Digital Signal Processing - Assignment 8

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Exercise 1

1.1

We can use the error function:

$$e(n) = s(n) - \hat{s}(n) = s(n) - h(n) * x(n)$$

converted to the frequency domain to find $J(\omega)$. Using the properties of additivity, and that convolution in the time domain is multiplication in the frequency domain, this is:

$$J(\omega) = S(\omega) - H(\omega)X(\omega)$$

1.2

$$E[J(\omega)^2] = E[(S(\omega) - H(\omega)X(\omega))^2]$$

$$= E[(S(\omega))^2 - 2S(\omega)H(\omega)X(\omega) + (H(\omega)X(\omega))^2]$$

$$\frac{\partial J}{\partial H(\omega)} = E[-2S(\omega)X(\omega) + 2H(\omega)X(\omega)] = 0$$

TODO: finish this

1.3

From $X(\omega) = S(\omega) + N(\omega)$, we have:

$$H(\omega) = \frac{\Phi_{sx}(\omega)}{\Phi_{xx}(\omega)}$$

$$= \frac{E[X(\omega)S(\omega)] - E[X(\omega)]E[S(\omega)]}{E[X(\omega)^2] - E[X(\omega)]^2}$$

$$= \frac{E[(S(\omega) + N(\omega))S(\omega)] - E[(S(\omega) + N(\omega))]E[S(\omega)]}{E[(S(\omega) + N(\omega))^2] - E[(S(\omega) + N(\omega))]^2}$$

$$= \frac{E[(S(\omega)^2] + E[N(\omega)S(\omega)] - (E(S(\omega)]^2 + E[N(\omega)S(\omega)])}{E[S(\omega)^2] + 2E[S(\omega)]E[N(\omega)] + E[N(\omega)^2] - (E[S(\omega)]^2 + 2E[S(\omega)]E[N(\omega)] + E[N(\omega)]^2)}$$

$$= \frac{\Phi_{ss}(\omega)}{\Phi_{ss}(\omega) + \Phi_{nn}(\omega)}$$

1.4

We can multiply the frequency response $H(\omega)$ by the Fourier transform of the input $X(\omega)$ to get the result in the frequency domain, then take the inverse Fourier transform to find the denoised signal in the time domain $\hat{s}(n)$.

1.5

Using the equation from question 1.3, this is:

$$H(\omega) = \frac{\Phi_{ss}(\omega)}{\Phi_{ss}(\omega) + \Phi_{nn}(\omega)}$$

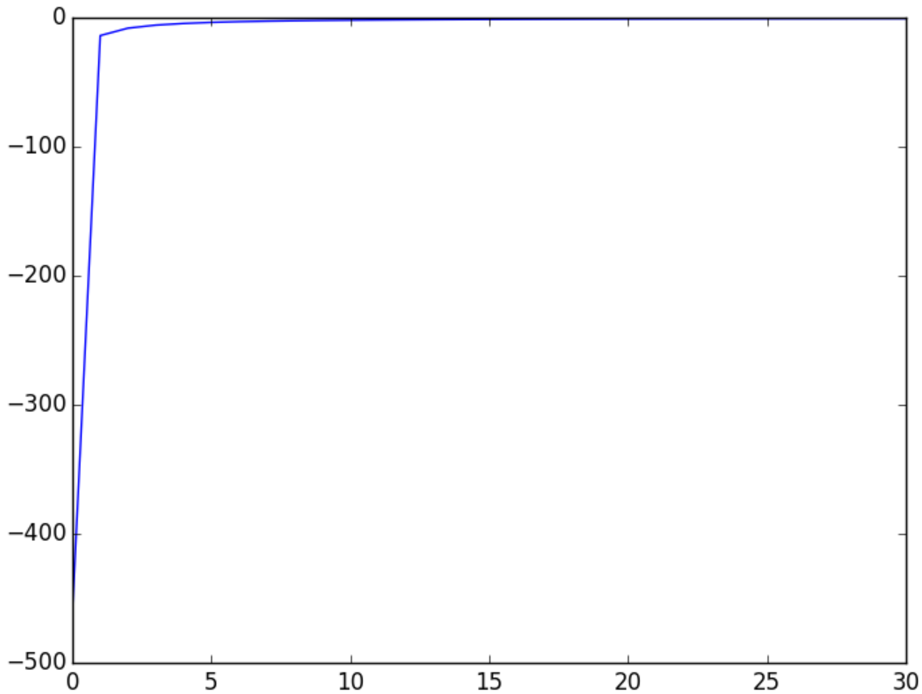
$$= \frac{\Phi_{ss}(\omega)/\Phi_{nn}(\omega)}{\Phi_{ss}(\omega)/\Phi_{nn}(\omega) + \Phi_{nn}(\omega)/\Phi_{nn}(\omega)}$$

$$= \frac{SNR_k}{SNR_k + 1}$$

1.6

For SNR_k in the range $[0, 30]$, using the above equation we get the frequency responses:
$[0, 0.5, 0.66666667, 0.75, 0.8, 0.83333333, 0.85714286, 0.875, 0.88888889, 0.9, 0.90909091,$
$0.91666667, 0.92307692, 0.92857143, 0.93333333, 0.9375, 0.94117647, 0.94444444, 0.94736842,$
$0.95, 0.95238095, 0.95454545, 0.95652174, 0.95833333, 0.96, 0.96153846, 0.96296296, 0.96428571,$
$0.96551724, 0.96666667, 0.96774194]$

If we take $20 * log(H(\omega) + \epsilon)$ (where ϵ is a small number to prevent taking the log of 0) and plot it against SNR_k, we get the following graph:


0
−100
−200
−300
−400
−500
0
5
10
15
20
25
30