Digital Signal Processing - Assignment 2

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Stephanie Lund (2555914)
Aljoscha Dietrich(2557976)

Exercise 1

a) $$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\ &= \sum_{n=-\infty}^{-1} x[n]e^{-j\omega n} + \sum_{n=0}^{3} x[n]e^{-j\omega n} + \sum_{n=4}^{\infty} x[n]e^{-j\omega n} \\ &= \sum_{n=0}^{3} x[n]e^{-j\omega n} \\ &= 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} \end{aligned}$$

b) $$\begin{aligned} X[k] &= \sum_{n=0}^{N-1} x[n]W_N^{kn} \\ &= \sum_{n=0}^{N-1} x[n](e^{-j\frac{2\pi}{N}})^{kn} \\ &= \sum_{n=0}^{3} x[n](e^{-j\frac{2\pi}{N}})^{kn} \\ &= 1 + (e^{-j\frac{2\pi}{N}})^{k} + (e^{-j\frac{2\pi}{N}})^{2k} + (e^{-j\frac{2\pi}{N}})^{3k} \end{aligned}$$

c) If we take N samples from the formula in part a), then $\omega = \frac{2\pi}{N}k$

$$\begin{aligned} X(e^{j\omega}) &= 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} \\ &= 1 + (e^{-j\frac{2\pi}{N}})^{k} + (e^{-j\frac{2\pi}{N}})^{2k} + (e^{-j\frac{2\pi}{N}})^{3k} \end{aligned}$$

Exercise 2

a) The highest frequency, given by $5cos(50\pi t)$, is 25. The minimum sampling rate must be at least twice the highest frequency, so it would need to be greater than 50.

b) $f_s = 25$. Then $t = \frac{k}{25}$ (where k is an integer), and:

$$\begin{aligned} y(t) &= 10cos(20\pi t - \frac{\pi}{4}) - 5cos(50\pi t) \\ &= 10cos(20\pi t - \frac{\pi}{4}) - 5cos(50\pi \frac{k}{25}) \\ &= 10cos(20\pi t - \frac{\pi}{4}) - 5cos(2\pi k) \\ &= 10cos(20\pi t - \frac{\pi}{4}) - 5 \end{aligned}$$

c) As shown above, $A = -5$

Exercise 3

W_2^0				W_2^1			
W_4^0		W_4^1		W_4^2		W_4^3	
W_8^0	W_8^1	W_8^2	W_8^3	W_8^4	W_8^5	W_8^6	W_8^7
1	$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$	$-i$	$-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$	-1	$-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$	i	$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	p_{17}	p_{18}
2	0	2	0	2	0	2	0

p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	p_{26}	p_{27}	p_{28}
4	0	0	0	4	0	0	0

S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
8	0	0	0	0	0	0	0

Exercise 4

a) $y[0] = h[0]x[0] = 0.5 \cdot 1.0 = 0.5$
$y[1] = h[0]x[1] + h[1]x[0] = 0.5 \cdot 2.0 + 0.5 \cdot 1.0 = 1.5$
$y[2] = h[0]x[2] + h[1]x[1] + h[2]x[0] = 0.5 \cdot 2.0 = 1.0$
$y[n] = [0.5 \quad 1.5 \quad 1.0]$

b) $H(e^{-j\omega}) = \sum_{n=-\infty}^{\infty} h(n) \cdot e^{-i\omega n} = h(0) + h(1) \cdot e^{-i\omega} = \frac{1}{2} \cdot [1 + e^{-j\omega}]$

c)
$$
\begin{aligned}
A(\omega) &= |H(e^{-j\omega})| \\
&= |\frac{1}{2} \cdot [1 + e^{-j\omega}]| \\
&= \frac{1}{2}|1 + cos(-\omega) + isin(-\omega)| \\
&= \frac{1}{2}\sqrt{(1 + cos(\omega))^2 + sin(\omega)^2} \\
&= \frac{1}{2}\sqrt{1 + cos^2(\omega) + 2cos(\omega) + sin^2(\omega)} \\
&= \frac{1}{2}\sqrt{2 + 2cos(\omega)} \\
&= \frac{1}{2}\sqrt{4(\frac{1 + cos(\omega)}{2})} \\
&= cos(\frac{\omega}{2})
\end{aligned}
$$

Then, for $\omega = 2\pi f$, the amplitude response as a function of f is $cos(\frac{2\pi f}{2})$ and looks as follows: