Kunnskap for en bedre verden

TDT4171 - Artificial Intelligence Methods

Assignment 3 - Making Decisions

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1 Decision Network

Problems

Problem a

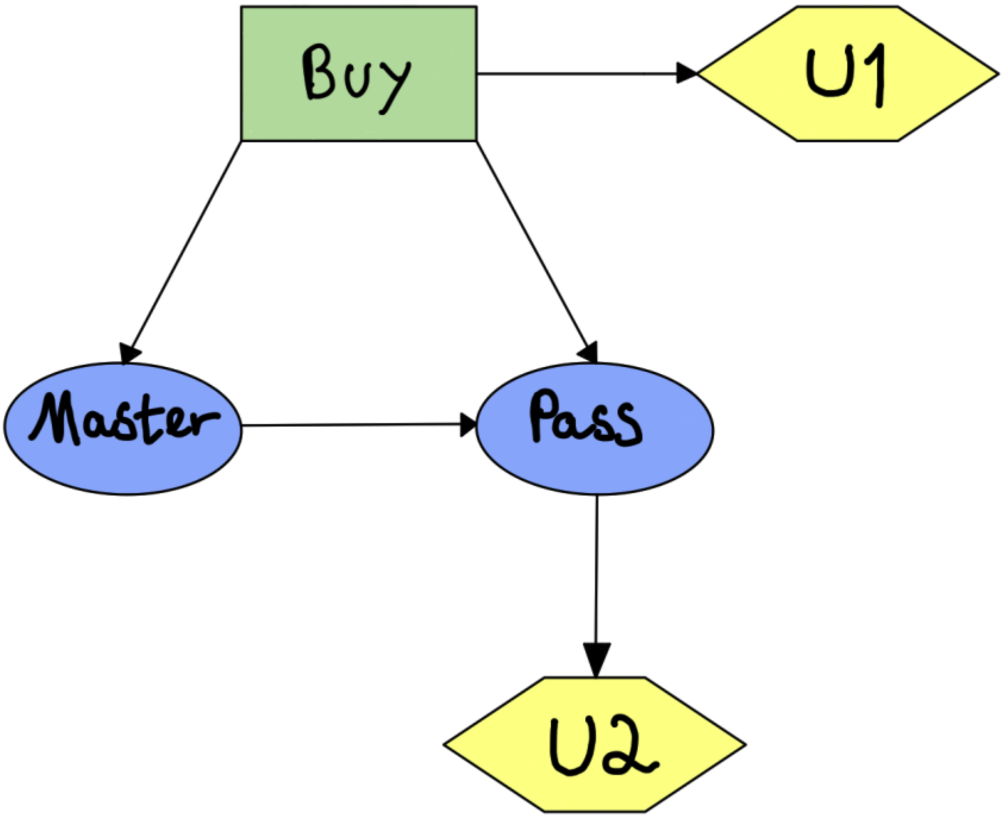

Figure 1: Decision network for problem 1a

Problem b

Compute the expected utility of the two decision alternatives $B = true$ (buying the book) and $B = false$ (not buying it) (show the calculations). What should Geir do?

First, I need to find the probability distributions:

	$Master$	$\overline{Master}$
Buy	0.9	0.1
$\overline{Buy}$	0.65	0.35

Table 1: Probability distribution for $P(master)$

	Buy		$\overline{Buy}$	
	$Master$	$\overline{Master}$	$Master$	$\overline{Master}$
$Pass$	0.9	0.4	0.7	0.2
$\overline{Pass}$	0.1	0.6	0.3	0.8

Table 2: Probability distribution for $P(pass)$

Then, by using the conditional probability rule and chain rule to know that

$$\begin{aligned}
P(x_1, x_2|Y) &= \frac{P(x_1, x_2, Y)}{P(Y)} && \text{Conditional probability rule} \\
&= \frac{P(x_1|\{x_2, Y\}) \cdot P(x_2, Y)}{P(Y)} && \text{Chain rule} \\
&= P(x_1|\{x_2, Y\}) \cdot \frac{P(x_2, Y)}{P(Y)} && \text{Rewrite} \\
&= P(x_1|\{x_2,\, Y\}) \cdot P(x_2|Y) && \text{Conditional probability rule}
\end{aligned}$$

