

# Special multiplication of two digit numbers

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6<sup>th</sup> March 2017

# 1 Introduction

The aim of this document is to explain to the reader a fast way to multiply two, two digit numbers, given that they meet certain criteria. Some random changes to test make. Some more mods.

We'll be going over the algorithm [1] and also the correctness of the algorithm [2].

## 2 Example

Let us first look at examples <sup>1</sup>, to understand what is happening.

### 2.1 Case 1

The first case is when the first(tens') digits are same and the last ones add to ten. If we take the numbers to be as  $10x + y$  and  $10x + z$  and  $y + z = 10$  In such case product becomes<sup>2</sup>

$$x(x + 1) : yz$$

The figure given below will clear the scheme of things:

$$\begin{array}{r} 66 \\ \times 64 \\ \hline 284 \\ 3960 \\ \hline 4224 \end{array} \quad \begin{array}{c} \text{4 + 6 = 10} \\ \text{⑥4} \times \text{⑥6} \\ \swarrow \quad \searrow \\ \text{⑥} \times (\text{⑥} + 1) \quad : \quad \underline{4} * \underline{6} = 42:24 \end{array}$$

### 2.2 Case 2

The second case is when the first digits add to ten and the last ones are same. If we take the numbers to be as  $10x + y$  and  $10z + y$  and  $x + z = 10$  In such case product becomes

$$xz + y : y^2$$

The figure given below will clear the scheme of things:

$$\begin{array}{r} 34 \\ \times 74 \\ \hline 136 \\ 2380 \\ \hline 2516 \end{array} \quad \begin{array}{c} \text{3 + 7 = 10} \\ \text{3④} \times \text{7④} \\ \swarrow \quad \searrow \\ \underline{3} \times \underline{7} + \text{④} \quad : \quad \text{④} \times \text{④} = 25:16 \end{array}$$

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<sup>1</sup>Pretty Amazing, Eh?

<sup>2</sup>  $x : y$  means  $100x + y$

### 3 Why this Rocks

The above method is simply amazing because we have to spend very less time and effort in calculating the product of two 2-digit numbers, given that they satisfy one of the criteria. This is very clear from the examples given in the Section 2.

## 4 Proof of Correctness

### 4.1 Case 1

We highlight the proof for Case 1.

Say we have two numbers, one is  $xy$  and the other is  $xz$  such that  $y + z = 10$ . Therefore the result of multiplication of these numbers, say  $a$ , is going to be:

$$\begin{aligned}a &= (10x + y) * (10x + z) \\&= 100x^2 + 10xz + 10xy + yz \\&= 100x^2 + 10xy + 10x(10 - y) + y(10 - y) \\&= 100x^2 + 10xy + 100x - 10xy + 10y - y^2 \\&= 100x^2 + 100x + 10y - y^2 \\a &= 100x(x + 1) + yz\end{aligned}$$

This is precisely what the algorithm gives. It says take the first part as  $x(x + 1)$  and take the second part as  $yz$ . That is, the product is  $x(x + 1) : yz$ .

### 4.2 Case 2

Now, the proof for Case 2.

Let the two numbers be such that one is  $xy$  and the other is  $zy$ . Let the product of these numbers be  $a$ . Then,

$$\begin{aligned}a &= (10x + y)(10z + y) \\&= 100xz + 10xy + 10zy + y^2 \\&= 100xz + 10y(x + z) + y^2 \\&= 100xz + 10y(10) + y^2 && [\text{Substituting } x + z = 10] \\&= 100xz + 100y + y^2 \\&= 100(xz + y) + y^2\end{aligned}$$

This is exactly what the algorithm tells us to do. It says take the first part as  $xz + y$  and take the second part as  $y^2$ . That is, the product is  $xz + y : y^2$ .

