Quantile Regression Estimates of Body Weight at Length for Walleye

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1   Management Brief:

2   Quantile Regression Estimates of Body Weight at Length in Walleye

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24   <A>Abstract

25   Quantile regression is a method of estimating fish weight at length for alternate portions

26   of a probability distribution that has not received much attention in fisheries literature. Previous

27   research has demonstrated that quantile regression can provide estimates of 75th quantile weight

28   at length without bias, which is advantageous compared to previously-defined standard weight

29   (W_s) equations derived from linear or quadratic regression methods. The goal of this study was to

30   demonstrate the utility of quantile regression as a tool to assess fish weight at length at various

31   portions of the probability distribution without bias using walleye *Sander vitreus* as a case study.

32   Quantile regression models at the 75th quantile were developed for three randomly selected

33   walleye populations from Georgia and South Dakota and compared to a large (n = 33,589)

34   reference population. Bootstrap resampling procedures indicated that only one population from

35   the state of Georgia had an intercept and slope similar to the reference population. For the one

36   population that had similar intercept and slope to the reference population, predictions of weight

37   at various lengths still fell below the 95% confidence intervals for predicted weights of the

38   reference population, suggesting that slight differences in intercepts and slopes in allometric

39   relationships can result in predicted weights that still differ at some lengths. Predicted weights of

40   walleye derived from the 10th, 25th, 50th, 75th, and 90th quantiles were used to demonstrate how

41   individuals and populations may be compared at different management targets. Overall, this

42   study demonstrates the relative ease with which quantile regression may be used to compare fish

43   body condition between populations without bias.

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47   <A>Introduction

48   Standard weight (W_s) equations and the concept of relative weight (W_r) were developed

49   so that fisheries managers would have a "quick, inexpensive, and useful way of obtaining and

50   interpreting fishery data for management purposes" (Wege and Anderson 1978). However, many

51   W_s equations exhibit length-related biases, hampering evaluations of fish body condition which

52   influences their ability to accurately assess fish body condition at all lengths (Gerow et al. 2005;

53   Ranney et al. 2010; Ranney et al. 2011). Additionally, there are a number of statistical concerns

54   regarding estimating W_s equations, estimating the length-related biases associated with W_s

55   equations (e.g., Gerow 2011; Ranney et al. 2011), and interpreting W_r (Brendan et al. 2003; Pope

56   and Kruse 2007). Despite these concerns, the use of W_r as an evaluation tool has expanded

57   beyond the scope envisioned by the originators (Cade et al. 2008; Ranney et al. 2011). Standard

58   weight equations and W_r have been used to test whether body condition differs among or within

59   fish populations and parametric tests are frequently used to compare W_r data (Murphy et al.

60   1990; Hyatt and Hubert 2001; Brendan et al. 2003). However, Cade et al. (2008) suggested that

61   other means of comparing fisheries populations may provide a higher level of statistical rigor and

62   be more relevant to the questions being asked. One tool to better compare changes in weight-

63   length relationships is quantile regression (Cade and Noon 2003; Cade et al. 2008; Cade et al.

64   2011; Crane et al. 2015; Crane and Farrell 2017).

65   Quantile regression is a method to estimate different quantiles of a response variable

66   distribution (Koenker and Bassett 1978; Cade and Noon 2003). In most regression applications,

67   the mean response variable is estimated as a function of the predictor variable (Cade and Noon

68   2003). For example, the linear weight-as-a-function-of-length model, common in fisheries

69   applications,

70
$$\log_{10}(W) = a + b \cdot \log_{10}(TL)$$

71   is an estimate of the mean response of $\log_{10}$-transformed weight (W) as a function of total $\log_{10}$-

72   transformed total length (TL). Quantile regression estimates use individual quantiles of the

73   response variable (i.e., W) as a function of the predictor variable (i.e., TL). Thus, for a given

74   quantile τ, $Q_W(\tau|TL)$ is the τth quantile Q as a function of TL (Cade et al. 2008). Quantile

75   regression allows for estimation of all quantiles of W as a function of TL from $\tau = 0.01\text{-}0.99$

76   (Cade et al. 2008).

