The Graph Algorithms Course

Pre-Requisite

- Basic Programming (we will code in C++)
- Basic - Intermediate Recursion
- Arrays, HM, LL, Trees

Outcome Expectation

- Course will be starting from very basics
  but we will cover adv CP topics
- Beneficial with Interview perspective too.

(21 classes)

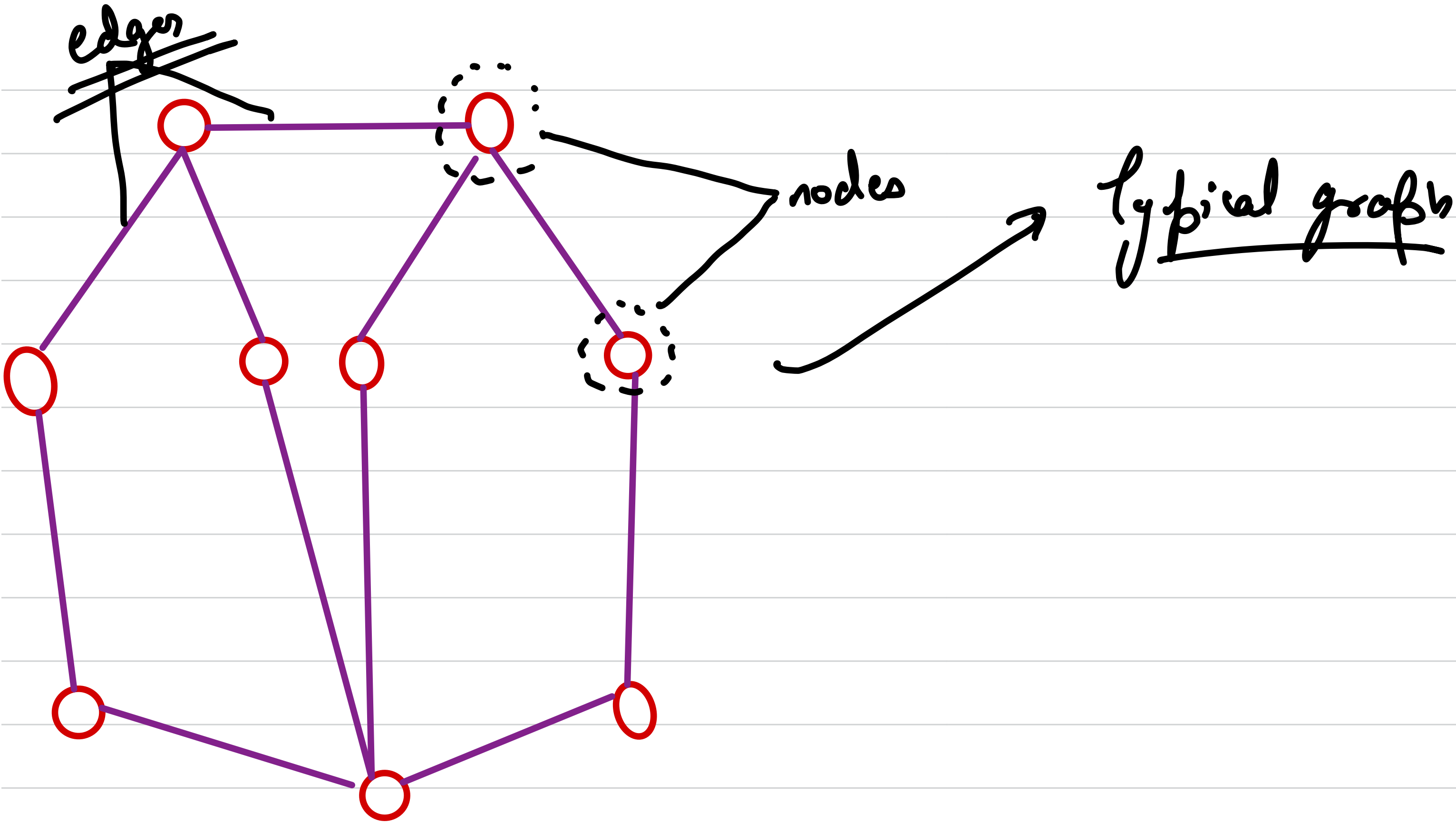
Topics

- Basic graphs & terminologies
- Traversal techniques BFS / DFS
- DP on graph (DP on DAGs)
- Matrix BFS / DFS, 0-1 BFS etc
- Trees (n-ary), DP on trees
- Shortest path, MST
- DSU
- SCC, Bridges / articulation point

Graphs

Q→ What is a graph ??

