

## Exercise 1

### 1.1

This is the Method of Least Squares, which can be solved as follows. First, find the partial derivatives by  $a$  and  $b$ , and set them to 0:

$$\frac{\partial \epsilon}{\partial a} = \sum_{i=1}^N 2x_i(ax_i + b - y_i) = 0$$

$$\frac{\partial \epsilon}{\partial b} = \sum_{i=1}^N 2(ax_i + b - y_i) = 0$$

Next, rewrite them as a series of linear equations:

$$\left[ \sum_{i=1}^N x_i^2 \right] a + \left[ \sum_{i=1}^N x_i \right] b = \sum_{i=1}^N x_i y_i$$

$$\left[ \sum_{i=1}^N x_i \right] a + \left[ \sum_{i=1}^N 1 \right] b = \sum_{i=1}^N y_i$$

Then solve the equation for  $a$  and  $b$  (the matrix is invertible as long as all of the  $x_i$  are not equal):

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^N x_i^2 & \sum_{i=1}^N x_i \\ \sum_{i=1}^N x_i & \sum_{i=1}^N 1 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^N x_i y_i \\ \sum_{i=1}^N y_i \end{pmatrix}$$

## 1.2

TODO

## Exercise 2

TODO