and that

$$EU(a) = \sum_x \left(P(x, a) \cdot \sum_{i=1}^{n} U(x_i, a) \right) \tag{1}$$

I can calculate $EU(B)$ (Buy the book) and $EU(\overline{B})$ (Not buy the book). In my calculations below, the a's in equation 1 is the set $\{B,\, \overline{B}\}$, $x \in \{(P, M, B), (\overline{P}, M, B), (P, \overline{M}, B), (\overline{P}, \overline{M}, B)\}$ for $EU(B)$ and $x \in \{(P, M, \overline{B}), (\overline{P}, M, \overline{B}), (P, \overline{M}, \overline{B}), (\overline{P}, \overline{M}, \overline{B})\}$ for $EU(\overline{B})$.

$$\begin{aligned}
EU(B) &= \sum_x \left(P(x, B) \cdot \sum_{i=1}^{n} U(x_i, B) \right) \\
&= P(P, M|B) \cdot U(P, M, B) + P(\overline{P}, M|B) \cdot U(\overline{P}, M, B) \\
&+ P(P, \overline{M}|B) \cdot U(P, \overline{M}, B) + P(\overline{P}, \overline{M}|B) \cdot U(\overline{P}, \overline{M}, B) \\
\\
&= P(P|\{B, M\}) \cdot P(M|B) \cdot (U(P) + U(M) + U(B)) \\
&+ P(\overline{P}|\{B, M\}) \cdot P(M|B) \cdot (U(\overline{P}) + U(M) + U(B)) \\
&+ P(P|\{B, \overline{M}\}) \cdot P(\overline{M}|B) \cdot (U(P) + U(\overline{M}) + U(B)) \\
&+ P(\overline{P}|\{B, \overline{M}\}) \cdot P(\overline{M}|B) \cdot (U(\overline{P}) + U(\overline{M}) + U(B)) \\
\\
&= 0.9 \cdot 0.9 \cdot (2100 + 0 + (-150)) + 0.1 \cdot 0.9 \cdot (0 + 0 + (-150)) \\
&+ 0.4 \cdot 0.1 \cdot (2100 + 0 + (-150)) + 0.6 \cdot 0.1 \cdot (0 + 0 + (-150)) \\
&= 1635
\end{aligned}$$

$$\begin{aligned}
EU(\overline{B}) &= \sum_x \left(P(x, \overline{B}) \cdot \sum_{i=1}^{n} U(x_i, \overline{B}) \right) \\
&= P(P, M|\overline{B}) \cdot U(P, M, \overline{B}) + P(\overline{P}, M|\overline{B}) \cdot U(\overline{P}, M, \overline{B}) \\
&+ P(P, \overline{M}|\overline{B}) \cdot U(P, \overline{M}, \overline{B}) + P(\overline{P}, \overline{M}|\overline{B}) \cdot U(\overline{P}, \overline{M}, \overline{B}) \\
\\
&= P(P|\{\overline{B}, M\}) \cdot P(M|\overline{B}) \cdot (U(P) + U(M) + U(\overline{B})) \\
&+ P(\overline{P}|\{\overline{B}, M\}) \cdot P(M|\overline{B}) \cdot (U(\overline{P}) + U(M) + U(\overline{B})) \\
&+ P(P|\{\overline{B}, \overline{M}\}) \cdot P(\overline{M}|\overline{B}) \cdot (U(P) + U(\overline{M}) + U(\overline{B})) \\
&+ P(\overline{P}|\{\overline{B}, \overline{M}\}) \cdot P(\overline{M}|\overline{B}) \cdot (U(\overline{P}) + U(\overline{M}) + U(\overline{B})) \\
\\
&= 0.7 \cdot 0.65 \cdot (2100 + 0 + 0) + 0.3 \cdot 0.65 \cdot (0 + 0 + 0) \\
&+ 0.2 \cdot 0.35 \cdot (2100 + 0 + 0) + 0.8 \cdot 0.35 \cdot (0 + 0 + 0) \\
&= 1102.5
\end{aligned}$$

Based on these expectations, Geir would get a higher utility from buying the book, so he should buy the book.

