Physics 468: Computing Project 7

January 24, 2020

Monte-Carlo modeling of a two-level laser

The Einstein A and B coefficients are a great way to understand the basic operation of a laser. In this project we'll develop a very crude but useful model of a laser without getting into the super grungy details of a *real* laser, but still including the basic idea of stimulated absorption/emission and spontaneous emission. In Einstein's original formulation the number of atoms in the upper state N_b as a time rate of change that depends on the intensity of the radiation field *and* the population of the two states:

$$\frac{dN_b}{dt} = -AN_b - IBN_b + IBN_a \tag{1}$$

where the ratio of A to B depends on the wavelength of the light whose energy is equal to the difference between the two energy levels. To make a laser there needs to be some way to pump atoms into the upper state so that the radiation field can be built up to much higher levels than would be likely at room temperature. To accommodate that, we need to add a "pump" term that is independent of the light intensity in the cavity:

$$\frac{dN_b}{dt} = -AN_b - IBN_b + IBN_a + PN_a \tag{2}$$

Single Mode Laser

We're thinking of a single mode laser so that we only need to keep track of the number of photons in one mode. We'll populate the laser cavity with "two state" atoms that have a lower state a and an upper state b. The cavity may have other modes as well, but we'll assume those modes have too great a loss rate (e.g., wall reflectivity) to sustain laser action. In this way, when a photon is emitted by *spontaneous* emission it is equally likely to go into any of the available modes. However, when an atom is *stimulated* to emit a photon, the emitted photon joins the mode that produced the stimulation! In this way, once the laser mode gets a significant number of photons, the overwhelming probability is for stimulated emission into that mode. However, even the one cavity mode with low loss will have *some* losses, so we need to account for that as well. The easiest way is to define a “decay time” during which the light intensity would drop by a factor of e without any atoms in the cavity at all. We can call this the cavity lifetime τ_c.

From rates to probabilities

In a Monte-Carlo simulation the idea is to use probabilities for various events to drive the simulation. Take spontaneous emission as an example. If there are no photons in the cavity, and there is no pump mechanism, Eq. 2 becomes:

$$\frac{dN_b}{dt} = -AN_b \tag{3}$$

We can rewrite this as:

$$dN_b = -N_b(A\,dt) \tag{4}$$

In other words, if each atom as a probability $P_{\text{spon}} = A\,dt$ of spontaneously emitting in a time dt then the expected number of atoms to emit a photon spontaneously in a time dt is $N_b P_{\text{spon}} = N_b(A\,dt)$. This basic strategy can be applied to *all* the terms in Eq. 2 and to similar equations for N_a and the number of photons in the mode. Of course the energy density will be related to the number of photons, the size of the cavity and the energy of each photon. In the end we'll wind up with probabilities that look something like:

$$\begin{aligned}
P_{\text{spon}} &= \alpha\,dt && \text{per atom}\\
P_{\text{stim}} &= N_{\text{photons}}\beta\,dt && \text{per atom}\\
P_{\text{pump}} &= \gamma\,dt && \text{per atom}\\
P_{\text{abs}} &= dt/\tau_c && \text{per photon}
\end{aligned} \tag{5}$$

Where α, β and γ are rates of spontaneous emission per atom, stimulated emission or absorption per atom per photon, and pump rate per atom respectively. P_{spon}, P_{stim}, P_{pump} and P_{abs} are probabilities of spontaneous emission, stimulated absorption or emission, pumping from a to b and photon loss (in the cavity either through wall absorption or output coupling) respectively. These probabilities are approximate, but get better for smaller time steps. Therefore, during the simulation the time step needs to be adjusted so that each of these probabilities remain small (i.e., $P \ll 1$).

So, what are we supposed to do?

Starting with a time step of 1 unit and all of the atoms starting out in the ground state, simulate the operation of a laser with various values of the parameters that seem interesting. For my values of $\alpha = 0.0001$, $\beta = 0.001$, $\gamma = 0.01$ using 1000 atoms, a τ_c of 10.0 time units and 10 modes (so the probability of spontaneous emission into the laser mode is 1/10) I get the following result:

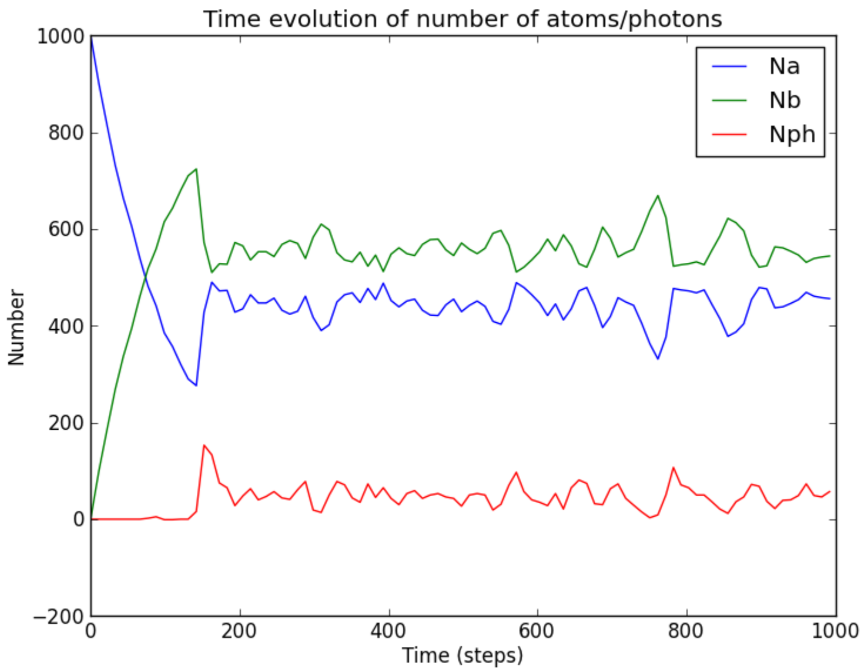

Figure 1: N_a, N_b and N_{ph} for 1000 time steps

Questions

1) How does increasing the number of non-laser modes affect the onset of lasing? Does it have an affect once stimulated emission begins?

2) What parameters would need to change to increase the average number of photons in the cavity once lasing starts?

3) How could you improve this program so that the simulation would be more realistic? Don't go crazy, but maybe a suggestion or two would be great.

Please answer these questions at the end of your report.