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Info 235

Laboratorul 3

(1.) i) Pentru curba Bézier de gradul m dată de vectorii de poziție $b_0, \dots, b_m$, se poate construi curba identică de grad $(m+1)$, dată de punctele de control $c_0, \dots, c_{m+1}$, a.i.

$$\begin{cases} c_0 = b_0 \\ c_k = \frac{m+1-k}{m+1} b_k + \frac{k}{m+1} b_{k-1}, \quad k = \overline{1,m} \\ c_{m+1} = b_m \end{cases}$$

În particular, pentru $m = 3$, $b_0 = (-3,1)$, $b_1 = (-4,4)$, $b_2 = (4,4)$, $b_3 = (3,1)$, găsim:

$$c_0 = b_0 = (-3,1)$$

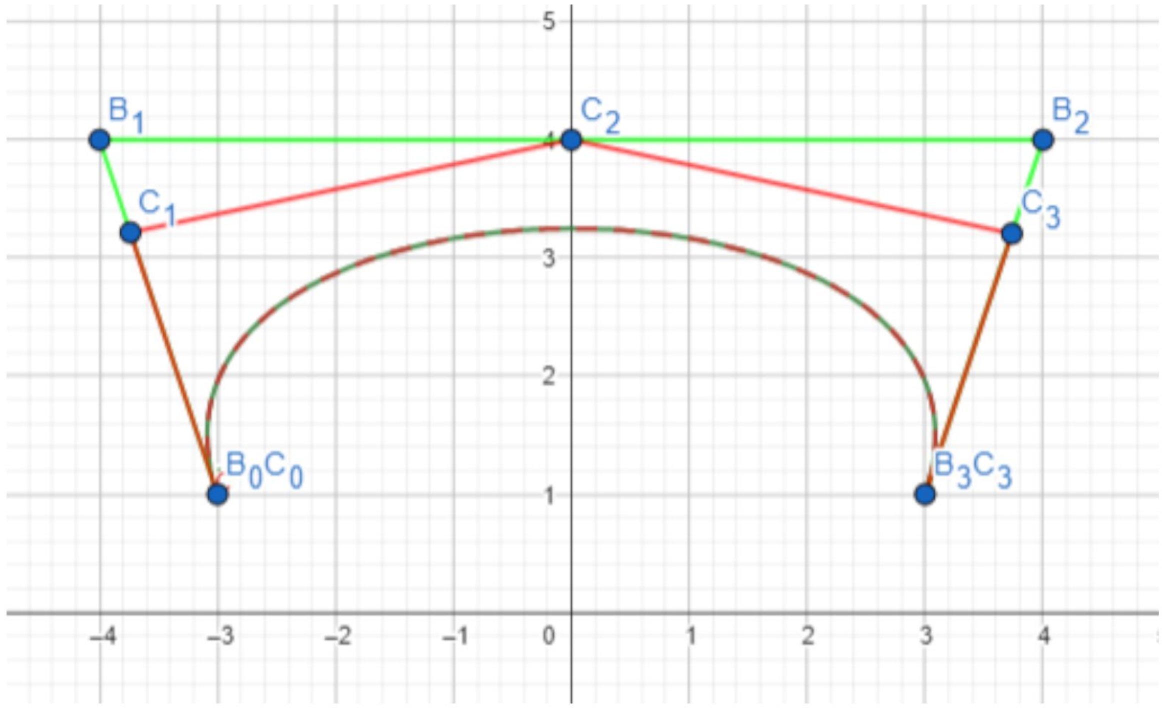
$$c_1 = \frac{3}{4} b_1 + \frac{1}{4} b_0 = \left(\frac{-12-3}{4}, \frac{12+1}{4}\right) = \left(-\frac{15}{4}, \frac{13}{4}\right)$$

$$c_2 = \frac{2}{4} b_2 + \frac{2}{4} b_1 = \left(\frac{8-8}{4}, \frac{8+8}{4}\right) = (0,4)$$

$$c_3 = \frac{1}{4} b_3 + \frac{3}{4} b_2 = \left(\frac{3+12}{4}, \frac{1+12}{4}\right) = \left(\frac{15}{4}, \frac{13}{4}\right)$$

$$c_4 = b_3 = (3,1)$$

ii)



② Pentru curba Bézier dată de punctele de control $b_0 \dots b_n$ și un interval $[a,b] \subsetneq [0,1]$, curba Bézier care descrie curba segmentată pe intervalul $[a,b]$ are pct. de control

$$c_k = C(a)^{n-k} \cdot C(b)^k \cdot \begin{pmatrix} b_0 \\ \vdots \\ b_n \end{pmatrix}.$$

Aplicăm pentru $n = 3$ și $b_0 - b_3$ de la ①, de două ori pentru (I) $[a,b] = [0, \frac{1}{3}]$, respectiv (II) $[a,b] = [\frac{1}{3}, 1]$.

