Recipes for multilevel imputation

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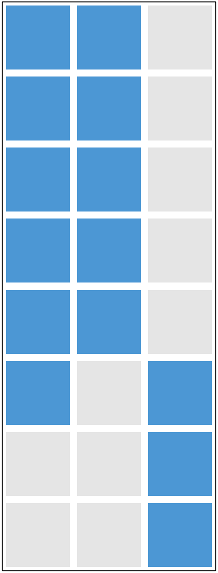
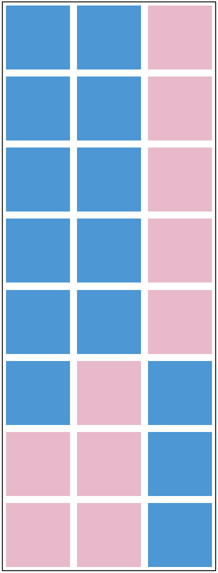
Main question

Can we use `mice` for multilevel data, and if so, how?

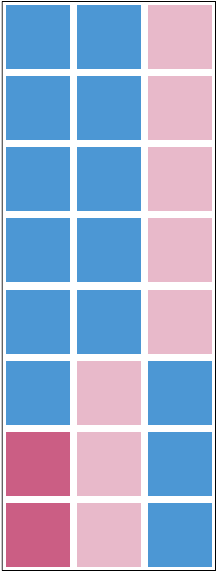
Imputation by fully conditional specification



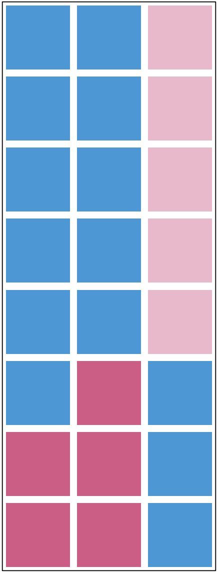
Imputation by fully conditional specification



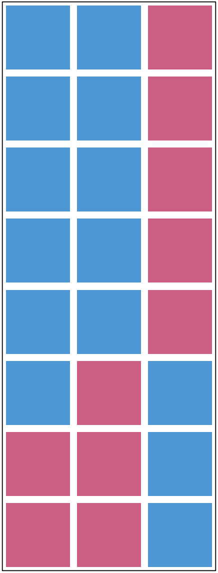
Imputation by fully conditional specification



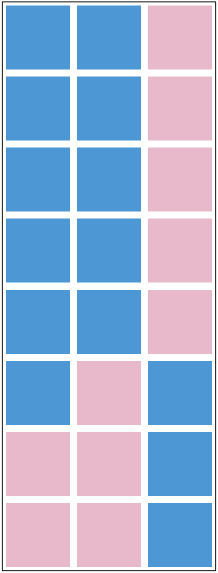
Imputation by fully conditional specification



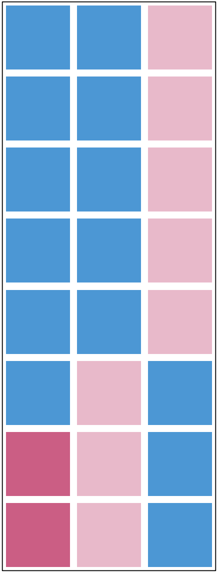
Imputation by fully conditional specification



Imputation by fully conditional specification - next iteration



Imputation by fully conditional specification - next iteration



brandsma data

- Brandsma and Knuver, Int J Ed Res, 1989.
- Extensively discussed in Snijders and Bosker (2012), 2nd ed.
- 4106 pupils, 216 schools, about 4% missing values

brandsma data subset

```r
library(mice)
d <- brandsma[, c("sch", "lpo", "sex", "den")]
head(d, 2)
```

```
##   sch lpo sex den
## 1   1  NA   1   1
## 2   1  50   1   1
```

- `sch`: School number, cluster variable, $C = 216$;
- `lpo`: Language test post, outcome at pupil level;
- `sex`: Sex of pupil, predictor at pupil level (0-1);
- `den`: School denomination, predictor at school level (1-4).