2 Decision Support System

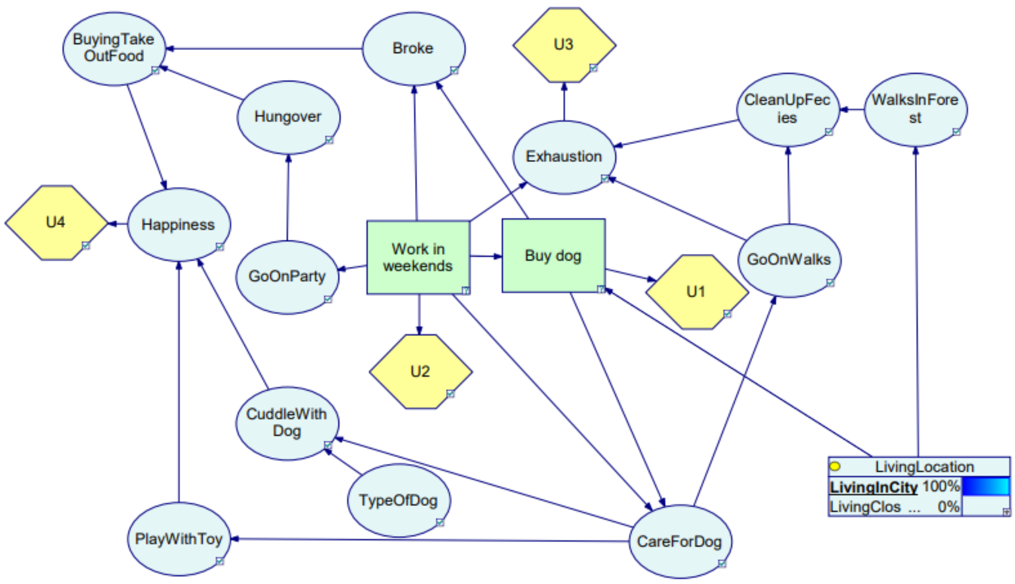

Figure 2: Decision network of whether or not to work in the weekends and buy a bog

2.1 Choosing a problem

The first step in any Decision support system modelling is to decide what to model. The problem I chose for my decision support system is whether or not I should work in the weekends and whether or not I should buy a dog from an animal shelter based on this.

2.2 Defining the variables

After deciding what to model, I had to define variables to take into account. What variables can influence my decision to work in weekends and what variables can influence my decision to buy a dog? I came up with the 14 following variables:

GoOnParty
This variable holds my probability to go on a party. True/False

Hungover
The probability of me being hungover. True/False

BuyingTakeOutFood
The probability of me buying takeout food. True/False

Broke
The probability of me being broke. True/False

Happiness
The probability of me being not happy, neutral and very happy.

PlayWithToy
The probability of playing with a dog toy. True/False

CuddleWithDog
: The probability of me cuddling with a dog. True/False

TypeOfDog
: The probability of the dog being a Golden retriever or a Chihuahua

CareForDog
: The probability of me caring for a dog. True/False

Exhaustion
: The probability of me being exhausted. True/False

GoOnWalks
: The probability of me going on walks. True/False

CleanUpFeces
: The probability of me having to clean up feces. True/False

WalksInForest
: The probability of me going on walks in the forest. True/False

LivingSituation
: The probability of my living situation. In the city or close to a forest. This variable is known.

There are of course many other variables that could have been taken into account, but these are the ones I found most important. They all either directly or indirectly influence each other and the final decisions.

GoOnParty
: **GoOnParty** depends on the decision to work in the weekends. If I work in the weekends, my probability of going to parties will decrease, and increase if not. This variable will eventualle influcene my happiness utility, which in turn will influence the decision of working in the weekends.

Hungover
: As **GoOnParty** is a variable I decided to include, it was natural that **hungover** had to be a variable as well, since hangovers will affect my happiness, and are, for the most part, induced by going to parties. As I tend to not be relatively happy when I'm hungover, this variable will eventually influence my happiness utility.

Broke
: The **broke** variable denotes the probability of me being broke. This has a huge impact on whether or not I should buy a dog or work in the weekends, as my decision depends on how much money I got. Dogs are expensive, and if I have no money I can't buy a dog. Likewise, if I am broke, my probability of wanting to work in the weekends gets higher.

BuyingTakeOutFood
: To buy takeout food or not will depend on whether I'm broke and whether or not I'm hungover. It will directly influence my happiness, since, subjectively, I am more happy when I don't have to make my own food.

