

Special multiplication of two digit numbers

Siddharth Mittal

6th March 2017

1 Introduction

The aim of this document is to explain to the reader a fast way to multiply two, two digit numbers, given that they meet certain criteria. Some random changes to test make. Some more mods.

We'll be going over the algorithm [1] and also the correctness of the algorithm [2].

2 Example

Let us first look at examples ¹, to understand what is happening.

2.1 Case 1

The first case is when the first(tens') digits are same and the last ones add to ten. If we take the numbers to be as $10x + y$ and $10x + z$ and $y + z = 10$ In such case product becomes²

$$x(x + 1) : yz$$

The figure given below will clear the scheme of things:

$$\begin{array}{r} 66 \\ \times 64 \\ \hline 284 \\ 3960 \\ \hline 4224 \end{array} \quad \begin{array}{c} \text{4 + 6 = 10} \\ \text{⑥4} \times \text{⑥6} \\ \swarrow \quad \searrow \\ \text{⑥} \times (\text{⑥} + 1) \quad : \quad \underline{4} * \underline{6} = 42:24 \end{array}$$

2.2 Case 2

The second case is when the first digits add to ten and the last ones are same. If we take the numbers to be as $10x + y$ and $10z + y$ and $x + z = 10$ In such case product becomes

$$xz + y : y^2$$

The figure given below will clear the scheme of things:

$$\begin{array}{r} 34 \\ \times 74 \\ \hline 136 \\ 2380 \\ \hline 2516 \end{array} \quad \begin{array}{c} \text{3 + 7 = 10} \\ \text{3④} \times \text{7④} \\ \swarrow \quad \searrow \\ \underline{3} \times \underline{7} + \text{④} \quad : \quad \text{④} \times \text{④} = 25:16 \end{array}$$

¹Pretty Amazing, Eh?

² $x : y$ means $100x + y$

3 Why this Rocks

The above method is simply amazing because we have to spend very less time and effort in calculating the product of two 2-digit numbers, given that they satisfy one of the criteria. This is very clear from the examples given in the Section 2.

4 Proof of Correctness

4.1 Case 1

We highlight the proof for Case 1.

Say we have two numbers, one is xy and the other is xz such that $y + z = 10$. Therefore the result of multiplication of these numbers, say a , is going to be:

$$\begin{aligned}a &= (10x + y) * (10x + z) \\&= 100x^2 + 10xz + 10xy + yz \\&= 100x^2 + 10xy + 10x(10 - y) + y(10 - y) \\&= 100x^2 + 10xy + 100x - 10xy + 10y - y^2 \\&= 100x^2 + 100x + 10y - y^2 \\a &= 100x(x + 1) + yz\end{aligned}$$

This is precisely what the algorithm gives. It says take the first part as $x(x + 1)$ and take the second part as yz . That is, the product is $x(x + 1) : yz$.

4.2 Case 2

Now, the proof for Case 2.

Let the two numbers be such that one is xy and the other is zy . Let the product of these numbers be a . Then,

$$\begin{aligned}a &= (10x + y)(10z + y) \\&= 100xz + 10xy + 10zy + y^2 \\&= 100xz + 10y(x + z) + y^2 \\&= 100xz + 10y(10) + y^2 && [\text{Substituting } x + z = 10] \\&= 100xz + 100y + y^2 \\&= 100(xz + y) + y^2\end{aligned}$$

This is exactly what the algorithm tells us to do. It says take the first part as $xz + y$ and take the second part as y^2 . That is, the product is $xz + y : y^2$.

