Analysis of intrinsic vowel duration in Northwestern Italian

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Attach packages

```
library(tidyverse)
theme_set(theme_light())
library(magrittr)
library(coretta2018itaegg)
library(brms)
library(posterior)
library(tidybayes)
library(marginaleffects)
library(ggdist)
library(mgcv)
library(tidygam)

my_seed <- 9899
```

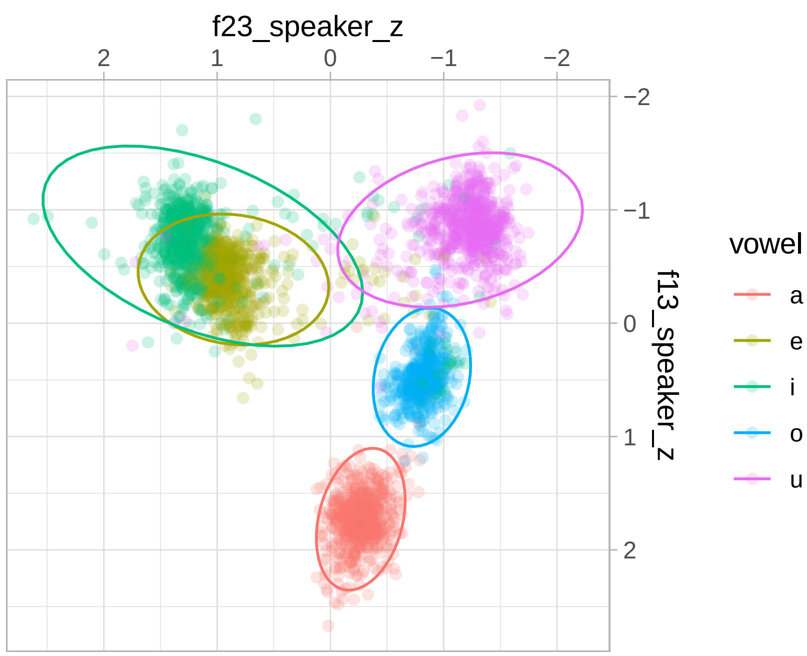
Read data

```
data("formants")

formants %<>% mutate(
  duration = duration * 1000,
  vowel = as.factor(label),
  duration_z = as.vector(scale(duration)),
  duration_log = log(duration),
  duration_logz = as.vector(scale(log(duration))),
```

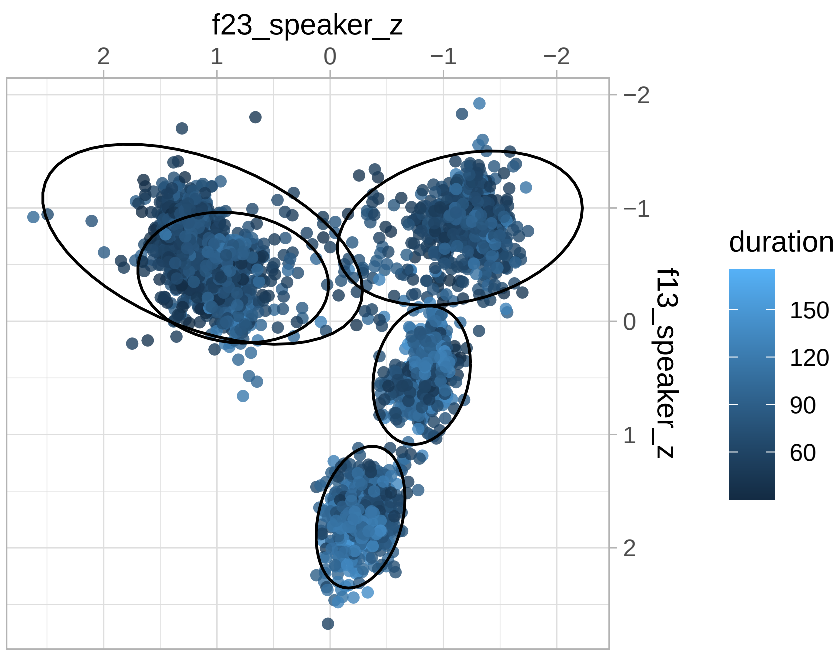
```
  f13_z = as.vector(scale(f13)),
  f23_z = as.vector(scale(f23)),
  speaker = as.factor(speaker)
)
contrasts(formants$vowel) <- "contr.sum"
```

Plotting

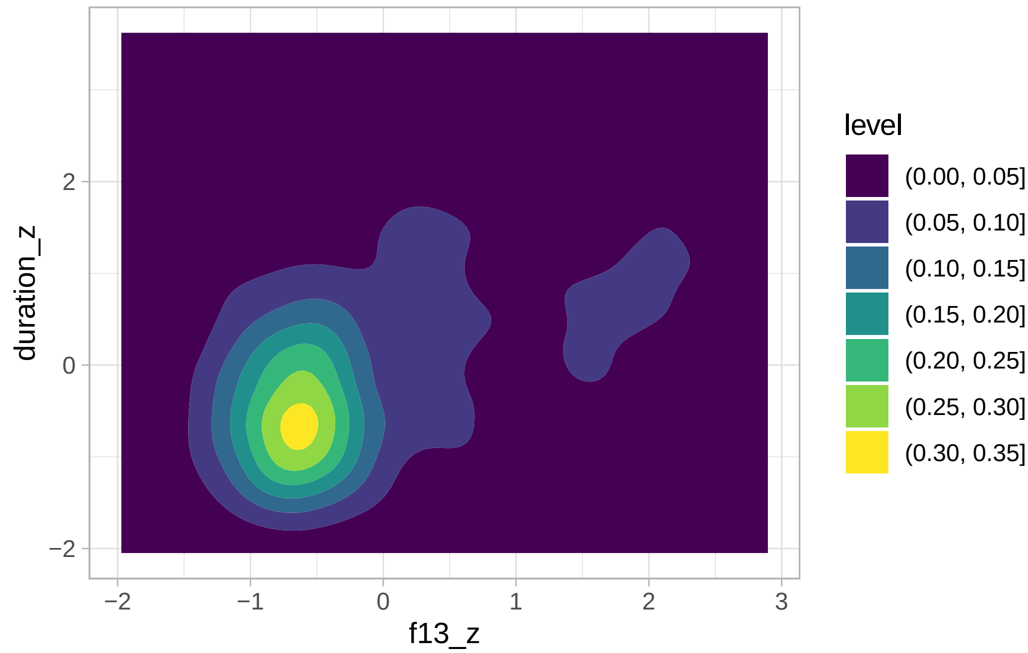
```
formants %>%
  group_by(speaker) %>%
  mutate(
    f13_speaker_z = as.vector(scale(f13)),
    f23_speaker_z = as.vector(scale(f23))
  ) %>%
  ggplot(aes(f23_speaker_z, f13_speaker_z, colour = vowel)) +
  geom_point(alpha = 0.2) +
  stat_ellipse(type = "norm") +
  scale_x_reverse(position = "top") + scale_y_reverse(position = "right") +
  coord_fixed()
```