Happiness
: My happiness will be influenced by a great number of variables, however, I decided to only have a *direct* influence from three; **BuyingTakeOutFood**, **CuddleWithDog** and **PlayWithToy**. As I see it, all other variables that influence these three, will indirectly influence happiness as well. Happiness will have a direct influence on the "happiness utility", which will be taken into consideration when deciding whether or not to work in the weekends and buy a dog or not.

PlayWithToy
The probability to play with a toy depends, in my opinion, on how much you care for your dog (or dogs in general). This is also one of the three variables that directly influence my happiness. Again this is purely subjective, but I think I am more happy when I play with a toy (or the dog in general) than if I don't play with the dog.

TypeOfDog
What type of dog I can get is important in my decision of buying a dog or not. Since I am buying a dog from an animal shelter, there is a given probability that they have a Golden retriever available and that they have a Chihuahua available. My personal preference given these two breeds are Golden retrievers, and what type of dog I can get will therefore affect my decision.

CuddleWithDog
The probability of cuddling with the dog will depend on my probability of caring for the dog, and what type of dog it is. Based on empirical data, Golden retrievers are more open to cuddling than Chihuahuas. Cuddling with the dog will make me happier than not cuddling with the dog, therefore **CuddleWithDog** directly influences my happiness. Since **CuddleWithDog** also depends on what type of dog it is, **TypeOfDog** indirectly affects my happiness.

CareForDog
There are a lot of things that can give a measure of how much you care for your dog, however, I decided the only things that would influence the variable **CareForDog** is whether you work in the weekends and if you buy a dog. If you work in the weekends, you do not make time for your dog, and the probability for **CareForDog** will decrease. If you buy a dog on the other hand, it will increase your probability of caring for dogs. I think a care for dogs is an extremely important variable for deciding if you are going to buy a dog or not.

GoOnWalks
The probability of going on walks will be influenced by how much I care for a dog. Higher care for a dog means a higher probability of going on walks. However, if I do not get a dog, my probability of going on walks is still not 0. I regularly go on walks. This variable will directly influence the probability of having to clean up feces or not (see **CleanUpFeces**). It will also directly influence the variable **Exhaustion** (see **Exhaustion**), which in turn will influence my decision of buying a dog or not.

CleanUpFeces
To clean up feces or not will depend on whether or not I am going on walks, and whether or not the walks are in the forest. If the dog is loose in the forest, the probability of me cleaning up it's feces are lower than if we are walking in the city. Furthermore, if I am not going on walks, the dog is certain to poop inside, and the probability of cleaning up feces are 100% (if I don't want my apartment to smell like dogshit ofc).

LivingSituation
My living situation is not a random variable, it is known beforehand. It is set to "Living in city". It influences my probability to go on walks in the forest.

WalksInForest
My probability of taking walks in a forest depends purely on my living situation. If I am living in the city, it is less likely that I go on walks in the forest, and more likely if I live close to the forest (as I am doing in my hometown). **WalksInForest** influences **CleanUpFeces** as described above.

Exhaustion
Exhaustion is, as previously mentioned, my probability of being exhausted. It depends directly on the decision of working in the weekends, **CleanUpFeces** and **GoOnWalks**, where the two latter are indirectly influenced by buying a dog. **Exhaustion** influences the "exhaustion utility" U3, which in turn will influence my decision to work in the weekends and/or buy a dog.

2.3 Probabilities

According to Russel and Norvig 2010, the probabilities of a decision support system can come from "Patient databases, literature studies or the expert's subjective assessments". In my decision support system, all my probabilities come from my (the expert) subjective assessment.

Work in weeke...	WorkInWeekends		NotWorkInWeekends	
Buy dog	BuyDog	NotBuyDog	BuyDog	NotBuyDog
Broke	0.3	0.2	0.7	0.9
NotBroke	0.7	0.8	0.3	0.1

Figure 3: Probability of being broke, given WorkInWeekend and BuyDog

Broke	Broke		NotBroke	
Hungover	Hungover	NotHungover	Hungover	NotHungover
BuyingTakeOutFood	0.4	0.1	0.8	0.3
NotBuyingTakeOutFood	0.6	0.9	0.2	0.7

