  0.800084093
## 50-54     3.08802264  1.442450719  1.645571926
## 55-59     4.55643503  1.908891692  2.647543341
## 60-64     5.94447983  2.509440007  3.435039824
## 65-69     7.94402675  3.094838412  4.849188338
## 70-74     9.81464282  4.004770837  5.809871980
## 75-79    11.41790149  4.511095621  6.906805870
## 80-84    11.04869359  4.364023689  6.684669896
## 85-89     7.83804137  3.032863378  4.805177991
## 90-94     3.74919759  1.497975829  2.251221765
## 95-99     0.99245394  0.398302792  0.594151143
## 100-104   0.13069329  0.051727293  0.078965993
## 105-109   0.00845995  0.003311803  0.005148147
##          86.31551794 36.707649587 49.607868357
##          tae.comp.q.m   tae.lq.q.m tae.diff.q.m
## 0        1.522463392 1.541778e+00 -0.0193146311
## 1-4      7.641210211 1.546716e+00  6.0944942571
## 5-9      2.212398818 8.656519e-01  1.3467468819
## 10-14     1.033069197 4.864013e-01  0.5466679154
## 15-19     1.002918310 8.426238e-01  0.1602944674
## 20-24     1.854624869 1.619755e+00  0.2348701542
## 25-29     1.729233067 1.504629e+00  0.2246041962
## 30-34     1.627440364 1.457767e+00  0.1696735963
## 35-39     1.692017226 1.621177e+00  0.0708404335

```

```

## 40-44    1.974881556 1.895655e+00  0.0792269319
## 45-49    2.784572526 2.316191e+00  0.4683815895
## 50-54    4.259596141 2.923593e+00  1.3360029713
## 55-59    6.378550406 3.593629e+00  2.7849217656
## 60-64    8.410207743 4.439985e+00  3.9702222533
## 65-69    10.144636897 4.918807e+00  5.2258302135
## 70-74    10.420683920 5.135102e+00  5.2855819548
## 75-79    9.414432550 4.563438e+00  4.8509942026
## 80-84    6.814196421 3.282934e+00  3.5312628209
## 85-89    3.739018307 1.766515e+00  1.9725034807
## 90-94    1.433720148 7.064783e-01  0.7272418454
## 95-99    0.283417233 1.440494e-01  0.1393678438
## 100-104   0.030178140 1.559056e-02  0.0145875788
## 105-109   0.001855555 9.597548e-04  0.0008958003
##          86.405322997 4.718942e+01  39.2158985231
##
## $tae.e
##           tae.comp.e.f   tae.lq.e.f tae.diff.e.f
## 0          7.627039e+02 4.147746e+02 3.479293e+02
## 1-4        8.068862e+02 4.594468e+02 3.474393e+02
## 5-9        5.575243e+02 4.257798e+02 1.317445e+02
## 10-14      5.231429e+02 4.044524e+02 1.186905e+02
## 15-19      5.223195e+02 3.889475e+02 1.333721e+02
## 20-24      5.302867e+02 3.668358e+02 1.634509e+02
## 25-29      5.549217e+02 3.433747e+02 2.115469e+02
## 30-34      5.869899e+02 3.236770e+02 2.633130e+02
## 35-39      6.148764e+02 3.084361e+02 3.064403e+02
## 40-44      6.296619e+02 2.936671e+02 3.359948e+02
## 45-49      6.230954e+02 2.762443e+02 3.468510e+02
## 50-54      5.885393e+02 2.548594e+02 3.336799e+02
## 55-59      5.365671e+02 2.294080e+02 3.071591e+02
## 60-64      4.696153e+02 2.005822e+02 2.690330e+02
## 65-69      3.980527e+02 1.678420e+02 2.302107e+02
## 70-74      3.190440e+02 1.327151e+02 1.863289e+02
## 75-79      2.387816e+02 9.527098e+01 1.435106e+02
## 80-84      1.583294e+02 6.014278e+01 9.818659e+01
## 85-89      8.548431e+01 3.091626e+01 5.456806e+01
## 90-94      3.437061e+01 1.235616e+01 2.201445e+01
## 95-99      8.855386e+00 3.197597e+00 5.657789e+00
## 100-104     1.283821e+00 4.550265e-01 8.287947e-01
## 105-109     9.815337e-02 3.425943e-02 6.389394e-02
## 110+        5.822578e-03 2.470463e-03 3.352115e-03
##           tae.comp.e.m   tae.lq.e.m tae.diff.e.m
## 0          9.072944e+02 6.306460e+02 2.766485e+02
## 1-4        9.540774e+02 6.605873e+02 2.934902e+02
## 5-9        7.019789e+02 6.336571e+02 6.832173e+01
## 10-14      7.029391e+02 6.149495e+02 8.798963e+01
## 15-19      7.164021e+02 6.045474e+02 1.118547e+02
## 20-24      7.127801e+02 5.820842e+02 1.306959e+02
## 25-29      7.056740e+02 5.445539e+02 1.611201e+02
## 30-34      7.016987e+02 5.120736e+02 1.896251e+02
## 35-39      6.973163e+02 4.799306e+02 2.173857e+02
## 40-44      6.876381e+02 4.441654e+02 2.434727e+02

```

```

## 45-49   6.649454e+02 4.020256e+02 2.629197e+02
## 50-54   6.189124e+02 3.532945e+02 2.656179e+02
## 55-59   5.493874e+02 2.979205e+02 2.514669e+02
## 60-64   4.557993e+02 2.383392e+02 2.174600e+02
## 65-69   3.511330e+02 1.764317e+02 1.747013e+02
## 70-74   2.450164e+02 1.193255e+02 1.256909e+02
## 75-79   1.539049e+02 7.092950e+01 8.297539e+01
## 80-84   8.383266e+01 3.653155e+01 4.730111e+01
## 85-89   3.726925e+01 1.578827e+01 2.148098e+01
## 90-94   1.249236e+01 5.459889e+00 7.032468e+00
## 95-99   2.551372e+00 1.150808e+00 1.400564e+00
## 100-104 3.066342e-01 1.404794e-01 1.661549e-01
## 105-109 2.235596e-02 1.022130e-02 1.213466e-02
## 110+    2.868997e-03 1.001968e-03 1.867030e-03
##           1.066338e+04 7.424544e+03 3.238832e+03
##
## $tot.tae.e0
##           [,1]      [,2]      [,3]
## Female  11388.96 6193.56 5195.403
## Male    12599.88 8757.98 3841.905

# create a table with results for 1 components
capture.output(file = "../tables/ageCompTotC-1.txt", print.xtable(xtable(age.comps.1$tot.tae.e0,
  booktabs = TRUE, digits = 2), only.contents = TRUE,
  include.rownames = TRUE, include.colnames = FALSE, hline.after = NULL,
  format.args = list(big.mark = ",")))

# 2 components re-run models with 1 component
adult.2 <- FALSE
smooth.2 <- FALSE
N.2 <- 1
S.2 <- 1
C.2 <- 2
# base model
mod.1_0.m.2 <- svdMod(q1logit.m, Qlogit.m, N.2, S.2, 10,
  TRUE, adult.2, TRUE, smooth.2, C.2)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "2 components"

mod.1_0.f.2 <- svdMod(q1logit.f, Qlogit.f, N.2, S.2, 10,
  TRUE, adult.2, TRUE, smooth.2, C.2)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "2 components"

# calculate age-specific comparisons in prediction
# errors use the lt.q,q,l, and e from above

```

```

age.comps.2 <- ageSpecificErrorComparisons(mod.1_0.f.2,
  mod.1_0.m.2, lt.lq, q, l, e, a1.f, a1.m)
# have a look
cat("\n\n")

```

```
age.comps.2
```

```

## $tae.q
##      tae.comp.q.f   tae.lq.q.f   tae.diff.q.f
## 0     1.285808191  1.309623169 -0.0238149773
## 1-4    1.511146077  1.297656974  0.2134891026
## 5-9     0.808544141  0.739540137  0.0690040039
## 10-14   0.526631577  0.488058117  0.0385734600
## 15-19   0.651270698  0.704215885 -0.0529451868
## 20-24   0.804282231  0.856790241 -0.0525080104
## 25-29   0.785431675  0.847677326 -0.0622456506
## 30-34   0.764372635  0.801390372 -0.0370177376
## 35-39   0.848389503  0.835957359  0.0124321432
## 40-44   0.976435675  0.918270079  0.0581655961
## 45-49   1.132657678  1.088777855  0.0438798237
## 50-54   1.483978917  1.442450719  0.0415281982
## 55-59   1.930368991  1.908891692  0.0214772991
## 60-64   2.551170467  2.509440007  0.0417304598
## 65-69   3.135437380  3.094838412  0.0405989679
## 70-74   4.139780124  4.004770837  0.1350092866
## 75-79   4.810746714  4.511095621  0.2996510921
## 80-84   4.502584955  4.364023689  0.1385612657
## 85-89   3.085540322  3.032863378  0.0526769446
## 90-94   1.460562863  1.497975829 -0.0374129663
## 95-99   0.376024852  0.398302792 -0.0222779403
## 100-104 0.047565413  0.051727293 -0.0041618803
## 105-109 0.003015721  0.003311803 -0.0002960819
##            37.621746799 36.707649587  0.9140972120
##      tae.comp.q.m   tae.lq.q.m   tae.diff.q.m
## 0     1.522463392  1.541778e+00 -1.931463e-02
## 1-4    1.994024064  1.546716e+00  4.473081e-01
## 5-9     0.934275058  8.656519e-01  6.862312e-02
## 10-14   0.548839260  4.864013e-01  6.243798e-02
## 15-19   0.874673949  8.426238e-01  3.205011e-02
## 20-24   1.662009166  1.619755e+00  4.225445e-02
## 25-29   1.551894271  1.504629e+00  4.726540e-02
## 30-34   1.506337223  1.457767e+00  4.857046e-02
## 35-39   1.669623278  1.621177e+00  4.844649e-02
## 40-44   1.971951950  1.895655e+00  7.629733e-02
## 45-49   2.360615129  2.316191e+00  4.442419e-02
## 50-54   2.889954239  2.923593e+00 -3.363893e-02
## 55-59   3.510055132  3.593629e+00 -8.357351e-02
## 60-64   4.298700276  4.439985e+00 -1.412852e-01
## 65-69   4.805026438  4.918807e+00 -1.137802e-01
## 70-74   4.973187843  5.135102e+00 -1.619141e-01
## 75-79   4.517732719  4.563438e+00 -4.570563e-02
## 80-84   3.169823587  3.282934e+00 -1.131100e-01
## 85-89   1.742350857  1.766515e+00 -2.416397e-02
## 90-94   0.685338361  7.064783e-01 -2.113994e-02

```

```

## 95-99      0.135349685 1.440494e-01 -8.699704e-03
## 100-104    0.014268151 1.559056e-02 -1.322411e-03
## 105-109    0.000868547 9.597548e-04 -9.120786e-05
##          47.339362576 4.718942e+01  1.499381e-01
##
## $tae.e
##           tae.comp.e.f   tae.lq.e.f   tae.diff.e.f
## 0          4.255861e+02 4.147746e+02 10.811450934
## 1-4        4.628845e+02 4.594468e+02 3.437701494
## 5-9        4.247085e+02 4.257798e+02 -1.071290624
## 10-14      4.035180e+02 4.044524e+02 -0.934434396
## 15-19      3.901472e+02 3.889475e+02 1.199746458
## 20-24      3.715156e+02 3.668358e+02 4.679756175
## 25-29      3.517933e+02 3.433747e+02 8.418537502
## 30-34      3.350725e+02 3.236770e+02 11.395510908
## 35-39      3.196560e+02 3.084361e+02 11.219856223
## 40-44      3.038892e+02 2.936671e+02 10.222005694
## 45-49      2.855039e+02 2.762443e+02 9.259510559
## 50-54      2.639106e+02 2.548594e+02 9.051173440
## 55-59      2.385241e+02 2.294080e+02 9.116143171
## 60-64      2.099611e+02 2.005822e+02 9.378879900
## 65-69      1.769739e+02 1.678420e+02 9.131848597
## 70-74      1.402972e+02 1.327151e+02 7.582166138
## 75-79      9.969742e+01 9.527098e+01 4.426444631
## 80-84      6.147056e+01 6.014278e+01 1.327783839
## 85-89      3.116552e+01 3.091626e+01 0.249262028
## 90-94      1.235848e+01 1.235616e+01 0.002314198
## 95-99      3.228173e+00 3.197597e+00 0.030576126
## 100-104    4.687723e-01 4.550265e-01 0.013745838
## 105-109    3.667028e-02 3.425943e-02 0.002410852
## 110+       5.822578e-03 2.470463e-03 0.003352115
##           5.312373e+03 5.193418e+03 118.954451800
##           tae.comp.e.m   tae.lq.e.m   tae.diff.e.m
## 0          6.288541e+02 6.306460e+02 -1.791864e+00
## 1-4        6.667792e+02 6.605873e+02 6.191908e+00
## 5-9        6.180367e+02 6.336571e+02 -1.562046e+01
## 10-14      5.989305e+02 6.149495e+02 -1.601905e+01
## 15-19      5.883242e+02 6.045474e+02 -1.622323e+01
## 20-24      5.663719e+02 5.820842e+02 -1.571230e+01
## 25-29      5.284402e+02 5.445539e+02 -1.611370e+01
## 30-34      4.961633e+02 5.120736e+02 -1.591030e+01
## 35-39      4.645619e+02 4.799306e+02 -1.536864e+01
## 40-44      4.296944e+02 4.441654e+02 -1.447105e+01
## 45-49      3.898204e+02 4.020256e+02 -1.220522e+01
## 50-54      3.437115e+02 3.532945e+02 -9.583031e+00
## 55-59      2.912965e+02 2.979205e+02 -6.623917e+00
## 60-64      2.335753e+02 2.383392e+02 -4.763917e+00
## 65-69      1.734086e+02 1.764317e+02 -3.023166e+00
## 70-74      1.177275e+02 1.193255e+02 -1.597942e+00
## 75-79      7.069097e+01 7.092950e+01 -2.385335e-01
## 80-84      3.673754e+01 3.653155e+01  2.059942e-01
## 85-89      1.625377e+01 1.578827e+01  4.654948e-01
## 90-94      5.683582e+00 5.459889e+00  2.236930e-01
## 95-99      1.209757e+00 1.150808e+00  5.894916e-02

```

```

## 100-104 1.486950e-01 1.404794e-01 8.215649e-03
## 105-109 1.100558e-02 1.022130e-02 7.842851e-04
## 110+    2.868997e-03 1.001968e-03 1.867030e-03
##          7.266434e+03 7.424544e+03 -1.581094e+02
##
## $tot.tae.e0
##           [,1]      [,2]      [,3]
## Female 6355.000 6193.56 161.44038
## Male   8733.096 8757.98 -24.88418

# create a table with results for 2 components
capture.output(file = "../tables/ageCompTotC-2.txt", print.xtable(xtable(age.comps.2$tot.tae.e0,
  booktabs = TRUE, digits = 2), only.contents = TRUE,
  include.rownames = TRUE, include.colnames = FALSE, hline.after = NULL,
  format.args = list(big.mark = ",")))

# 3 components re-run models with 1 component
adult.3 <- FALSE
smooth.3 <- FALSE
N.3 <- 1
S.3 <- 1
C.3 <- 3
# base model
mod.1_0.m.3 <- svdMod(q1logit.m, Qlogit.m, N.3, S.3, 10,
  TRUE, adult.3, TRUE, smooth.3, C.3)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "3 components"

mod.1_0.f.3 <- svdMod(q1logit.f, Qlogit.f, N.3, S.3, 10,
  TRUE, adult.3, TRUE, smooth.3, C.3)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "3 components"

# calculate age-specific comparisons in prediction
# errors use the lt.q,q,l, and e from above
age.comps.3 <- ageSpecificErrorComparisons(mod.1_0.f.3,
  mod.1_0.m.3, lt.lq, q, l, e, a1.f, a1.m)
# have a look
cat("\n\n")

age.comps.3

## $tae.q
##           tae.comp.q.f  tae.lq.q.f  tae.diff.q.f
## 0          1.285808191  1.309623169 -0.0238149773
## 1-4        1.448076298  1.297656974  0.1504193237
## 5-9        0.833974494  0.739540137  0.0944343568

```

