Applied Linear Algebra in Data Analysis

Vector spaces: A set of vectors that is closed under vector scaling and vector addition. E.g. $\mathbb{R}^n$, $\mathbb{C}^n$. *A vector space will always contain the zero vector.*

Subspace: A subset of a vector space $\mathcal{V}$ which is also a vector space.

Span of a set (of vectors): The set of all linear combinations of a set of vectors $\mathcal{S} = \{\mathbf{s}_i\}_{i=1}^p$ from the vector space $\mathcal{V}$.

$$\operatorname{span} \mathcal{S} = \left\{ \sum_{i=1}^{p} \alpha_i \mathbf{s}_i \;\middle|\; \alpha_i \in \mathbb{R} \right\} \subseteq \mathcal{V}$$

Linear Independence: A set $\mathcal{S}$ is linearly independent if and only if, $\sum_{i=1}^{p} \alpha_i \mathbf{s}_i = \mathbf{0} \implies \alpha_i = 0, \forall i$. *If the set has* $\mathbf{0}$, *then the set is linearly dependent.*

Basis: A set of vectors $\mathcal{B}$ is a basis for a vector space $\mathcal{V}$ if and only if, $\mathcal{B}$ is linearly independent and $\operatorname{span} \mathcal{B} = \mathcal{V}$. The elements of $\mathcal{B}$ are called basis vectors of $\mathcal{V}$. *There are infinitely many bases for a vector space. Every vector in $\mathcal{V}$ can be written as a **unique** linear combination of the basis vectors.*

Dimension: The number of basis vectors in a basis of a vector space $\mathcal{V}$ is called the dimension of $\mathcal{V}$.

Inner product: $\mathbf{x}^\top \mathbf{y} = \sum_{i=1}^{n} x_i y_i, \ \mathbf{x}, \mathbf{y} \in \mathbb{R}^n$.

Norm: Measure of the length of a vector. $\|\mathbf{x}\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}, \ \mathbf{x} \in \mathbb{R}^n$. $\|\mathbf{x}\|_2^2 = \mathbf{x}^\top \mathbf{x}$.

Cauchy-Schwarz Inequality: $|\mathbf{x}^\top \mathbf{y}| \leq \|\mathbf{x}\|_2 \|\mathbf{y}\|_2$.

Orthogonality: Two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ are orthogonal if and only if, $\mathbf{x}^\top \mathbf{y} = 0$.

Orthonormal basis: A basis $\mathcal{B} = \{\mathbf{b}_i\}_{i=1}^n$ is orthonormal if and only if, $\mathbf{b}_i^\top \mathbf{b}_j = \delta_{ij}$, where δ_{ij} is the Kronecker delta function.

Linear Function: A function $f : \mathbb{R}^n \to \mathbb{R}$ that satisfies superposition. All linear functions f can be represented as $f(\mathbf{x}) = \mathbf{w}^\top \mathbf{x}$, where $\mathbf{w} \in \mathbb{R}^n$.