TDT4171 Artificial Intelligence Methods

Assignment 2

January 2022

1 Information

- **Delivery deadline: February 11, 2022, by 23:59**. No late delivery will be graded! Deadline extensions will only be considered for extraordinary situations such as family or health-related circumstances. However, these circumstances must be documented, e.g., with a doctor's note ("legeerklæring"). Having a lot of work in other classes is not a legitimate excuse for late delivery.
- Students can NOT work in groups. Each student can only submit a solution individually.
- This homework counts for 4% of the final grade.
- Cribbing("koking") from other students is not accepted, and if detected, will lead to immediate failure of the course. The consequence will apply to both the source and the one cribbing.
- Required reading for this assignment: Chapter 14 (the parts in the curriculum found on Blackboard "Sources and syllabus" → "Preliminary syllabus") in Artificial Intelligence: A Modern Approach, Global Edition 4th Edition, Russell & Norvig. All the page and equation references follow this version of the book.
- For help and questions related to the assignment, ask the student assistants during the guidance hours or use Piazza. The guidance hours and link to Piazza can be found under "Course work" on Blackboard. For other inquires, an email can be sent to tdt4171@idi.ntnu.no.
- Deliver your solution on Blackboard. Please upload your assignment as one PDF report and one source file containing the code (e.g., a .py file). Please do not put the files into an archive.

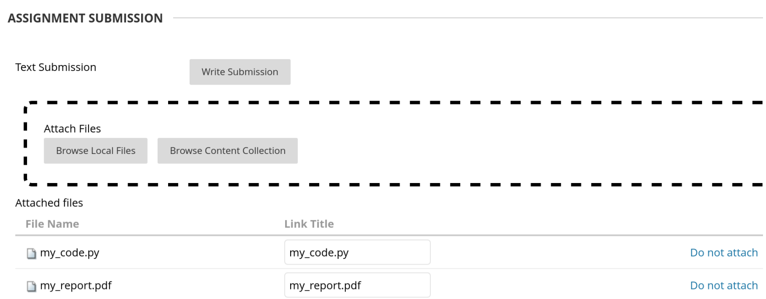

Figure 1: Delivery Example

2 Assignment

There are three tasks in this assignment. The main task is to implement the Forward-Backward algorithm for the Hidden Markov (HMM) model by programming, and try it out on the book's *umbrella world* described in Figure 14.2 on Page 482. Using another programming language other than Python is possible, but less help might be provided. No matter which programming language, it is beneficial to choose one that offers linear algebra out of the box.

We recommend using Python with the NumPy library for this assignment. Although it is possible to do the assignment without it, it is beneficial to learn how to use NumPy for cleaner and more elegant code. Moreover, other courses on artificial intelligence might use NumPy too. On how to use NumPy, check out the NumPy: the absolute basics for beginners tutorial and the NumPy documentation.

For the programming part of the assignment, the code must be runnable without any modifications after delivery. The code must be readable and contain explaining comments where appropriate. The commenting is especially important if the code does not work.

Make sure to read the whole assignment text before starting to solve them.

2.1 Time and Uncertainty

Describe the *umbrella world* as an HMM:

- What is the set of unobserved variable(s) for a given time-slice t (denoted $\mathbf{X}_t$ in the book)?
- What is the set of observable variable(s) for a given time-slice t (denoted $\mathbf{E}_t$ in the book)?
- Present the *dynamic model* $\mathbf{P}(\mathbf{X}_t|\mathbf{X}_{t-1})$ and the *observation model* $\mathbf{P}(\mathbf{E}_t|\mathbf{X}_t)$ as matrices.
- Which assumptions are encoded in this model? Are the assumptions reasonable for this particular domain? (See 14.1 Time and Uncertainty on Page 479).

2.2 Inference in Temporal Models - Filtering

Implement filtering using the *Forward* operation (see Equation 14.5 on Page 485 and Equation 14.12 on Page 492) by programming. The forward operation can be done with matrix operations in the HMM.

- Verify your implementation by calculating $\mathbf{P}(\mathbf{X}_2|\mathbf{e}_{1:2})$, where $\mathbf{e}_{1:2}$ is the evidence that the umbrella was used both on day 1 and day 2. The desired result is that the probability of rain at day 2 (after the observations) is 0.883.
- Use your program to calculate the probability of rain at day 5 given the following sequence of observations:

$$\mathbf{e}_{1:5} = \{\texttt{Umbrella}_1 = \texttt{true},\ \texttt{Umbrella}_2 = \texttt{true},\\ \texttt{Umbrella}_3 = \texttt{false},\ \texttt{Umbrella}_4 = \texttt{true},\ \texttt{Umbrella}_5 = \texttt{true}\}.$$

  Document your answer by showing all *normalized* forward messages (in the book, the unnormalized forward messages are denoted $\mathbf{f}_{1:k}$ for $k = 1, 2, \ldots, 5$) in the PDF report.

2.3 Inference in Temporal Models - Smoothing

Implement smoothing using the *Forward-Backward* algorithm (Figure 14.4 on Page 488) by programming. The *Forward-Backward* algorithm can be done with matrix operations.

- Verify your implementation by calculating $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:2})$ where $\mathbf{e}_{1:2}$ is the evidence that the umbrella was used the first two days, as in Section 2.2. The desired result is $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:2}) = \langle 0.883, 0.117\rangle$.
- Use your *Forward-Backward* algorithm to calculate the probability of rain at day 1 given the following sequence of observations:

$$\begin{aligned}\mathbf{e}_{1:5} = \{&\texttt{Umbrella}_1 = \texttt{true},\ \texttt{Umbrella}_2 = \texttt{true},\\ &\texttt{Umbrella}_3 = \texttt{false},\ \texttt{Umbrella}_4 = \texttt{true},\ \texttt{Umbrella}_5 = \texttt{true}\},\end{aligned}$$

  i.e., $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:5})$. Document your answer by showing all backward messages ($\mathbf{b}_{k+1:t}$ for $k = 1, 2, \ldots, 5$) in the PDF report.