```

## 10-14  0.507410728  0.488058117  0.0193526114
## 15-19  0.649044412  0.704215885 -0.0551714728
## 20-24  0.803617792  0.856790241 -0.0531724492
## 25-29  0.798313035  0.847677326 -0.0493642910
## 30-34  0.768152161  0.801390372 -0.0332382111
## 35-39  0.841069874  0.835957359  0.0051125147
## 40-44  0.949653313  0.918270079  0.0313832338
## 45-49  1.105073480  1.088777855  0.0162956252
## 50-54  1.446536064  1.442450719  0.0040853452
## 55-59  1.888410119  1.908891692 -0.0204815725
## 60-64  2.464142167  2.509440007 -0.0452978397
## 65-69  3.070790072  3.094838412 -0.0240483395
## 70-74  4.089763507  4.004770837  0.0849926694
## 75-79  4.794356682  4.511095621  0.2832610601
## 80-84  4.506306943  4.364023689  0.1422832532
## 85-89  3.129507401  3.032863378  0.0966440228
## 90-94  1.474359746  1.497975829 -0.0236160834
## 95-99  0.373297609  0.398302792 -0.0250051834
## 100-104 0.046076748  0.051727293 -0.0056505451
## 105-109 0.002866161  0.003311803 -0.0004456413
##           37.276606997 36.707649587  0.5689574097
##           tae.comp.q.m   tae.lq.q.m   tae.diff.q.m
## 0          1.522463392 1.541778e+00 -1.931463e-02
## 1-4        2.007524189 1.546716e+00  4.608082e-01
## 5-9        0.905395356 8.656519e-01  3.974342e-02
## 10-14       0.519360559 4.864013e-01  3.295928e-02
## 15-19       0.880295096 8.426238e-01  3.767125e-02
## 20-24       1.678246787 1.619755e+00  5.849207e-02
## 25-29       1.572030317 1.504629e+00  6.740145e-02
## 30-34       1.512880998 1.457767e+00  5.511423e-02
## 35-39       1.675786492 1.621177e+00  5.460970e-02
## 40-44       1.939066821 1.895655e+00  4.341220e-02
## 45-49       2.321138426 2.316191e+00  4.947489e-03
## 50-54       2.867337843 2.923593e+00 -5.625533e-02
## 55-59       3.516199893 3.593629e+00 -7.742875e-02
## 60-64       4.306662489 4.439985e+00 -1.333230e-01
## 65-69       4.793216499 4.918807e+00 -1.255902e-01
## 70-74       4.934318236 5.135102e+00 -2.007837e-01
## 75-79       4.439414651 4.563438e+00 -1.240237e-01
## 80-84       3.117173782 3.282934e+00 -1.657598e-01
## 85-89       1.697061581 1.766515e+00 -6.945325e-02
## 90-94       0.669393339 7.064783e-01 -3.708496e-02
## 95-99       0.133539462 1.440494e-01 -1.050993e-02
## 100-104      0.014235445 1.559056e-02 -1.355116e-03
## 105-109      0.000874022 9.597548e-04 -8.573278e-05
##           47.023615674 4.718942e+01 -1.658088e-01
##
## $tae.e
##           tae.comp.e.f   tae.lq.e.f   tae.diff.e.f
## 0          4.214281e+02 4.147746e+02  6.6534501569
## 1-4        4.588743e+02 4.594468e+02 -0.5724678570
## 5-9        4.230239e+02 4.257798e+02 -2.7558128299
## 10-14      4.018217e+02 4.044524e+02 -2.6307177179
## 15-19      3.883738e+02 3.889475e+02 -0.5736875035

```

```

## 20-24 3.693614e+02 3.668358e+02 2.5255434316
## 25-29 3.481562e+02 3.433747e+02 4.7814898259
## 30-34 3.296990e+02 3.236770e+02 6.0220860254
## 35-39 3.136621e+02 3.084361e+02 5.2259297440
## 40-44 2.981869e+02 2.936671e+02 4.5197854368
## 45-49 2.808038e+02 2.762443e+02 4.5594915240
## 50-54 2.602204e+02 2.548594e+02 5.3609591087
## 55-59 2.359192e+02 2.294080e+02 6.5111921459
## 60-64 2.081710e+02 2.005822e+02 7.5887648590
## 65-69 1.764887e+02 1.678420e+02 8.6466497382
## 70-74 1.406060e+02 1.327151e+02 7.8908802064
## 75-79 1.005492e+02 9.527098e+01 5.2781959467
## 80-84 6.241651e+01 6.014278e+01 2.2737379848
## 85-89 3.173310e+01 3.091626e+01 0.8168444363
## 90-94 1.245448e+01 1.235616e+01 0.0983177656
## 95-99 3.207747e+00 3.197597e+00 0.0101498818
## 100-104 4.571742e-01 4.550265e-01 0.0021477532
## 105-109 3.478501e-02 3.425943e-02 0.0005255724
## 110+ 5.822578e-03 2.470463e-03 0.0033521155
## 5.265655e+03 5.193418e+03 72.2368077507
## tae.comp.e.m tae.lq.e.m tae.diff.e.m
## 0 6.263207e+02 6.306460e+02 -4.325224e+00
## 1-4 6.641657e+02 6.605873e+02 3.578452e+00
## 5-9 6.160276e+02 6.336571e+02 -1.762948e+01
## 10-14 5.984997e+02 6.149495e+02 -1.644984e+01
## 15-19 5.881790e+02 6.045474e+02 -1.636841e+01
## 20-24 5.662205e+02 5.820842e+02 -1.586372e+01
## 25-29 5.284677e+02 5.445539e+02 -1.608621e+01
## 30-34 4.962605e+02 5.120736e+02 -1.581308e+01
## 35-39 4.644070e+02 4.799306e+02 -1.552354e+01
## 40-44 4.293127e+02 4.441654e+02 -1.485272e+01
## 45-49 3.892972e+02 4.020256e+02 -1.272843e+01
## 50-54 3.427845e+02 3.532945e+02 -1.051002e+01
## 55-59 2.897293e+02 2.979205e+02 -8.191171e+00
## 60-64 2.315641e+02 2.383392e+02 -6.775124e+00
## 65-69 1.711955e+02 1.764317e+02 -5.236269e+00
## 70-74 1.157736e+02 1.193255e+02 -3.551848e+00
## 75-79 6.919474e+01 7.092950e+01 -1.734759e+00
## 80-84 3.599701e+01 3.653155e+01 -5.345370e-01
## 85-89 1.584599e+01 1.578827e+01 5.772126e-02
## 90-94 5.561485e+00 5.459889e+00 1.015960e-01
## 95-99 1.192985e+00 1.150808e+00 4.217678e-02
## 100-104 1.478994e-01 1.404794e-01 7.420066e-03
## 105-109 1.102810e-02 1.022130e-02 8.067996e-04
## 110+ 2.868997e-03 1.001968e-03 1.867030e-03
## 7.246159e+03 7.424544e+03 -1.783844e+02
##
## $tot.tae.e0
## [,1] [,2] [,3]
## Female 6292.912 6193.56 99.35165
## Male 8697.914 8757.98 -60.06576
# create a table with results for 3 components
capture.output(file = "../tables/ageCompTotC-3.txt", print.xtable(xtable(age.comps.3$tot.tae.e0,

```

```

booktabs = TRUE, digits = 2), only.contents = TRUE,
include.rownames = TRUE, include.colnames = FALSE, hline.after = NULL,
format.args = list(big.mark = ",")))

# 4 components re-run models with 1 component
adult.4 <- FALSE
smooth.4 <- FALSE
N.4 <- 1
S.4 <- 1
C.4 <- 4
# base model
mod.1_0.m.4 <- svdMod(q1logit.m, Qlogit.m, N.4, S.4, 10,
TRUE, adult.4, TRUE, smooth.4, C.4)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

mod.1_0.f.4 <- svdMod(q1logit.f, Qlogit.f, N.4, S.4, 10,
TRUE, adult.4, TRUE, smooth.4, C.4)

##
## [1] "Adult mortality is direct input to predictions: FALSE"
## [1] "SVD model is smoothed: FALSE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

# calculate age-specific comparisons in prediction
# errors use the lt.q, q, l, and e from above
age.comps.4 <- ageSpecificErrorComparisons(mod.1_0.f.4,
mod.1_0.m.4, lt.lq, q, l, e, a1.f, a1.m)
# have a look
cat("\n\n")

age.comps.4

## $tae.q
##          tae.comp.q.f  tae.lq.q.f  tae.diff.q.f
## 0      1.285808191  1.309623169 -0.0238149773
## 1-4    1.405605466  1.297656974  0.1079484913
## 5-9    0.774404676  0.739540137  0.0348645388
## 10-14   0.509533972  0.488058117  0.0214758555
## 15-19   0.631768929  0.704215885 -0.0724469560
## 20-24   0.776761164  0.856790241 -0.0800290770
## 25-29   0.767838972  0.847677326 -0.0798383535
## 30-34   0.759797072  0.801390372 -0.0415933000
## 35-39   0.835094853  0.835957359 -0.0008625060
## 40-44   0.949378962  0.918270079  0.0311088829
## 45-49   1.104900947  1.088777855  0.0161230918
## 50-54   1.446209914  1.442450719  0.0037591953
## 55-59   1.890110030  1.908891692 -0.0187816618
## 60-64   2.426462671  2.509440007 -0.0829773354

```

```

## 65-69    2.975484183  3.094838412 -0.1193542288
## 70-74    3.859023820  4.004770837 -0.1457470169
## 75-79    4.452347552  4.511095621 -0.0587480699
## 80-84    4.256483071  4.364023689 -0.1075406180
## 85-89    3.032195850  3.032863378 -0.0006675275
## 90-94    1.466339018  1.497975829 -0.0316368113
## 95-99    0.373584075  0.398302792 -0.0247187170
## 100-104   0.045990476  0.051727293 -0.0057368173
## 105-109   0.002843693  0.003311803 -0.0004681095
##           36.027967560  36.707649587 -0.6796820277
##           tae.comp.q.m   tae.lq.q.m   tae.diff.q.m
## 0         1.522463e+00  1.541778e+00 -0.0193146311
## 1-4       2.006411e+00  1.546716e+00  0.4596952560
## 5-9       8.794502e-01  8.656519e-01  0.0137982698
## 10-14     5.178406e-01  4.864013e-01  0.0314393559
## 15-19     8.854434e-01  8.426238e-01  0.0428195914
## 20-24     1.691406e+00  1.619755e+00  0.0716517282
## 25-29     1.5677782e+00  1.504629e+00  0.0631531799
## 30-34     1.502594e+00  1.457767e+00  0.0448277045
## 35-39     1.674806e+00  1.621177e+00  0.0536296841
## 40-44     1.951835e+00  1.895655e+00  0.0561803576
## 45-49     2.351121e+00  2.316191e+00  0.0349304345
## 50-54     2.893663e+00  2.923593e+00 -0.0299296940
## 55-59     3.446941e+00  3.593629e+00 -0.1466873652
## 60-64     4.120316e+00  4.439985e+00 -0.3196690361
## 65-69     4.537801e+00  4.918807e+00 -0.3810053609
## 70-74     4.686842e+00  5.135102e+00 -0.4482603087
## 75-79     4.272008e+00  4.563438e+00 -0.2914298592
## 80-84     3.078585e+00  3.282934e+00 -0.2043487757
## 85-89     1.700815e+00  1.766515e+00 -0.0657000210
## 90-94     6.657873e-01  7.064783e-01 -0.0406910103
## 95-99     1.285520e-01  1.440494e-01 -0.0154973660
## 100-104   1.334220e-02  1.559056e-02 -0.0022483617
## 105-109   8.105134e-04  9.597548e-04 -0.0001492414
##           4.609662e+01  4.718942e+01 -1.0928054694
##
## $tae.e
##           tae.comp.e.f   tae.lq.e.f   tae.diff.e.f
## 0         4.108094e+02  4.147746e+02 -3.965212e+00
## 1-4       4.502406e+02  4.594468e+02 -9.206167e+00
## 5-9       4.164232e+02  4.257798e+02 -9.356524e+00
## 10-14     3.954913e+02  4.044524e+02 -8.961155e+00
## 15-19     3.815325e+02  3.889475e+02 -7.414935e+00
## 20-24     3.618813e+02  3.668358e+02 -4.954584e+00
## 25-29     3.401536e+02  3.433747e+02 -3.221157e+00
## 30-34     3.209064e+02  3.236770e+02 -2.770591e+00
## 35-39     3.042722e+02  3.084361e+02 -4.163924e+00
## 40-44     2.879651e+02  2.936671e+02 -5.702007e+00
## 45-49     2.700467e+02  2.762443e+02 -6.197647e+00
## 50-54     2.487578e+02  2.548594e+02 -6.101545e+00
## 55-59     2.240441e+02  2.294080e+02 -5.363838e+00
## 60-64     1.956326e+02  2.005822e+02 -4.949684e+00
## 65-69     1.648870e+02  1.678420e+02 -2.955035e+00
## 70-74     1.308905e+02  1.327151e+02 -1.824629e+00

```

```

## 75-79   9.421929e+01 9.527098e+01 -1.051684e+00
## 80-84   5.958329e+01 6.014278e+01 -5.594867e-01
## 85-89   3.100424e+01 3.091626e+01  8.798583e-02
## 90-94   1.240199e+01 1.235616e+01  4.582874e-02
## 95-99   3.209910e+00 3.197597e+00  1.231316e-02
## 100-104 4.565116e-01 4.550265e-01  1.485138e-03
## 105-109 3.451248e-02 3.425943e-02  2.530459e-04
## 110+    5.822578e-03 2.470463e-03  3.352115e-03
##           5.104850e+03 5.193418e+03 -8.856859e+01
##           tae.comp.e.m  tae.lq.e.m  tae.diff.e.m
## 0        6.189620e+02 6.306460e+02 -1.168393e+01
## 1-4      6.577537e+02 6.605873e+02 -2.833600e+00
## 5-9      6.085742e+02 6.336571e+02 -2.508288e+01
## 10-14    5.910655e+02 6.149495e+02 -2.388406e+01
## 15-19    5.808574e+02 6.045474e+02 -2.368999e+01
## 20-24    5.590702e+02 5.820842e+02 -2.301396e+01
## 25-29    5.200264e+02 5.445539e+02 -2.452747e+01
## 30-34    4.849374e+02 5.120736e+02 -2.713621e+01
## 35-39    4.506845e+02 4.799306e+02 -2.924610e+01
## 40-44    4.143479e+02 4.441654e+02 -2.981748e+01
## 45-49    3.740160e+02 4.020256e+02 -2.800958e+01
## 50-54    3.277359e+02 3.532945e+02 -2.555868e+01
## 55-59    2.759549e+02 2.979205e+02 -2.196555e+01
## 60-64    2.203616e+02 2.383392e+02 -1.797765e+01
## 65-69    1.637741e+02 1.764317e+02 -1.265764e+01
## 70-74    1.119345e+02 1.193255e+02 -7.391022e+00
## 75-79    6.791521e+01 7.092950e+01 -3.014291e+00
## 80-84    3.590373e+01 3.653155e+01 -6.278165e-01
## 85-89    1.588130e+01 1.578827e+01  9.303009e-02
## 90-94    5.518531e+00 5.459889e+00  5.864214e-02
## 95-99    1.154423e+00 1.150808e+00  3.615189e-03
## 100-104  1.399408e-01 1.404794e-01 -5.385160e-04
## 105-109  1.030230e-02 1.022130e-02  8.100027e-05
## 110+     2.868997e-03 1.001968e-03  1.867030e-03
##           7.086583e+03 7.424544e+03 -3.379612e+02
##
## $tot.tae.e0
##           [,1]      [,2]      [,3]
## Female  6134.350 6193.56 -59.20994
## Male    8595.722 8757.98 -162.25837
# create lines for table with e0 total errors for
# log-quad and SVD-Comp with components 1-4 start with
# log-quad
capture.output(file = "../tables/ageCompLQ.txt", cat(paste("Log-Quad & ",
  paste(format(round(age.comps.1$tot.tae.e0[, 2], 0),
    big.mark = ",", nsmall = 0, trim = TRUE), collapse = " & ",
    sep = "")), "\\\\""))
# SVD-Comp components 1-4
capture.output(file = "../tables/ageCompSVD-Comp.txt", cat(paste("SVD-Comp, C=1 & ",
  paste(format(round(age.comps.1$tot.tae.e0[, 1], 0),
    big.mark = ",", nsmall = 0, trim = TRUE), collapse = " & ",
    sep = "")), "\\\\\n", paste("SVD-Comp, C=2 & ",
  paste(format(round(age.comps.2$tot.tae.e0[, 1], 0),

```