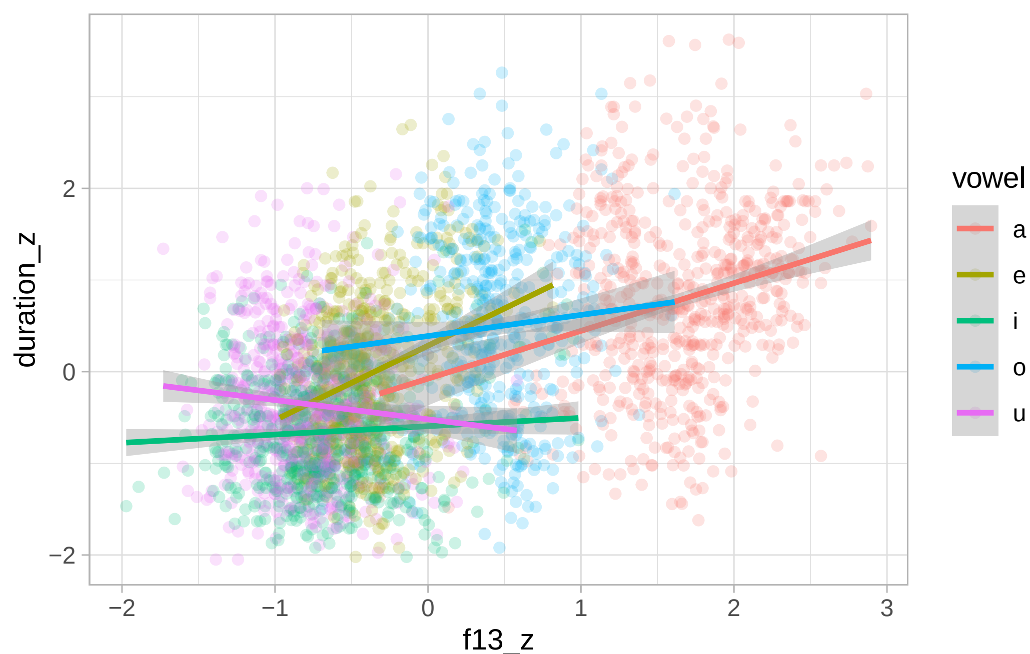
```r
formants %>%
  group_by(speaker) %>%
  mutate(
    f13_speaker_z = as.vector(scale(f13)),
    f23_speaker_z = as.vector(scale(f23))
  ) %>%
  ggplot(aes(f23_speaker_z, f13_speaker_z)) +
  geom_point(aes(colour = duration), alpha = 0.8) +
  stat_ellipse(aes(group = vowel), type = "norm") +
  scale_x_reverse(position = "top") + scale_y_reverse(position = "right") +
  coord_fixed()
```




```r
formants %>%
  ggplot(aes(f13_z, duration_z)) +
  geom_density_2d_filled()
```




```r
formants %>%
  ggplot(aes(f13_z, duration_z, colour = vowel)) +
  geom_point(alpha = 0.2) +
  geom_smooth(method = "lm", formula = y ~ x)
```



Linear modelling

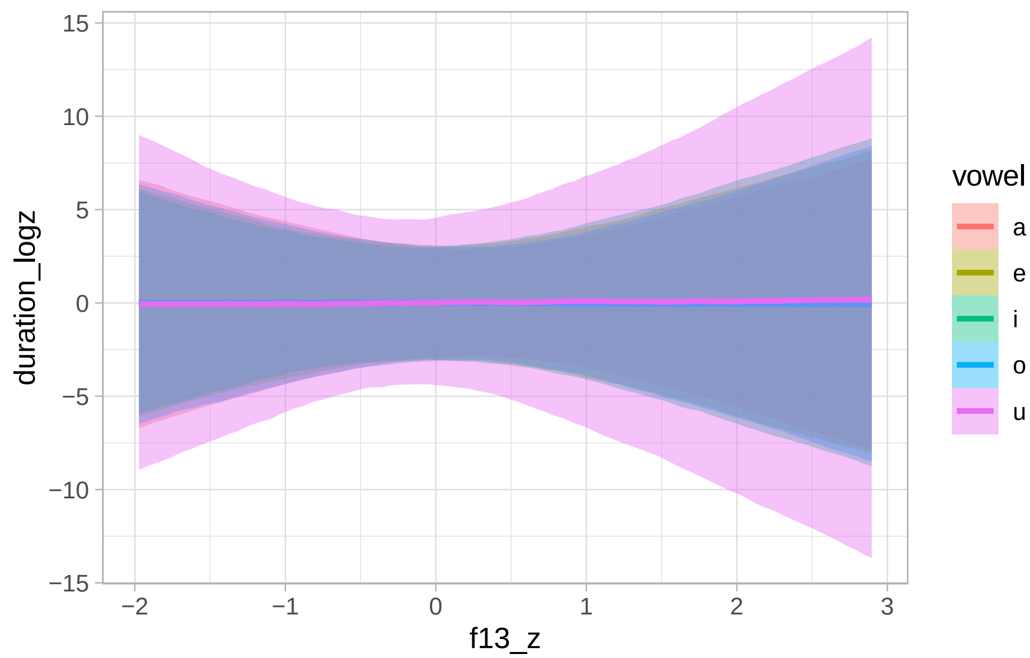
Prior predictive checks

The outcome `duration_logz` and predictor `f13_z` are z-scored and `vowel` is sum coded so that `Intercept` is the grand mean.

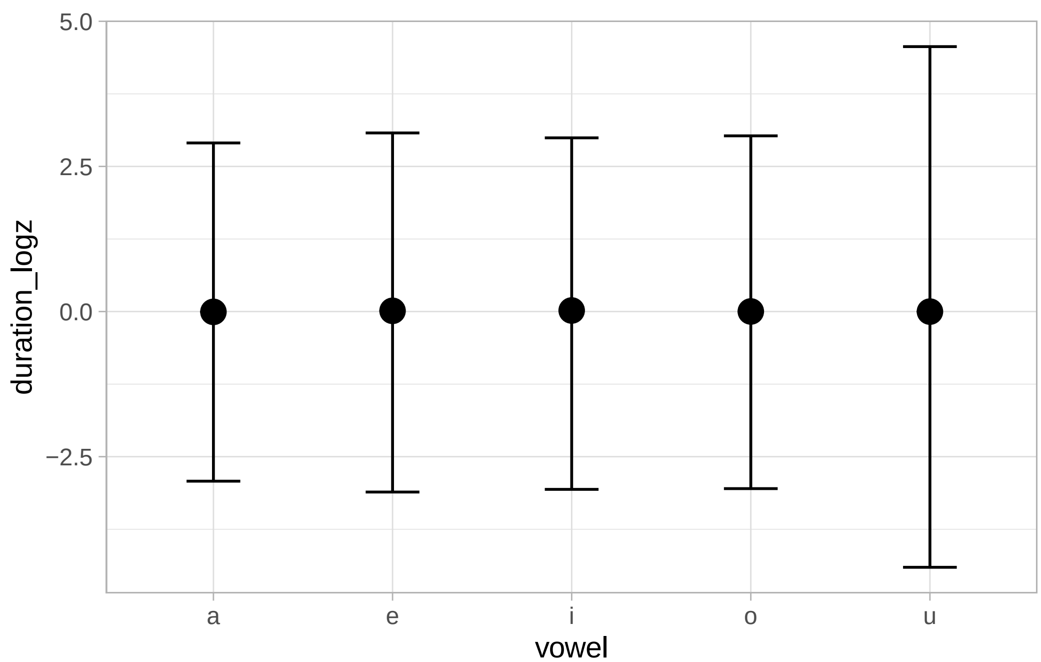
I am using relatively weakly informative priors.

```
priors <- c(
  prior(normal(0, 1), class = Intercept),
  prior(normal(0, 1), class = b),
  prior(cauchy(0, 0.1), class = sigma),
  prior(lkj(2), class = cor),
  prior(cauchy(0, 0.1), class = sd)
)

bm_1_priors <- brm(
  duration_logz ~
    vowel * f13_z +
    (vowel * f13_z | speaker),
  family = gaussian,
  data = formants,
  prior = priors,
  cores = 4,
  threads = threading(2),
  backend = "cmdstanr",
  sample_prior = "only",
  file = "data/cache/bm_1_priors",
)
```

```
conditional_effects(bm_1_priors, "f13_z:vowel")
```




```
conditional_effects(bm_1_priors, "vowel")
```



Model fit

```
bm_1 <- brm(
  duration_logz ~
    vowel * f13_z +
    (vowel * f13_z | speaker),
  family = gaussian,
  data = formants,
  prior = priors,
  cores = 4,
  threads = threading(2),
  backend = "cmdstanr",
  file = "data/cache/bm_1",
)
```

```
fixef(bm_1, probs = c(0.05, 0.95))
```

```
                Estimate  Est.Error          Q5         Q95
Intercept    -0.15456092 0.14810441 -0.39139975  0.09485949
vowel1       -0.28607100 0.13267976 -0.51491880 -0.07680009
vowel2        0.35434327 0.05833619  0.25963235  0.45007120
vowel3       -0.32030985 0.05730238 -0.41276840 -0.22539825
vowel4        0.47457028 0.07210259  0.35954355  0.59458675
f13_z         0.36700826 0.05277162  0.28164785  0.45174305
vowel1:f13_z  0.35767128 0.08984698  0.21301775  0.50970635
vowel2:f13_z  0.12961894 0.09920732 -0.03739698  0.28873440
vowel3:f13_z -0.05189318 0.06790845 -0.16322630  0.06080566
vowel4:f13_z -0.01187440 0.10306594 -0.18354195  0.15770550
```

