```python
"""Script for estimating the order parameter."""

import numpy as np
from hopfield import StochasticHopfieldNetwork
from pattern_utilities import generate_n_random_patterns


def main():
    n_neurons = 200
    # Change n_patterns to 45 for the second question
    n_patterns = 7
    noise_parameter = 2
    n_time_steps = int(2e5)
    n_iterations = 100

    patterns = generate_n_random_patterns(n_patterns, n_neurons)

    network = StochasticHopfieldNetwork()
    network.set_patterns(patterns)
    network.set_diagonal_weights_rule("zero")
    network.set_noise_parameter(noise_parameter)
    network.generate_weights()

    pattern1 = patterns[0, :]
    updated_pattern = pattern1.copy()

    m = np.zeros(n_iterations)
    for i in range(n_iterations):
        for _ in range(n_time_steps):
            updated_pattern = network.update_random_neuron(updated_pattern)
            m[i] += np.inner(updated_pattern, pattern1)
        m[i] /= n_neurons
        m[i] /= n_time_steps

    m_estimate = sum(m) / n_iterations
    print("{:.3f}".format(m_estimate))


if __name__ == "__main__":
    main()
```

```python
"""Implementation of a Hopfield network using Hebb's rule."""

from abc import ABC, abstractmethod
import numpy as np
from scipy import stats
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6
7
8   class HopfieldNetwork(ABC):
9       """Abstract Hopfield net using Hebb's rule to compute weights."""
10
11      def set_diagonal_weights_rule(self, diagonal_weights_rule):
12          """Specify if the diagonal weights should be zero or not.
13
14          diagonal_weights_rule must equal "zero" or "non-zero".
15          """
16          if diagonal_weights_rule == "zero":
17              self.diagonal_weights_equal_zero = True
18          elif diagonal_weights_rule == "non-zero":
19              self.diagonal_weights_equal_zero = False
20          else:
21              raise KeyError(
22                  diagonal_weights_rule
23                  + " not a valid diagonal_weights_rule"
24                  )
25
26      def set_patterns(self, patterns):
27          """Specify the stored patterns.
28
29          patterns must be a 1D- or 2D-array where each row is a pattern.
30          """
31          self.patterns = patterns
32
33      def generate_weights(self):
34          """Generate the weights for the network."""
35          _, n_neurons = self.patterns.shape
36          self.weights = np.zeros((n_neurons, n_neurons))
37          for pattern in self.patterns:
38              self.weights += np.outer(pattern, pattern)
39          self.weights /= n_neurons
40
41          if self.diagonal_weights_equal_zero:
42              np.fill_diagonal(self.weights, 0)
43
44      def update_random_neuron(self, pattern):
45          n_neurons = pattern.shape
46          neuron_index = np.random.randint(n_neurons)
47          return self.update_neuron(pattern, neuron_index)
48
49      def update_neuron(self, pattern, neuron_index):
50          """Returns updated pattern after update of neuron_index."""
51          weights_i = self.weights[neuron_index, :]
52          local_field = np.inner(weights_i, pattern)
53          updated_neuron = self.get_state_of_local_field(local_field)
54          updated_pattern = pattern.copy()
```

```
        updated_pattern[neuron_index] = updated_neuron
        return updated_pattern

    @abstractmethod
    def get_state_of_local_field(self, local_field):
        pass


class DeterministicHopfieldNetwork(HopfieldNetwork):
    """Concrete Hopfield net with deterministic updating."""

    def get_state_of_local_field(self, local_field):
        """Update rule when updating a neuron."""
        return 1 if local_field >= 0 else -1


class StochasticHopfieldNetwork(HopfieldNetwork):
    """Concrete Hopfield net with stochastic updating."""

    def get_state_of_local_field(self, local_field):
        """Update rule when updating a neuron.

        Returns 1 with probability
        1 / (1+exp(-2*noise_parameter*local_field),
        else -1.
        """
        p = 1/(1 + np.exp(-2*self.noise_parameter*local_field))
        rand = stats.bernoulli.rvs(p)
        return 1 if rand else -1

    def set_noise_parameter(self, noise_parameter):
        self.noise_parameter = noise_parameter
```

```
"""Functions for working with patterns.

A pattern is defined as a 1D-array and several patterns are stored as
a 2D-array, where each row corresponds to one pattern. Each element can
take the values -1 or +1 only.
"""

import numpy as np
from scipy import stats


def generate_n_random_patterns(n_patterns, n_neurons):
    """Returns n_patterns random patterns, each of length n_neurons.

    If several patterns are generated, each row corresponds to one
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    pattern.
    """
    random_0s_and_1s = stats.bernoulli.rvs(0.5, size=(n_patterns, n_neurons))
    random_minus_1s_and_1s = 2*random_0s_and_1s - 1
    return random_minus_1s_and_1s


def get_index_of_equal_pattern(pattern_to_match, patterns):
    """Returns the row index of patterns corresponding to the pattern
    that is equal to pattern_to_match.

    Returns -1 if no matching pattern is found.
    """
    for index, pattern in enumerate(patterns):
        n_different_neurons = get_n_different_neurons(pattern_to_match, pattern)
        if n_different_neurons == 0:
            return index
    return -1


def get_n_different_neurons(pattern1, pattern2):
    return sum(pattern1 != pattern2)


def vector_to_typewriter(vector, n_columns):
    """Returns 2D-array of vector.

    The first row in the returned array consists of the first n_columns
    elements in vector and so on.
    """
    return np.reshape(vector, (-1, n_columns))


def print_typewriter_pattern(pattern, n_columns):
    """Prints a pattern in a typewriter scheme."""
    print_pattern(vector_to_typewriter(pattern, n_columns))


def print_pattern(pattern):
    np.set_printoptions(formatter={"float_kind": lambda x: "%.4f" % x})
    print(repr(pattern), sep=", ")
```