```

    big.mark = "", nsmall = 0, trim = TRUE), collapse = " & ",
    sep = "")), "\\\\\\n", paste("SVD-Comp, C=3 & ",
paste(format(round(age.comps.3$tot.tae.e0[, 1], 0),
    big.mark = "", nsmall = 0, trim = TRUE), collapse = " & ",
    sep = "")), "\\\\\\n", paste("SVD-Comp, C=4 & ",
paste(format(round(age.comps.4$tot.tae.e0[, 1], 0),
    big.mark = "", nsmall = 0, trim = TRUE), collapse = " & ",
    sep = "")), "\\\\\\"))
# differences between SVD-Comp components 1-4 and
# log-quad
capture.output(file = "../tables/ageCompSVD-CompLogQuadDiffs.txt",
  cat(paste("SVD-Comp, C=1 - Log-Quad & ", paste(format(round(age.comps.1$tot.tae.e0[, 1] - age.comps.1$tot.tae.e0[, 2], 0), big.mark = "", nsmall = 0, trim = TRUE), collapse = " & ", sep = "")),
  "\\\\\\n", paste("SVD-Comp, C=2 - Log-Quad & ", paste(format(round(age.comps.2$tot.tae.e0[, 1] - age.comps.1$tot.tae.e0[, 2], 0), big.mark = "", nsmall = 0, trim = TRUE), collapse = " & ", sep = "")), "\\\\\\n", paste("SVD-Comp, C=3 - Log-Quad & ",
  paste(format(round(age.comps.3$tot.tae.e0[, 1] - age.comps.1$tot.tae.e0[, 2], 0), big.mark = "", nsmall = 0, trim = TRUE), collapse = " & ", sep = "")), "\\\\\\n", paste("SVD-Comp, C=4 - Log-Quad & ",
  paste(format(round(age.comps.4$tot.tae.e0[, 1] - age.comps.1$tot.tae.e0[, 2], 0), big.mark = "", nsmall = 0, trim = TRUE), collapse = " & ", sep = "")))

```

7 Test on Other Countries

SVD-Comp is tested on two different countries that are not part of the HMD and are not developed countries but for which reasonable data exist: Mexico and South Africa. Example life tables for Mexico (1983–1985) from the Human Life Table Database (www.lifetable.de) [<https://www.lifetable.de/data/hld.zip>] and South Africa (2005) come from the WHO’s Global Health Observatory [<http://apps.who.int/gho/data/?theme=main&vid=61540>]. The life tables are converted to standard five-year age groups ending at ages 80-84, the oldest second-to-last age group that is common accross both examples. Predictions are made with both SVD-Comp and Log Quad using both child and adult mortality as predictors, and both the data and those predictions are plotted.

```

# Mexico read 1983-1985 Mexican life tables from Human
# Life Table Database
mex <- read.csv("../data/non-HMD life tables/Mexico1983-1985.csv",
  header = TRUE)
# female
mex.f.q <- mex[, 15][97:191]
mex.f.q <- standardFiveYear(mex.f.q)[1:18]
# male
mex.m.q <- mex[, 15][1:95]
mex.m.q <- standardFiveYear(mex.m.q)[1:18]

# South Africa read 2005 life tables for South Africa
# from the WHO Global Health Observatory
rsa <- read.csv("../data/non-HMD life tables/SouthAfrica2005.csv",
  header = TRUE)

```

```

# female
rsa.f.q <- rsa[, 3][1:18]
# male
rsa.m.q <- rsa[, 2][1:18]

# Now have standard 5qx through ages 80-84, i.e. not
# including 1.0 at age 85

# logits
mex.f ql <- logit(mex.f.q)
mex.m ql <- logit(mex.m.q)
rsa.f ql <- logit(rsa.f.q)
rsa.m ql <- logit(rsa.m.q)

# child and adult Mx

# Mexico female
mex.f.Q <- rep(0, 2)
# child mx
mex.f.Q[1] <- childQ5(mex.f.q)
# adult mx
mex.f.Q[2] <- adultQ5(mex.f.q)
# mmale
mex.m.Q <- rep(0, 2)
# child mx
mex.m.Q[1] <- childQ5(mex.m.q)
# adult mx
mex.m.Q[2] <- adultQ5(mex.m.q)

# RSA female
rsa.f.Q <- rep(0, 2)
# child mx
rsa.f.Q[1] <- childQ5(rsa.f.q)
# adult mx
rsa.f.Q[2] <- adultQ5(rsa.f.q)
# mmale
rsa.m.Q <- rep(0, 2)
# child mx
rsa.m.Q[1] <- childQ5(rsa.m.q)
# adult mx
rsa.m.Q[2] <- adultQ5(rsa.m.q)

# have a look
mex.f.Q

## [1] 0.05336201 0.13518150
mex.m.Q

## [1] 0.06264442 0.23507692
rsa.f.Q

## [1] 0.0688960 0.4660984

```

```

rsa.m.Q

## [1] 0.0815180 0.5427579

# Predictions

# models
adult <- TRUE
smooth <- TRUE
N <- 1
S <- 1
C <- 4
mod.1_0.sm.m <- svdMod(q1logit.m, Qlogit.m, N, S, 10, TRUE,
    adult, TRUE, TRUE, C)

## 
## [1] "Adult mortality is direct input to predictions: TRUE"
## [1] "SVD model is smoothed: TRUE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

mod.1_0.sm.f <- svdMod(q1logit.f, Qlogit.f, N, S, 10, TRUE,
    adult, TRUE, TRUE, C)

## 
## [1] "Adult mortality is direct input to predictions: TRUE"
## [1] "SVD model is smoothed: TRUE"
## [1] "1 iterations"
## [1] "100% sample fraction"
## [1] "4 components"

### predictions with child and adult

#### Mexico female
mex.f.p.ca <- ltPredict(mod.1_0.sm.f, TRUE, logit(mex.f.Q[1]),
    logit(mex.f.Q[2]))
mex.f.p5.ca <- standardFiveYear(expit(mex.f.p.ca[, 1]))
# male
mex.m.p.ca <- ltPredict(mod.1_0.sm.m, TRUE, logit(mex.m.Q[1]),
    logit(mex.m.Q[2]))
mex.m.p5.ca <- standardFiveYear(expit(mex.m.p.ca[, 1]))

## RSA female
rsa.f.p.ca <- ltPredict(mod.1_0.sm.f, TRUE, logit(rsa.f.Q[1]),
    logit(rsa.f.Q[2]))
rsa.f.p5.ca <- standardFiveYear(expit(rsa.f.p.ca[, 1]))
# male
rsa.m.p.ca <- ltPredict(mod.1_0.sm.m, TRUE, logit(rsa.m.Q[1]),
    logit(rsa.m.Q[2]))
rsa.m.p5.ca <- standardFiveYear(expit(rsa.m.p.ca[, 1]))

# predictions with Log-Quad

# Source functions file
source("../R/logQuad/DataProgramsExamples/R/functions.R")

```

```

# Create labels for age vectors
ages.5x1 <- c("0", "1-4", paste(seq(5, 105, 5), seq(9, 109,
  5), sep = "-"), "110+")
sexes <- c("Female", "Male", "Total")

# Import matrix of model coefficients
tmp1 <- read.csv("../R/logQuad/DataProgramsExamples/Data/coefs.logquad.HMD719.csv")
tmp2 <- array(c(as.matrix(tmp1[, 3:6])), dim = c(24, 3,
  4), dimnames = list(ages.5x1, sexes, c("ax", "bx", "cx",
  "vx")))
coefs <- aperm(tmp2, c(1, 3, 2))

### Mexico female
mex.f.LQp.ca <- lthat.any2.logquad(coefs, "Female", Q5 = mex.f.Q[1],
  QQa = mex.f.Q[2])$lt[1:23, 2] # with adult
# male
mex.m.LQp.ca <- lthat.any2.logquad(coefs, "Male", Q5 = mex.m.Q[1],
  QQa = mex.m.Q[2])$lt[1:23, 2] # with adult

### RSA female
rsa.f.LQp.ca <- lthat.any2.logquad(coefs, "Female", Q5 = rsa.f.Q[1],
  QQa = rsa.f.Q[2])$lt[1:23, 2] # with adult
# male
rsa.m.LQp.ca <- lthat.any2.logquad(coefs, "Male", Q5 = rsa.m.Q[1],
  QQa = rsa.m.Q[2])$lt[1:23, 2] # with adult

### Plots, logit scale

### Mexico female child and adult only
plot(mex.f.q1)
points(logit(mex.f.p5.ca), type = "l", col = "blue")
points(logit(mex.f.LQp.ca), type = "l")
# male child and adult only
plot(mex.m.q1)
points(logit(mex.m.p5.ca), type = "l", col = "blue")
points(logit(mex.m.LQp.ca), type = "l")

### RSA female child and adult only
plot(rsa.f.q1)
points(logit(rsa.f.p5.ca), type = "l", col = "blue")
points(logit(rsa.f.LQp.ca), type = "l")
# male child and adult only
plot(rsa.m.q1)
points(logit(rsa.m.p5.ca), type = "l", col = "blue")
points(logit(rsa.m.LQp.ca), type = "l")

### ggplot

# data

ages <- ages.5x1[1:18]
# data only
q.logit.data <- data.frame(rbind(cbind(c(0, 1, seq(5, 80,

```

```

5)), mex.f.q1, "Mexico", "Female", "Data"), cbind(c(0,
1, seq(5, 80, 5)), mex.m.q1, "Mexico", "Male", "Data"),
cbind(c(0, 1, seq(5, 80, 5)), rsa.f.q1, "South Africa",
"Female", "Data"), cbind(c(0, 1, seq(5, 80, 5)),
rsa.m.q1, "South Africa", "Male", "Data")))
colnames(q.logit.data) <- c("Age", "Value", "Country", "Sex",
"Source")
rownames(q.logit.data) <- seq(1, 18 * 4, 1)
q.logit.data$Age <- as.numeric(as.character(q.logit.data$Age))
q.logit.data$Value <- as.numeric(as.character(q.logit.data$Value))
# predicted values
q.logit.pred <- data.frame(rbind(cbind(c(0, 1, seq(5, 80,
5)), logit(mex.f.p5.ca)[1:18], "Mexico", "Female", "Predicted by SVD-Comp"),
cbind(c(0, 1, seq(5, 80, 5)), logit(mex.m.p5.ca)[1:18],
"Mexico", "Male", "Predicted by SVD-Comp"), cbind(c(0,
1, seq(5, 80, 5)), logit(mex.f.LQp.ca)[1:18], "Mexico",
"Female", "Predicted by Log-Quad"), cbind(c(0, 1,
seq(5, 80, 5)), logit(mex.m.LQp.ca)[1:18], "Mexico",
"Male", "Predicted by Log-Quad"), cbind(c(0, 1,
seq(5, 80, 5)), logit(rsa.f.p5.ca)[1:18], "South Africa",
"Female", "Predicted by SVD-Comp"), cbind(c(0, 1,
seq(5, 80, 5)), logit(rsa.m.p5.ca)[1:18], "South Africa",
"Male", "Predicted by SVD-Comp"), cbind(c(0, 1,
seq(5, 80, 5)), logit(rsa.f.LQp.ca)[1:18], "South Africa",
"Female", "Predicted by Log-Quad"), cbind(c(0, 1,
seq(5, 80, 5)), logit(rsa.m.LQp.ca)[1:18], "South Africa",
"Male", "Predicted by Log-Quad")))
colnames(q.logit.pred) <- c("Age", "Value", "Country", "Sex",
"Source")
rownames(q.logit.pred) <- seq(1, 18 * 8, 1)
q.logit.pred$Age <- as.numeric(as.character(q.logit.pred$Age))
q.logit.pred$Value <- as.numeric(as.character(q.logit.pred$Value))
q.logit.pred

```

	Age	Value	Country	Sex
## 1	0	-3.15840049	Mexico	Female
## 2	1	-4.75010506	Mexico	Female
## 3	5	-5.54514497	Mexico	Female
## 4	10	-5.90047153	Mexico	Female
## 5	15	-5.51757062	Mexico	Female
## 6	20	-5.31771387	Mexico	Female
## 7	25	-5.13573474	Mexico	Female
## 8	30	-4.87492640	Mexico	Female
## 9	35	-4.53510615	Mexico	Female
## 10	40	-4.17305212	Mexico	Female
## 11	45	-3.80190195	Mexico	Female
## 12	50	-3.39906683	Mexico	Female
## 13	55	-2.98999854	Mexico	Female
## 14	60	-2.50951800	Mexico	Female
## 15	65	-1.99926277	Mexico	Female
## 16	70	-1.41427648	Mexico	Female
## 17	75	-0.80067082	Mexico	Female
## 18	80	-0.15839475	Mexico	Female
## 19	0	-2.96380393	Mexico	Male

## 20	1	-4.43705975	Mexico	Male
## 21	5	-5.14818928	Mexico	Male
## 22	10	-5.39399881	Mexico	Male
## 23	15	-4.71576987	Mexico	Male
## 24	20	-4.38881740	Mexico	Male
## 25	25	-4.38598993	Mexico	Male
## 26	30	-4.27276668	Mexico	Male
## 27	35	-4.02682889	Mexico	Male
## 28	40	-3.68549466	Mexico	Male
## 29	45	-3.29474684	Mexico	Male
## 30	50	-2.87371372	Mexico	Male
## 31	55	-2.44874386	Mexico	Male
## 32	60	-1.98870195	Mexico	Male
## 33	65	-1.52112879	Mexico	Male
## 34	70	-1.00228620	Mexico	Male
## 35	75	-0.44950413	Mexico	Male
## 36	80	0.16057489	Mexico	Male
## 37	0	-3.14378162	Mexico	Female
## 38	1	-4.36588948	Mexico	Female
## 39	5	-5.65752532	Mexico	Female
## 40	10	-6.04630623	Mexico	Female
## 41	15	-5.69006250	Mexico	Female
## 42	20	-5.47445718	Mexico	Female
## 43	25	-5.25989363	Mexico	Female
## 44	30	-4.97604526	Mexico	Female
## 45	35	-4.61769092	Mexico	Female
## 46	40	-4.23856979	Mexico	Female
## 47	45	-3.84040530	Mexico	Female
## 48	50	-3.42644951	Mexico	Female
## 49	55	-3.00770496	Mexico	Female
## 50	60	-2.48413340	Mexico	Female
## 51	65	-1.93595423	Mexico	Female
## 52	70	-1.31650115	Mexico	Female
## 53	75	-0.68048081	Mexico	Female
## 54	80	-0.05337267	Mexico	Female
## 55	0	-2.95041291	Mexico	Male
## 56	1	-4.28376855	Mexico	Male
## 57	5	-5.08349945	Mexico	Male
## 58	10	-5.33390921	Mexico	Male
## 59	15	-4.69779670	Mexico	Male
## 60	20	-4.38907757	Mexico	Male
## 61	25	-4.38742733	Mexico	Male
## 62	30	-4.27169186	Mexico	Male
## 63	35	-4.02638839	Mexico	Male
## 64	40	-3.70442732	Mexico	Male
## 65	45	-3.30003467	Mexico	Male
## 66	50	-2.85954531	Mexico	Male
## 67	55	-2.40497650	Mexico	Male
## 68	60	-1.92954455	Mexico	Male
## 69	65	-1.45194116	Mexico	Male
## 70	70	-0.94569706	Mexico	Male
## 71	75	-0.39864474	Mexico	Male
## 72	80	0.18129494	Mexico	Male
## 73	0	-2.92286464	South Africa	Female

