```
"""Script for computing the one-step error probability."""
2
    import numpy as np
3
    from hopfield import DeterministicHopfieldNetwork
    from pattern_utilities import generate_n_random_patterns, print_pattern
    def main():
8
        n_neurons = 120
        n_{patterns\_vector} = [12, 24, 48, 70, 100, 120]
10
        # Set diagonal_weights_rule to "non-zero" in second task
        diagonal_weights_rule = "zero"
12
        n_iterations = int(1e5)
13
        one_step_error_probability = np.zeros(len(n_patterns_vector))
14
15
        for n_patterns_i, n_patterns in enumerate(n_patterns_vector):
16
            n_errors = 0
17
            for _ in range(n_iterations):
18
                network = DeterministicHopfieldNetwork()
19
20
                patterns = generate_n_random_patterns(n_patterns, n_neurons)
21
                network.set_patterns(patterns)
22
                network.set_diagonal_weights_rule(diagonal_weights_rule)
23
                network.generate_weights()
24
25
                pattern_to_feed_index = np.random.randint(n_patterns)
26
                original_pattern = patterns[pattern_to_feed_index, :]
27
                neuron_to_update_index = np.random.randint(n_neurons)
28
29
                updated_pattern = network.update_neuron(
30
                     original_pattern,
31
                     neuron_to_update_index
32
                     )
                updated_neuron = updated_pattern[neuron_to_update_index]
34
                original_neuron = original_pattern[neuron_to_update_index]
35
                updated_neuron = updated_pattern[neuron_to_update_index]
36
37
                if original_neuron != updated_neuron:
38
                     n_errors += 1
39
            one_step_error_probability[n_patterns_i] = n_errors/n_iterations
41
42
        print_pattern(one_step_error_probability)
43
44
45
    if __name__ == "__main__":
46
        main()
47
```

```
"""Implementation of a Hopfield network using Hebb's rule."""
1
2
    from abc import ABC, abstractmethod
3
    import numpy as np
    from scipy import stats
    class HopfieldNetwork(ABC):
8
        """Abstract Hopfield net using Hebb's rule to compute weights."""
10
        def set_diagonal_weights_rule(self, diagonal_weights_rule):
11
             """Specify if the diagonal weights should be zero or not.
12
13
            diagonal_weights_rule must equal "zero" or "non-zero".
14
            .....
            if diagonal_weights_rule == "zero":
16
                 self.diagonal_weights_equal_zero = True
17
            elif diagonal_weights_rule == "non-zero":
18
                 self.diagonal_weights_equal_zero = False
19
            else:
20
                raise KeyError(
21
                     diagonal_weights_rule
22
                     + " not a valid diagonal_weights_rule"
23
                     )
24
25
        def set_patterns(self, patterns):
26
            """Specify the stored patterns.
27
28
            patterns must be a 1D- or 2D-array where each row is a pattern.
29
            .....
30
            self.patterns = patterns
31
32
        def generate_weights(self):
33
            """Generate the weights for the network."""
34
            _, n_neurons = self.patterns.shape
35
            self.weights = np.zeros((n_neurons, n_neurons))
36
            for pattern in self.patterns:
37
                 self.weights += np.outer(pattern, pattern)
            self.weights /= n_neurons
39
40
            if self.diagonal_weights_equal_zero:
41
                np.fill_diagonal(self.weights, 0)
42
43
        def update_random_neuron(self, pattern):
            n_neurons = pattern.shape
45
            neuron_index = np.random.randint(n_neurons)
46
            return self.update_neuron(pattern, neuron_index)
47
```

```
48
        def update_neuron(self, pattern, neuron_index):
49
            """Returns updated pattern after update of neuron_index."""
            weights_i = self.weights[neuron_index, :]
51
            local_field = np.inner(weights_i, pattern)
52
            updated_neuron = self.get_state_of_local_field(local_field)
            updated_pattern = pattern.copy()
54
            updated_pattern[neuron_index] = updated_neuron
55
            return updated_pattern
57
        @abstractmethod
58
        def get_state_of_local_field(self, local_field):
59
            pass
60
61
62
    class DeterministicHopfieldNetwork(HopfieldNetwork):
63
        """Concrete Hopfield net with deterministic updating."""
64
65
        def get_state_of_local_field(self, local_field):
            """"Update rule when updating a neuron."""
67
            return 1 if local field >= 0 else -1
68
69
70
    class StochasticHopfieldNetwork(HopfieldNetwork):
71
        """Concrete Hopfield net with stochastic updating."""
72
73
        def get_state_of_local_field(self, local_field):
74
            """"Update rule when updating a neuron.
75
76
            Returns 1 with probability
77
            1 / (1+exp(-2*noise_parameter*local_field),
78
            else -1.
79
            .....
80
            p = 1/(1 + np.exp(-2*self.noise_parameter*local_field))
81
            rand = stats.bernoulli.rvs(p)
            return 1 if rand else -1
83
84
        def set_noise_parameter(self, noise_parameter):
            self.noise_parameter = noise_parameter
86
```

```
"""Functions for working with patterns.

A pattern is defined as a 1D-array and several patterns are stored as
a 2D-array, where each row corresponds to one pattern. Each element can
take the values -1 or +1 only.
"""

import numpy as np
```

```
from scipy import stats
10
    def generate_n_random_patterns(n_patterns, n_neurons):
12
        """Returns n_patterns random patterns, each of length n_neurons.
13
        If several patterns are generated, each row corresponds to one
15
        pattern.
16
        .....
17
        random_0s_and_1s = stats.bernoulli.rvs(0.5, size=(n_patterns, n_neurons))
18
        random_minus_1s_and_1s = 2*random_0s_and_1s - 1
19
        return random_minus_1s_and_1s
20
21
22
    def get_index_of_equal_pattern(pattern_to_match, patterns):
23
        """Returns the row index of patterns corresponding to the pattern
24
        that is equal to pattern_to_match.
25
26
        Returns 1 if no matching pattern is found.
27
28
        for index, pattern in enumerate(patterns):
29
            n_different_neurons = get_n_different_neurons(pattern_to_match, pattern)
30
            if n_different_neurons == 0:
31
                return index
32
        return -1
33
34
35
    def get_n_different_neurons(pattern1, pattern2):
36
        return sum(pattern1 != pattern2)
37
38
39
    def vector_to_typewriter(vector, n_columns):
40
        """Returns 2D-array of vector.
41
42
        The first row in the returned array consists of the first n_columns
        elements in vector and so on.
44
45
        return np.reshape(vector, (-1, n_columns))
47
48
    def print_typewriter_pattern(pattern, n_columns):
49
        """Prints a pattern in a typewriter scheme."""
50
        print_pattern(vector_to_typewriter(pattern, n_columns))
51
52
53
    def print_pattern(pattern):
54
        np.set_printoptions(formatter={"float_kind": lambda x: "%.4f" % x})
55
        print(repr(pattern), sep=", ")
```