

Refining Theories for Projectile

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March 4, 2024

1 Context Theories

CT:realArith

CT:functions

CT:nDimSpace(n)

CT:trigonometry

CT:vectors

CT:CartCoordSyst

CT:Differentiation

CT:Integration

2 Assumptions

The fields for the assumptions are the text description, the relevant mathematical relation and the rationale (intention?).

A:oneD = (“The motion of the body is one dimensional.”, $v_2(t) = v_3(t) = 0$, “The body can be modelled as moving in a straight line.”)

A:constAccel = (“The acceleration is constant”, $\frac{da}{dt} = 0$, “The body undergoes constant acceleration, like when a body is in free fall with no external force acting on it, or a charged particle in a constant electric field.”)

A:timeStartZero = (“Time starts at zero.”, $t = 0$, “The time that the modelling starts is an arbitrary decision, so the choice is made to start at zero to simplify the equations”)

3 Theories and Theory Refinement

3.1 Theory for TM:acceleration

$$\text{TM:acceleration} = (a(t) = \frac{d}{dt}v(t))$$

$$, \{ \text{CT:realArith}, \text{CT:vectors}, \text{CT:CartCoordSyst}, \text{CT:Differentiation} \}, \{ \text{MD:cartSyst}, \text{A:threeD} \})$$

$$\text{TM:acceleration} = \left(\begin{bmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix}, \{ \text{CT:realArith}, \text{CT:vectors}, \text{CT:CartCoordSyst}, \text{CT:Differentiation} \}, \{ \text{MD:cartSyst}, \text{A:threeD} \} \right)$$

3.2 Refinement for GD:rectVel

$$\text{TM:acceleration} = \left(\begin{bmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix}, \{ \text{CT:realArith}, \dots \}, \{ \text{MD:cartSyst}, \text{A:threeD} \} \right)$$

Replace A:threeD by A:oneD. $v_2(t) = v_3(t) = 0$ therefore $a_2(t) = a_3(t) = 0$.

$$\text{TM:acceleration}' = (a_1(t) = \frac{d}{dt}v_1(t), \{ \text{CT:realArith}, \dots \}, \{ \text{MD:cartSyst}, \text{A:oneD} \})$$

Apply the assumption A:constAccel.

$$\text{TM:acceleration}'' = (a_1 = \frac{d}{dt}v_1(t), \{ \text{CT:realArith}, \dots \}, \{ \text{MD:cartSyst}, \text{A:oneD}, \text{A:constAccel} \})$$

Relabel a_1 as a^c , the constant acceleration.

$$\text{TM:acceleration}''' = (a^c = \frac{d}{dt}v_1(t), \{ \text{CT:realArith}, \dots \}, \{ \text{MD:cartSyst}, \text{A:oneD}, \text{A:constAccel} \})$$

Relabel $v_1(t)$ as $v(t)$.

$$\text{TM:acceleration}'''' = (a^c = \frac{d}{dt}v(t), \{ \text{CT:realArith}, \dots \}, \{ \text{MD:cartSyst}, \text{A:oneD}, \text{A:constAccel} \})$$

Assume that at $t = 0$ (A:timeStartZero) the velocity $v(0)$ is v^i and integrate using CT:integration.

$$\text{GD:rectVel} = (v(t) = v^i + a^c t, \{ \text{CT:realArith}, \dots, \text{CT:integration} \}, \{ \text{MD:cartSyst}, \text{A:oneD}, \text{A:constAccel}, \text{A:timeStartZero} \})$$

4 Projectile Project

$$\text{Project} = (\text{StrategyP}, \{ \text{CT}, \text{BT}, \text{HT}, \text{GT}, \text{PT}, \text{FT}, \text{RT} \})$$

$$\text{CT} = (\{ \text{CT:realArith}, \text{CT:trigonometry}, \text{CT:vectors}, \text{CT:CartCoordSyst}, \text{CT:Differentiation}, \text{CT:Integration} \})$$

$$\text{BT} = (\text{StrategyBT}, \{ \text{TM:acceleration}, \text{TM:velocity}, \text{TM:directionCosines} \})$$

HT = (StrategyHT, {GD:rectVel, GD:rectPos})

GT = (StrategyGT, {GD:velVec, GD:posVec, GD:magAngleToCompRep})

PT = (StrategyPT, {PT:coordSyst, DD:speedIX, DD:speedIY, PT:velVecInitMagAndAngle, PT:posVecInitMagAndAngle, PT:posVecInitPos, PT:velVecPlanetaryGrav, PT:posVecPlanetaryGrav})

FT = (StrategyFT, {IM:calOfLandingTime, IM:calOfLandingDist, IM:offsetIM, IM:messageIM})

RT = (StrategyRT, {RT:lngDstErr})