```

## 74      1 -3.94001820 South Africa Female
## 75      5 -5.68994613 South Africa Female
## 76     10 -5.86930272 South Africa Female
## 77     15 -4.34277224 South Africa Female
## 78     20 -3.97842681 South Africa Female
## 79     25 -3.70866357 South Africa Female
## 80     30 -3.47328726 South Africa Female
## 81     35 -3.16479440 South Africa Female
## 82     40 -2.92672816 South Africa Female
## 83     45 -2.61782732 South Africa Female
## 84     50 -2.29711356 South Africa Female
## 85     55 -1.96394228 South Africa Female
## 86     60 -1.58089976 South Africa Female
## 87     65 -1.22238041 South Africa Female
## 88     70 -0.79991350 South Africa Female
## 89     75 -0.37225634 South Africa Female
## 90     80  0.12045668 South Africa Female
## 91      0 -2.71449321 South Africa   Male
## 92      1 -3.64257827 South Africa   Male
## 93      5 -4.66587492 South Africa   Male
## 94     10 -4.80731817 South Africa   Male
## 95     15 -3.60968229 South Africa   Male
## 96     20 -2.94164554 South Africa   Male
## 97     25 -2.82535790 South Africa   Male
## 98     30 -2.75596246 South Africa   Male
## 99     35 -2.63361974 South Africa   Male
## 100    40 -2.46573490 South Africa   Male
## 101    45 -2.26323428 South Africa   Male
## 102    50 -2.01570814 South Africa   Male
## 103    55 -1.74905626 South Africa   Male
## 104    60 -1.38535229 South Africa   Male
## 105    65 -1.01668881 South Africa   Male
## 106    70 -0.57988938 South Africa   Male
## 107    75 -0.08694849 South Africa   Male
## 108    80  0.48876482 South Africa   Male
## 109      0 -2.90412553 South Africa Female
## 110      1 -4.00631550 South Africa Female
## 111      5 -3.68628735 South Africa Female
## 112     10 -3.73116611 South Africa Female
## 113     15 -3.04774334 South Africa Female
## 114     20 -2.74891422 South Africa Female
## 115     25 -2.68677932 South Africa Female
## 116     30 -2.71393627 South Africa Female
## 117     35 -2.71744123 South Africa Female
## 118     40 -2.71604827 South Africa Female
## 119     45 -2.63324876 South Africa Female
## 120     50 -2.43892804 South Africa Female
## 121     55 -2.17063112 South Africa Female
## 122     60 -1.87705073 South Africa Female
## 123     65 -1.47428624 South Africa Female
## 124     70 -1.02804138 South Africa Female
## 125     75 -0.51873534 South Africa Female
## 126     80  0.04001734 South Africa Female
## 127      0 -2.69594596 South Africa   Male

```

```

## 128   1 -3.91555820 South Africa Male
## 129   5 -4.20696450 South Africa Male
## 130  10 -4.52232347 South Africa Male
## 131  15 -3.78742645 South Africa Male
## 132  20 -3.17384325 South Africa Male
## 133  25 -2.96299314 South Africa Male
## 134  30 -2.77415274 South Africa Male
## 135  35 -2.56119479 South Africa Male
## 136  40 -2.33738148 South Africa Male
## 137  45 -2.11351664 South Africa Male
## 138  50 -1.85815711 South Africa Male
## 139  55 -1.60725550 South Africa Male
## 140  60 -1.26893860 South Africa Male
## 141  65 -0.93059509 South Africa Male
## 142  70 -0.53962853 South Africa Male
## 143  75 -0.10991051 South Africa Male
## 144  80  0.38891300 South Africa Male
##
##                               Source
## 1 Predicted by SVD-Comp
## 2 Predicted by SVD-Comp
## 3 Predicted by SVD-Comp
## 4 Predicted by SVD-Comp
## 5 Predicted by SVD-Comp
## 6 Predicted by SVD-Comp
## 7 Predicted by SVD-Comp
## 8 Predicted by SVD-Comp
## 9 Predicted by SVD-Comp
## 10 Predicted by SVD-Comp
## 11 Predicted by SVD-Comp
## 12 Predicted by SVD-Comp
## 13 Predicted by SVD-Comp
## 14 Predicted by SVD-Comp
## 15 Predicted by SVD-Comp
## 16 Predicted by SVD-Comp
## 17 Predicted by SVD-Comp
## 18 Predicted by SVD-Comp
## 19 Predicted by SVD-Comp
## 20 Predicted by SVD-Comp
## 21 Predicted by SVD-Comp
## 22 Predicted by SVD-Comp
## 23 Predicted by SVD-Comp
## 24 Predicted by SVD-Comp
## 25 Predicted by SVD-Comp
## 26 Predicted by SVD-Comp
## 27 Predicted by SVD-Comp
## 28 Predicted by SVD-Comp
## 29 Predicted by SVD-Comp
## 30 Predicted by SVD-Comp
## 31 Predicted by SVD-Comp
## 32 Predicted by SVD-Comp
## 33 Predicted by SVD-Comp
## 34 Predicted by SVD-Comp
## 35 Predicted by SVD-Comp
## 36 Predicted by SVD-Comp

```

```
## 37 Predicted by Log-Quad
## 38 Predicted by Log-Quad
## 39 Predicted by Log-Quad
## 40 Predicted by Log-Quad
## 41 Predicted by Log-Quad
## 42 Predicted by Log-Quad
## 43 Predicted by Log-Quad
## 44 Predicted by Log-Quad
## 45 Predicted by Log-Quad
## 46 Predicted by Log-Quad
## 47 Predicted by Log-Quad
## 48 Predicted by Log-Quad
## 49 Predicted by Log-Quad
## 50 Predicted by Log-Quad
## 51 Predicted by Log-Quad
## 52 Predicted by Log-Quad
## 53 Predicted by Log-Quad
## 54 Predicted by Log-Quad
## 55 Predicted by Log-Quad
## 56 Predicted by Log-Quad
## 57 Predicted by Log-Quad
## 58 Predicted by Log-Quad
## 59 Predicted by Log-Quad
## 60 Predicted by Log-Quad
## 61 Predicted by Log-Quad
## 62 Predicted by Log-Quad
## 63 Predicted by Log-Quad
## 64 Predicted by Log-Quad
## 65 Predicted by Log-Quad
## 66 Predicted by Log-Quad
## 67 Predicted by Log-Quad
## 68 Predicted by Log-Quad
## 69 Predicted by Log-Quad
## 70 Predicted by Log-Quad
## 71 Predicted by Log-Quad
## 72 Predicted by Log-Quad
## 73 Predicted by SVD-Comp
## 74 Predicted by SVD-Comp
## 75 Predicted by SVD-Comp
## 76 Predicted by SVD-Comp
## 77 Predicted by SVD-Comp
## 78 Predicted by SVD-Comp
## 79 Predicted by SVD-Comp
## 80 Predicted by SVD-Comp
## 81 Predicted by SVD-Comp
## 82 Predicted by SVD-Comp
## 83 Predicted by SVD-Comp
## 84 Predicted by SVD-Comp
## 85 Predicted by SVD-Comp
## 86 Predicted by SVD-Comp
## 87 Predicted by SVD-Comp
## 88 Predicted by SVD-Comp
## 89 Predicted by SVD-Comp
## 90 Predicted by SVD-Comp
```

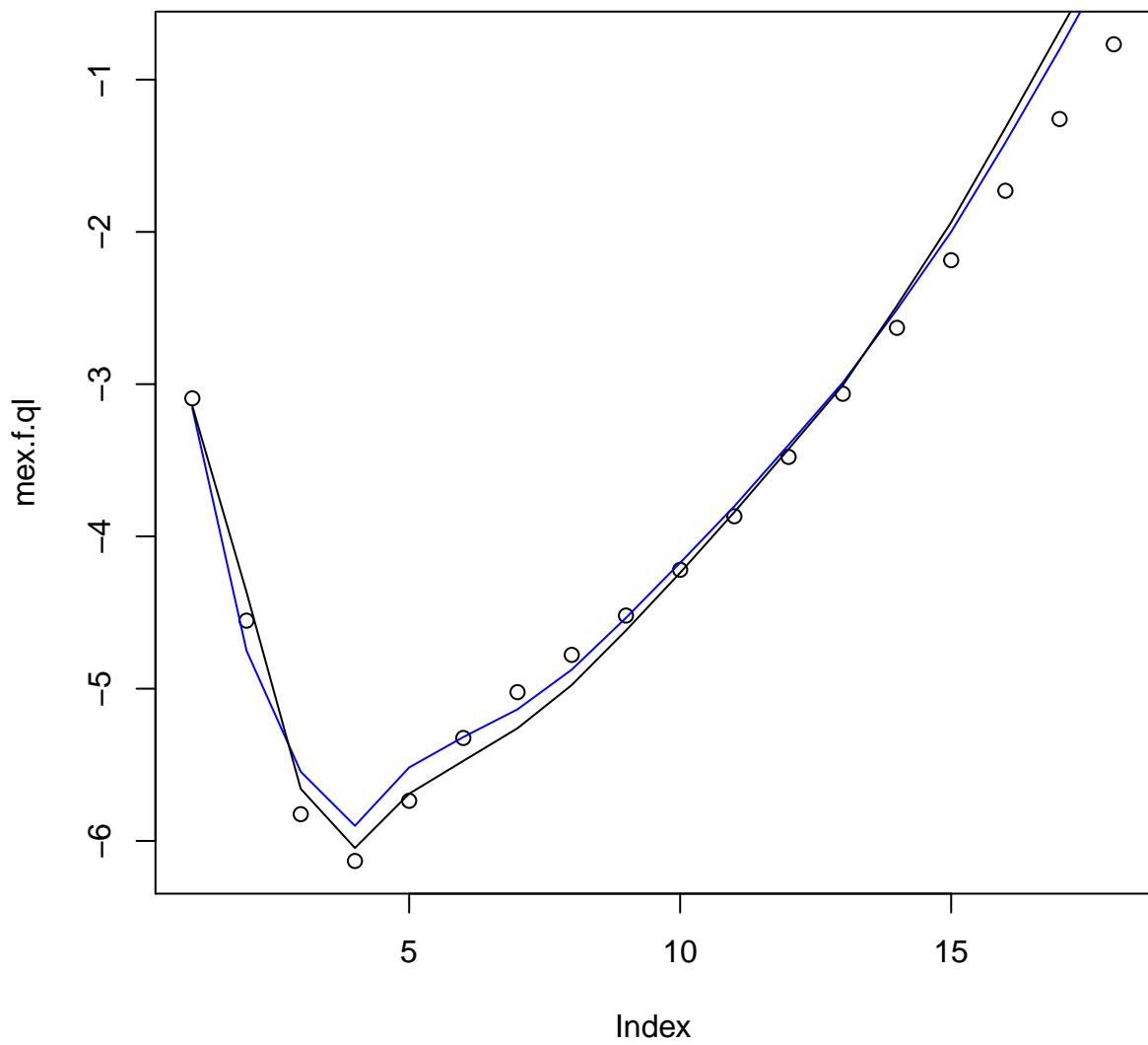
```
## 91 Predicted by SVD-Comp
## 92 Predicted by SVD-Comp
## 93 Predicted by SVD-Comp
## 94 Predicted by SVD-Comp
## 95 Predicted by SVD-Comp
## 96 Predicted by SVD-Comp
## 97 Predicted by SVD-Comp
## 98 Predicted by SVD-Comp
## 99 Predicted by SVD-Comp
## 100 Predicted by SVD-Comp
## 101 Predicted by SVD-Comp
## 102 Predicted by SVD-Comp
## 103 Predicted by SVD-Comp
## 104 Predicted by SVD-Comp
## 105 Predicted by SVD-Comp
## 106 Predicted by SVD-Comp
## 107 Predicted by SVD-Comp
## 108 Predicted by SVD-Comp
## 109 Predicted by Log-Quad
## 110 Predicted by Log-Quad
## 111 Predicted by Log-Quad
## 112 Predicted by Log-Quad
## 113 Predicted by Log-Quad
## 114 Predicted by Log-Quad
## 115 Predicted by Log-Quad
## 116 Predicted by Log-Quad
## 117 Predicted by Log-Quad
## 118 Predicted by Log-Quad
## 119 Predicted by Log-Quad
## 120 Predicted by Log-Quad
## 121 Predicted by Log-Quad
## 122 Predicted by Log-Quad
## 123 Predicted by Log-Quad
## 124 Predicted by Log-Quad
## 125 Predicted by Log-Quad
## 126 Predicted by Log-Quad
## 127 Predicted by Log-Quad
## 128 Predicted by Log-Quad
## 129 Predicted by Log-Quad
## 130 Predicted by Log-Quad
## 131 Predicted by Log-Quad
## 132 Predicted by Log-Quad
## 133 Predicted by Log-Quad
## 134 Predicted by Log-Quad
## 135 Predicted by Log-Quad
## 136 Predicted by Log-Quad
## 137 Predicted by Log-Quad
## 138 Predicted by Log-Quad
## 139 Predicted by Log-Quad
## 140 Predicted by Log-Quad
## 141 Predicted by Log-Quad
## 142 Predicted by Log-Quad
## 143 Predicted by Log-Quad
## 144 Predicted by Log-Quad
```

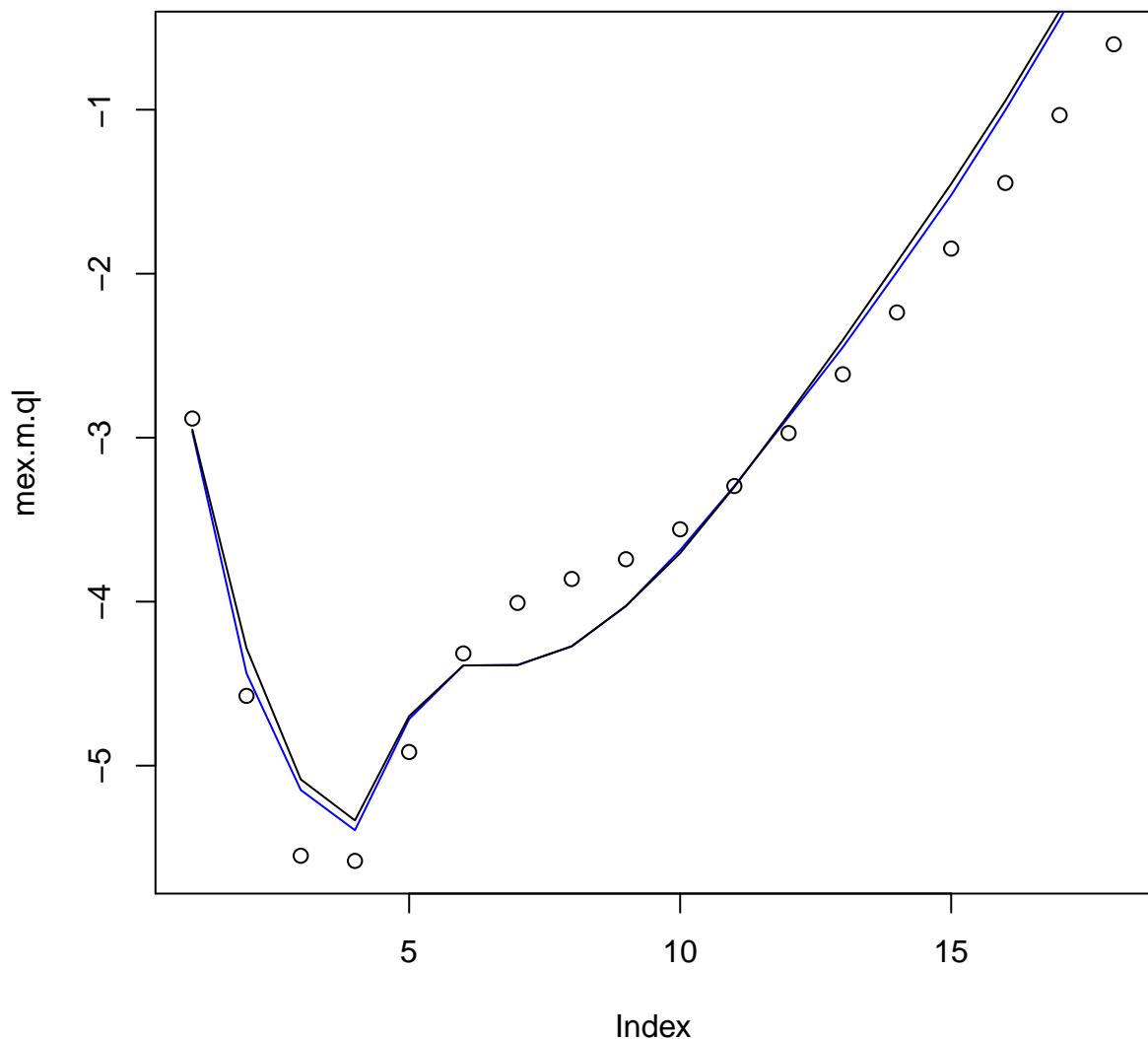
```