Figure 4: Probability of buying takeout food, given Broke and Hungover

Buy dog	BuyDog		NotBuyDog	
Work in weekends	WorkInWee...	NotWorkIn...	WorkInWee...	NotWorkIn...
CareForDogs	0.3	0.8	0.4	0.7
NotCareForDogs	0.7	0.2	0.6	0.3

Figure 5: Probability of caring for dogs, given BuyDog and WorkInWeekends

GoOnWalks	GoOnWalks		NotGoOnWalks	
WalksInForest	WalksInForest	NotWalksIn...	WalksInForest	NotWalksIn...
CleanUpFecies	0.2	0.8	1	1
NotCleanUpFecies	0.8	0.2	0	0

Figure 6: Probability of cleaning up feces, given GoOnWalks and WalksInForest

CareForDog	CareForDogs		NotCareForDogs	
TypeOfDog	Golden	Chihuahua	Golden	Chihuahua
CuddleWithDog	0.7	0.6	0.6	0.4
NotCuddleWithDog	0.3	0.4	0.4	0.6

Figure 7: Probability of cuddling with the dog, given CareForDog and TypeOfDog

CleanUpFecies	CleanUpFecies				NotCleanUpFecies			
GoOnWalks	GoOnWalks		NotGoOnWalks		GoOnWalks		NotGoOnWalks	
Work in weeke...	WorkInWee...	NotWorkIn...	WorkInWee...	NotWorkIn...	WorkInWee...	NotWorkIn...	WorkInWee...	NotWorkIn...
Exhausted	0.8	0.6	0.7	0.3	0.7	0.4	0.6	0.2
NotExhausted	0.2	0.4	0.3	0.7	0.3	0.6	0.4	0.8

Figure 8: Probability of being exhaused, given CleanUpFeces, GoOnWalks and WorkInWeekends

Work in weekends	WorkInWee...	NotWorkIn...
GoOnParty	0.2	0.8
NotGoOnParty	0.8	0.2

Figure 9: Probability of going to a party, given WorkInWeekends

CareForDog	CareForDogs	NotCareForDogs
GoOnWalks	0.8	0.4
NotGoOnWalks	0.2	0.6

Figure 10: Probability of going on walks, given CareForDog

PlayWithToy	PlayWithToy				NotPlayWithToy			
CuddleWithDog	CuddleWithDog		NotCuddleWithDog		CuddleWithDog		NotCuddleWithDog	
BuyingTakeOut...	BuyingTake...	NotBuyingT...	BuyingTake...	NotBuyingT...	BuyingTake...	NotBuyingT...	BuyingTake...	NotBuyingT...
NotHappy	0.1	0.1	0.2	0.5	0.1	0.2	0.4	0.8
Neutral	0.1	0.2	0.5	0.4	0.2	0.2	0.5	0.1
VeryHappy	0.8	0.7	0.3	0.1	0.7	0.6	0.1	0.1

Figure 11: Probability of my happiness, given PlayWithToy, CuddleWithDog and BuyingTakeOutFood

GoOnParty	GoOnParty	NotGoOnPa...
Hungover	0.9	0.3
NotHungover	0.1	0.7

Figure 12: Probability of being hungover, given GoOnParty

LivingInCity	0.75
LivingCloseToNature	0.25

Figure 13: Probability of my living location. It is actually known, but set to .25 of close to nature, because I am at home place 3 months each year

CareForDog	CareForDogs	NotCareFor...
PlayWithToy	0.85	0.3
NotPlayWithToy	0.15	0.7

Figure 14: Probability of playing with toy, given CareForDog

Golden	0.8
Chihuahua	0.2

Figure 15: Probability of Golden or Chihuahua

LivingLocation	LivingInCity	LivingCloseToNature
WalksInForest	0.3	0.7
NotWalksInForest	0.7	0.3