Model of scientific interest

Predict `lpo` from the

- level-1 predictor `sex`
- level-2 predictor `den`

Level notation - Bryk and Raudenbush (1992)

$$
\begin{aligned}
\texttt{lpo}_{ic} &= \beta_{0c} + \beta_{1c}\texttt{sex}_{ic} + \epsilon_{ic} && (1)\\
\beta_{0c} &= \gamma_{00} + \gamma_{01}\texttt{den}_{c} + u_{0c} && (2)\\
\beta_{1c} &= \gamma_{10} && (3)
\end{aligned}
$$

- lpo_{ic} is the test score of pupil i in school c
- sex_{ic} is the sex of pupil i in school c
- den_{c} is the religious denomination of school c
- β_{0c} is a random intercept that varies by cluster
- β_{1c} is a sex effect, assumed to be the same across schools.
- $\epsilon_{ic} \sim N(0, \sigma_\epsilon^2)$ is the within-cluster random residual at the pupil level

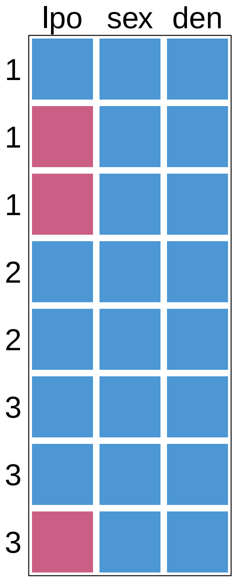
Where are the missings?

In single level data, missingness may be in the outcome and/or in the predictors

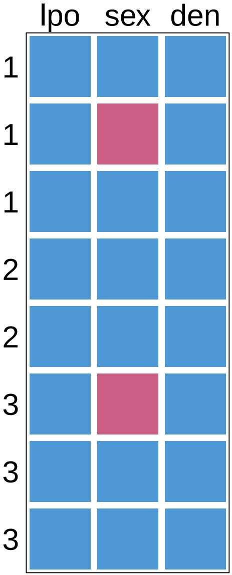
With multilevel data, missingness may be in:

1. the outcome variable;
2. the level-1 predictors;
3. the level-2 predictors;
4. the class variable.

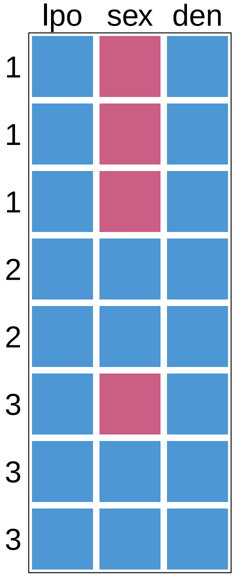
Univariate missing, level-1 outcome



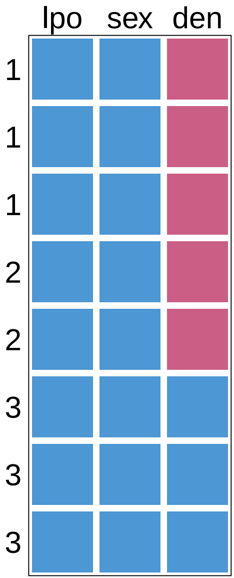
Univariate missing, level-1 predictor, sporadically missing



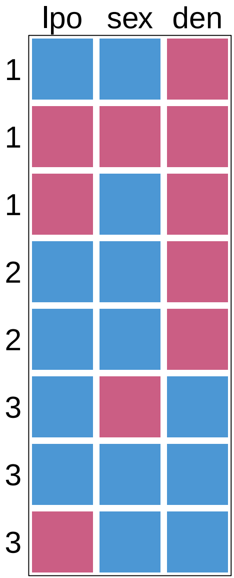
Univariate missing, level-1 predictor, systematically missing



Univariate missing, level-2 predictor



Multivariate missing



Fully conditional specification for multilevel data

$$\dot{\texttt{lpo}}_{ic} \sim N(\beta_0 + \beta_1 \texttt{den}_c + \beta_2 \texttt{sex}_{ic} + u_{0c}, \sigma_\epsilon^2) \tag{4}$$

$$\dot{\texttt{sex}}_{ic} \sim N(\beta_0 + \beta_1 \texttt{den}_c + \beta_2 \texttt{lpo}_{ic} + u_{0c}, \sigma_\epsilon^2) \tag{5}$$

Theoretical problem with FCS

Conditional expectation of $\texttt{sex}_{ic}$ in a random effects model depends on

- $\texttt{lpo}_{ic}$,
- $\overline{\texttt{lpo}}_i$, the mean of cluster i, and
- n_i, the size of cluster i.

Resche-Rigon & White (2018) suggest the imputation model

- should incorporate the cluster means of level-1 predictors
- be heteroscedastic if cluster sizes vary

General imputation/modeling sequence - START SIMPLE

1. Pick a simple complete-data model
2. Create imputations using an imputation template
3. Check the imputes (convergence/plausibility)
4. Estimate parameters
5. Make complete-data model more realistic, go to 1.