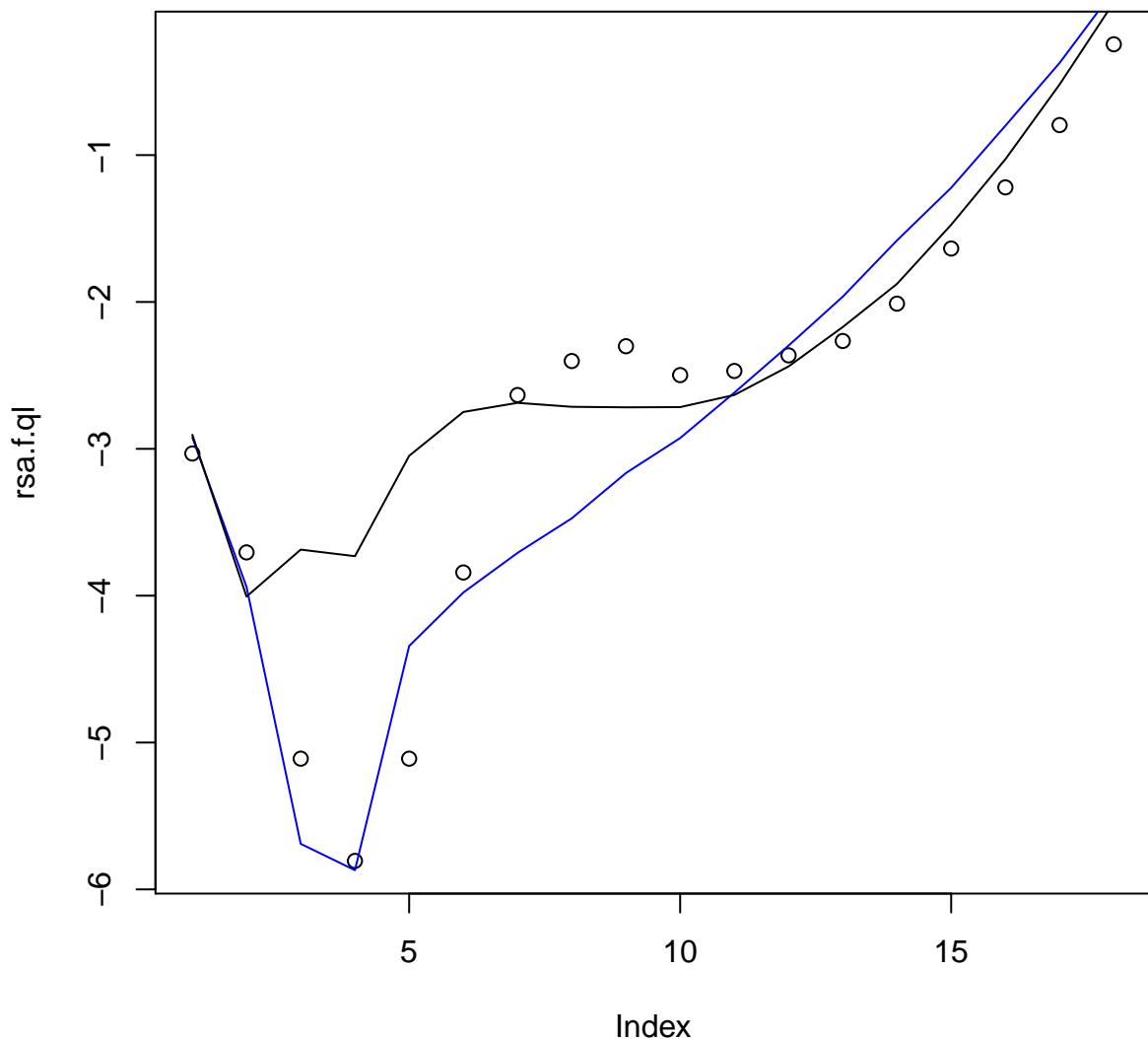
# plot data and predictions
ggplot(data = q.logit.data, aes(x = Age, y = Value, colour = Source)) +
  geom_line(data = q.logit.pred, aes(x = Age, y = Value,
    colour = Source), size = 1) + scale_x_continuous(breaks = c(0,
  1, seq(5, 80, 5)), labels = c("0,1-4", "", paste(seq(5,
  80, 5), c(seq(9, 84, 5))), sep = "-")), minor_breaks = c()) +
  theme(axis.text.x = element_text(angle = 60, hjust = 1)) +
  geom_point(size = 1.5) + # geom_line(aes(x=Age, y=Value, colour=Source),
# size=0.5) +
  labs(y = expression(""+[bolditalic(n)] * bolditalic("q")*[bolditalic(x)] *
    bold(" (logit scale)")), x = expression(bold("Age (years)")))) +
  facet_wrap(~interaction(Sex, Country, sep = ", "), ncol = 2) +
  # facet_wrap(~Sex + Country,ncol=2) +
  theme(legend.title = element_blank(), legend.position = c(0.15,
  0.91)) + theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(strip.text = element_text(face = "bold")) + ggsave("../figures/fig7.pdf",
  width = 6.5, height = 6.5, units = c("in"))

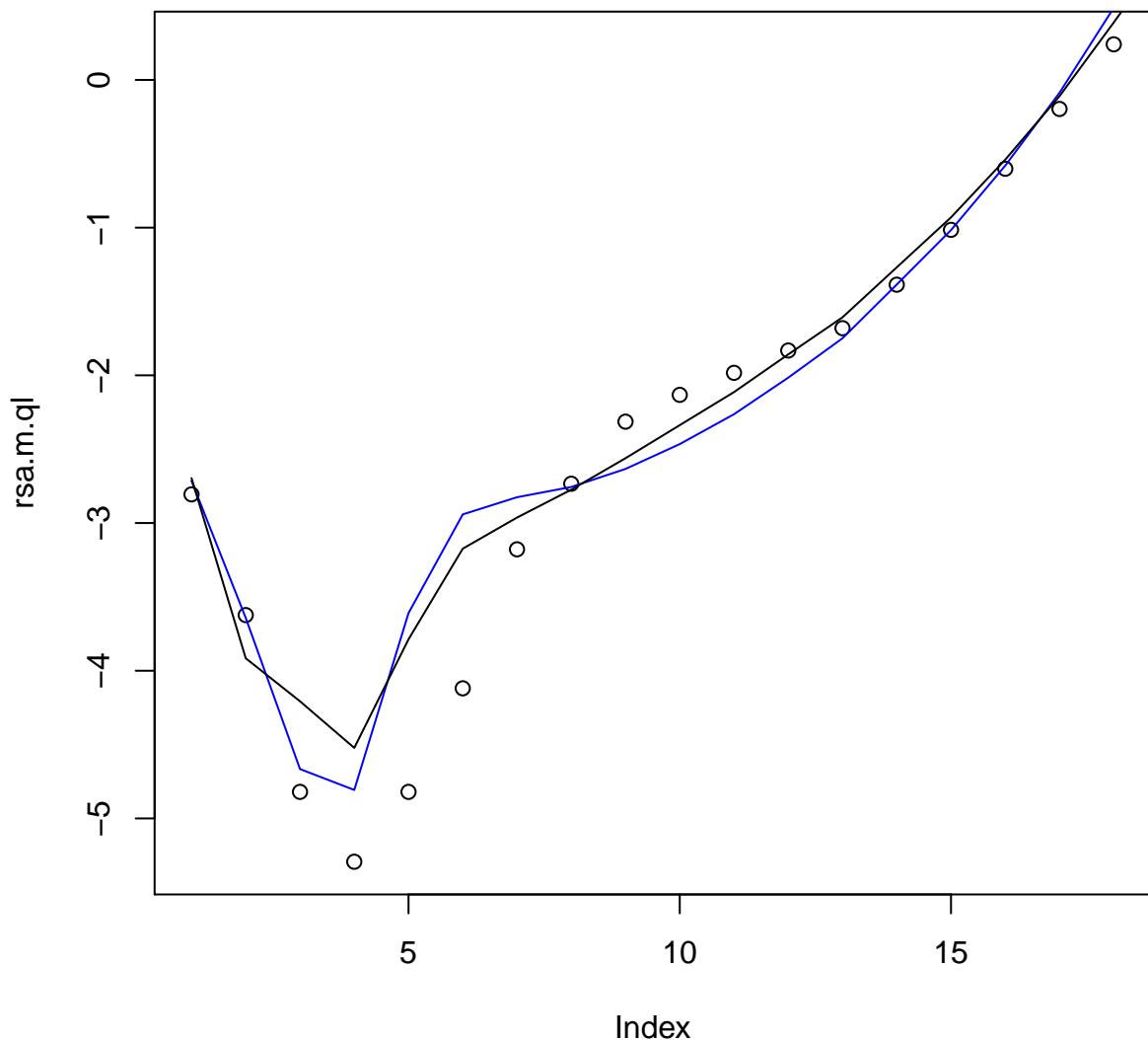
# grayscale
ggplot(data = q.logit.data, aes(x = Age, y = Value, colour = Source)) +
  geom_line(data = q.logit.pred, aes(x = Age, y = Value,
    colour = Source), size = 1) + scale_x_continuous(breaks = c(0,
  1, seq(5, 80, 5)), labels = c("0,1-4", "", paste(seq(5,
  80, 5), c(seq(9, 84, 5))), sep = "-")), minor_breaks = c()) +
  theme_bw() + theme(axis.text.x = element_text(angle = 60,
  hjust = 1)) + geom_point(size = 1.5) + # geom_line(aes(x=Age, y=Value, colour=Source),
# size=0.5) +
  labs(y = expression(""+[bolditalic(n)] * bolditalic("q")*[bolditalic(x)] *
    bold(" (logit scale)")), x = expression(bold("Age (years)")))) +
  facet_wrap(~interaction(Sex, Country, sep = ", "), ncol = 2) +
  # facet_wrap(~Sex + Country,ncol=2) +
  theme(legend.title = element_blank(), legend.position = c(0.15,
  0.91)) + theme(legend.position = "bottom", legend.box = "horizontal") +
  theme(strip.text = element_text(face = "bold")) + scale_colour_grey(start = 0,
  end = 0.8) + ggsave("../figures/fig7-BW.pdf", width = 6.5,
  height = 6.5, units = c("in"))

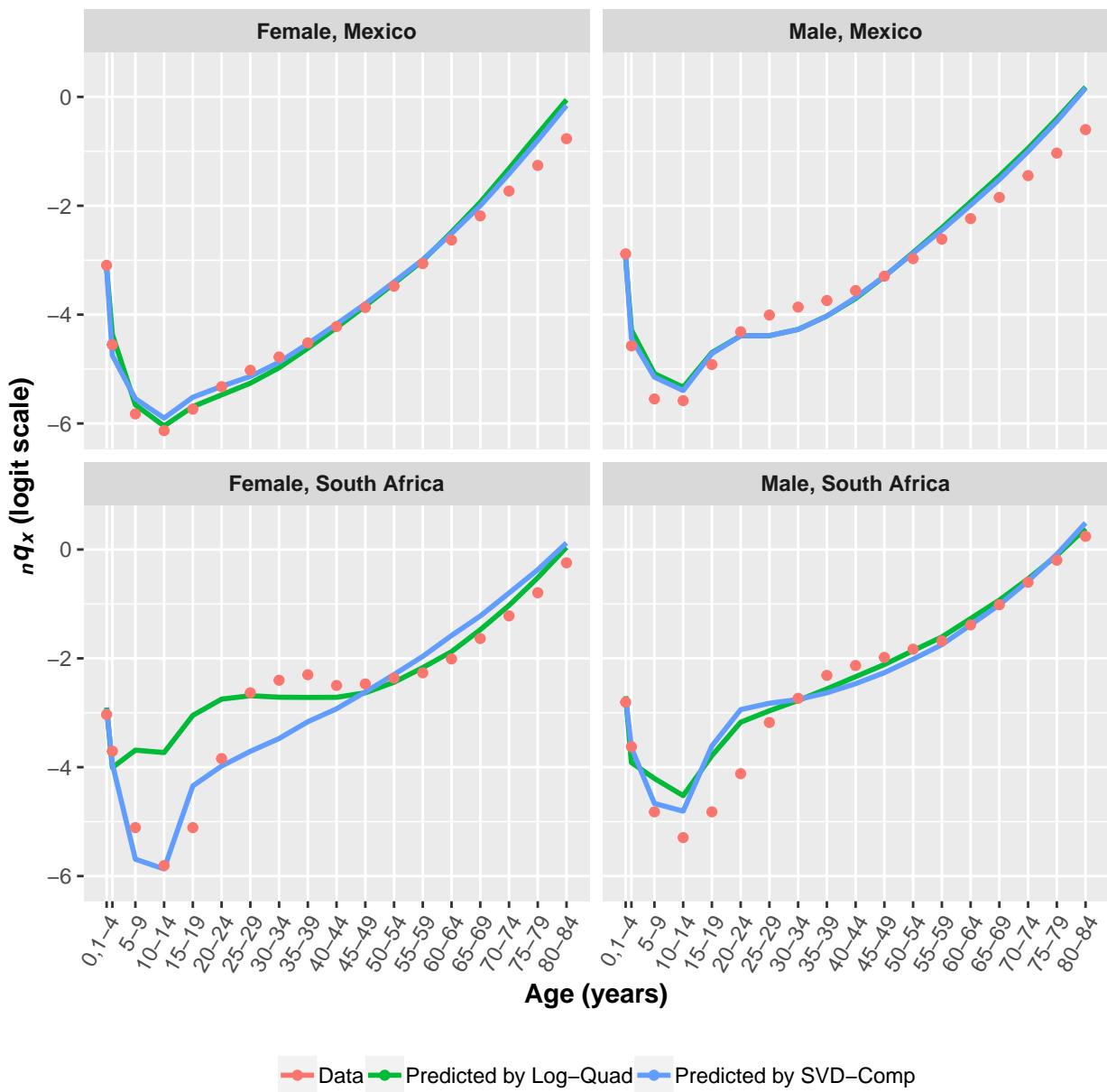
```

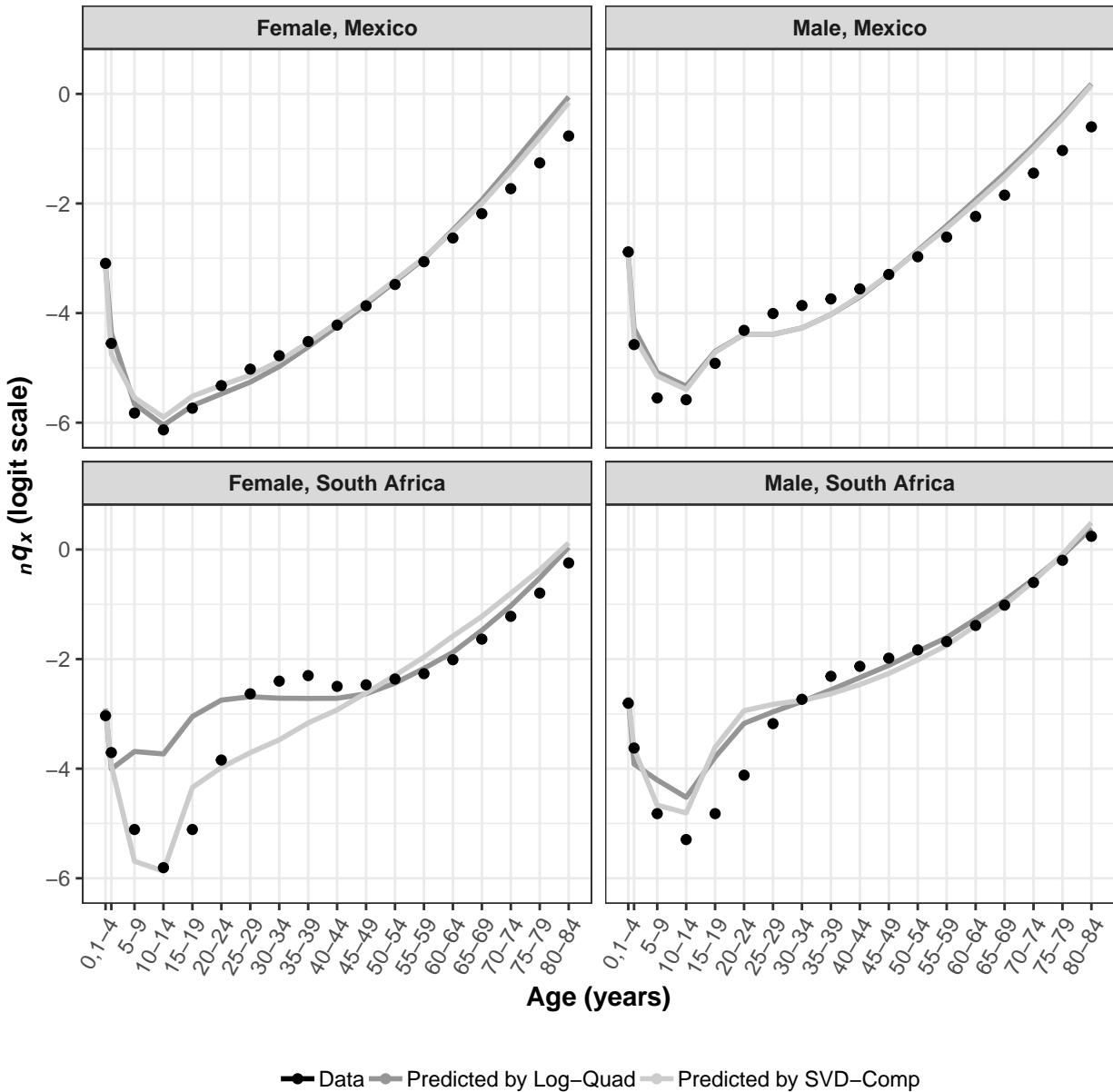












8 Make Compressed Models Object for Package

The SVD-Comp package predicts life tables using child or (child,adult) mortality as inputs. To do this calibrated by HMD, it needs all the component values and models calculated above. This piece of code wraps all that into a nice list and saves it in very compact form.