Figure 16: Probability of going on walks in forest, given LivingLocation

All of the variables probabilities are listed in figure 3 through figure 16. Many of them are not that exiting and most of them are pretty intuitive (like that you have a higher probability of getting a hangover (figure 12) from going to a party than if you don't go to a party). Some that are interesting however, are the probabilities in figure 15. As stated in section 2.1, I am buying a dog from an animal shelter. Therefore, the probability of me getting a Golden retriever (which is what I actually want) is not 100%, since the shelter may be "empty" of Golden retrievers. Further interesting probabilities are the ones in figure 7. Cuddle with dog are independent of **CareForDog** and, interestingly **TypeOfDog**. It's pretty obvious that your probability to care for your dog will influence your probability of cuddling with the dog, however, it may not be as intuitive as to why what type of dog it is would have an impact on whether or not you cuddle with the dog. Because of my personal experience, that Golden retrievers are more cuddly than Chihuahuas, the probability of cuddling with the dog will be slightly higher if the dog is a Golden retriever. Another interesting thing about this probability is that there are almost no difference between **CuddleWithDog** given **CareForDogs** and **Golden/Chihuahua**. This is because I think if you *really* care for your dog, you will cuddle with it even though it is not that cuddly (as the Chihuahua). The last probability table I want to talk about is the one in figure 6. The probability $P(CleanUpFeces|\{NotGoOnWalks, WalksInForest\})$ and $P(CleanUpFeces|\{NotGoOnWalks, NotWalksInForest\})$ are both 100%. This is because I believe that if you do not go on walks with your dog, it will, with a 100% certainty, shit inside and you'll have to clean it up.

2.4 Utilities

In my network, I have four utility measures. $U1$, $U2$, $U3$ and $U4$. $U1$ denotes the utility of buying a dog, $U2$ the utility of working in the weekends, $U3$ is a utility measure based on my exhaustion level and $U4$ is a measure of what utility I will get based on my happiness. At first, I wanted to have the utilities reflect one year, but this meant having over $200k$ utility for working in the weekends (because of the student loan and payment for working half of the weekends in one year) and then a significantly lower utility for happiness and exhaustion ($[-1000, 3000]$ or something like that), making the happiness and exhaustion utilities negligible. They had almost no impact on my decisions, since $U1$ and $U2$ were so big in contrast to $U3$ and $U4$. Therefore, I decided to scale all utilities down to the range $[-10, 10]$, so that $U1$ and $U2$ would not have "monopoly" on my decisions.

All utilities can seen in figures 17 to 20 below.

Buy dog	BuyDog	NotBuyDog
Value	-2	1

Figure 17: U1

Work in weeke...	WorkInWee...	NotWorkIn...
Value	7	3

Figure 18: U2

Exhaustion	Exhausted	NotExhausted
Value	-3	5

Figure 19: U3

Happiness	NotHappy	Neutral	VeryHappy
Value	-10	0	10

Figure 20: U4

2.5 Expected utilities

Work in weeke...	WorkInWee...	NotWorkIn...
BuyDog	5.9452168	5.3235488
NotBuyDog	**9.234688**	**7.9846352**

Figure 21: Expected overall utility

The expected utility for my decision support system can be seen in figure 21. As can be seen in the figure, I will get a higher utility when working in the weekends and not buying a dog, so that's what I should do.

In figures 22 to 25 are the expected utilities for each utility. Based on my probabilities and "common sense", I think these makes sense. Figures 22 and 23 are just the same as their original utilities, because no probabilities are included. Figure 24 and 25 however, have taken my probabilities into account. Based on these expected utilities, I would be least exhausted if I do not buy a dog and do not work in the weekends, which makes sense. I would not have to go on walks, play with toys, work in the weekends, clean up feces, etc..., giving more time to relax and "charge my batteries". As for the happiness utility in figure 25, I would be more happy if I do not work in the weekends and buy a dog, which also makes sense. I would not have to work in the weekends, and I would have a dog to play with and to keep me company.

Buy dog	BuyDog	NotBuyDog
Exp. utility	-2	**1**

Figure 22: Expected utility U1

Work in weeke...	WorkInWee...	NotWorkIn...
Exp. utility	**7**	3

Figure 23: Expected utility U2

Work in weeke...	WorkInWeekends		NotWorkInWeekends	
Buy dog	BuyDog	NotBuyDog	BuyDog	NotBuyDog
Exp. utility	-0.85792	-0.87776	1.30976	**1.38144**

Figure 24: Expected utility U3

Work in weeke...	WorkInWeekends		NotWorkInWeekends	
Buy dog	BuyDog	NotBuyDog	BuyDog	NotBuyDog
Exp. utility	1.8031368	2.112448	**3.0137888**	2.6031952

Figure 25: Expected utility U4

Bibliography

Russel, Stuart and Peter Norvig (2010). *Artificial Intelligence: A Modern Approach*. 3rd ed. s.n.