Let’s get the estimated effect of vowel quality for /u/.

```
bm_1_draws <- as_draws_df(bm_1) %>%
  mutate(
    vowel5 = b_Intercept - b_vowel1 - b_vowel2 - b_vowel3 - b_vowel4,
    b_vowel5 = vowel5 - b_Intercept
  )

quantile(bm_1_draws$b_vowel5, probs = c(0.05, 0.95))
```

```
         5%         95%
-0.37541152 -0.07203285
```

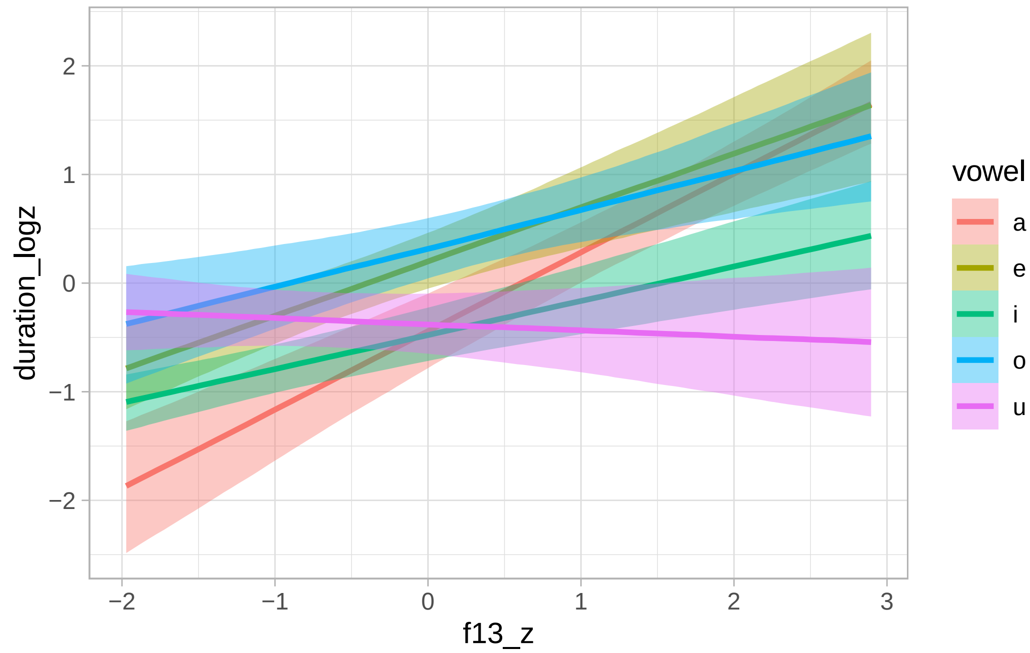
```r
bm_1 %>%
  as_draws_df() %>%
  select(b_vowel1:b_vowel4) %>%
  pivot_longer(b_vowel1:b_vowel4) %>%
  group_by(name) %>%
  summarise(
      cri95 = list(round(quantile2(value, probs = c(0.025, 0.975)), 2)),
      cri90 = list(round(quantile2(value, probs = c(0.05, 0.95)), 2)),
      cri80 = list(round(quantile2(value, probs = c(0.1, 0.9)), 2)),
      cri60 = list(round(quantile2(value, probs = c(0.2, 0.8)), 2))
  ) %>%
  knitr::kable(format = "latex") %>% cat(sep = "\n")
```

```
Warning: Dropping 'draws_df' class as required metadata was removed.
```

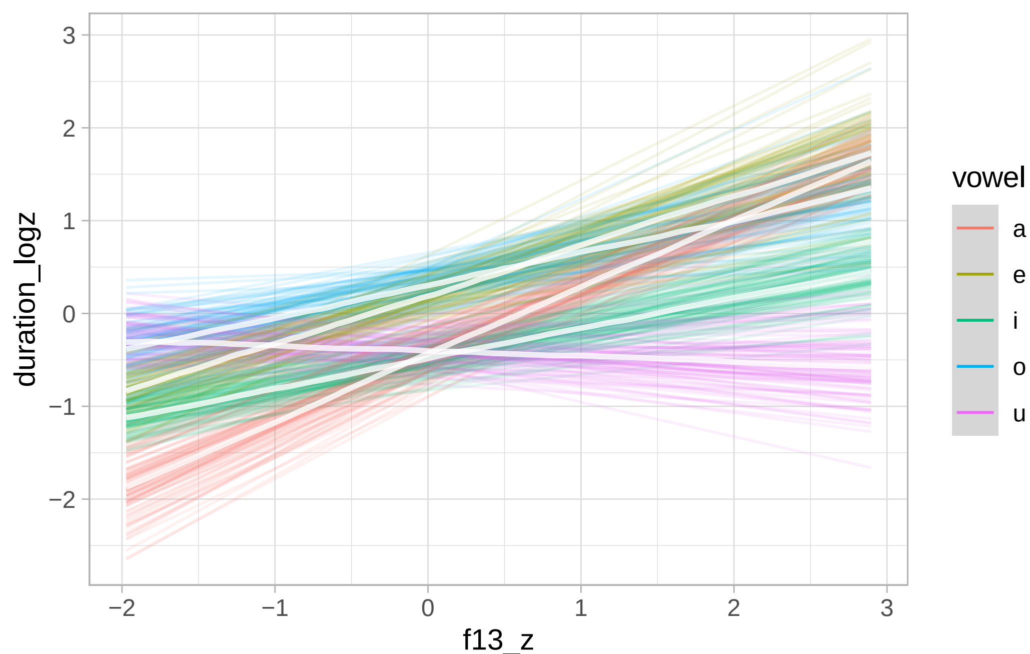
```
\begin{tabular}{l|l|l|l|l}
\hline
name & cri95 & cri90 & cri80 & cri60\\
\hline
b\_vowel1 & -0.55, -0.04 & -0.51, -0.08 & -0.46, -0.12 & -0.40, -0.17\\
\hline
b\_vowel2 & 0.24, 0.47 & 0.26, 0.45 & 0.28, 0.43 & 0.3, 0.4\\
\hline
b\_vowel3 & -0.43, -0.21 & -0.41, -0.23 & -0.39, -0.25 & -0.37, -0.27\\
\hline
b\_vowel4 & 0.33, 0.62 & 0.36, 0.59 & 0.38, 0.57 & 0.42, 0.53\\
\hline
\end{tabular}
```

Model plotting

```r
conditional_effects(bm_1, "f13_z:vowel", prob = 0.9)
```




```r
conditional_effects(bm_1, "f13_z:vowel", spaghetti = TRUE, ndraws = 100, prob = 0.9)
```



We need to get the predicted draws to convert duration and F1 back to ms and hz. Note that duration was logged then scaled.

```
seq_minmax <- function(x, by = 1) {
  seq(min(x), max(x), by = by)
}

bm_1_grid <- expand_grid(
  vowel = levels(formants$vowel),
  f13_z = seq_minmax(formants$f13_z, 0.5)
)

bm_1_preds <- epred_draws(bm_1, newdata = bm_1_grid, re_formula = NA) %>%
  mutate(
    duration_log = .epred * sd(formants$duration_log) + mean(formants$duration_log),
    duration = exp(duration_log),
    f13 = f13_z * sd(formants$f13) + mean(formants$f13)
  )
```

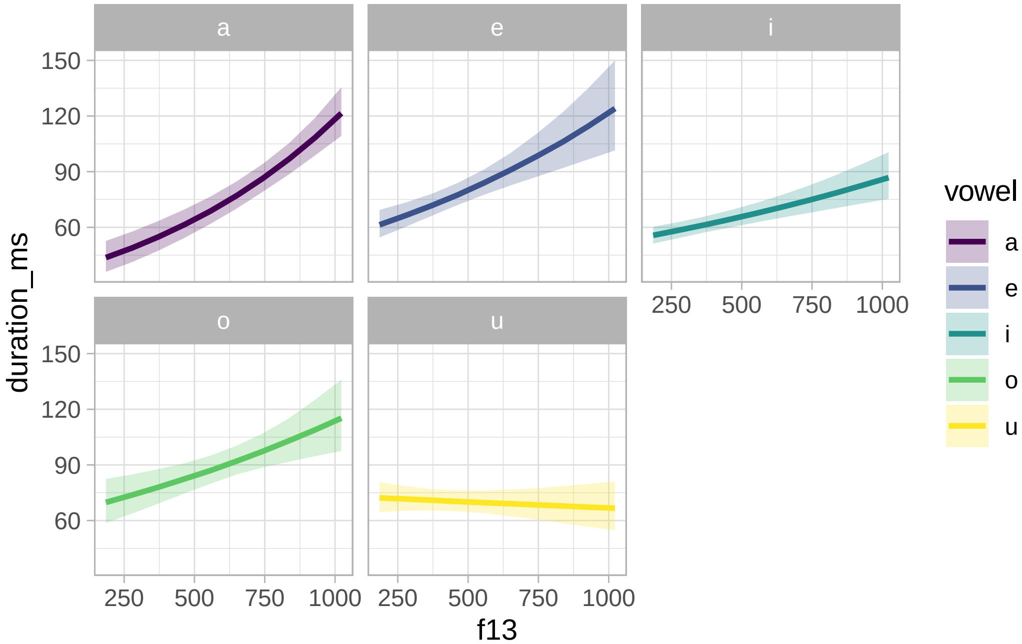
Let's also calculate the mean F1 values for each vowel, to be added in the plot below.

```
vmean_f13 <- formants %>%
  group_by(vowel) %>%
  summarise(f13_mean = mean(f13))
vmean_f13z <- formants %>%
  group_by(vowel) %>%
  summarise(f13z_mean = mean(f13_z))
```

