# Refining Theories for Projectile

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February 19, 2024

## 1 Projectile Project

Project = (StrategyP, {CT, BT, HT, GT, PT, FT, RT})

CT = ({CT:realArith, CT:trigonometry, CT:vectors, CT:CartCoordSyst, CT:Differentiation, CT:Integration})
*generated from the CTs used by the other theories*

BT = (StrategyBT, {TM:acceleration, TM:velocity, TM:directionCosines})

HT = (StrategyHT, {GD:rectVel, GD:rectPos})

GT = (StrategyGT, {GD:velVec, GD:posVec, GD:magAngleToCompRep})

PT = (StrategyPT, {PT:coordSyst, DD:speedIX, DD:speedIY, PT:velVecInitMagAndAngle, PT:posVecInitMagAndAngle, PT:posVecInitPos, PT:velVecPlanetaryGrav, PT:posVecPlanetaryGrav})

FT = (StrategyFT, {IM:calOfLandingTime, IM:calOfLandingDist, IM:offsetIM, IM:messageIM})

RT = (StrategyRT, {RT:lngDstErr})

## 2 Assumptions

The fields for the assumptions are the text description, the relevant mathematical relation and the rationale (intention?).

A:oneD = (“The motion of the body is one dimensional.”, $v_2(t) = v_3(t) = 0$, “The body can be modelled as moving in a straight line.”)

A:constAccel = (“The acceleration is constant”, $\frac{da}{dt} = 0$, “The body undergoes constant acceleration, like when a body is in free fall with no external force acting on it, or a charged particle in a constant electric field.”)

A:timeStartZero = (“Time starts at zero.”, $t = 0$, “The time that the modelling starts is an arbitrary decision, so the choice is made to start at zero to simplify the equations”)

# 3 Theories and Theory Refinement

## 3.1 Theory for TM:acceleration

TM:acceleration = ( $\begin{bmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix}$, {CT:realArith, CT:vectors, CT:CartCoodSyst, CT:Differentiation}, {MD:cartSyst, A:threeD})

## 3.2 Refinement for GD:rectVel

TM:acceleration = ( $\begin{bmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix}$, {CT:realArith, ...}, {MD:cartSyst, A:threeD})

Replace A:threeD by A:oneD. $v_2(t) = v_3(t) = 0$ therefore $a_2(t) = a_3(t) = 0$.

TM:acceleration’ = ($a_1(t) = \frac{d}{dt} v_1(t)$, {CT:realArith, ...}, {MD:cartSyst, A:oneD})

Apply the assumption A:constAccel.

TM:acceleration” = ($a_1 = \frac{d}{dt} v_1(t)$, {CT:realArith, ...}, {MD:cartSyst, A:oneD, A:constAccel})

Relabel $a_1$ as $a^c$, the constant acceleration.

TM:acceleration”’ = ($a^c = \frac{d}{dt} v_1(t)$, {CT:realArith, ...}, {MD:cartSyst, A:oneD, A:constAccel})

Relabel $v_1(t)$ as $v(t)$.

TM:acceleration”” = ($a^c = \frac{d}{dt} v(t)$, {CT:realArith, ...}, {MD:cartSyst, A:oneD, A:constAccel})

Assume that at $t = 0$ (A:timeStartZero) the velocity $v(0)$ is $v^i$ and integrate using CT:integration.

GD:rectVel = ($v(t) = v^i + a^c t$, {CT:realArith, ..., CT:integration}, {MD:cartSyst, A:oneD, A:constAccel, A:timeStartZero})