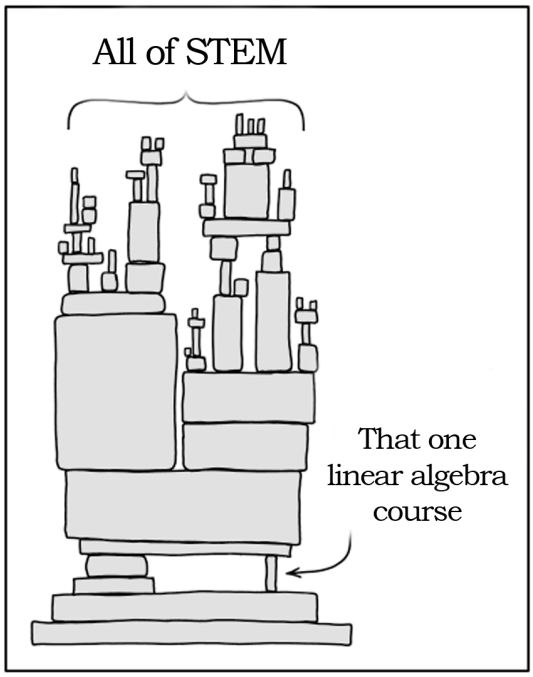
Applied Linear Algebra in Data Analysis



Source: https://twitter.com/Quasilocal/status/1664701458200121351

The above figure is not that far away from the truth. Linear algebra today is as important as calculus, if not more.

Linear algebra serves as the first step towards high-dimensional thinking, a prerequisite skill for modern data analysis. Paired with probability theory and optimization, it comprises a potent trio of tools driving the current machine learning revolution.

In this course, our mission is to lay a solid foundation in applied linear algebra, along with basic probability theory and optimization. But why these three topics?

- **Linear Algebra:** Think in high dimensions & visualize complex structures.
- **Probability Theory:** Navigate uncertainty with precision. Probability theory provides a principled approach to handling the unpredictable nature of real-world data.
- **Optimization:** Seek the best solutions. Optimization allows us to navigate the vast solution space and pinpoint the most efficient paths to solving problems.

Throughout this course, we will:

a) Introduce fundamental concepts in linear algebra, matrix methods, probability theory, and optimization.
b) Demonstrate the practical application of these concepts in real-world scenarios.
c) Provide hands-on experience for applying these concepts to solve real-world problems.

Course Content

(Modules in orange cover concepts and Modules in blue are applications)

Vectors and Vector Spaces; Matrices; Solving Linear Equations; Orthogonality; Matrix Inverses; Signal processing: Fourier and Wavelet Transforms; Eigenvectors and Eigenvalues; Systems theory: Linear Dynamical Systems; Positive Definiteness & Semi-definiteness; Matrix Norms; Singular Value Decomposition; Dimensionality Reduction: Principal Component Analysis; Image compression; Probability and Statistics: Gaussian distribution; Optimization: A brief introduction; Linear Least Squares and its Variants; Nonlinear least squares; Linear program; Convex Optimization; Signal processing: Filtering; Machine learning: Cross-validation; Linear Regression Models; Linear Models for Classification; Neuromechanics: Linear Programming applied to neuromuscular control, and radiation planning.

Assignments

- This is a hands-on course heavy on pen-and-paper and programming assignments. Students are encouraged to use Python or Julia for their programming assignments.
- All assignments will need to be submitted through Teams.
- There will be a total of 13 assignments, and the best 10 will be used for your final grade.
- All assignments will be released after the tutorial session on a Thursday, and they will be due before 6 PM the following Thursday.

- You have a cumulative 5-day late submission grace period for your assignments for the entire course. If you miss the 6 PM deadline for an assignment, a submission made before 6 PM the following day will be considered 1-day late submission.
- If you use up your 5-day grace period, all subsequent late assignments will receive 0 marks.

Resources

The topics covered in this course are quite mature, so it's no surprise that there are numerous wonderful resources. Here are some resources that cover the topics discussed in this course. There are several wonderful resources:

1. G Strang, Introduction to Linear Algebra. Wellesley, *MA: Wellesley-Cambridge Press*, 1993.
2. CD Meyer, Matrix Analysis and Applied Linear Algebra. *Siam*; 2000 Jun 1.
3. S Boyd and L Vandenberghe, Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares. Online Book.
4. FJ Valero-Cuevas. Fundamentals of Neuromechanics. *Berlin: Springer,* 2016.
5. CM Bishop, and MN Nasrabadi. Pattern recognition and machine learning. *New York: Springer*, 2006.
6. Online lectures: Linear Algebra by Prof. Gilbert Strang.
7. Selected online lectures: Matrix Methods in Data Analysis, Signal Processing, and Machine Learning, by Prof. Gilbert Strang.
8. Selected online lectures: Linear Dynamical Systems by Prof. Stephen Boyd.
9. Selected online lectures: Convex Optimization by Prof. Stephen Boyd.

Grading

The final grade for the course will consider the performance on the assignments, quizzes, mid-term, and the final exam.