```
# function to make a list with all the information necessary to do predictions
#   using the HMD-calibrated SVD-Comp
make.models <- function(allModels.f,allModels.sm.f,allModels.m,allModels.sm.m) {

  # initialize array to hold components
  components <- array(
    data=rep(0,880),
    dim=c(2,4,110),
    dimnames=list(
      # child mortality
      sex=c("M","F"),
      age=c("0-14","5-9","10-14","15-19","20-24","25-29","30-34","35-39","40-44","45-49","50-54","55-59","60-64","65-69","70-74","75-79","80-84"),
      # adult mortality
      sex=c("M","F"),
      age=c("0-14","5-9","10-14","15-19","20-24","25-29","30-34","35-39","40-44","45-49","50-54","55-59","60-64","65-69","70-74","75-79","80-84")
    )
  )
```

```

    c("female","male"),
    c("1","2","3","4"),
    c(paste(seq(0,109,1),sep=' ',"))
)
)

# initialize array to hold smoothed components
components.sm <- array(
  data=rep(0,880),
  dim=c(2,4,110),
  dimnames=list(
    c("female","male"),
    c("1","2","3","4"),
    c(paste(seq(0,109,1),sep=' ',")))
)
)

# grab the component values
for (s in 1:2) {
  for (v in 1:4) {
    ifelse(
      s==1,
      components[s,v,] <- allModels.f$svd$s1$d[v]*allModels.f$svd$s1$u[,v],
      components[s,v,] <- allModels.m$svd$s1$d[v]*allModels.m$svd$s1$u[,v]
    )
  }
}

# store female and male components separately
components.f <- components[1,,]
components.m <- components[2,,]

# same for smooth components
for (s in 1:2) {
  for (v in 1:4) {
    ifelse(
      s==1,
      components.sm[s,v,] <- allModels.sm.f$svd$s1$d[v]*allModels.sm.f$svd.sm$s1$u[,v],
      components.sm[s,v,] <- allModels.sm.m$svd$s1$d[v]*allModels.sm.m$svd.sm$s1$u[,v]
    )
  }
}

components.sm.f <- components.sm[1,,]
components.sm.m <- components.sm[2,,]

# # plot the components to be sure you have the right ones
# par(mfrow=c(4,2))
# pdf(file="../figures/components.pdf")
# {
#   plot(components[1,1,])
#   points(components[2,1,],col="red")
#   points(components.sm[1,1,],type="l")
}

```

```

#   points(components.sm[2, 1, ], type="l", col="red")
#
#   plot(components[1, 2, ])
#   points(components[2, 2, ], col="red")
#   points(components.sm[1, 2, ], type="l")
#   points(components.sm[2, 2, ], type="l", col="red")
#
#   plot(components[1, 3, ])
#   points(components[2, 3, ], col="red")
#   points(components.sm[1, 3, ], type="l")
#   points(components.sm[2, 3, ], type="l", col="red")
#
#   plot(components[1, 4, ])
#   points(components[2, 4, ], col="red")
#   points(components.sm[1, 4, ], type="l")
#   points(components.sm[2, 4, ], type="l", col="red")
#
#   plot(components.sm[1, 1, ], type="l", ylim=c(-1250, 75))
#   abline(h=0, lwd=0.5)
#   points(components.sm[1, 2, ], type="l", col="red")
#   points(components.sm[1, 3, ], type="l", col="green")
#   points(components.sm[1, 4, ], type="l", col="blue")
#
#   plot(components.sm[1, 2, ], type="l", col="red")
#   abline(h=0, lwd=0.5)
#   points(components.sm[1, 3, ], type="l", col="green")
#   points(components.sm[1, 4, ], type="l", col="blue")
#
#   plot(components.sm[2, 1, ], type="l", ylim=c(-1250, 75))
#   abline(h=0, lwd=0.5)
#   points(components.sm[2, 2, ], type="l", col="red")
#   points(components.sm[2, 3, ], type="l", col="green")
#   points(components.sm[2, 4, ], type="l", col="blue")
#
#   plot(components.sm[2, 2, ], type="l", col="red")
#   abline(h=0, lwd=0.5)
#   points(components.sm[2, 3, ], type="l", col="green")
#   points(components.sm[2, 4, ], type="l", col="blue")
# } # test that I got the right SVD stuff
# dev.off()

# strip unnecessary stuff from an object
cleanModel = function(cm) {
  cm$y = c()
  cm$model = c()
  cm$residuals = c()
  cm$fitted.values = c()
  cm$effects = c()
  cm$qr$qqr = c()
  cm$linear.predictors = c()
  cm$weights = c()
  cm$prior.weights = c()
  cm$data = c()
}

```

```

attr(cm$terms,".Environment") = c()
attr(cm$formula,".Environment") = c()
return(cm)
}

# # have a look at the massive compression
# object.size(allModels.m$mods$s1$v1)
# object.size(cleanModel(allModels.m$mods$s1$v1))
# as.numeric(object.size(cleanModel(allModels.m$mods$s1$v1))) / as.numeric(object.size(allModels.m$mo

# create the female models list
mods.f <- list(
  components = components.f, # components
  components.sm = components.sm.f, # smooth components
  aml = cleanModel(allModels.f$mods$s1$aml), # adult mx model
  v1 = cleanModel(allModels.f$mods$s1$v1), # v1 model
  v2 = cleanModel(allModels.f$mods$s1$v2), # v2 model
  v3 = cleanModel(allModels.f$mods$s1$v3), # v3 model
  v4 = cleanModel(allModels.f$mods$s1$v4), # v4 model
  offset = allModels.f$offset, # offset
  q0 = cleanModel(allModels.f$mods$s1$q0), # 1q0 model
  rownames = rownames(allModels.f$ql.samp$s1) # row names = age groups
)
# mods.f
# object.size(mods.f)

# make male models list
mods.m <- list(
  components = components.m,
  components.sm = components.sm.m,
  aml = cleanModel(allModels.m$mods$s1$aml),
  v1 = cleanModel(allModels.m$mods$s1$v1),
  v2 = cleanModel(allModels.m$mods$s1$v2),
  v3 = cleanModel(allModels.m$mods$s1$v3),
  v4 = cleanModel(allModels.m$mods$s1$v4),
  offset = allModels.m$offset,
  q0 = cleanModel(allModels.m$mods$s1$q0),
  rownames = rownames(allModels.m$ql.samp$s1)
)
# mods.m
object.size(mods.m)

# make a list of both female and male model lists
mods <- list(
  female = mods.f,
  male = mods.m
)
# mods

# have a look at the size of the finished models list
# format(object.size(mods),units="Mb")

# library(dplyr)

```

```

# d <- ldply(names(mods), function(v) {
#   v.size <- format(object.size(mods[[v]]), unit="Mb")
#   data.frame(variable=v, size=v.size)
# })
# # have a look at the sizes of the components of the models list
# d[order(as.numeric(d$size), decreasing=TRUE),]

return(mods)

}

# create and same the models object for the package
mods <- make.models(mod.1_0.f,mod.1_0.sm.f,mod.1_0.m,mod.1_0.sm.m)
# have a look at it
mods

## $female
## $female$components
##          0         1         2         3
## 1 -934.81217 -1078.100301 -1121.21604 -1145.2916297
## 2  -32.99025    -69.490925   -67.86207   -63.7837818
## 3  -11.69773     -8.068989   -3.52183    0.9974696
## 4   13.71181    17.132887   15.05006   20.7077312
##          4         5         6         7
## 1 -1160.685642 -1173.2862553 -1182.517755 -1193.76699
## 2   -58.178780    -56.6680603   -53.996840   -53.36262
## 3   -5.378499     0.2667628    8.425027   14.22613
## 4   16.321638    14.9561594   15.176928   13.44546
##          8         9         10        11
## 1 -1201.25817 -1206.91829 -1209.423345 -1210.303262
## 2   -48.84598    -46.73611   -45.755033   -46.033797
## 3   21.30969    19.06144    23.700518   19.916801
## 4   14.18258    10.83815    5.474541   -1.091755
##          12        13        14        15
## 1 -1206.6571539 -1200.033331 -1189.916024 -1178.670393
## 2   -42.3177904    -40.445241   -35.589607   -30.443038
## 3   15.4238493    13.901605    8.618961    8.440697
## 4   -0.4554301    -3.078721   -8.145599   -15.368778
##          16        17        18
## 1 -1167.2730792 -1160.905888 -1154.0006870
## 2   -25.7805487    -24.877492   -21.4763153
## 3    0.8121733     0.419798   -0.5214893
## 4   -13.2525635    -10.980000   -14.2932031
##          19        20        21
## 1 -1151.148324 -1148.9962600 -1148.103284
## 2   -22.068752    -23.5686739   -24.816738
## 3   -2.141245     0.8982334    1.858938
## 4   -14.187194    -12.8942770   -14.460371
##          22        23        24
## 1 -1146.3264382 -1145.4395426 -1143.612718
## 2   -26.4021749    -27.8075888   -27.735071
## 3   -0.3666085    -0.0724522    -1.069955
## 4   -13.5375389    -14.5051234   -13.787198
##          25        26        27        28

```

```

## 1 -1140.9366149 -1138.436012 -1136.115358 -1134.004769
## 2 -25.7864157 -24.480313 -23.619273 -24.710907
## 3 -0.6622387 -3.263604 -3.563435 -3.267511
## 4 -12.4497726 -11.002969 -8.983937 -12.587832
## 29 30 31 32
## 1 -1129.114003 -1125.269436 -1124.115213 -1119.206619
## 2 -18.996073 -18.274580 -17.806027 -16.925832
## 3 -6.374302 -5.008698 -3.315137 -4.515220
## 4 -5.784267 -5.499891 -7.849819 -8.234091
## 33 34 35 36
## 1 -1115.835515 -1110.861879 -1105.619430 -1102.105419
## 2 -15.818446 -12.585437 -9.143346 -8.588348
## 3 -2.707198 -7.266547 -7.227161 -6.115746
## 4 -10.073368 -7.927899 -5.249192 -5.615115
## 37 38 39 40
## 1 -1098.488362 -1091.854493 -1088.599365 -1083.0097720
## 2 -7.286434 -2.879493 -2.964686 -0.7508958
## 3 -5.223041 -8.734254 -7.505259 -7.2698574
## 4 -5.595850 -1.871233 -3.847943 1.5920128
## 41 42 43
## 1 -1080.116203 -1073.0626654 -1069.1299698
## 2 1.694397 4.6252393 7.3838375
## 3 -5.540805 -7.8055573 -7.2161259
## 4 -4.871465 -0.8718879 -0.7767959
## 44 45 46
## 1 -1064.541365 -1058.8514420 -1054.889348
## 2 9.283018 11.1644946 12.109123
## 3 -6.800455 -8.6333907 -7.465745
## 4 -2.855260 -0.7720526 -2.393644
## 47 48 49
## 1 -1049.4542090 -1043.6547412 -1038.4110186
## 2 15.5908287 16.3279039 18.5009792
## 3 -7.5838034 -8.0044147 -7.6368042
## 4 0.1816466 0.9119338 0.6607888
## 50 51 52
## 1 -1031.9139423 -1029.035524 -1021.971725
## 2 17.7204241 19.212011 18.838770
## 3 -9.1163164 -6.450327 -8.249205
## 4 0.2965171 -0.476073 1.553808
## 53 54 55 56
## 1 -1017.7797202 -1012.146580 -1007.318725 -1001.531402
## 2 19.6590513 20.243928 21.085839 21.654670
## 3 -7.2491736 -7.832601 -7.392123 -8.324445
## 4 0.5107027 1.356850 1.789300 2.007953
## 57 58 59 60
## 1 -996.652526 -990.375377 -985.025744 -977.255251
## 2 22.451681 21.817291 21.749096 20.038136
## 3 -7.515985 -7.510561 -6.747312 -8.076162
## 4 2.270948 2.482399 2.128034 3.456917
## 61 62 63 64
## 1 -973.208784 -965.410193 -959.818653 -953.261444
## 2 22.113236 20.869301 20.506518 20.535917
## 3 -6.899889 -8.005847 -5.715749 -6.667228
## 4 3.077265 3.937888 3.805540 4.441357

```

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##          65          66          67          68
## 1 -946.294415 -941.186624 -934.309492 -927.107105
## 2  20.510389  20.985521  21.504532  20.829420
## 3  -7.376701  -5.357044  -5.349904  -5.715573
## 4   4.646478   4.700423   4.802689   5.381330
##          69          70          71          72
## 1 -920.772636 -911.668892 -907.156339 -897.890510
## 2  21.359732  19.736063  21.176998  20.171163
## 3  -4.202347  -6.183550  -2.999161  -4.232993
## 4   4.931949   6.168962   5.366502   6.342689
##          73          74          75          76
## 1 -891.034010 -883.263194 -875.739129 -868.738524
## 2  20.725812  20.991569  20.879042  21.699420
## 3  -3.327413  -4.051487  -3.342029  -2.697316
## 4   6.281325   6.467125   6.676577   6.746863
##          77          78          79          80
## 1 -862.35064847 -853.669232 -846.9606410 -838.549113
## 2  22.38557812  22.297605  23.2402296  23.682562
## 3   0.07064408  -2.099756  -0.5147411  -1.819591
## 4   6.19439775   6.567227   5.7278639   6.121445
##          81          82          83          84
## 1 -833.41544021 -824.38599294 -817.1344828 -809.213014
## 2  25.78987663  25.26542519  25.9917358  26.078457
## 3  -0.09811858  -0.02852641  0.9519132  1.226091
## 4   4.91597936   5.26318351   4.8370870   4.791055
##          85          86          87          88
## 1 -802.531725 -795.342441 -788.901990 -782.192583
## 2  27.023658  28.099464  29.077736  29.707654
## 3   2.120207   2.952962   3.933364   4.559833
## 4   4.273231   4.053887   3.331666   2.951848
##          89          90          91          92
## 1 -775.803406 -768.100896 -763.489981 -756.266707
## 2  31.065477  30.577440  32.806112  32.865273
## 3   5.703415   5.383927   7.183681   7.389361
## 4   2.334745   2.385219   1.189876   1.191874
##          93          94          95          96
## 1 -750.2708398 -744.0903099 -737.3014974 -731.38674180
## 2  33.7329875  34.2424604  34.4189738  35.07435303
## 3   8.0981488   8.5103744   9.0876925   9.65858311
## 4   0.7817512   0.5256476   0.3003499  -0.05316081
##          97          98          99          100
## 1 -725.6289774 -720.0377564 -714.622373 -709.391720
## 2  35.7125796  36.3304793  36.924762  37.492161
## 3  10.2023140  10.7170435  11.200832  11.652157
## 4  -0.3979676  -0.7325309  -1.055445  -1.365091
##          101         102         103         104
## 1 -704.353888 -699.516069 -694.884620 -690.464524
## 2  38.029442  38.533610  39.001871  39.431837
## 3  12.069480  12.451542  12.797393  13.106330
## 4  -1.660054  -1.938819  -2.200237  -2.443102
##          105         106         107         108
## 1 -686.259436 -682.271699 -678.501955 -674.949544
## 2  39.821570  40.169652  40.475220  40.738110
## 3  13.378040  13.612565  13.810361  13.972177