See https://stefvanbuuren.name/fimd/sec-mlguidelines.html

Seven imputation templates, increasing complexity

1. *Intercept-only model, missing outcomes*
2. *Random intercepts, missing level-1 predictor*
3. Random intercepts, contextual model
4. *Random intercepts, missing level-2 predictor*
5. Random intercepts, interactions
6. Random slopes, missing outcomes and predictors
7. Random slopes, interactions

1 Intercept-only model, missing outcomes (model)

$$\texttt{lpo}_{ic} = \beta_{0c} + \epsilon_{ic} \tag{6}$$

$$\beta_{0c} = \gamma_{00} + u_{0c} \tag{7}$$

1 Intercept-only model, missing outcomes (imputation)

```
d <- brandsma[, c("sch", "lpo")]
pred <- make.predictorMatrix(d)
pred["lpo", "sch"] <- -2
imp <- mice(d, pred = pred, meth = "2l.pmm", m = 10, maxit
            print = FALSE, seed = 152)
```

1 Intercept-only model, missing outcomes (analysis)

```
library(lme4)
```

```
## Loading required package: Matrix
```

```
fit <- with(imp, lmer(lpo ~ (1 | sch), REML = FALSE))
summary(pool(fit))
```

```
##             estimate std.error statistic   df p.value
## (Intercept)     40.9     0.322       127 3368       0
```

1 Intercept-only model, missing outcomes (variances)

```
library(mitml)
testEstimates(as.mitml.result(fit), var.comp = TRUE)$var.co
```

```
##                                Estimate
## Intercept~~Intercept|sch         18.021
## Residual~~Residual               63.306
## ICC|sch                           0.222
```

2 Random intercepts, missing level-1 (model)

$$\texttt{lpo}_{ic} = \beta_{0c} + \beta_{1c}\texttt{iqv}_{ic} + \epsilon_{ic} \quad (8)$$

$$\beta_{0c} = \gamma_{00} + u_{0c} \quad (9)$$

$$\beta_{1c} = \gamma_{10} \quad (10)$$

- Missing values in both `lpo` and `iqv`

2 Random intercepts, missing level-1 (imputation)

```
d <- brandsma[, c("sch", "lpo", "iqv")]
pred <- make.predictorMatrix(d)
pred["lpo", ] <- c(-2, 0, 3)
pred["iqv", ] <- c(-2, 3, 0)
imp <- mice(d, pred = pred, meth = "2l.pmm", seed = 919,
            m = 10, print = FALSE)
```

- Impute `lpo` from `iqv` *and* the cluster means of `iqv`
- Impute `iqv` from `lpo` *and* the cluster means of `lpo`
- Alternative: Use `mitml::panImpute()` or `mitml::jomoImpute()`

2 Random intercepts, missing level-1 (predictorMatrix)

```
pred
```

```
##     sch lpo iqv
## sch   0   1   1
## lpo  -2   0   3
## iqv  -2   3   0
```

2 Random intercepts, missing level-1 (analysis)

```
fit <- with(imp, lmer(lpo ~  iqv + (1 | sch), REML = FALSE
summary(pool(fit))
```

```
##             estimate std.error statistic   df p.value
## (Intercept)    40.96    0.2378       172 3337       0
## iqv             2.52    0.0525        48 2127       0
```

```
testEstimates(as.mitml.result(fit), var.comp = TRUE)$var.co
```

```
##                              Estimate
## Intercept~~Intercept|sch        9.479
## Residual~~Residual             40.862
## ICC|sch                         0.188
```

4 Random intercepts, missing level-2 predictor (model)

$$\texttt{lpo}_{ic} = \beta_{0c} + \beta_{1c}\texttt{iqv}_{ic} + \epsilon_{ic} \tag{11}$$

$$\beta_{0c} = \gamma_{00} + \gamma_{01}\texttt{den}_c + u_{0c} \tag{12}$$

$$\beta_{1c} = \gamma_{10} \tag{13}$$

- Missing values in `lpo`, `iqv` and `den`
- For `den` the imputation model uses school level aggregates