Graph is a collection of nodes and edges where each node might point to/connected to other nodes. The nodes represent real life entities and are connected by edges representing relationship between the nodes.


edges
nodes
typical graph

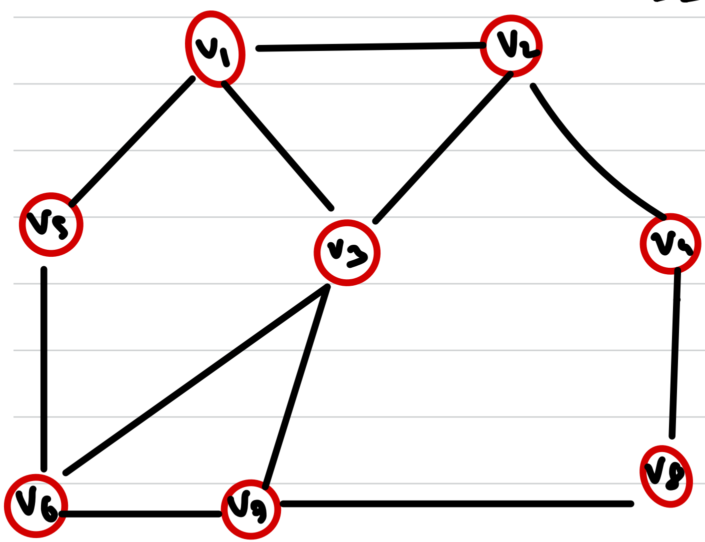
Applications of graph

Biology → PPI graph
→ Metabolic network graph

Electrical → Circuit org graph

Computer Science → Shortest path (map)
→ gsm frequency assignment
→ Routing mechanism
→ social media

Formal definition of graph



V → set of vertices

$V = \{ v_1, v_2, v_3 \ldots\ldots v_n \}$

Edge set

$E = \{ \{v_1, v_2\}$
$\{v_2, v_3\}$
$\vdots$
$\}$

→ unordered set

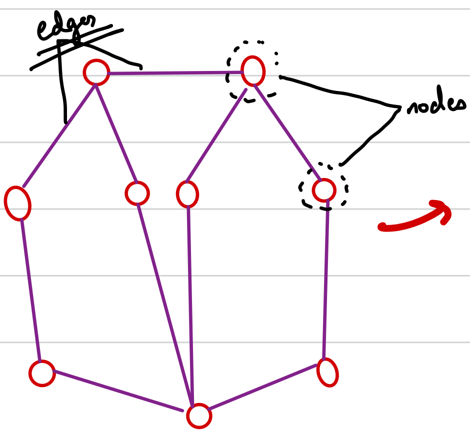
$G = (V, E)$ → mathematical notation of graph

A graph G is an ordered /unordered pair of a set V vertices & E, edges.