77   Length-biased W_s equations may lead to low biased estimates of W_r at the upper length

78   ranges for many species. Indeed, Ranney et al. (2010 and 2011) found that W_s equations

79   developed for walleye *Sander vitreus* using either the regression-line percentile (RLP) technique

80   (Murphy et al. 1990) or the empirical percentile (EmP) method (Gerow et al. 2005) were biased

81   low. Regressing the 75th quantile of walleye weight as a function of length can generate unbiased

82   predictions of walleye weight at the 75th quantile of weight (Cade et al. 2008). Using direct

83   estimates from any quantile of fish weight—from 0.01 to 0.99—would provide a more

84   statistically-valid means of comparing fish populations or individual fish to a reference

85   population (Cade et al. 2008). However, little attention has been given to this method in the

86   fisheries literature to date.

87   The goal of this study was demonstrate the utility of quantile regression as a tool to assess

88   fish weight at length without bias using walleye as a case study. Direct comparison of

89   population-specific quantile regression lines—based on well-established statistical theory—show

90   how managers can directly compare fish populations to each other or to a reference population.

91   Predicted weights of walleye derived from $\tau = 0.10, 0.25, 0.50, 0.75,$ and 0.90 across 10

92   millimeter (mm) length classes demonstrate how populations may be compared at different

93 management targets. Walleye were considered an ideal candidate species for this study based on

94 previous analyses (see Ranney et al. 2010, 2011), availability of data from a large geographic

95 area, and their recreational and economic importance (Aiken 2011).

96

97 <A>Methods

98   I used walleye data from Ranney et al. (2010, 2011) to generate regression models. I

99 filtered these data prior to analyses with the filtering methods described in Ranney et al. (2010)

100 because data quality can influence parameter estimation, model fit, and predictive ability

101 (Belsley et al. 1980). I refer to these data and any quantile regressions from this dataset as the

102 "reference" dataset and regressions. I solicited additional walleye weight and length data from

103 state fisheries management agencies in Georgia and South Dakota. I randomly selected from

104 each state three walleye populations. Sample size was not a consideration when randomly

105 selecting populations. I refer to these data and the quantile regressions from these datasets as the

106 "state" datasets and regressions.

107   I used the linear quantile regression function rq() in the quantile regression package

108 "quantreg" (Koenker 2017) in R version 3.3.3 (R Development Core Team 2017) to regress

109 $\log_{10}W$ as a function of $\log_{10}TL$ to estimate 75th quantile intercepts (β_0) and slopes (β_1) from the

110 reference and state population data. I used $\tau = 0.75$ because this is similar to the established value

111 used in W_s equations and could be considered an estimator of fish with "above average" body

112 weight. Previous quantile regression analyses (i.e., Cade et al. 2008; Cade et al. 2011) have

113 shown that a single linear model can provide estimates for separate population-level β_0 and β_1

114 when a categorical factor (i.e., reference and each state population identifier) and its interaction

115 with the continuous predictor variable (i.e., $\log_{10}TL$) are included in the same model. I estimated

116   the standard error of β_0 and β_1 by resampling with replacement 5,000 $\log_{10}W$-$\log_{10}TL$ pairs 1,000

117   times. I calculated 95% confidence intervals around the β_0 and β_1 with the t statistic = 1.960.

118   I used the same linear model described above to estimate which state populations had

119   different β_0 and β_1 from the reference population. To do this, I set the reference population as the

120   base level then set the contrasts to be among the rest of the categorical factors (population

121   identifier and its interaction with the continuous predictor variable). With the contrasts set, the

122   coefficient estimates from the model then become the estimated differences in β_0 and β_1 of each

123   population from the reference population rather than the actual estimates of β_0 and β_1. Similar to

124   estimating the standard error of the β_0 and β_1 for each population-level model, I estimated the

125   standard error of the differences in β_0 and β_1 from the reference population with resampling. I

126   resampled 5,000 $\log_{10}W$-$\log_{10}TL$ pairs 1,000 times and calculated the 95% confidence intervals

127   around the differences in β_0 and β_1 with the t statistic = 1.960. Alpha for all statistical tests was

128   set to 0.05.

129   I compared quantiles of weight at specific lengths from each state population to the

130   reference population by regressing $\log_{10}W$ as a function of $\log_{10}TL$ with τ = 0.05, 0.10, …, 0.95

131   in 0.05 increments for each state populations. I then estimated weight and 95% confidence

132   intervals of weight for the midpoints of the length categories for walleye [midpoints: substock =

133   125 mm; stock-quality = 315 mm; quality-preferred = 445; preferred-memorable = 570 mm;

134   memorable-trophy = 695 mm (Gabelhouse 1984)].