Înainte de asta, rescriem ecuațiile lui c_k din teoremă particularizate pentru $n = 3$:

$$c_0 = C(a)^3 \cdot C(b)^0 \cdot \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = C_1(a) C_2(a) C_3(a) \begin{pmatrix} b_0 \\ \vdots \\ b_3 \end{pmatrix}$$

$$c_1 = C(a)^2 \cdot C(b)^1 \cdot \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = C_1(a) C_2(a) \cdot C_3(b) \begin{pmatrix} b_0 \\ \vdots \\ b_3 \end{pmatrix}$$

$$c_2 = C(a) \cdot C(b)^2 \cdot \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = C_1(a) \cdot C_2(b) \cdot C_3(b) \begin{pmatrix} b_0 \\ \vdots \\ b_3 \end{pmatrix}$$

$$c_3 = C(a)^0 C(b)^3 \cdot \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = C_1(b) \cdot C_2(b) \cdot C_3(b) \cdot \begin{pmatrix} b_0 \\ \vdots \\ b_3 \end{pmatrix}$$

$$\Rightarrow c_0 = \{1-a \quad a\} \begin{Bmatrix} 1-a & a & 0 \\ 0 & 1-a & a \end{Bmatrix} \left\{ \begin{bmatrix} 1-a & a & 0 & 0 \\ 0 & 1-a & a & 0 \\ 0 & 0 & 1-a & a \end{bmatrix} \right\} \begin{Bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

$$= \{1-a \quad a\} \begin{bmatrix} 1-a & a & 0 \\ 0 & 1-a & a \end{bmatrix} \begin{bmatrix} (1-a)b_0 + ab_1 \\ (1-a)b_1 + ab_2 \\ (1-a)b_2 + ab_3 \end{bmatrix}$$

$$= \{1-a \quad a\} \begin{bmatrix} (1-a)((1-a)b_0 + ab_1) + a((1-a)b_1 + ab_2) \\ (1-a)((1-a)b_1 + ab_2) + a((1-a)b_2 + ab_3) \end{bmatrix}$$

$$c_1 = [(1-a) \quad a] \begin{bmatrix} (1-a)((1-b)b_0 + bb_1) + a((1-b)b_1 + bb_2) \\ (1-a)((1-b)b_1 + bb_2) + a((1-b)b_2 + bb_3) \end{bmatrix}$$

$$c_2 = \{1-a \quad a\} \begin{bmatrix} (1-b)((1-b)b_0 + bb_1) + b((1-b)b_1 + bb_2) \\ (1-b)((1-b)b_1 + bb_2) + b((1-b)b_2 + bb_3) \end{bmatrix}$$

$$c_3 = [1-b \quad b] \begin{bmatrix} (1-b)((1-b)b_0 + bb_1) + b((1-b)b_1 + bb_2) \\ (1-b)((1-b)b_1 + bb_2) + b((1-b)b_2 + bb_3) \end{bmatrix}$$

I. $a = 0, \; b = \frac{1}{3}$

$$\Rightarrow c_0 = [1 \cdot 0] \cdot \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = b_0 = (-3, 1)$$

$$c_1 = \left(1 - \frac{1}{3}\right) b_0 + \frac{1}{3} b_1 = \frac{2b_0 + b_1}{3} = \left(-\frac{10}{3}, 2\right)$$

$$c_2 = \frac{2}{3}\left(\frac{2}{3} b_0 + \frac{1}{3} b_1\right) + \frac{1}{3}\left(\frac{2}{3} b_1 + \frac{1}{3} b_2\right) = \left(-\frac{8}{3}, \frac{8}{3}\right)$$

$$c_3 = \frac{2}{3}\left(\frac{2}{3}\left(\frac{2b_0 + b_1}{3}\right) + \frac{1}{3}\left(\frac{2b_1 + b_2}{3}\right)\right) + \frac{1}{3}\left(\frac{2}{3} \cdot \frac{2b_1 + b_2}{3} + \frac{1}{3} \cdot \frac{2b_2 + b_3}{3}\right)$$

$$= \left(-\frac{15}{9}, \frac{27}{9}\right) = \left(-\frac{5}{3}, 3\right)$$

II $\quad a = \frac{1}{3}, b = 1$

$$c_0 = \frac{2}{3}\left(\frac{2}{3}\cdot\frac{2b_0+b_1}{3} + \frac{1}{3}\cdot\frac{2b_1+b_2}{3}\right) + \frac{1}{3}\left(\frac{2}{3}\cdot\frac{2b_1+b_2}{3} + \frac{1}{3}\cdot\frac{2b_2+b_3}{3}\right) = \left(-\frac{5}{3}, \frac{27}{9}\right) = \left(-\frac{5}{3}, 3\right)$$

$$c_1 = \frac{2}{3}\left(\frac{2}{3}b_1 + \frac{1}{3}b_2\right) + \frac{1}{3}\left(\frac{2}{3}b_2 + \frac{1}{3}b_3\right) = \left(\frac{1}{3}, \frac{11}{3}\right)$$

$$c_2 = \frac{2}{3}(b_2) + \frac{1}{3}(b_3) = \left(\frac{11}{3}, 3\right)$$

$$c_3 = b_3 = (3, 1)$$

Prin urmare, cele două curbe sunt date de punctele de control:

c: $\quad c_0(-3, 1);\ c_1\left(-\frac{10}{3}, 2\right);\ c_2\left(-\frac{8}{3}, \frac{8}{3}\right);\ c_3\left(-\frac{5}{3}, 3\right)$

c': $\quad c_0'\left(-\frac{5}{3}, 3\right);\ c_1\left(\frac{1}{3}, \frac{11}{3}\right);\ c_2\left(\frac{11}{3}, 3\right);\ c_3(3, 1).$

(Mi-am amintit târziu că puteam calcula $\frac{2b_i+b_{i+1}}{3}$ separat și nu mai ajungeam la formule atât de lungi...)

ii)