

## Exercise 1.1

## Exercise 2.1

a)

If the highest frequency in the signal is  $f_{max} = 5kHz$ , then the minimum sample rate must be  $f_s = 10kHz$ . This also gives us the maximum value of  $T$ :  $T_{max} = f_s^{-1} = 0.0001s$

b)

If the samplerate is  $f_s = 10kHz$ , then  $T = f_s^{-1} = 0.0001s$ . We use this to convert the angular frequency to an ordinary one as follows:

$$\omega = \frac{\pi/8}{T} = 2\pi f_{cutoff}$$

If we solve the above equation for  $f_{cutoff}$  with  $T = 0.0001s$ , we get  $f_{cutoff} = 625Hz$

c)

Same as above with  $T = 0.00005 \implies f_{cutoff} = 1250Hz$

## Exercise 2.2

We can assume that upsampling by factor 3 and then downsampling by factor 3 has no effect on the signal, so it always holds  $x[n] = x_e[n]$ .

To determine whether  $x[n]$  and  $x_r[n]$  are equal we must thus only check whether the filter between  $x_e[n]$  and  $x_r[n]$  has any effect on  $x_e[n]$ . This is the case when  $x_e[n]$  (or effectively  $x[n]$ ) has impulses outside the range  $[-\frac{\pi}{3}, \frac{\pi}{3}]$ , which can be checked by looking at the Fourier transform of the corresponding signals.

Doing this, one can see that only the signal  $x[n]$  from part b) will be affected by the filter, since the FT of a) and c) show impulses within the range of  $[-\frac{\pi}{3}, \frac{\pi}{3}]$ , while the FT of b) clearly has impulses at  $-\frac{\pi}{2}, \frac{\pi}{2}$ .

So, in short:

a)

$$x[n] = \cos\left(\frac{\pi n}{4}\right) \implies x[n] = x_r[n]$$

**b)**

$$x[n] = \cos\left(\frac{\pi n}{2}\right) \implies x[n] \neq x_r[n]$$

**c)**

$$x[n] = \left[\frac{\sin\left(\frac{\pi n}{8}\right)}{\pi n}\right]^2 \implies x[n] = x_r[n]$$