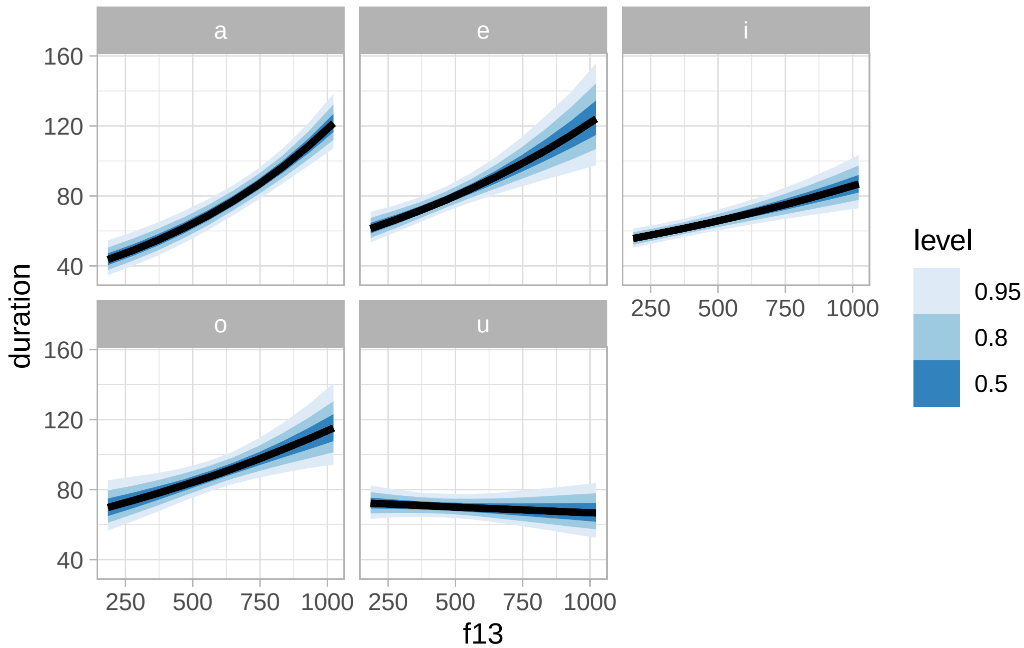
We can now plot the model predictions in the original scale.

```
bm_1_preds %>%
  group_by(vowel, f13) %>%
  summarise(
    duration_ms = median(duration),
    # Get the 90% CrI
    q0.05 = quantile(duration, probs = 0.05),
    q0.90 = quantile(duration, probs = 0.95),
    .groups = "drop"
  ) %>%
  ggplot(aes(f13, duration_ms)) +
  geom_ribbon(aes(ymin = q0.05, ymax = q0.90, fill = vowel), alpha = 0.25) +
  geom_line(aes(colour = vowel), linewidth = 1) +
  facet_wrap(~vowel) +
```

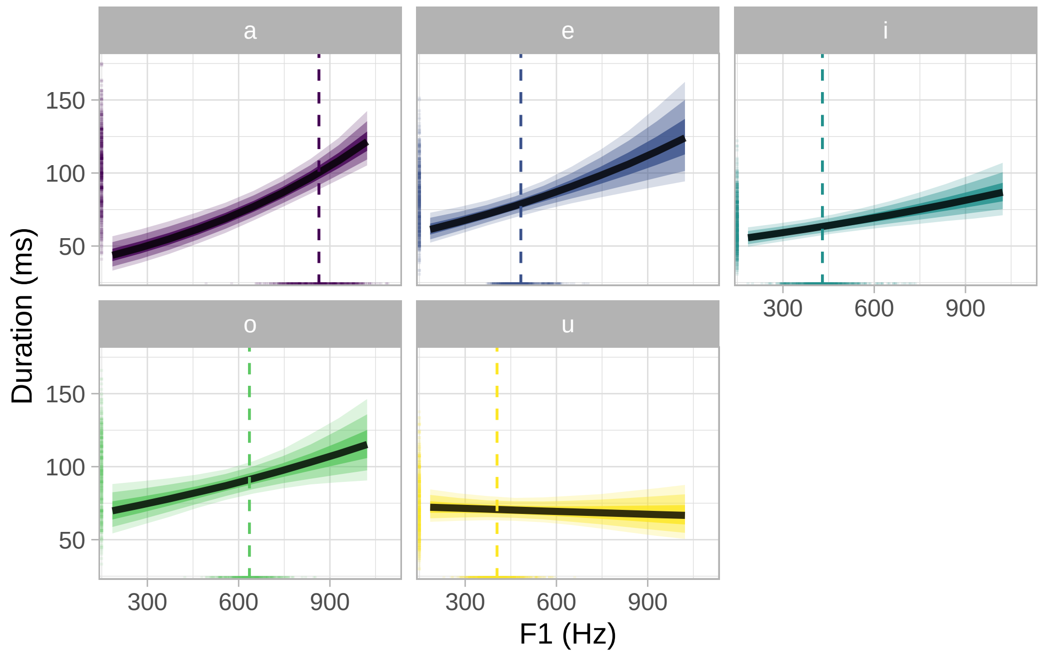
```
  scale_colour_viridis_d() +
  scale_fill_viridis_d()
```




```
bm_1_preds %>%
  group_by(vowel, f13) %>%
  ggplot(aes(f13, duration)) +
  stat_lineribbon() +
  facet_wrap(~vowel) +
  scale_fill_brewer()
```



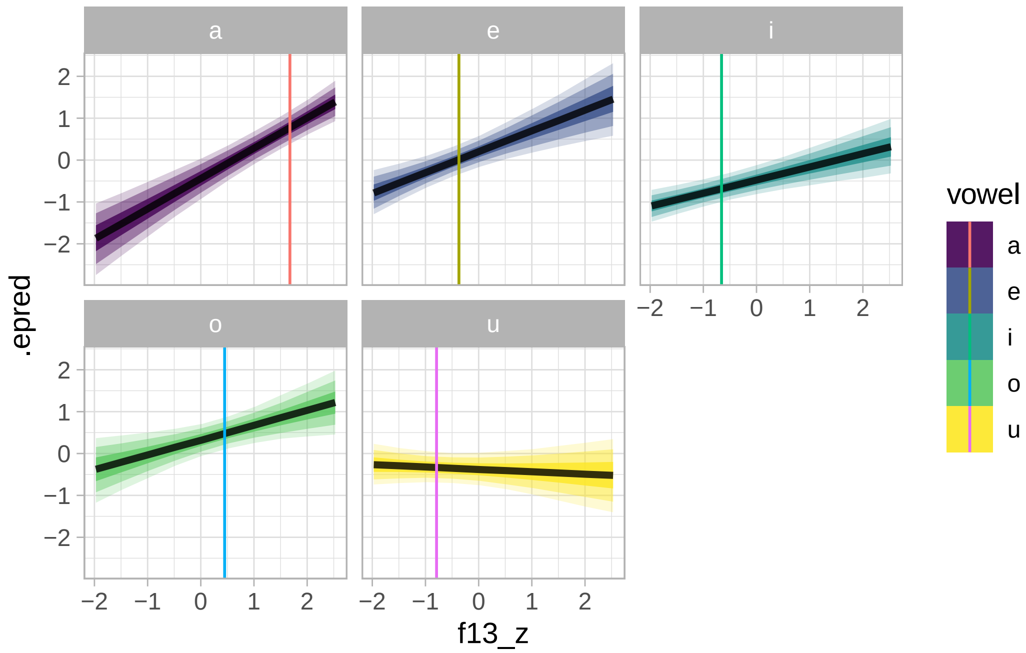

```
bm_1_preds %>%
  group_by(vowel, f13) %>%
  ggplot(aes(f13, duration, fill = vowel)) +
  stat_ribbon(.width = 0.98, alpha = 0.2) +
  stat_ribbon(.width = 0.9, alpha = 0.4) +
  stat_lineribbon(.width = 0.6, alpha = 0.8) +
  geom_vline(data = vmean_f13, aes(xintercept = f13_mean, colour = vowel), linetype = "das
  geom_rug(data = formants, alpha = 0.1, length = unit(0.015, "npc"), aes(colour = vowel))
  facet_wrap(~vowel) +
  labs(
    x = "F1 (Hz)", y = "Duration (ms)"
  ) +
  scale_fill_viridis_d() +
  scale_colour_viridis_d() +
  theme(legend.position = "none")
```




```
ggsave("img/bm1-pred-plot-ms-hz.png", width = 7, height = 5)
```