4 Random intercepts, missing level-2 (imputation)

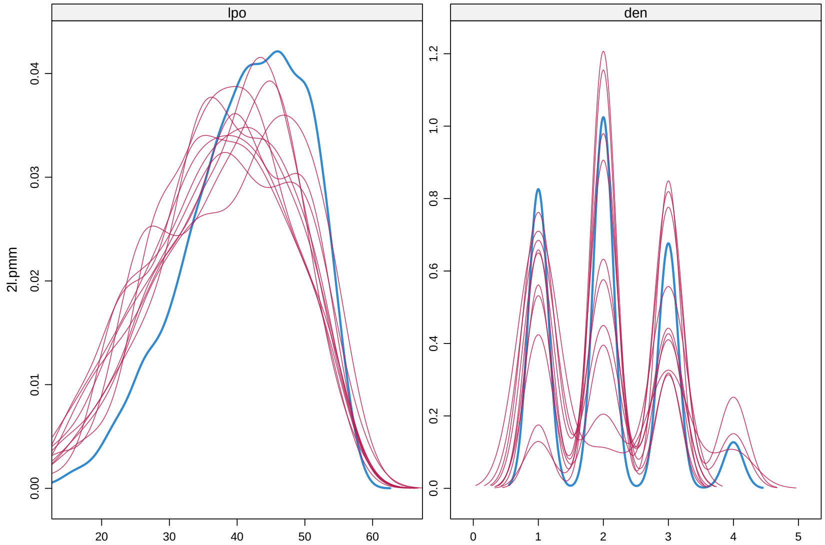
```r
d <- brandsma[, c("sch", "lpo", "iqv", "den")]
meth <- make.method(d)
meth[c("lpo", "iqv", "den")] <- c("2l.pmm", "2l.pmm",
                                   "2lonly.pmm")
pred <- make.predictorMatrix(d)
pred["lpo", ] <- c(-2, 0, 3, 1)
pred["iqv", ] <- c(-2, 3, 0, 1)
pred["den", ] <- c(-2, 1, 1, 0)
imp <- mice(d, pred = pred, meth = meth, seed = 418,
            m = 10, print = FALSE)
```

4 Random intercepts, missing level-2 (predictorMatrix)

```
pred
```

```
##     sch lpo iqv den
## sch   0   1   1   1
## lpo  -2   0   3   1
## iqv  -2   3   0   1
## den  -2   1   1   0
```

4 Random intercepts, missing level-2 (density)



4 Random intercepts, missing level-2 (analysis)

```
##                     estimate std.error statistic   df  p.val
## (Intercept)           40.071    0.4549     88.09  187 0.0000
## iqv                    2.516    0.0532     47.34 1242 0.0000
## as.factor(den)2        2.041    0.5925      3.45  430 0.0005
## as.factor(den)3        0.234    0.6519      0.36  285 0.7192
## as.factor(den)4        1.843    1.1642      1.58 1041 0.1137

##                            Estimate
## Intercept~~Intercept|sch      8.621
## Residual~~Residual           40.761
## ICC|sch                       0.175
```

Classic recipe for single-level data: Which predictors?

1. Include all variables that appear in the complete-data model
2. Include variables related to the nonresponse
3. Include variables that explain a considerable amount of variance
4. Remove from variables selected in steps 2 and 3 those variables that have too many missing values within the subgroup of incomplete cases

Does this recipe also apply to multilevel data?

Recipe: Missing level-1

	Recipe for a level-1 target
1.	Define the most general analytic model
2.	Select a `2l` method that imputes close to the data
3.	Include all level-1 variables
4.	Include the disaggregated cluster means of level-1 variables
5.	Include all level-1 interactions implied by analytic model
6.	Include all level-2 predictors
7.	Include all level-2 interactions implied by analytic model
8.	Include all cross-level interactions implied by analytic model
9.	Include predictors related to the missingness and the target
10.	Exclude any terms involving the target

Recipe: Missing level-2

	Recipe for a level-2 target
1.	Define the most general analytic model
2.	Select a `2lonly` method that imputes close to the data
3.	Include the cluster means of all level-1 variables
4.	Include the cluster means of all level-1 interactions
5.	Include all level-2 predictors
6.	Include all interactions of level-2 variables
7.	Include predictors related to the missingness and target
8.	Exclude any terms involving the target

Conclusion

Can we use `mice` for multilevel data, and if so, how?

- Hot spot of current research
- Multilevel imputation: more complex, but doable
- Start simple, take small steps
- Build upon templates and modeling recipes
- Study https://stefvanbuuren.name/fimd/sec-mlguidelines.html
- Gain confidence at each step
- Start playing around...