135   To demonstrate how individual fish may be compared at different management targets, I

136   used the reference dataset to create linear quantile regressions of $\log_{10}W$ as a function of $\log_{10}TL$

137   at τ = 0.10, 0.25, 0.50, 0.75, and 0.90 and predicted total weight values of each τ at 10-mm

138   length classes from 155 mm to 745 mm. I pooled data from 102 populations (N = 33,597

139   walleye) and assigned equal weights to all populations regardless of contributed sample size

140   (Cade et al. 2008). These values of τ are appropriate benchmarks of comparison for population

141   weight data. Estimating the 90th quantile of weight as a function of length corresponds to a fish

142   of "excellent" body weight; the 75th quantile allows for a comparison of fish to the reference

143   population "above average" body weight; the 50th quantile allows for comparison of fish to a

144   reference population "average" body weight; the 25th quantile allows for comparison of fish to a

145   reference population "below average" body weight; the 10th quantile allows for comparison of

146   fish to a reference population "poor" body weight. I exponentiated the predicted values of

147    $\log_{10}W$ to convert weight back to g [e.g., $10^{(\log_{10}W)}$].

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149   <A>Results

150   The filtered reference data set included 102 populations and 33,597 individual

151   observations of weight and length. The three randomly sampled populations from each state

152   contained $N = 313$, 795, and 199 for the GA1, GA2, and GA3 populations, respectively, and $N =$

153   392, 280, and 140 for the SD1, SD2, and SD3 populations, respectively. Individuals from

154   populations in Georgia were collected with fall gillnetting and late-winter electrofishing.

155   Individuals from South Dakota populations were collected with summer gillnets. Collection of

156   individuals in the reference dataset was of unknown methods.

157   The 75th quantile regression model β_0 and β_1 estimates for the reference dataset were

158   -5.702 and 3.277, respectively (Table 1). The 95% confidence interval for the reference data set

159   was $\beta_0 = -5.717$-5.687 and $\beta_1 = 3.271$-3.282. The lowest β_0 was in the GA 1 population (-5.713)

160   and the largest was in the SD 2 population (-5.174; Table 1). The lowest β_1 value in the state

161   population data was in the SD 2 population (3.070) and the largest in the GA 1 population

162   (3.275; Table 1).

163   Bootstrapped 95% confidence intervals of 75th quantile β_0 and β_1 for the reference dataset

164   overlapped the 75th quantile estimates of slope and intercept for only one of three Georgia

165   populations and none of the South Dakota populations (Table 2). Estimates and 95% confidence

166   intervals of weight at total length equal to the midpoints of length categories across all quantiles

167   from $\tau = 0.05, \ldots, 0.95$ by 0.05 increments overlapped considerably for all three populations from

168   Georgia and the reference population, especially at TL = 125 mm (Figure 1, Panel A). Overlap in

169   the confidence intervals for all Georgia populations and the reference population is evident

170   across the remaining four weight-at-total-length estimates, though becomes less pronounced at

171   the upper length ranges for GA2. There was less overlap in the confidence interval bands for the

172   South Dakota populations at lower lengths (TL = 125, 315, and 445 mm) than at higher lengths

173   (TL = 570 and 695 mm; Figure 1, Panel B). Estimates of weight at $\tau = 0.10, 0.25, 0.50, 0.75,$ and

174   0.90 from the reference population for each 10-mm length class ranged from 23.6, 25.3, 27.3,

175   29.8, and 32.1 g at the 155 mm length class to 4,107.1, 4.385.1, 4.730.0, 5,114.9, and 5,532.0 g

176   at the 745 mm length class (Table 3).

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178   <A>Discussion

179   Quantile regression of fisheries weight-length data is a means to compare individual fish

180   weights and population regression data without the biases and statistical limitations inherent in

181   W_s equations and W_r values (Cade et al 2008). While W_r is an appropriate tool for evaluating how

182   a management action may impact fish condition, quantile regression of a reference dataset(s)

183   provide fisheries scientists and managers a means by which conspecific fish populations and

184   individuals can be compared with statistical rigor (Cade et al. 2008; Ranney et al. 2011).