But let's also plot this in the standardised logged duration scale.

```
bm_1_preds %>%
  group_by(vowel, f13) %>%
  ggplot(aes(f13_z, .epred, fill = vowel)) +
  stat_ribbon(.width = 0.98, alpha = 0.2) +
  stat_ribbon(.width = 0.9, alpha = 0.4) +
  stat_lineribbon(.width = 0.6, alpha = 0.8) +
  geom_vline(data = vmean_f13z, aes(xintercept = f13z_mean, colour = vowel)) +
  facet_wrap(~vowel) +
  scale_fill_viridis_d()
```



Average predictions and comparisons

```
avg_comparisons(bm_1, variables = "f13_z", conf_level = 0.9)
```

```

  Term Contrast Estimate 5.0 % 95.0 %
 f13_z       +1    0.357 0.287  0.427

Columns: term, contrast, estimate, conf.low, conf.high
```

```
avg_comparisons(bm_1, variables = "f13_z", by = "vowel", conf_level = 0.9)
```

```

  Term Contrast vowel Estimate  5.0 % 95.0 %
 f13_z mean(+1)     o   0.3583  0.163 0.5559
 f13_z mean(+1)     a   0.7207  0.575 0.8842
 f13_z mean(+1)     e   0.5006  0.320 0.6735
 f13_z mean(+1)     u  -0.0566 -0.201 0.0906
 f13_z mean(+1)     i   0.3164  0.219 0.4172

Columns: term, contrast, vowel, estimate, conf.low, conf.high, predicted, predicted_hi, predi
```

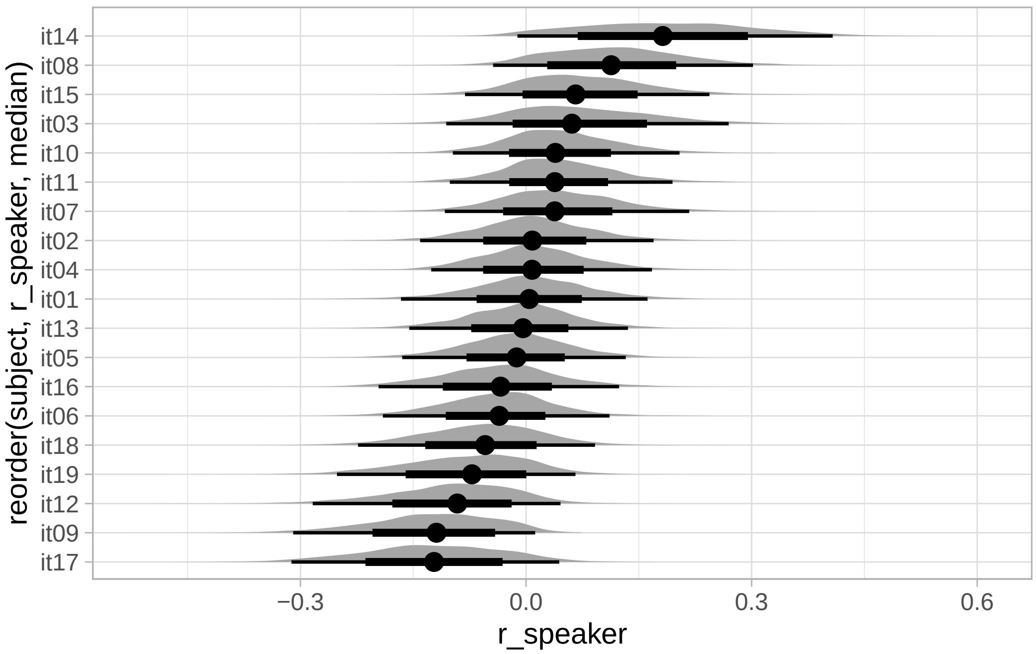
```r
avg_predictions(bm_1, by = "vowel", conf_level = 0.9) %>%
  as_tibble() %>%
  mutate_if(
    is.numeric, function (x) {exp(x * sd(formants$duration_log) + mean(formants$duration_l
  )
```

```
# A tibble: 5 x 4
  vowel estimate conf.low conf.high
  <fct>    <dbl>    <dbl>     <dbl>
1 a         99.5     98.4     101.
2 e         79.2     78.3      80.1
3 i         63.7     63.0      64.4
4 o         91.3     90.1      92.5
5 u         70.8     70.0      71.5
```

Group-level effects

```r
bm_1_ranef <- bm_1 %>%
  spread_draws(b_Intercept, r_speaker[subject,var]) %>%
  mutate(condition_mean = b_Intercept + r_speaker)
```

```r
bm_1_ranef %>%
  filter(var == "f13_z") %>%
  ggplot(aes(y = reorder(subject, r_speaker, median), x = r_speaker)) +
  stat_halfeye()
```



Non-linear modelling

F1

```
gam_1 <- bam(
  duration_logz ~
    vowel +
    s(f13_z) +
    s(f13_z, speaker, by = vowel, bs = "fs", m = 1),
  data = formants
)
```

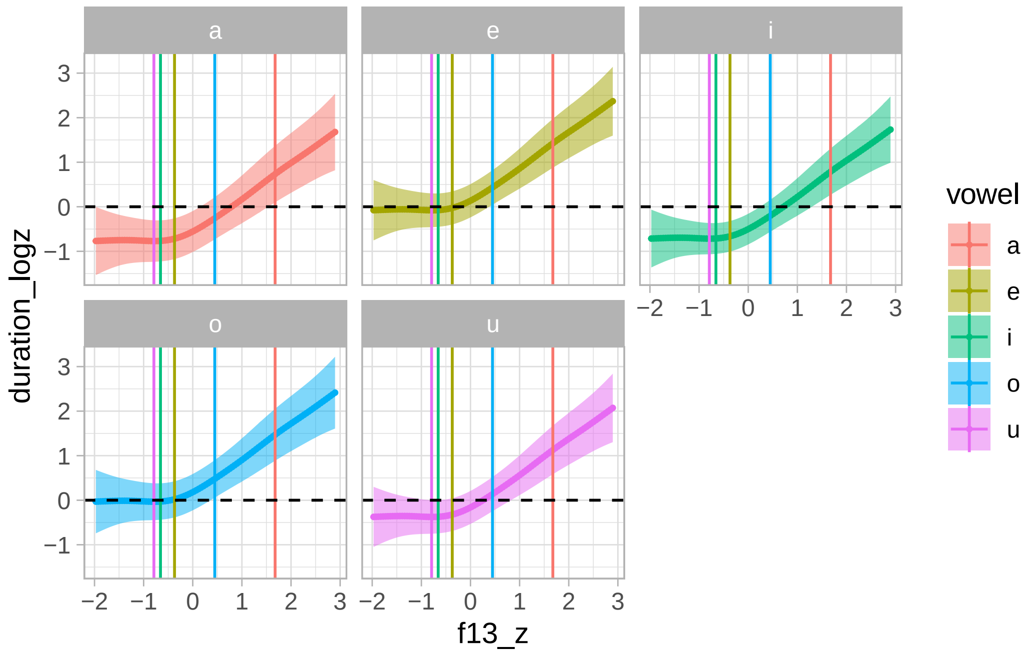
```
summary(gam_1)
```

```
Family: gaussian
Link function: identity

Formula:
duration_logz ~ vowel + s(f13_z) + s(f13_z, speaker, by = vowel,
    bs = "fs", m = 1)
```

```
Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.01644    0.07064   0.233   0.8160
vowel1      -0.37509    0.18756  -2.000   0.0456 *
vowel2       0.31573    0.14358   2.199   0.0280 *
vowel3      -0.32102    0.12877  -2.493   0.0127 *
vowel4       0.36146    0.16098   2.245   0.0248 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
                           edf  Ref.df      F p-value
s(f13_z)                 4.522   5.439 16.791  <2e-16 ***
s(f13_z,speaker):vowela 25.638 145.000  6.699  <2e-16 ***
s(f13_z,speaker):vowele 24.058 137.000  6.514  <2e-16 ***
s(f13_z,speaker):voweli 27.487 157.000  4.448  <2e-16 ***
s(f13_z,speaker):vowelo 34.267 149.000  6.074  <2e-16 ***
s(f13_z,speaker):vowelu 31.521 151.000  6.066  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.712   Deviance explained = 72.7%
fREML = 2663.4  Scale est. = 0.28762   n = 3053
```

```r
vmean <- aggregate(formants$f13_z, list(formants$vowel), mean)
# fs_terms <- c("s(f13_z,speaker)")
fs_terms <- c("s(f13_z,speaker):vowela", "s(f13_z,speaker):vowele", "s(f13_z,speaker):vowe

predict_gam(gam_1, exclude_terms = fs_terms, length_out = 100) %>%
  plot(series = "f13_z", comparison = "vowel") +
  geom_vline(data = vmean, aes(xintercept = x, colour = Group.1)) +
  geom_hline(yintercept = 0, linetype = "dashed") +
  facet_wrap(~vowel)
```

```
Warning: There was 1 warning in `dplyr::mutate()`.
i In argument: `fit = rowSums(dplyr::across())`.
Caused by warning:
! Using `across()` without supplying `.cols` was deprecated in dplyr 1.1.0.
i Please supply `.cols` instead.
```

