Applied Linear Algebra in Data Analaysis

Linear Dynamical Systems & Positive Definite Matrices Assignment

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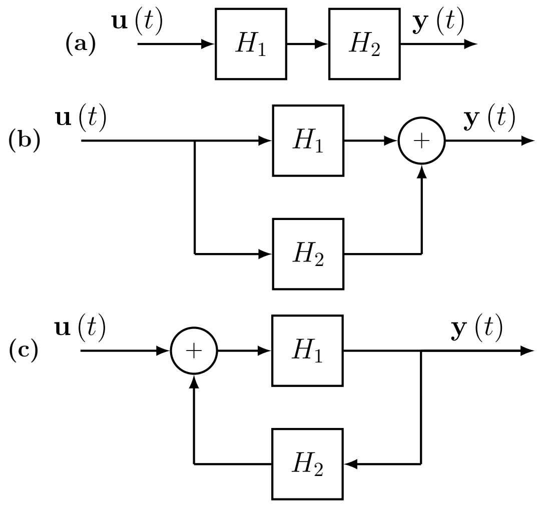
Marks: 26

1. Derive the state and measurement equations for the following composite systems, assuming the system H_i to have the parameters $(\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i, \mathbf{D}_i)$. **[Marks: 6]**



2. **[Programming]** Write a python program to simulate a continuous-time mass, spring, damper system, described by the follloing differential equation.

$$M\ddot{y}(t) + B\dot{y}(t) + Ky(t) = u(t)$$

Assuming the states of the system to be $\mathbf{x}(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix}$, find out the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and $\mathbf{D}$. **[Marks: 2]**

Assuming that the input $u(t) = 0, \forall t \geq 0$, and assuming an initial condition of $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, numerically solve the state compute the evolution of the state and the output of the system using the following procedure. Let Δ be the time step used for the integration, then the time is divided into discrete time instants $n\Delta$, where $n \in \mathbb{Z}_{\geq 0}$. Assuming that we know the value of the state at time $n\Delta$, the rate of change of the state $\dot{\mathbf{x}}$ and the output $\mathbf{y}(n\Delta)$ at a time $n\Delta$ are given by,

$$\dot{\mathbf{x}}(n\Delta) = \mathbf{A}\mathbf{x}(n\Delta) + \mathbf{B}\mathbf{u}(n\Delta)$$
$$\dot{\mathbf{y}}(n\Delta) = \mathbf{C}\mathbf{x}(n\Delta) + \mathbf{D}\mathbf{u}(n\Delta)$$

We can compute the state at time $(n+1)\Delta$ from $\dot{\mathbf{x}}(n\Delta)$,

$$\mathbf{x}((n+1)\Delta) \approxeq \mathbf{x}(n\Delta) + \mathbf{x}(n\Delta) \cdot \Delta$$

Starting from the value of the start at time 0, $\mathbf{x}(0)$, we can numerically compute the evolution of the state for a given input $\mathbf{u}(t)$.

Compute the states and the output of the system from time $t = 0s$ to $t = 10s$ for following values of the parameters M, B, K, **[Marks: 3]**

(a) M = 1, B = 3, K = 1

(b) M = 1, B = 1, K = 1

(c) M = 0, B = 0, K = 1

Carry out the simulations for different values of $\Delta = 0.1, 0.01, 0.001$. Compute the states and plot them as function of time. **[Marks: 2]**

What differences do you find for the three systems for the different parameters and when using different step times? What do you think is the reason for the differences? **[Marks: 2]**