## 5 Pseudocode

$n1$  and  $n2$  are the two 2-digit numbers that we are going to multiply.

```

int Multiply2(int n1,int n2){
   $x_1 \leftarrow n1/10$ ;
   $y_1 \leftarrow n1\%10$ ;
   $x_2 \leftarrow n2/10$ ;
   $y_2 \leftarrow n2\%10$ ;

```

```

if  $x_1 == x_2, y_1 + y_2 == 10$  then
   $f1 \leftarrow x_1 * (x_1 + 1)$ ;
   $f2 \leftarrow y_1 * y_2$ ;
   $prod \leftarrow 100 * f1 + f2$ ;
end if

```

▷ Case1

```

if  $x_1 + x_2 == 10, y_1 == y_2$  then
   $f1 \leftarrow x_1 * x_2 + y_1$ ;
   $f2 \leftarrow y_1^2$ ;
   $prod \leftarrow 100 * f1 + f2$ ;
end if
}

```

▷ Case2

## 6 Generalization

We can generalize this. Let us look at how to do it. The arithmetic proofs remain similar to the 2-digit case. We consider two  $r$ -digit numbers.

Case	What is same	What adds to what	Product
Case1	The first digit	Last $r - 1$ add to $10^{r-1}$	$10^{2r-2}x(x+1) + y_1 * y_2$
Case2	The last digit	First $r - 1$ add to $10^{r-1}$	$10^{2r-2}(y_1 * y_2 + x) + x^2$
Case3	First $r$ digits	Last digits add to 10	$10^{2r-2}x(x+1) + yz$
Case4	Last $r$ digits	First digits add to 10	$10^{2r-2}(xz + y) + y^2$

### 6.1 Case 1

Let the first digit(the one at  $r_{th}$  place) be  $x$ . Let the two remaining parts be  $y_1$  and  $y_2$ . Then the product is  $10^{2r-2}x(x+1) + y_1 * y_2$

### 6.2 Case 2

Let the last digit(the one at ones' place) be  $x$ . Let the two remaining parts be  $y_1$  and  $y_2$ . Then the product is  $10^{2r-2}(y_1 * y_2 + x) + x^2$

### 6.3 Case 3

Let the first  $r - 1$  digits be  $x$ . The last digits of two numbers be  $y$  and  $z$ . Then the product is  $10^{2r-2}x(x+1) + yz$

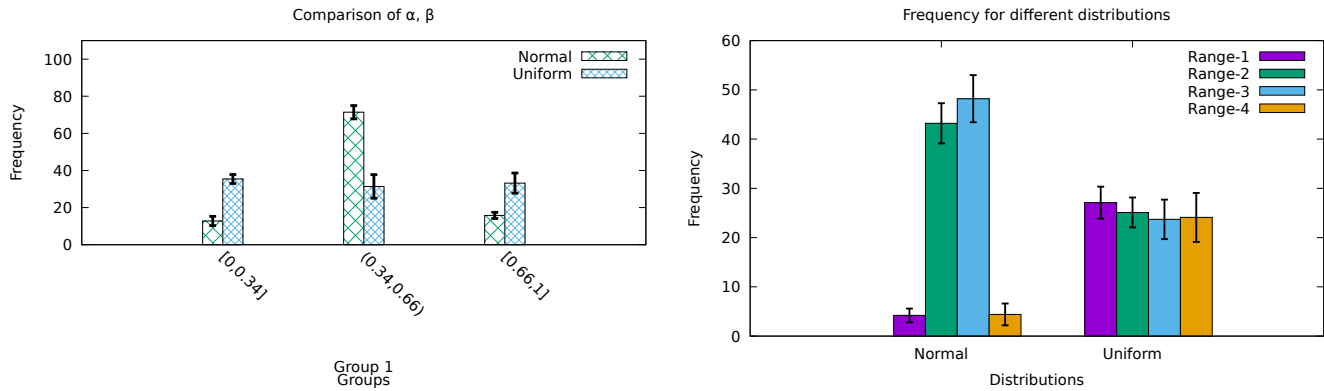
### 6.4 Case 4

Let the last  $r - 1$  digits be  $y$ . The first digits of two numbers be  $x$  and  $z$ . Then the product is  $10^{2r-2}(xz + y) + y^2$

## 7 Miscellaneous

You can find an amazing tool to visualize and understand this kind of fast multiplication at my site<sup>3</sup> : `home.iitk.ac.in/~sidm/`

## 8 Some random graphs to bless the world



## References

- [1] Brahma. Atharveda, 1100 BC.
- [2] Siddharth Mittal. Faster multiplication techniques, 1997.

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<sup>3</sup>Just Kidding.