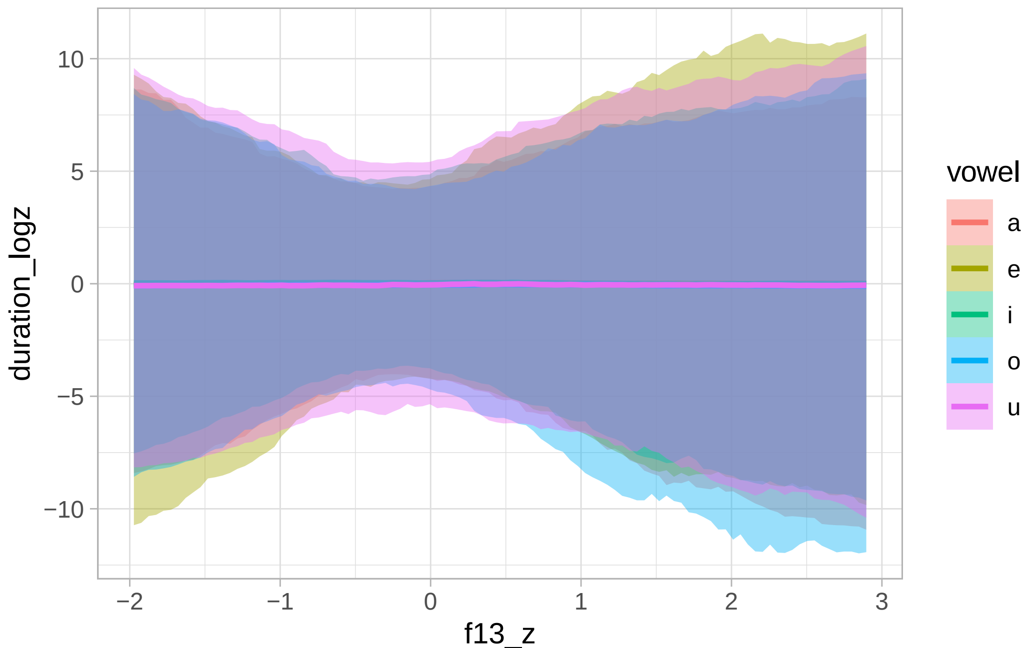


BRM

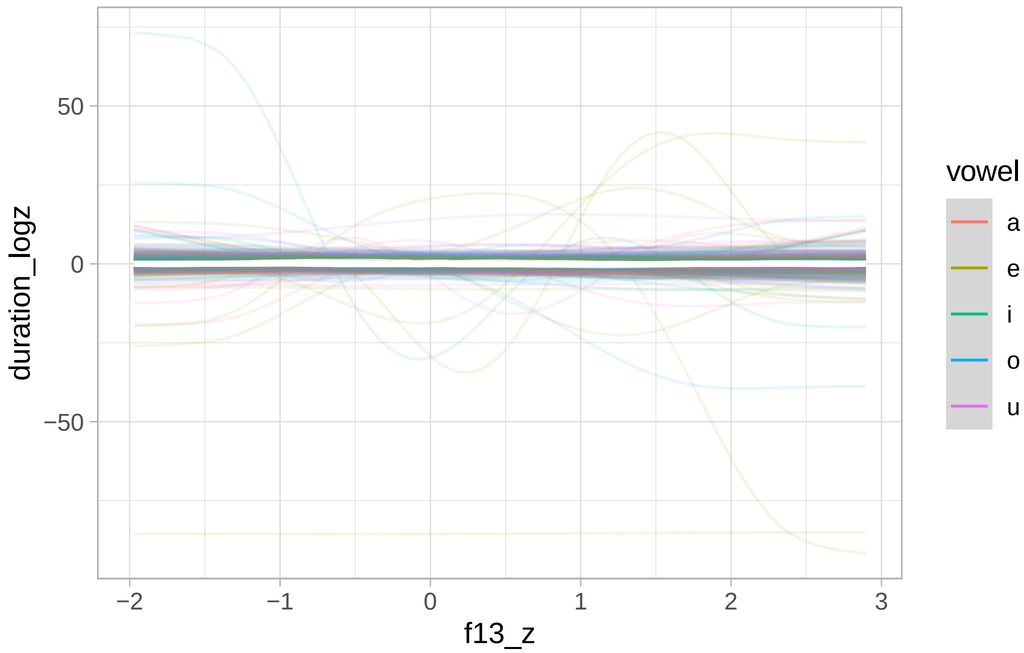
```
priors_s <- c(
  prior(normal(0, 1), class = Intercept),
  prior(normal(0, 1), class = b),
  prior(cauchy(0, 0.01), class = sigma),
  prior(cauchy(0, 1), class = sds)
)

bms_1_priors <- brm(
  duration_logz ~
    vowel +
    s(f13_z, k = 5) +
    s(f13_z, speaker, by = vowel, bs = "fs", m = 1, k = 5),
  family = gaussian,
  data = formants,
  prior = priors_s,
  sample_prior = "only",
  cores = 4,
  threads = threading(2),
  backend = "cmdstanr",
  file = "data/cache/bms_1_priors",
```

```
  seed = my_seed
)
```

```
conditional_effects(bms_1_priors, "f13_z:vowel")
```




```
conditional_effects(bms_1_priors, "f13_z:vowel", spaghetti = TRUE, ndraws = 100)
```



We specify `k = 5` based on the mgcv modelling above. Reducing `k` speeds up estimation (because there are less basis functions, hence less parameters to estimate).