Assignments	20
Quizzes	10
Mid-term	15
Final	55
Total	100

An absolute grading scheme will be followed for the course:

S: >=90; **A**: >=80; **B**: >= 70; **C**: >= 60; **D**: >=50; **E**: >=40; **F**: <40

Lectures Schedule (January – April 2024)

Lecture timings:
Monday, Tuesday: 6:00 – 7:15 PM
Thursday: 6:00 – 7:30 PM

Week	Day	Date	Day	Details	Details
1	1	08/01	Mon	Concepts in Vector Spaces - 1	Vector spaces, Subspace, Span, Linear Independence
	2	09/01	Tues	Concepts in Vector Spaces - 2	Norm, Inner Products, Basis, Orthonormal Basis, Linear Functions
	3	11/01	Thurs	Tutorial 1: Setting up the programming environment.	
2	4	15/01	Mon	Matrices – 1	Matrices, Matrix Multiplication
	5	16/01	Tues	Matrices – 2	Rank of. Matrix, Matrix Inverse, Complex Vectors & Matrices
	6	18/01	Thurs	Tutorial 2	
3	7	22/01	Mon	Solving linear equations 1	Linear Equations, Understanding Solutions to Linear Equations
	8	23/01	Tues	Solving linear equations 2	General Solution to Linear Equations, Four Fundamental Subspaces of a Matrix
	9	25/01	Thurs	Tutorial 3	
4	10	29/01	Mon	Orthogonality	Orthogonality, Orthogonal Projections, Gram-Schmidt Procedure, QR Decomposition
	11	30/01	Tues	Matrix Inverses	Meaning of matrix inverses, Left and Right Inverses, Generalized Inverses
	12	01/02	Thurs	Tutorial 4	
5	13	05/02	Mon	**Application**: Signal Processing – Fourier & Wavelet Transform	
	14	06/02	Tues	Eigenvectors and Eigenvalues	Similarity Transformation, Eigenvectors and Eigenvalues, Diagonalization of a Matrix, Jordan Forms
	15	08/02	Thurs	Tutorial 5	
6	16	12/02	Mon	**Application**: Linear Dynamical Systems	
	17	13/02	Tues	Positive Definiteness and Matrix Norms	Positive Definite Matrices, Matrix Norms
	18	15/02	Thurs	Tutorial 6	
7	19	19/02	Mon	Singular Value Decomposition	Equivalent Matrices, Diagonalization of any Matrix
	20	20/02	Tues	**Application**: Dimensionality reduction, PCA and Image compression.	Introduction to PCA
	21	22/02	Thurs	Tutorial 7	
8	22	26/02	Mon	Review	
	23	27/02	Tues	Review	
	24	29/02	Thurs	**Mid-Semester Exam**	
9	24	04/03	Mon	Probability & Statistics - 1	Basic Probability Theory, Random Variables, Bayes Theorem
	25	05/03	Tues	Probability & Statistics - 2	Some Useful Probability Distributions/Densities
	26	07/03	Thurs	Tutorial 8	
10	27	11/03	Mon	Optimization - 1	Basic Concept, Steepest Descent
	28	12/03	Tues	Optimization - 2	Gradients with Momentum, Coordinate Descent
	29	14/03	Thurs	Tutorial 9	

11	30	18/03	Mon	Optimization - 3	Introduction to Constrained Optimization
	31	19/03	Tues	Convex Optimization: An Introduction	Convex Sets, Convex Functions, and their Properties
	32	21/03	Thurs	Tutorial 10	
12	33	25/03	Mon	Linear Least Squares and its Variants – 1	Linear Least Squares
	34	26/03	Tues	Linear Least Squares and its Variants – 2	Multiobjective Least Squares, Constrained Least Squares
	35	28/03	Thurs	Tutorial 11	
13	36	01/04	Mon	**Application**: Signal Processing – Filtering, CT Image reconstruction	
	37	02/04	Tues	**Application**: Machine learning – Cross-validation; Linear regression models	
	38	04/04	Thurs	Tutorial 12	
14	39	08/04	Mon	Non-linear Least Squares	Gauss-Newton Method, Levenberg-Marquardt Method
	40	09/04	Tues	Linear Programs	Definition, Basic Concepts
	41	11/04	Thurs	Tutorial 13	
15	42	15/04	Mon	Linear Programs	Simplex Algorithm
	43	16/04	Tues	**Application**: Neuromechanics - Neuromuscular control, Radiation planning	
	44	18/04	Thurs	Review	
16	45	22/04	Mon	**Final exam**	

Assignment Schedule

Assignment	Details	Released on	Due on
1	Vector spaces	11/01/24	18/01/24
2	Matrices	18/01/24	25/01/24
3	Solution to Linear Equations	25/01/24	01/02/24
4	Orthogonality & Matrix Inverses	01/02/24	08/02/24
5	Signal Processing, Eigenvectors and Eigenvalues	08/02/24	15/02/24
6	Linear Dynamical Systems and Positive Definite Matrices	15/02/24	22/02/24
7	SVD and Dimensionality Reduction	22/02/24	07/03/24
8	Probability and Statistics	07/03/24	14/03/24
9	Optimization	14/03/24	21/03/24
10	Convex optimization	21/03/24	28/03/24
11	Linear Least Squares	28/03/24	04/04/24
12	Signal Processing and Machine Learning	04/04/24	11/04/24
13	Nonlinear Least Squares & Linear Programming	11/04/24	18/04/24