```

```

## 4 -2.666487 -2.869801 -3.052633 -3.214914
##
## 109
## 1 -671.612112
## 2 40.958623
## 3 14.099196
## 4 -3.356866
##
## $female$components.sm
##          0           1           2           3
## 1 -934.81217 -1078.100301 -1121.216041 -1145.29163
## 2 -32.99025 -58.988543 -63.952014 -62.92926
## 3 -11.69773 -8.068989 -4.420903 -2.49848
## 4 13.71181 17.132887 15.050060 16.67386
##          4           5           6           7
## 1 -1160.6856424 -1173.28626 -1182.517755 -1193.76699
## 2 -59.5927582 -56.69477 -54.442565 -52.17201
## 3 -0.3463088 3.08090 7.950616 13.09219
## 4 16.3274600 15.60045 14.591220 13.23684
##          8           9          10          11
## 1 -1201.25817 -1206.918292 -1209.423345 -1210.303262
## 2 -49.61767 -47.456878 -46.052637 -44.652947
## 3 17.13726 19.409608 19.819670 18.526995
## 4 11.30154 8.633641 5.395408 1.958207
##          12          13          14          15
## 1 -1206.657154 -1200.033331 -1189.916024 -1178.670393
## 2 -42.449133 -39.323477 -35.295935 -30.906156
## 3 16.027441 12.975990 9.754004 6.510828
## 4 -1.423945 -4.698681 -7.738761 -10.204250
##          16          17          18
## 1 -1167.273079 -1160.905888 -1154.0006870
## 2 -27.183741 -24.572774 -23.0144380
## 3 3.539144 1.313592 0.1047249
## 4 -11.835304 -12.732258 -13.2098132
##          19          20          21
## 1 -1151.1483238 -1148.9962600 -1148.1032840
## 2 -22.7652227 -23.6169447 -24.9114030
## 3 -0.1670923 0.1023427 0.3151709
## 4 -13.4989272 -13.6821016 -13.7728009
##          22          23          24
## 1 -1146.3264382 -1145.4395426 -1143.6127180
## 2 -26.2018134 -27.0201361 -26.8890087
## 3 0.1386775 -0.3237634 -0.9224932
## 4 -13.7366688 -13.4918093 -12.9700040
##          25          26          27          28
## 1 -1140.936615 -1138.436012 -1136.115358 -1134.004769
## 2 -25.921240 -24.812255 -23.899539 -22.548987
## 3 -1.663064 -2.517414 -3.361038 -4.086176
## 4 -12.192964 -11.261069 -10.260475 -9.250140
##          29          30          31          32
## 1 -1129.114003 -1125.269436 -1124.115213 -1119.206619
## 2 -20.470269 -18.724776 -17.664982 -16.604355
## 3 -4.536586 -4.567794 -4.368567 -4.383002
## 4 -8.373666 -7.854489 -7.759797 -7.845213
##          33          34          35          36

```

```

## 1 -1115.835515 -1110.861879 -1105.619430 -1102.105419
## 2 -14.927856 -12.556837 -10.193297 -8.303077
## 3 -4.875847 -5.643484 -6.220978 -6.494627
## 4 -7.733662 -7.228478 -6.403372 -5.433533
## 37 38 39 40
## 1 -1098.488362 -1091.854493 -1088.599365 -1083.0097720
## 2 -6.355594 -4.227115 -2.369339 -0.4480196
## 3 -6.759788 -7.088604 -7.215446 -7.0831098
## 4 -4.435841 -3.491861 -2.717247 -2.2050497
## 41 42 43 44
## 1 -1080.116203 -1073.062665 -1069.129970 -1064.541365
## 2 1.912682 4.522952 7.008868 9.134674
## 3 -6.967912 -7.046700 -7.242703 -7.460153
## 4 -1.931044 -1.785256 -1.674898 -1.536339
## 45 46 47
## 1 -1058.851442 -1054.8893478 -1049.4542090
## 2 10.971973 12.8304163 14.7837874
## 3 -7.642565 -7.7363009 -7.7843457
## 4 -1.294883 -0.9048838 -0.4255002
## 48 49 50
## 1 -1.043655e+03 -1038.4110186 -1031.9139423
## 2 1.644659e+01 17.5767207 18.2414562
## 3 -7.857219e+00 -7.9175378 -7.8723615
## 4 -3.264720e-04 0.2703773 0.4182407
## 51 52 53
## 1 -1029.0355237 -1021.9717255 -1017.779720
## 2 18.7282635 19.1631351 19.673866
## 3 -7.7418830 -7.6444817 -7.625339
## 4 0.5552834 0.7574517 1.026850
## 54 55 56 57
## 1 -1012.146580 -1007.318725 -1001.531402 -996.652526
## 2 20.316425 21.001157 21.600090 21.916418
## 3 -7.658135 -7.709217 -7.712358 -7.616769
## 4 1.332252 1.641254 1.928165 2.186065
## 58 59 60 61
## 1 -990.375377 -985.025744 -977.255251 -973.208784
## 2 21.809150 21.421424 21.158835 21.159859
## 3 -7.476133 -7.392309 -7.359304 -7.265059
## 4 2.435812 2.708919 3.017273 3.345914
## 62 63 64 65
## 1 -965.410193 -959.818653 -953.261444 -946.294415
## 2 21.002902 20.716409 20.602375 20.708508
## 3 -7.049742 -6.794782 -6.581802 -6.330000
## 4 3.672025 3.980574 4.261861 4.510371
## 66 67 68 69
## 1 -941.186624 -934.309492 -927.107105 -920.772636
## 2 20.951169 21.111003 21.051087 20.825711
## 3 -5.965430 -5.587186 -5.282056 -5.004971
## 4 4.731291 4.940974 5.155349 5.380049
## 70 71 72 73
## 1 -911.668892 -907.156339 -897.890510 -891.034010
## 2 20.606057 20.567007 20.591332 20.700780
## 3 -4.663557 -4.264071 -3.930728 -3.703345
## 4 5.610771 5.839908 6.057889 6.249161

```

```

##          74          75          76          77
## 1 -883.263194 -875.739129 -868.738524 -862.350648
## 2  20.907890  21.206925  21.665073  22.153868
## 3  -3.435541  -2.951175  -2.272745  -1.659273
## 4   6.391252   6.461607   6.446782   6.347949
##          78          79          80          81
## 1 -853.669232 -846.960641 -838.5491133 -833.4154402
## 2  22.620847  23.245862  24.1050952  24.9611950
## 3  -1.300201  -1.095876  -0.8276495  -0.3898574
## 4   6.176367   5.946876   5.6793921   5.4006610
##          82          83          84          85
## 1 -824.3859929 -817.134483 -809.213014 -802.531725
## 2  25.5075344  25.895956  26.396231  27.137971
## 3   0.1765126   0.797772   1.462737   2.196057
## 4   5.1303203   4.863447   4.573529   4.234737
##          86          87          88          89
## 1 -795.342441 -788.901990 -782.192583 -775.803406
## 2  28.054617  28.975826  29.833036  30.594145
## 3   2.994833   3.810599   4.588651   5.311471
## 4   3.839939   3.402902   2.946287   2.487028
##          90          91          92          93
## 1 -768.100896 -763.489981 -756.266707 -750.2708398
## 2  31.345151  32.211863  32.991217  33.6125821
## 3   6.015441   6.724472   7.397274   7.9997300
## 4   2.034632   1.602465   1.208274   0.8587744
##          94          95          96          97
## 1 -744.0903099 -737.3014974 -731.3867418 -725.6289774
## 2  34.1233626  34.5807549  35.1053549  35.7050205
## 3   8.5547933   9.0979080   9.6403985   10.1729172
## 4   0.5414416   0.2349479  -0.0761416  -0.3952438
##          98          99         100         101
## 1 -720.0377564 -714.622373 -709.391720 -704.353888
## 2  36.3161628  36.908158  37.473582  38.009013
## 3  10.6835970  11.165305  11.615002  12.030968
## 4  -0.7170226  -1.033864  -1.340139  -1.632542
##          102         103         104         105
## 1 -699.516069 -694.884620 -690.464524 -686.259436
## 2  38.511479  38.978272  39.407050  39.795913
## 3  12.411983  12.757105  13.065700  13.336686
## 4  -1.909176  -2.168477  -2.408626  -2.626278
##          106         107         108         109
## 1 -682.271699 -678.501955 -674.949544 -671.612112
## 2  40.142924  40.441899  40.672841  40.818386
## 3  13.567416  13.751737  13.884708  13.970867
## 4  -2.815687  -2.970451  -3.087959  -3.171742
##
## $female$aml
##
## Call:
## lm(formula = aml ~ cm + cml + cmcls + cmclc)
##
## Coefficients:
## (Intercept)           cm            cml           cmcls
##      5.92112     -10.79701      4.00517      0.69463

```

```

##      cmlc
##  0.04597
##
##
## $female$v1
##
## Call:
## lm(formula = svd$v[, 1] ~ cm + cml + cmls + cmlc + am + amls +
##     amlc + cmlaml)
##
## Coefficients:
## (Intercept)          cm          cml          cmls
##  6.260e-03   1.633e-02  -4.751e-03  -7.956e-04
##     cmlc          am          amls          amlc
##  -6.122e-05  -2.820e-03   3.749e-04  -1.615e-05
##     cmlaml
##  -3.819e-04
##
##
## $female$v2
##
## Call:
## lm(formula = svd$v[, 2] ~ cm + cml + cmls + cmlc + am + amls +
##     amlc + cmlaml)
##
## Coefficients:
## (Intercept)          cm          cml          cmls
## -0.285395   0.500757  -0.156724  -0.029283
##     cmlc          am          amls          amlc
##  -0.002192  -0.002448   0.012689   0.002095
##     cmlaml
##  -0.006205
##
##
## $female$v3
##
## Call:
## lm(formula = svd$v[, 3] ~ cm + cml + cmls + cmlc + am + amls +
##     amlc + cmlaml)
##
## Coefficients:
## (Intercept)          cm          cml          cmls
##  0.345769  -0.796891   0.202197   0.022961
##     cmlc          am          amls          amlc
##  0.002058   0.079428  -0.023815   0.002820
##     cmlaml
##  0.043416
##
##
## $female$v4
##
## Call:
## lm(formula = svd$v[, 4] ~ cm + cml + cmls + cmlc + am + amls +
##     amlc + cmlaml)

```

```

## 
## Coefficients:
## (Intercept)          cm         cml        cmls
## -0.933533    1.934863   -0.537323   -0.106343
##      cmlc          am         amls        amlc
## -0.007025   -0.049005    0.014147    0.002104
##      cmlaml
## -0.003531
## 
## 
## $female$offset
## [1] 10
## 
## $female$q0
## 
## Call:
## lm(formula = as.numeric(ql[1, samp]) ~ cml + cmls)
## 
## Coefficients:
## (Intercept)          cml        cmls
## -0.9508     0.6592    -0.0377
## 
## 
## $female$rownames
## [1] "0"   "1"   "2"   "3"   "4"   "5"   "6"   "7"
## [9] "8"   "9"   "10"  "11"  "12"  "13"  "14"  "15"
## [17] "16"  "17"  "18"  "19"  "20"  "21"  "22"  "23"
## [25] "24"  "25"  "26"  "27"  "28"  "29"  "30"  "31"
## [33] "32"  "33"  "34"  "35"  "36"  "37"  "38"  "39"
## [41] "40"  "41"  "42"  "43"  "44"  "45"  "46"  "47"
## [49] "48"  "49"  "50"  "51"  "52"  "53"  "54"  "55"
## [57] "56"  "57"  "58"  "59"  "60"  "61"  "62"  "63"
## [65] "64"  "65"  "66"  "67"  "68"  "69"  "70"  "71"
## [73] "72"  "73"  "74"  "75"  "76"  "77"  "78"  "79"
## [81] "80"  "81"  "82"  "83"  "84"  "85"  "86"  "87"
## [89] "88"  "89"  "90"  "91"  "92"  "93"  "94"  "95"
## [97] "96"  "97"  "98"  "99"  "100" "101" "102" "103"
## [105] "104" "105" "106" "107" "108" "109"
## 
## 
## $male
## $male$components
##          0          1          2          3
## 1 -918.907400 -1067.7577497 -1106.9976489 -1130.482833
## 2  -46.762420   -80.3257730   -74.1620063   -72.859535
## 3   -6.093825   -4.9014561   -6.3117837   -13.775142
## 4    5.187333   -0.3900189   -0.6369774    4.633521
##          4          5          6          7
## 1 -1145.467156 -1156.215917 -1165.304184 -1171.972581
## 2   -65.772801   -61.863400   -61.348047   -57.246245
## 3   -12.212158   -11.915365   -13.959189   -14.145176
## 4    1.920031    3.085376    5.568228    5.782408
##          8          9         10         11
## 1 -1179.20131 -1185.634456 -1187.85386 -1188.719031

```

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## 2   -53.18097   -51.176119   -48.51613   -44.674823
## 3   -13.96148   -17.160869   -13.28159   -14.143095
## 4    8.64218    4.862689    2.09220    1.748089
##          12        13        14        15
## 1  -1185.3539556 -1179.367834 -1166.757982 -1153.069064
## 2   -40.4969505  -38.072395  -30.082990  -25.843765
## 3   -12.0386341  -10.708389  -4.838728  -3.742612
## 4    0.8798751   1.515544  -3.065599  -4.221248
##          16        17        18        19
## 1  -1.135088e+03 -1119.994406 -1104.976629 -1098.944408
## 2  -1.732023e+01  -12.582271  -6.059524  -7.945558
## 3  -4.035991e-02   3.040498   6.075331   8.197556
## 4  -6.865471e+00  -6.585074  -11.082816  -11.609534
##          20        21        22        23
## 1  -1094.974023 -1092.67707 -1091.18860 -1090.64323
## 2   -10.166749   -11.03025  -12.04597  -12.26401
## 3    8.094635   10.25060   11.11026   12.27288
## 4   -12.580588  -14.69842  -12.02623  -12.50604
##          24        25        26        27
## 1  -1091.05006 -1090.80533 -1091.11963 -1089.911264
## 2   -12.58721   -11.78948  -11.36607  -8.961508
## 3   12.20967   13.35735   13.50989   14.381489
## 4   -11.74958  -10.84258  -11.71841  -10.365331
##          28        29        30        31
## 1  -1088.835823 -1087.567047 -1086.160889 -1084.208020
## 2   -8.400243   -7.364397  -9.740522  -5.375643
## 3   15.094168   14.411176  13.953099  14.842239
## 4   -8.635942  -8.237436  -7.350837  -7.041318
##          32        33        34        35
## 1  -1081.093880 -1078.435798 -1075.317085 -1071.972152
## 2   -6.379372   -4.961203  -3.978003  -4.598939
## 3   14.136720   14.472116  14.890262  14.141483
## 4   -4.735706  -3.974359  -2.777733  -1.195722
##          36        37        38
## 1  -1068.508786 -1065.560257 -1060.0771915
## 2   -2.624597   -3.093011  -0.5091694
## 3   13.421643   12.879395  13.1689338
## 4   -1.119241  -1.850958   0.7122004
##          39        40        41
## 1  -1055.8759034 -1050.6962716 -1047.156414
## 2    0.8938587   0.4514354   3.789717
## 3   12.1851889   12.4535174  11.640551
## 4   1.0732119   3.4734569   2.537027
##          42        43        44        45
## 1  -1040.777161 -1036.222318 -1031.371167 -1025.179953
## 2    4.134943   6.420122   7.153389   8.515586
## 3   10.944900   10.832794   9.998354   9.460780
## 4   4.040377   4.103012   4.584914   5.313658
##          46        47        48        49
## 1  -1020.743318 -1015.329017 -1009.804937 -1004.188594
## 2   10.145274   11.621507  11.826108  13.383243
## 3   8.916410   8.485985   7.902708   7.268519
## 4   5.728307   6.331918   6.893515   6.999801
##          50        51        52        53

