Digital Signal Processing - Assignment 4

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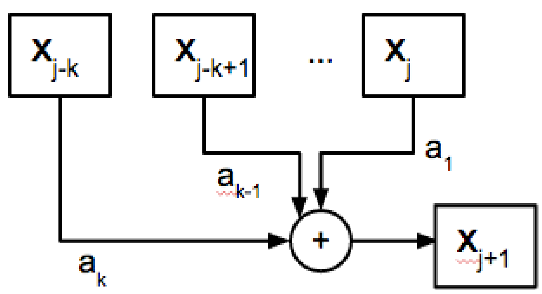
Exercise 1

1.1

The LPC technique predicts the next values from a given signal. It is based on the source-filter model of speech production, which states that speech is created by a source (the vocal cords) and an independent filter (the vocal tract) which creates resonances (formants). It is useful for encoding compressed speech.

1.2



$$x_{j+1} = \sum_{i=0}^{k} a_i x_{j-i}$$

1.3

The error equation for some measured x_n is: $e_n = x_n + \sum_{i=1}^{k} a_i x_{n-i}$

We want to minimize the squared error $\mathcal{E} = E[|e^2|]$ with respect to the a_k parameters.

Therefore, we want to minimize $\mathcal{E} = \frac{1}{N}\sum_{n=1}^{N}(x_n + \sum_{k=1}^{p} a_k x_{n-k})^2$

We can do this by taking the derivative to get $\mathcal{E} = -\frac{1}{N}\sum_{n=1}^{N} x_n x_{n-k}$.

1.4

We start with:

$$\begin{aligned} x(n) &= [-2, 0, 1, -1, 0, 2] \\ x(n+1) &= [0, 1, -1, 0, 2, 0] \\ x(n+2) &= [1, -1, 0, 2, 0, 0] \end{aligned}$$

Therefore, the autocorrelation matrix R_{xx} with order 2 and window size of 6 is:

$$\begin{bmatrix} r(0) & r(1) \\ r(1) & r(0) \end{bmatrix} = \begin{bmatrix} 10/6 & -1/6 \\ -1/6 & 10/6 \end{bmatrix}$$

We want to solve for:

$$R_{xx}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} r(1) \\ r(2) \end{bmatrix} = \begin{bmatrix} -1/6 \\ -4/6 \end{bmatrix}$$

Solving this system, we get:

$$a_1 = -\frac{14}{99} \qquad a_2 = -\frac{41}{99}$$

Therefore we have the prediction:

$$f[6] = a_1 * f[5] + a_2 * f[4] = -\frac{28}{99}$$

1.5

$$\begin{aligned} f[2] = a_1 * f[1] + a_2 * f[0] = &\frac{82}{99} \\ f[3] = -&\frac{14}{99} \\ f[4] = -&\frac{27}{99} \\ f[5] = &\frac{41}{99} \end{aligned}$$

The errors are:

$$\begin{aligned} e_2 = &\frac{17}{99} \\ e_3 = &-\frac{85}{99} \\ e_4 = &\frac{27}{99} \\ e_5 = &\frac{157}{99} \end{aligned}$$

1.6

TODO