Digital Signal Processing - Assignment 4

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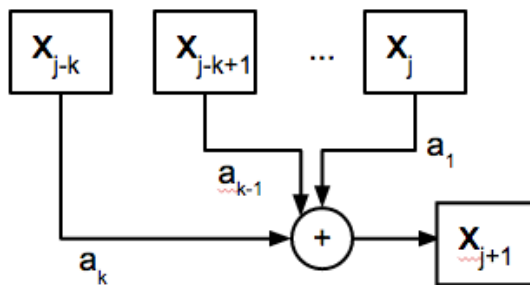Aljoscha Dietrich(2557976)

Exercise 1

1.1

The LPC technique predicts the next values from a given signal. It is based on the source-filter model of speech production, which states that speech is created by a source (the vocal cords) and an independent filter (the vocal tract) which creates resonances (formants). It is useful for encoding compressed speech.

1.2

$$x_{j+1} = \sum_{i=0}^{k} a_i x_{j-i}$$

1.3

The error equation for some measured x_n is: $e_n = x_n + \sum_{i=1}^{k} a_i x_{n-i}$

We want to minimize the squared error $\mathcal{E} = E[|e^2|]$ with respect to the a_k parameters.

Therefore, we want to minimize $\mathcal{E} = \frac{1}{N}\sum_{n=1}^{N}(x_n + \sum_{k=1}^{p} a_k x_{n-k})^2$

We can do this by taking the derivative to get $\mathcal{E} = -\frac{1}{N}\sum_{n=1}^{N} x_n x_{n-k}$.

1.4

We start with:

$$x(n) = [-2, 0, 1, -1, 0, 2]$$
$$x(n+1) = [0, 1, -1, 0, 2, 0]$$
$$x(n+2) = [1, -1, 0, 2, 0, 0]$$

Therefore, the autocorrelation matrix R_{xx} with order 2 and window size of 6 is:

$$\begin{bmatrix} x(n) * x(n) & x(n) * x(n+1) \\ x(n) * x(n+1) & x(n) * x(n) \end{bmatrix} = \begin{bmatrix} 10/6 & -1/6 \\ -1/6 & 10/6 \end{bmatrix}$$

We want to solve for:

$$R_{xx}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} x(n) * x(n+1) \\ x(n+1) * x(n+2) \end{bmatrix} = \begin{bmatrix} -1/6 \\ -4/6 \end{bmatrix}$$

Solving this system, we get:

$$a_1 = -\frac{14}{99} \qquad a_2 = -\frac{41}{99}$$

Therefore we have the prediction:

$$f[6] = a_1 * f[5] + a_2 * f[4] = -\frac{28}{99}$$

1.5

$$f[2] = a_1 * f[1] + a_2 * f[0] = \frac{82}{99}$$

$$f[3] = -\frac{14}{99}$$

$$f[4] = -\frac{27}{99}$$

$$f[5] = \frac{41}{99}$$

The errors are:

$$e_2 = \frac{17}{99}$$

$$e_3 = -\frac{85}{99}$$

$$e_4 = \frac{27}{99}$$

$$e_5 = \frac{157}{99}$$

1.6

TODO