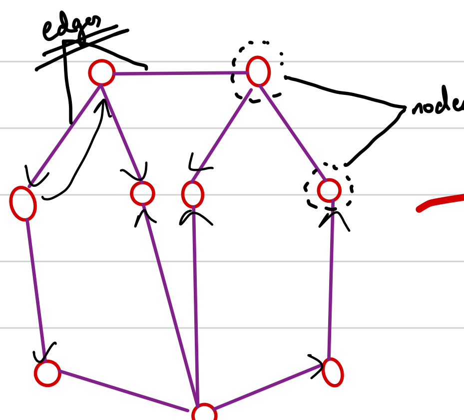
Types of graph

Based On direction

→ Directed → water flow, road network,
→ Undirected → facebook



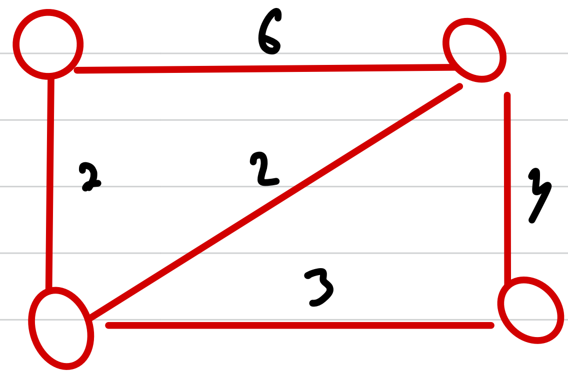
→ undirected



→ directed

Based on edge property

1. Weighted
2. Unweighted



Based on edge density

1) Sparse

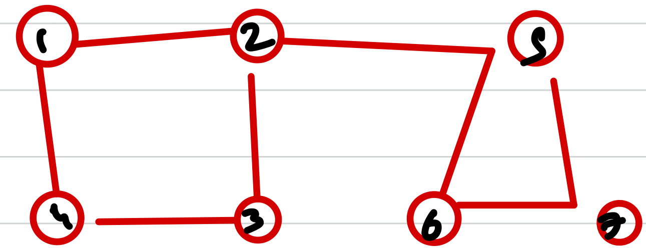
2) Dense

Representing graphs as a data structure

1) Adjacency Matrix
2) Adjacency List → adjacency set/map
   → edge list
3) Incidence Matrix

adjacency matrix

$$A_{ij} = \begin{cases} 1 \rightarrow \text{if there is an edge from } i \text{ to } j \\ 0 \rightarrow \text{else} \end{cases}$$



$A =$

	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	1	0	1	0	1	0	0
3	0	1	0	1	0	0	0
4	1	0	1	0	0	0	0
5	0	1	0	0	0	1	1
6	0	0	0	0	1	0	1
7	0	0	0	0	1	1	0

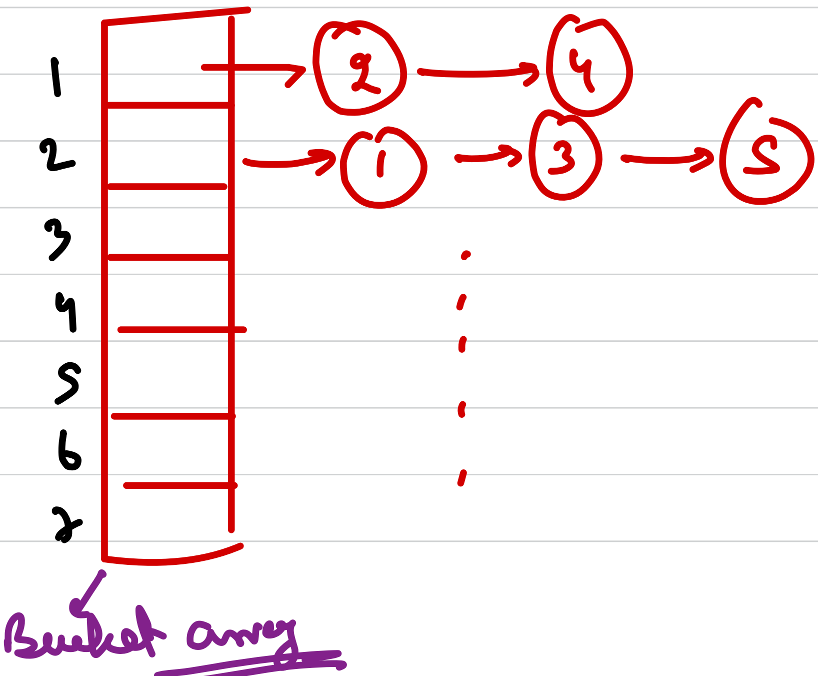
$\longrightarrow \underline{\underline{O(V^2)}}$

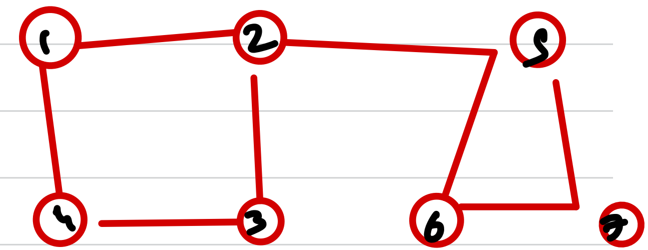
$\underline{\underline{N \times N}}$

adjacency List

{v, w*}

Array of LL





O(V + E) → space

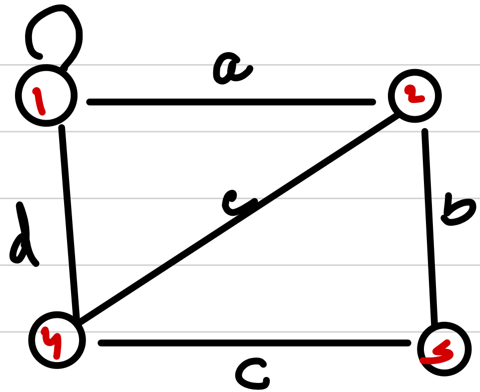
adjacency map/set

array of hashmap

Incidence Matrix

5min

$$M = \begin{array}{c|ccccc|c} & a & b & c & d & e & \\ \hline & 1 & 0 & 0 & 1 & 0 & 1 \\ & 1 & 1 & 0 & 0 & 1 & 2 \\ & 0 & 1 & 1 & 0 & 0 & 3 \\ & 0 & 0 & 1 & 1 & 1 & 4 \end{array}$$



$(V \times E)$

col → edges

rows → vertex

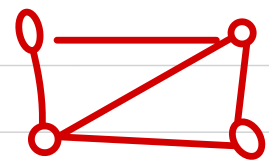
$$M_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ vertex belongs to the } j^{th} \text{ edge} \\ 0 & \text{else} \end{cases}$$

col sum → 2

row sum → degree

Qn What is a degree ??

Degree of a vertex in a graph G, is the total no. of edges incident/associated with it.

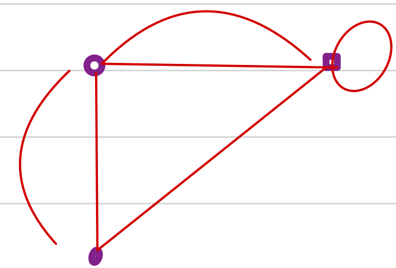


directed graph → Indegree/outdegree

outdegree → no. of outgoing edges
indegree → no. of incoming edges

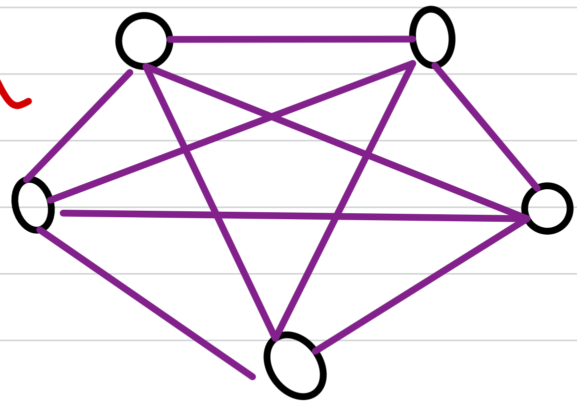
Multi graph → an undirected graph with multiple edges btw vertices & self loops allowed.

2 multi graph



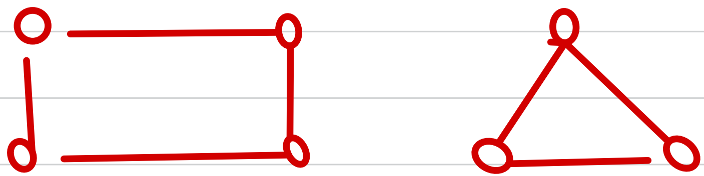
Simple graph → An undirected graph in which both multiple edges & loops are not allowed.

Complete graph → A graph in which every vertex is directly connected to every other vertex.



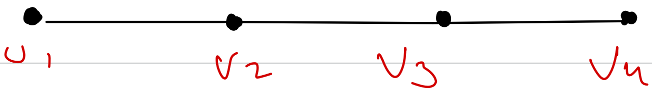
Connected graph → A graph in which there is some path between 2 vertices but not necessarily direct

Disconnected graph → atleast 2 vertices do not have a path to every other vertex.



Component → a subset of a disconnected/connected graph which is connected.

path → A path P_n is a graph whose vertices can be arranged in some sequence such that



$V = \{v_1, v_2, v_3, v_4\}$

edge set of the graph is

$$E = \{ v_i v_{i+1} \ \forall \ i \in [1, n-1] \}$$

Cycle → A cycle C_n is a graph whose vertices can be arranged in a cyclic sequence such that edge set is

$$E = \{v_i v_{i+1} \ \forall\, i \in [1, n-1]\} \cup \{v_1 v_n\}$$