5 Pseudocode

$n1$ and $n2$ are the two 2-digit numbers that we are going to multiply.

```
int Multiply2(int n1,int n2){
   $x_1 \leftarrow n1/10$ ;
   $y_1 \leftarrow n1\%10$ ;
   $x_2 \leftarrow n2/10$ ;
   $y_2 \leftarrow n2\%10$ ;
```

```
  if  $x_1 == x_2, y_1 + y_2 == 10$  then
     $f1 \leftarrow x_1 * (x_1 + 1)$ ;
     $f2 \leftarrow y_1 * y_2$ ;
     $prod \leftarrow 100 * f1 + f2$ ;
  end if
```

▷ Case1

```
  if  $x_1 + x_2 == 10, y_1 == y_2$  then
     $f1 \leftarrow x_1 * x_2 + y_1$ ;
     $f2 \leftarrow y_1^2$ ;
     $prod \leftarrow 100 * f1 + f2$ ;
  end if
}
```

▷ Case2

6 Generalization

We can generalize this. Let us look at how to do it. The arithmetic proofs remain similar to the 2-digit case. We consider two r -digit numbers.

Case	What is same	What adds to what	Product
Case1	The first digit	Last $r - 1$ add to 10^{r-1}	$10^{2r-2}x(x + 1) + y_1 * y_2$
Case2	The last digit	First $r - 1$ add to 10^{r-1}	$10^{2r-2}(y_1 * y_2 + x) + x^2$
Case3	First r digits	Last digits add to 10	$10^{2r-2}x(x + 1) + yz$
Case4	Last r digits	First digits add to 10	$10^{2r-2}(xz + y) + y^2$

6.1 Case 1

Let the first digit(the one at r_{th} place) be x . Let the two remaining parts be y_1 and y_2 . Then the product is $10^{2r-2}x(x + 1) + y_1 * y_2$

6.2 Case 2

Let the last digit(the one at ones' place) be x . Let the two remaining parts be y_1 and y_2 . Then the product is $10^{2r-2}(y_1 * y_2 + x) + x^2$

6.3 Case 3

Let the first $r - 1$ digits be x . The last digits of two numbers be y and z . Then the product is $10^{2r-2}x(x + 1) + yz$

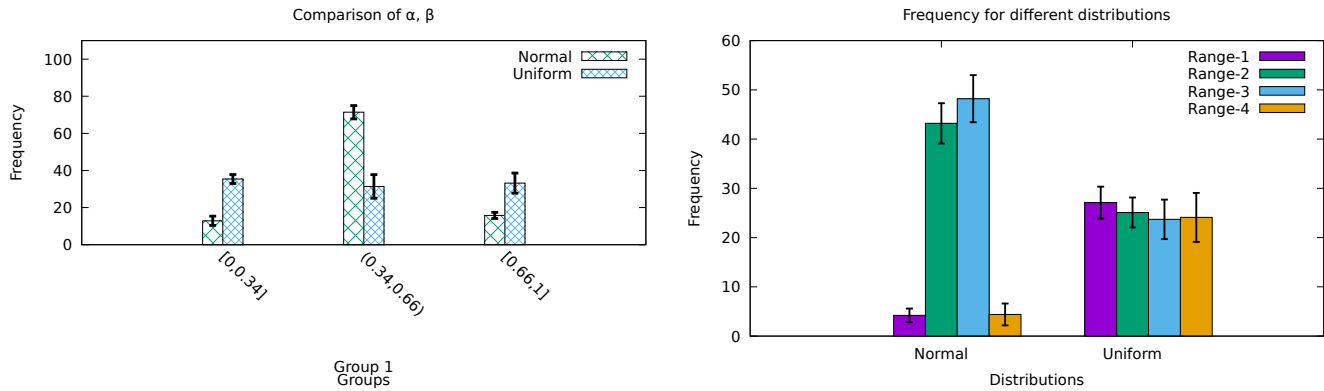
6.4 Case 4

Let the last $r - 1$ digits be y . The first digits of two numbers be x and z . Then the product is $10^{2r-2}(xz + y) + y^2$

7 Miscellaneous

You can find an amazing tool to visualize and understand this kind of fast multiplication at my site³ : `home.iitk.ac.in/~sidm/`

8 Some random graphs to bless the world



References

- [1] Brahma. Atharveda, 1100 BC.
- [2] Siddharth Mittal. Faster multiplication techniques, 1997.

³Just Kidding.