The model takes about 4-5 hours to run on 8 cores.

```
bms_1 <- brm(
  duration_logz ~
    vowel +
    s(f13_z, k = 5) +
    s(f13_z, speaker, by = vowel, k = 5, bs = "fs", m = 1),
  family = gaussian,
  data = formants,
  prior = priors_s,
  cores = 4,
  iter = 4000,
  control = list(adapt_delta = 0.9999),
  threads = threading(2),
  backend = "cmdstanr",
  file = "data/cache/bms_1",
  seed = my_seed
)
```

```
summary(bms_1, prob = 0.9)
```

```
 Family: gaussian
  Links: mu = identity; sigma = identity
Formula: duration_logz ~ vowel + s(f13_z, k = 5) + s(f13_z, speaker, by = vowel, k = 5, bs =
   Data: formants (Number of observations: 3053)
  Draws: 4 chains, each with iter = 4000; warmup = 2000; thin = 1;
         total post-warmup draws = 8000

Smooth Terms:
                           Estimate Est.Error l-90% CI u-90% CI Rhat Bulk_ESS
sds(sf13_z_1)                  2.04      1.13     0.96     3.89 1.00     1391
sds(sf13_zspeakervowela_1)     0.31      0.22     0.03     0.71 1.01      544
sds(sf13_zspeakervowela_2)     1.88      1.72     0.09     5.25 1.02      486
sds(sf13_zspeakervowele_1)     0.34      0.22     0.03     0.73 1.00      374
sds(sf13_zspeakervowele_2)     1.69      1.59     0.08     4.91 1.01      493
sds(sf13_zspeakervoweli_1)     0.33      0.22     0.03     0.72 1.00      475
sds(sf13_zspeakervoweli_2)     1.82      1.68     0.09     5.16 1.00      591
sds(sf13_zspeakervowelo_1)     0.32      0.22     0.03     0.73 1.00      414
sds(sf13_zspeakervowelo_2)     1.75      1.68     0.08     5.15 1.02      455
sds(sf13_zspeakervowelu_1)     0.32      0.21     0.03     0.70 1.01      460
sds(sf13_zspeakervowelu_2)     1.66      1.57     0.09     4.88 1.01      654
                           Tail_ESS
sds(sf13_z_1)                  2370
sds(sf13_zspeakervowela_1)     1266
sds(sf13_zspeakervowela_2)     1255
sds(sf13_zspeakervowele_1)      973
sds(sf13_zspeakervowele_2)     1255
sds(sf13_zspeakervoweli_1)     1100
sds(sf13_zspeakervoweli_2)     1195
sds(sf13_zspeakervowelo_1)     1139
sds(sf13_zspeakervowelo_2)     1055
sds(sf13_zspeakervowelu_1)     1182
sds(sf13_zspeakervowelu_2)     1417

Population-Level Effects:
          Estimate Est.Error l-90% CI u-90% CI Rhat Bulk_ESS Tail_ESS
Intercept     0.06      0.15    -0.20     0.30 1.00     1639     2455
vowel1       -0.18      0.10    -0.35    -0.01 1.01     1300     2422
vowel2        0.29      0.04     0.22     0.36 1.01     1556     3821
vowel3       -0.40      0.04    -0.47    -0.34 1.01     1537     2909
vowel4        0.34      0.04     0.27     0.40 1.00     2819     3944
sf13_z_1      0.90      0.69    -0.23     2.04 1.00     2538     3869

Family Specific Parameters:
```

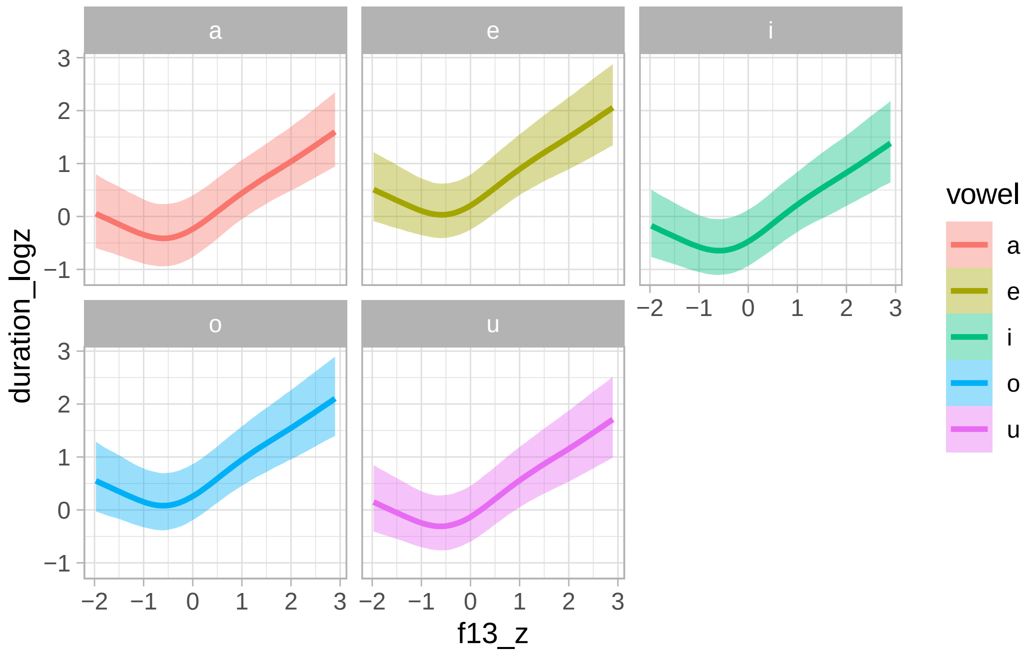
```
      Estimate Est.Error l-90% CI u-90% CI Rhat Bulk_ESS Tail_ESS
sigma     0.55      0.01     0.54     0.56 1.00     3441     4293

Draws were sampled using sample(hmc). For each parameter, Bulk_ESS
and Tail_ESS are effective sample size measures, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
```

```
bms_1 %>%
  as_draws_df() %>%
  select(b_vowel1:b_vowel4) %>%
  pivot_longer(b_vowel1:b_vowel4) %>%
  group_by(name) %>%
  summarise(
      cri95 = list(round(quantile2(value, probs = c(0.025, 0.975)), 2)),
      cri90 = list(round(quantile2(value, probs = c(0.05, 0.95)), 2)),
      cri80 = list(round(quantile2(value, probs = c(0.1, 0.9)), 2)),
      cri60 = list(round(quantile2(value, probs = c(0.2, 0.8)), 2))
  ) %>%
  knitr::kable(format = "latex") %>% cat(sep = "\n")
```

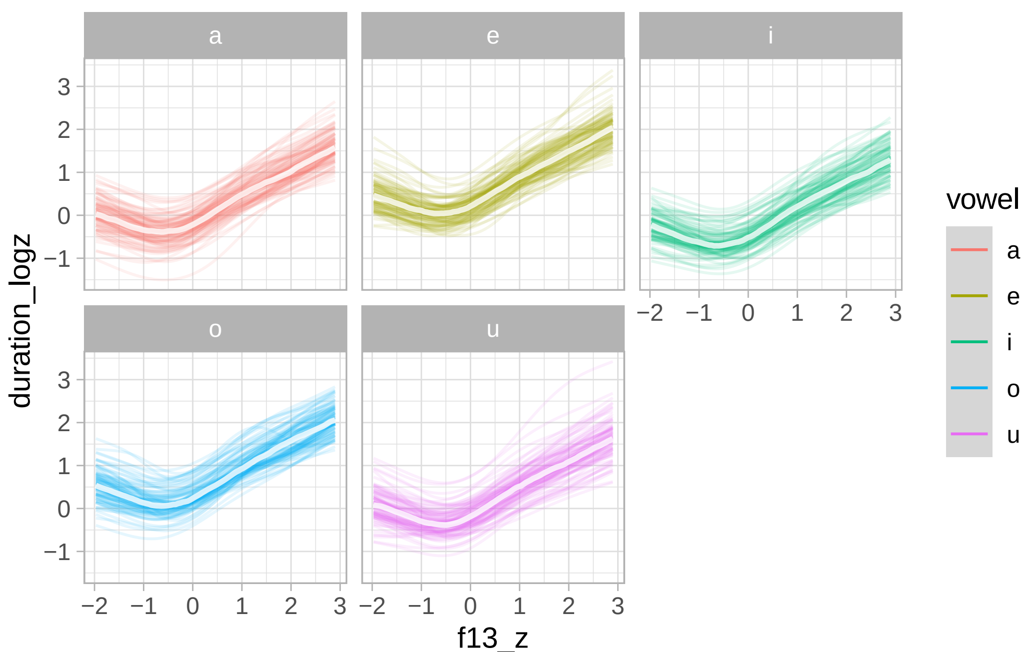
```
Warning: Dropping 'draws_df' class as required metadata was removed.
```

```
\begin{tabular}{l|l|l|l|l}
\hline
name & cri95 & cri90 & cri80 & cri60\\
\hline
b\_vowel1 & -0.38, 0.02 & -0.35, -0.01 & -0.31, -0.05 & -0.26, -0.09\\
\hline
b\_vowel2 & 0.21, 0.37 & 0.22, 0.36 & 0.24, 0.34 & 0.26, 0.32\\
\hline
b\_vowel3 & -0.48, -0.32 & -0.47, -0.34 & -0.45, -0.35 & -0.43, -0.37\\
\hline
b\_vowel4 & 0.26, 0.41 & 0.27, 0.40 & 0.29, 0.39 & 0.30, 0.37\\
\hline
\end{tabular}
```

```
plot(conditional_effects(bms_1, "f13_z:vowel"), plot = FALSE)[[1]] + facet_wrap(~vowel)
```