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## 1 -997.777654 -994.097103 -986.903491 -982.049441
## 2 13.437250 15.258691 15.740898 16.290281
## 3 6.983844 6.078865 5.851411 4.904395
## 4 7.701205 7.068229 7.901648 7.794469
##      54          55          56          57
## 1 -976.097212 -970.896291 -965.387386 -960.176928
## 2 17.535915 18.683344 19.276501 20.344859
## 3 4.567656 4.263172 3.289643 2.724659
## 4 8.105849 7.864041 8.292673 8.130057
##      58          59          60          61
## 1 -953.978910 -948.379380 -941.200060 -937.591799
## 2 20.275096 20.654850 20.048406 20.898099
## 3 2.291617 2.080595 2.618374 1.509016
## 4 8.946455 8.390194 9.337832 7.958720
##      62          63          64          65
## 1 -930.198506 -924.5725418 -918.5734711 -912.3624195
## 2 20.486635 20.5268522 20.6563287 20.6182449
## 3 1.416116 0.7290468 0.6334766 0.2099518
## 4 8.818968 8.5918179 8.6173659 8.8468299
##      66          67          68          69
## 1 -907.9145748 -901.7491004 -895.487712 -889.834177
## 2 21.1385531 21.1837263 21.110886 21.204106
## 3 -0.4054377 -0.7973464 -1.107968 -1.833113
## 4 7.3534470 7.6648716 7.563915 6.682618
##      70          71          72          73
## 1 -882.486582 -878.424221 -870.293626 -864.214496
## 2 19.837564 21.088000 20.026403 20.126150
## 3 -1.605840 -3.294001 -2.623511 -2.877187
## 4 7.300382 5.420135 6.543078 6.130091
##      74          75          76          77
## 1 -857.626694 -851.23161 -845.156607 -839.724954
## 2 19.851225 19.76794 20.049416 20.278227
## 3 -3.051375 -3.03863 -3.922084 -5.167379
## 4 6.203302 6.08607 5.587756 3.901878
##      78          79          80          81
## 1 -832.180196 -826.211813 -818.898210 -814.112678
## 2 19.936025 20.243795 20.234399 21.444922
## 3 -4.292642 -4.633387 -3.963119 -4.437799
## 4 4.935100 3.721218 4.060713 2.573278
##      82          83          84          85
## 1 -806.105990 -799.711084 -792.476835 -786.782192
## 2 20.670597 20.880805 20.559330 21.455172
## 3 -4.991629 -5.509210 -6.076071 -6.469185
## 4 2.477312 1.679605 1.477025 0.504247
##      86          87          88          89
## 1 -780.179766 -774.1536971 -768.109007 -762.318754
## 2 21.935225 22.6184392 22.925923 23.799227
## 3 -7.257010 -7.7368331 -8.113589 -8.662216
## 4 -0.039927 -0.9238375 -1.565932 -2.582696
##      90          91          92          93
## 1 -755.640320 -751.111137 -744.570987 -739.123298
## 2 23.702633 25.464396 25.658109 26.410372
## 3 -8.751635 -9.636846 -10.071138 -10.579257
## 4 -2.739763 -4.283337 -4.391203 -5.028195

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##          94          95          96          97
## 1 -733.670476 -727.775823 -722.53691 -717.44895
## 2  26.777044  26.067273  26.48468  26.89721
## 3 -11.032325 -11.104530 -11.44645 -11.76423
## 4 -5.390992 -6.081449 -6.59490 -7.07833
##          98          99         100         101
## 1 -712.518841 -707.753005 -703.157225 -698.736500
## 2  27.302846  27.699483  28.085049  28.457353
## 3 -12.057035 -12.324199 -12.565218 -12.779860
## 4 -7.530303 -7.949676 -8.335431 -8.686902
##          102         103         104         105
## 1 -694.49518 -690.436551 -686.562931 -682.875751
## 2  28.81432  29.154043  29.474681  29.774755
## 3 -12.96803 -13.129963 -13.266024 -13.376832
## 4 -9.00369 -9.285696 -9.533101 -9.746394
##          106         107         108         109
## 1 -679.375195 -676.06048 -672.92983 -669.98046
## 2  30.052932  30.30832  30.54016  30.74823
## 3 -13.463232 -13.52632 -13.56719 -13.58726
## 4 -9.926345 -10.07408 -10.19096 -10.27854
##
## $male$components.sm
##          0          1          2          3
## 1 -918.907400 -1067.7577497 -1106.9976489 -1130.482833
## 2 -46.762420 -69.2341449 -72.6404749 -70.859043
## 3 -6.093825 -4.9014561 -8.5777244 -10.392577
## 4  5.187333 -0.3900189 -0.6369774  2.428802
##          4          5          6          7
## 1 -1145.467156 -1156.215917 -1165.304184 -1171.972581
## 2 -66.855021 -63.168542 -60.235519 -57.151344
## 3 -11.820047 -12.720042 -13.409787 -14.042210
## 4  3.097681  3.917469  4.716861  5.215068
##          8          9         10         11
## 1 -1179.201313 -1185.634456 -1187.853858 -1188.719031
## 2 -53.925228 -50.982433 -47.975248 -44.526205
## 3 -14.518931 -14.602081 -14.112633 -13.046000
## 4  5.131929  4.413756  3.289659  2.040126
##          12         13         14
## 1 -1185.3539556 -1179.3678340 -1166.757982
## 2 -40.7030447 -36.2354082 -30.783267
## 3 -11.3658852 -9.0075460 -6.153619
## 4  0.7419566 -0.7128786 -2.396808
##          15         16         17         18
## 1 -1153.069064 -1135.0880529 -1119.994406 -1104.976629
## 2 -24.696539 -18.4497216 -12.924260 -9.416816
## 3 -3.137867 -0.1460607  2.703693  5.188621
## 4 -4.236941 -6.1078215 -7.923543 -9.601648
##          19         20         21         22
## 1 -1098.944408 -1094.974023 -1092.67707 -1091.18860
## 2 -8.714735 -9.730916 -10.90407 -11.71497
## 3  7.135411  8.621973  9.85696  10.92147
## 4 -11.006613 -11.985796 -12.45870 -12.47518
##          23         24         25         26
## 1 -1090.64323 -1091.05006 -1090.80533 -1091.11963

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## 2   -12.13514   -12.14085   -11.67920   -10.73773
## 3    11.78932    12.48805    13.10099    13.66756
## 4   -12.19109   -11.76598   -11.26945   -10.67723
##          27         28         29         30
## 1 -1089.911264 -1088.835823 -1087.567047 -1086.160889
## 2   -9.531846    -8.597743    -8.183067    -7.737077
## 3    14.134485   14.400310   14.453439   14.421459
## 4   -9.951616    -9.111299    -8.207944    -7.255760
##          31         32         33         34
## 1 -1084.208020 -1081.093880 -1078.435798 -1075.317085
## 2   -6.790102    -5.859306    -5.092387    -4.442810
## 3    14.413286   14.432099   14.427295   14.306123
## 4   -6.227857    -5.123182    -4.004588    -2.967544
##          35         36         37
## 1 -1071.972152 -1068.508786 -1065.5602574
## 2   -3.871715    -3.144514    -2.1577145
## 3    14.002229   13.582870   13.1761818
## 4   -2.071150    -1.281237    -0.4864325
##          38         39         40
## 1 -1060.0771915 -1055.8759034 -1050.696272
## 2   -0.8714882     0.3717158    1.575393
## 3    12.8220408   12.4744101   12.082568
## 4     0.3962924    1.3350227   2.221640
##          41         42         43         44
## 1 -1047.156414 -1040.777161 -1036.222318 -1031.371167
## 2    3.044126     4.563624    5.989119    7.311376
## 3    11.625727   11.123324   10.598970   10.053880
## 4    2.979836    3.613430    4.172191    4.702744
##          45         46         47         48
## 1 -1025.179953 -1020.743318 -1015.329017 -1009.804937
## 2     8.641552    10.009110   11.195475   12.145740
## 3     9.500172    8.958133    8.426695    7.893087
## 4     5.226928    5.740519    6.221817    6.643789
##          49         50         51         52
## 1 -1004.188594 -997.777654 -994.097103 -986.903491
## 2    13.022028   13.919975   14.866942   15.726080
## 3     7.353287    6.804032    6.240838    5.670760
## 4     6.988316    7.256865    7.470056    7.650322
##          53         54         55         56
## 1 -982.049441 -976.097212 -970.896291 -965.387386
## 2    16.555797   17.512781   18.485018   19.320786
## 3     5.110794    4.564851    4.007604    3.435937
## 4     7.807636    7.946138    8.078363    8.220175
##          57         58         59         60
## 1 -960.176928 -953.978910 -948.379380 -941.200060
## 2    19.944928   20.284986   20.398214   20.451616
## 3     2.918663    2.534908    2.274712    2.027688
## 4     8.371592    8.508498    8.598607    8.629542
##          61         62         63         64
## 1 -937.591799 -930.198506 -924.5725418 -918.5734711
## 2    20.543765   20.578746   20.5828469   20.6456150
## 3     1.700397    1.307874    0.9088496    0.5169059
## 4     8.619302    8.590433    8.5343504    8.4130432
##          65         66         67         68

```

```

## 1 -912.3624195 -907.9145748 -901.749100 -895.487712
## 2  20.7895793  20.9822574  21.098595  21.056273
## 3   0.1073272 -0.3277693 -0.767889 -1.203158
## 4   8.2025186  7.9230007  7.617468  7.309775
##      69          70          71          72
## 1 -889.83418 -882.486582 -878.424221 -870.293626
## 2  20.83221  20.584407  20.462067  20.287854
## 3  -1.64466 -2.090234 -2.479046 -2.743807
## 4   7.00245  6.708487  6.461757  6.280783
##      73          74          75          76
## 1 -864.214496 -857.626694 -851.231613 -845.156607
## 2  20.076642  19.939408  19.922118  20.017274
## 3  -2.923416 -3.140114 -3.485652 -3.924883
## 4   6.131462  5.939390  5.645681  5.253323
##      77          78          79          80
## 1 -839.724954 -832.180196 -826.211813 -818.898210
## 2  20.098854  20.133965  20.258074  20.542900
## 3  -4.285235 -4.434818 -4.435844 -4.467339
## 4   4.816835  4.380538  3.937194  3.454402
##      81          82          83          84
## 1 -814.112678 -806.105990 -799.711084 -792.476835
## 2  20.820798  20.871042  20.838985  20.985553
## 3  -4.665530 -5.042544 -5.525531 -6.050571
## 4   2.922995  2.360753  1.780777  1.174461
##      85          86          87          88
## 1 -786.782192 -780.1797662 -774.1536971 -768.109007
## 2  21.405292  21.9643634  22.5211466  23.056443
## 3  -6.593869 -7.1380894 -7.6525235 -8.121856
## 4   0.526544 -0.1655933 -0.8894405 -1.627376
##      89          90          91          92
## 1 -762.318754 -755.640320 -751.111137 -744.570987
## 2  23.601563  24.247543  25.024939  25.717589
## 3  -8.567092 -9.029921 -9.528676 -10.030396
## 4  -2.363510 -3.081786 -3.761365 -4.385396
##      93          94          95          96
## 1 -739.123298 -733.670476 -727.775823 -722.536912
## 2  26.200292  26.406184  26.429030  26.573736
## 3  -10.482818 -10.858492 -11.171536 -11.460680
## 4  -4.956984 -5.496836 -6.021736 -6.531891
##      97          98          99          100
## 1 -717.448947 -712.518841 -707.753005 -703.157225
## 2  26.904249  27.297915  27.692631  28.076869
## 3  -11.748829 -12.031292 -12.295922 -12.536393
## 4  -7.017838 -7.471945 -7.891601 -8.276815
##      101         102         103         104
## 1 -698.736500 -694.495183 -690.436551 -686.562931
## 2  28.447899  28.803681  29.142296  29.462002
## 3  -12.750973 -12.939355 -13.101715 -13.238452
## 4  -8.627748 -8.944388 -9.226527 -9.473817
##      105         106         107         108
## 1 -682.875751 -679.375195 -676.060481 -672.92983
## 2  29.761280  30.038382  30.287309  30.48773
## 3  -13.349962 -13.436524 -13.498416 -13.53790
## 4  -9.684991 -9.857712 -9.990453 -10.08562

```

```

##          109
## 1 -669.98046
## 2   30.61837
## 3  -13.56048
## 4  -10.15008
##
## $male$aml
##
## Call:
## lm(formula = aml ~ cm + cml + cmls + cmlc)
##
## Coefficients:
## (Intercept)          cm          cml          cmls
## -0.70346      2.88318      0.35930      0.10413
##          cmlc
##      0.01598
##
##
## $male$v1
##
## Call:
## lm(formula = svd$v[, 1] ~ cm + cml + cmls + cmlc + am + amls +
##      amlc + cmlaml)
##
## Coefficients:
## (Intercept)          cm          cml          cmls
## 9.171e-03     1.034e-02    -3.288e-03    -6.297e-04
##          cmlc          am          amls          amlc
## -4.522e-05    -1.981e-03     9.949e-05    -1.180e-05
##          cmlaml
##      -3.034e-05
##
##
## $male$v2
##
## Call:
## lm(formula = svd$v[, 2] ~ cm + cml + cmls + cmlc + am + amls +
##      amlc + cmlaml)
##
## Coefficients:
## (Intercept)          cm          cml          cmls
## -0.1937817     0.3200685    -0.1093803    -0.0209625
##          cmlc          am          amls          amlc
## -0.0015087    -0.0079042     0.0023952     0.0008903
##          cmlaml
##      -0.0006750
##
##
## $male$v3
##
## Call:
## lm(formula = svd$v[, 3] ~ cm + cml + cmls + cmlc + am + amls +
##      amlc + cmlaml)
##

```

```

## Coefficients:
## (Intercept)          cm         cml        cmls
##  0.1606100   -0.4248549   0.1161023   0.0261142
##      cmlc          am         amls        amlc
##  0.0016607   0.1096817  -0.0021355  -0.0009072
##      cmlaml
##  -0.0035825
##
##
## $male$v4
##
## Call:
## lm(formula = svd$v[, 4] ~ cm + cml + cmls + cmlc + am + amls +
##      amlc + cmlaml)
##
## Coefficients:
## (Intercept)          cm         cml        cmls
## -9.107e-01   1.880e+00  -5.268e-01  -1.002e-01
##      cmlc          am         amls        amlc
## -6.459e-03  -5.700e-02  -3.956e-03  -3.731e-05
##      cmlaml
##  -2.277e-03
##
##
## $male$offset
## [1] 10
##
## $male$q0
##
## Call:
## lm(formula = as.numeric(ql[1, samp]) ~ cml + cmls)
##
## Coefficients:
## (Intercept)          cml        cmls
## -0.82806    0.68947   -0.03693
##
##
## $male$rownames
## [1] "0"   "1"   "2"   "3"   "4"   "5"   "6"   "7"
## [9] "8"   "9"   "10"  "11"  "12"  "13"  "14"  "15"
## [17] "16"  "17"  "18"  "19"  "20"  "21"  "22"  "23"
## [25] "24"  "25"  "26"  "27"  "28"  "29"  "30"  "31"
## [33] "32"  "33"  "34"  "35"  "36"  "37"  "38"  "39"
## [41] "40"  "41"  "42"  "43"  "44"  "45"  "46"  "47"
## [49] "48"  "49"  "50"  "51"  "52"  "53"  "54"  "55"
## [57] "56"  "57"  "58"  "59"  "60"  "61"  "62"  "63"
## [65] "64"  "65"  "66"  "67"  "68"  "69"  "70"  "71"
## [73] "72"  "73"  "74"  "75"  "76"  "77"  "78"  "79"
## [81] "80"  "81"  "82"  "83"  "84"  "85"  "86"  "87"
## [89] "88"  "89"  "90"  "91"  "92"  "93"  "94"  "95"
## [97] "96"  "97"  "98"  "99"  "100" "101" "102" "103"
## [105] "104" "105" "106" "107" "108" "109"

```

```
save(file="../RData/mods.RData",compress=TRUE,list=c("mods"))
# the saved file should be around 15KB !!
```

9 Wrap up

Save the workspace and clear everything

```
save(file = paste("../Rdata/All-SVD-Comp_",
                 "%Y-%m-%d"), ".RData", sep = ""),
      compress = TRUE, list = ls())
rm(list = ls())
```