185   Fisheries scientists and managers may consider that W_r and W_s still have a use in fisheries

186   science; however, the biases inherent in W_s equations (see Ranney et al. 2010 and 2011 for a

187   detailed discussion) do not provide the statistical validity with which to compare individuals and

188   populations. Those working with weight-length data should think long and hard to consider

189   whether this tool is relevant to the questions they are asking. Linear quantile regression provides

190   the capability to compare unbiased estimates of quantiles and is rooted in standard linear model

191   procedures rather than the ad-hoc methods in use today to compare W_r values.

192   Estimates of weight at length from quantile regression can be used in place of W_r in

193   nearly all instances in which W_r is subject to statistical analysis. Linear quantile regression is a

194   more statistically rigorous tool that can be used to compare changes in weight-length

195   relationships pre- and posttreatment or to compare the weight-length relationships of two

196   separate populations (e.g., Cade et al. 2008; Cade et al. 2011; Crane et al. 2015; Crane and

197   Farrell 2017). Populations that are monitored on a yearly basis (e.g., trophy fisheries or

198   threatened and endangered populations) are also well-suited to quantile regression analysis.

199   Slight differences in β_0 and β_1 in allometric relationships can result in large differences in

200   predictions of weight at various lengths. For example, though GA1 and the reference population

201   had similar β_0 and β_1 (Table 1), the predictions of weight at the 75th quantile for the GA1

202   population at higher lengths (Figure 1, Panel A) fell outside the 95% confidence intervals for the

203   reference dataset. Similarly, differences in β_0 and β_1 in allometric relationships does not imply

204   that predicted weights will be different across length ranges. At TL = 315 (Figure 1, Panel B),

205   the SD1 and SD2 populations had almost identical predictions of weight at the 75th quantile

206   when compared to the reference population. This suggests that the joint effect of β_0 and β_1 is

207   what matters most and that it is still difficult to make conclusive statements about how

208   predictions (and populations) differ until actual predictions of weight have been compared with

209   each other. I could have achieved better precision in the confidence intervals around the

210   predictions of weight at length at various quantiles for both the state and reference populations

211   had I resampled > 5,000 weight-length pairs and bootstrapped the regressions > 1,000 times.

212   However, resampling pairs to the size of the entire dataset ($n = 35,716$) and running 10,000

213   bootstrap regressions would have required significant additional computing time.

214   With unbiased estimates of 75th quantile weight for 10-mm length categories, a fisheries

215   manager (or angler) can compare the length and weight of an individual fish to the reference

216   values in Table 3 to determine how that individual compares to the reference population. For

217   example, if a 580 mm walleye weighs 2,100 g, the fisheries manager can compare those values

218   to the 585 mm length class in Table 3. Thus, a fish that weighs 2,100 g and is 580 mm long is

219   somewhere between the 25th and 50th quantile estimate of weight for the reference population. If

220   interested, a fisheries manager could determine at which τ that 580 mm fish weighing 2,100 g

221   lies. Further, any additional quantile weights between $\tau = 0.01$ and 0.99 and their confidence

222   intervals may be readily calculated and included in a tabular form as a quick reference for fish

223   body weight to identify potential areas for growth bottlenecks. For example, a fisheries manager

224   who may be responsible for the SD1 and SD2 populations population can determine that the 75th

225   quantile of fish at the S-Q midpoint (Figure 1, Panel B, TL = 315) are tracking well with the

226   reference dataset. However, at the M-T midpoint (Figure 1, Panel B, TL = 695), walleye from

227   those same populations fall well below the species-wide standards at all estimated quantiles

228   except $\tau = 0.15$ and below. This could be an indication that prey availability at length categories

229   beyond the Q-P category may be limiting walleye growth. Similarly, the manager responsible for

230   all Georgia populations can see that the populations of walleye at the Q-P midpoint (Figure 1,

231   Panel A, TL = 445) are all tracking well with the species-wide reference populations. Table 3

232   could easily be converted to imperial measurements if so desired.