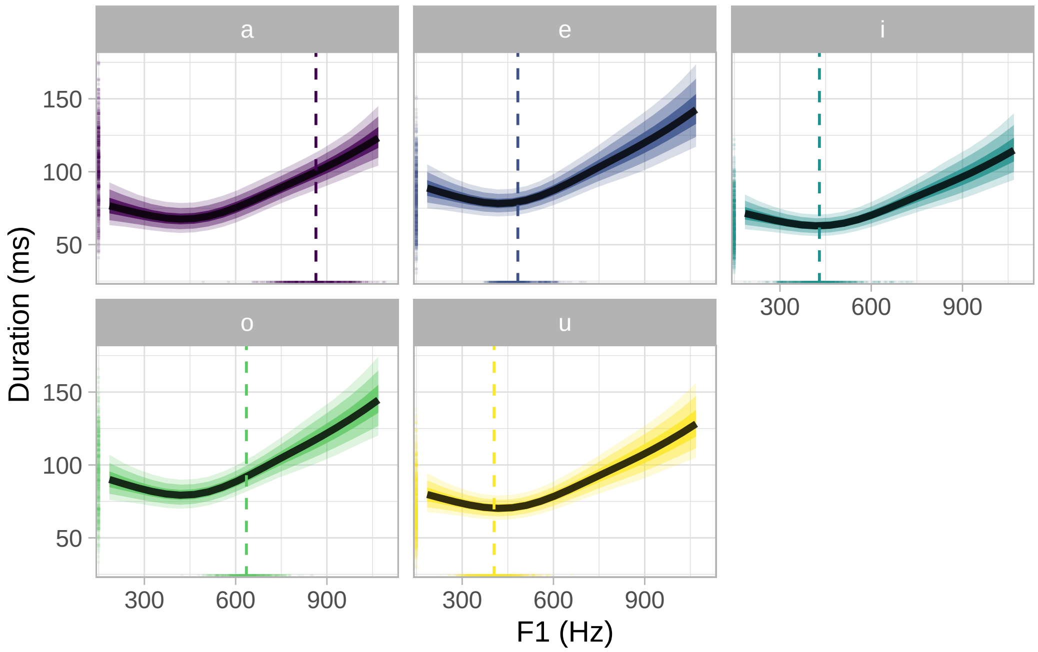

```
plot(conditional_effects(bms_1, "f13_z:vowel", spaghetti = TRUE, ndraws = 100), plot = FAL
```



Let's plot on the original scale.

```
bms_1_grid <- expand_grid(
  vowel = levels(formants$vowel),
  f13_z = seq_minmax(formants$f13_z, 0.25),
  speaker = NA
)

bms_1_preds <- epred_draws(bms_1, newdata = bms_1_grid, re_formula = NA) %>%
  mutate(
    duration_log = .epred * sd(formants$duration_log) + mean(formants$duration_log),
    duration = exp(duration_log),
    f13 = f13_z * sd(formants$f13) + mean(formants$f13)
  )
```

```
bms_1_preds %>%
  group_by(vowel, f13) %>%
  ggplot(aes(f13, duration, fill = vowel)) +
  stat_ribbon(.width = 0.98, alpha = 0.2) +
  stat_ribbon(.width = 0.9, alpha = 0.4) +
  stat_lineribbon(.width = 0.6, alpha = 0.8) +
  geom_vline(data = vmean_f13, aes(xintercept = f13_mean, colour = vowel), linetype = "das
  geom_rug(data = formants, alpha = 0.1, length = unit(0.015, "npc"), aes(colour = vowel))
  facet_wrap(~vowel) +
  labs(
    x = "F1 (Hz)", y = "Duration (ms)"
  ) +
  scale_fill_viridis_d() +
  scale_colour_viridis_d() +
  theme(legend.position = "none")
```




```
ggsave("img/bms1-pred-plot-ms-hz.png", width = 7, height = 5)
```

F1 and F2

```
gam_2 <- bam(
  duration_logz ~
    vowel +
    s(f13_z, f23_z) +
    s(f13_z, f23_z, speaker, bs = "fs", m = 1),
  data = formants
)
```

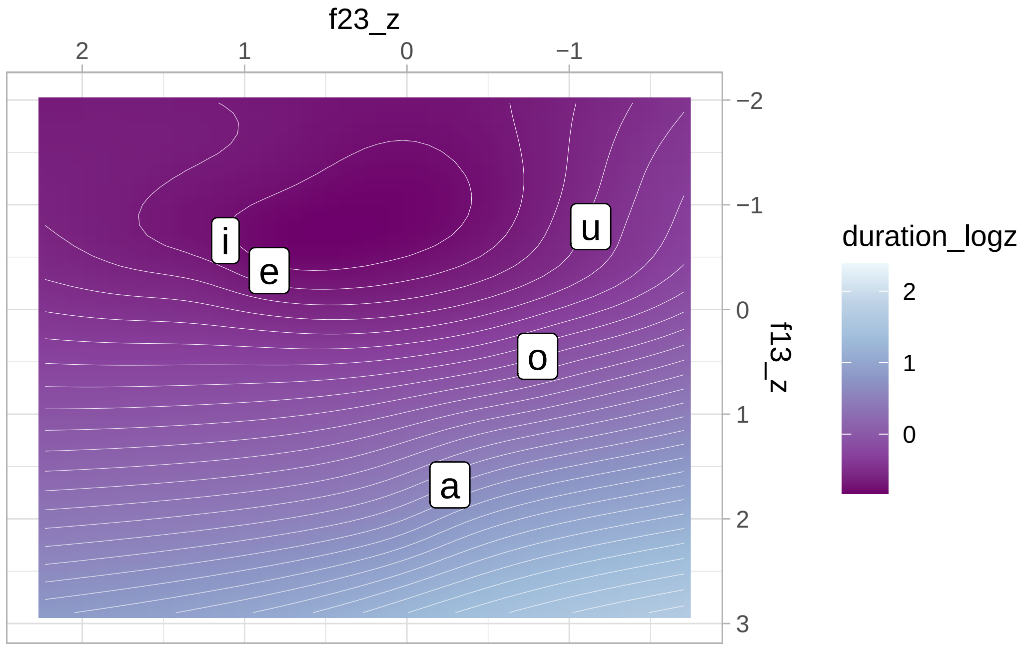
```
summary(gam_2)
```

```
Family: gaussian
Link function: identity

Formula:
duration_logz ~ vowel + s(f13_z, f23_z) + s(f13_z, f23_z, speaker,
```

```
    bs = "fs", m = 1)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.04815    0.14310   0.336    0.737
vowel1      -0.14641    0.09984  -1.466    0.143
vowel2       0.40133    0.05185   7.741 1.35e-14 ***
vowel3      -0.33404    0.05005  -6.675 2.95e-11 ***
vowel4       0.25247    0.04908   5.144 2.87e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
                         edf Ref.df     F p-value
s(f13_z,f23_z)          13.1  17.04 6.503  <2e-16 ***
s(f13_z,f23_z,speaker) 101.9 567.00 6.987  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.708   Deviance explained = 71.9%
fREML = 2606.4  Scale est. = 0.29247   n = 3053
```

```
gam_2_preds <- predict_gam(gam_2, length_out = 50, exclude_terms = "s(f13_z,f23_z,speaker)
```

```
vmeans <- formants %>%
  group_by(vowel) %>%
  summarise(
    f13_z = mean(f13_z), f23_z = mean(f23_z)
  )

gam_2_preds %>%
  ggplot(aes(f23_z, f13_z)) +
  geom_raster(aes(fill = duration_logz), interpolate = TRUE) +
  geom_contour(aes(z = duration_logz), bins = 40, colour = "white", linewidth = 0.05) +
  geom_label(data = vmeans, aes(label = vowel), size = 5) +
  scale_x_reverse(position = "top") +
  scale_y_reverse(position = "right") +
  scale_fill_distiller(palette = "BuPu")
```


f23_z
2
1
0
−1
−2
−1
0
1
2
3
f13_z
i
e
u
o
a
duration_logz
2
1
0