

Applied Linear Algebra in Data Analysis

Optimization: A brief introduction

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Optimization

- ▶ Optimization is the process of finding the best solution to a problem from a set of possible solutions.
- ▶ Optimization problems come up in many applications in engineering, science, economics, biology, medicine, operations research, etc.
- ▶ Optimization problems can be classified in different ways, but one major classification gives us: **unconstrained** and **constrained** optimization problems.
- ▶ In the context of data analysis, we are often interested in optimization problems of the following form: consider a set of observations $\{\mathbf{a}_i, y_i\}_{i=1}^m$. We are interested in identifying

$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}) \quad \text{subject to } g_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m$$

where, $f(\mathbf{x})$ is the **objective function** and $g_i(\mathbf{x})$ are the **constraint functions**.

Optimization in single variable

- Consider the function $f(x) = x^2 - 2x + 1$.

Some important definitions

- ▶ **Random experiment** – A experiment whose outcome is not predictable.
 - ▶ Tossing of a coin.
 - ▶ Voltage across a real resistor (R) for a known current.
 - ▶ Height and weight of 40 year old person randomly chosen from a population.
- ▶ The **outcome** of a random experiment is any observable variable of interest.
- ▶ **Sample space** of the experiment S is the universe of possible values we can observe for a random experiment's outcome.
- ▶ An **event** of an experiment is any subset of the sample space S .

Steepest descent algorithm

- ▶ Consider the experiment tossing a dice, and we observe the count of the dots that turn on the top face of the dice.
 - ▶ Observed outcome is an even number. $A = \{2, 4, 6\} \subset S$
 - ▶ Observed outcome is a positive number. $A = S \implies$ **Sure event**
 - ▶ Observed outcome is 0. $A = \{\} \implies$ **Impossible event**
- ▶ For discrete sample spaces and **elementary event** is an event with just single sample point.
- ▶ We can combine events to produce other events that might be of interest to us. Set operations can be used to perform algebra on events.