233   Though I have refrained from referring to quantile regressions predictions of weight as

234   estimates of "condition," I did select the quantiles shown in Table 3 for a reason. Historically, W_s

235   equations were estimated at the 75th percentile (Wege and Anderson 1978) and if the ratio of

236   individual fish weight to $W_s \times 100$ was greater than 100, then that individual is assumed to be in

237   good or 'above average' condition (Wege and Anderson 1978; Neumann et al. 2012). Here,

238   because quantile regression is estimating the response of $\log_{10}$-transformed fish weight as a

239   function of $\log_{10}$-transformed total length at the quantile specified, we can use phrases like

240   "excellent", "above average," "average", "below average", and "poor" weight for $\tau = 0.90, 0.75,$

241   0.50, 0.25, and 0.10, respectively, because that is one interpretation of what those quantiles

242   represent. Other quantiles and more-refined definitions could be established to identify

243   benchmarks for comparison in other species or for different management priorities. Lower values

244   of τ for non-game or threatened or endangered fishes may be a more appropriate benchmark for

245   comparison. State and regional fisheries scientists and managers can determine which quantile or

246   set of quantiles best fits the question they are asking or the species they are investigating. The

247   ease with which quantile models can be estimated and compared makes the need to set a

248   "standard" quantile (or set of quantiles) superfluous.

249   I have used a reference dataset that spans a large portion of the geographic range of

250   walleye. Given that this paper is meant as an introduction to using quantile regression to estimate

251   fish weight at length, this seemed like a reasonable first step. This is not meant to suggest,

252   however, that fisheries scientists and managers should always use such a robust data set to which

253 they can compare their own populations. Indeed, though fisheries managers in Georgia may want

254 to compare their populations to those of the upper Midwest United States periodically, it seems

255 more likely that managers in one region would be most interested in making within-region

256 comparisons. As a result, regional reference data sets could be established and used as reference

257 populations, depending upon the comparison of interest. Fisheries scientists and managers could

258 easily define a useful organization of geography and habitat (e.g., reservoirs, lakes, rivers, etc.)

259 so that consistency within a region is maintained. Making the decision as to which regions and

260 habitats should be grouped together is challenging; however, the well-developed statistical

261 method by which those comparisons can be made should allow fisheries scientists and managers

262 to spend more time considering the organization question rather than having to consider how to

263 conduct that analysis.

264 The minimum sample size needed for a reference dataset has not been investigated.

265 However, given that confidence interval estimates are made with the t statistic (Zar 1999), if

266 sample sizes are low, then confidence intervals of any range will be less precise. The minimum

267 number of populations that should be included in a reference dataset has also not been

268 investigated. Provided that the reference dataset contains enough populations to be representative

269 of the species across the spatial extent of the comparison(s) being made, the number of

270 populations in the reference dataset is likely of little concern because all populations are given

271 equal weights, regardless of contributed sample size (Cade et al. 2008).

272 Many state and provincial management agencies already have standard methods in place

273 to collect fisheries weight-length data. In these instances, I would not presume to suggest

274 alternative methods for "standardized" collections. For management agencies that do not have a

275 standard method by which to collect fisheries length-weight data for a given species, I would

276 recommend establishing protocols as soon as is reasonable (see Bonar et al. 2009 for guidelines).

277 Recognizing that time of year may impact weight of an individual fish (e.g., pre-spawn vs. post-

278 spawn), state reference datasets that are compiled from a standardized collection method would

279 be consistent. For larger, regional reference datasets, the variability inherent across standardized

280 collection methods from contributing management agencies (e.g., pre-spawn vs. late summer vs.

281 under ice) would likely negate any biases that could be inherent in the data (e.g., heavier fish

282 collected during pre-spawn, lighter fish collected in mid-winter). Additionally, resampling when

283 estimating the β_0 and β_1 coefficients of the quantile regression models as described above would

284 serve to reduce the influence of collection method on estimates of β_0, β_1, or the differences

285 between the β_0 and β_1 of the reference population and any other populations being compared.

286 Frequency of reference dataset compilation should be determined by fisheries scientists

287 and managers. Climate change may impact different geographic regions at different rates. As a

288 result, defining the frequency with which reference datasets should be established is beyond the

289 scope of this paper. Climate change could impact growth of fishes across geographic regions and

290 latitudes and reference datasets should reflect those changes. Regardless of how climate change

291 affects fish growth, the ease with which population comparisons can be made with quantile

292 regression will allow fisheries scientists and managers more time to think about their research

293 questions and management goals rather than the best way to conduct analyses of their data.

294 Quantile regressions of fisheries $\log_{10}$-transformed weight-length data provide a

295 statistically valid means of comparing any quantile estimates of weight. While I do not advocate

296 eliminating W_s and W_r as a means of evaluating fish condition in management-appropriate

297 situations, quantile regression provides a simple, tractable, and statistically-valid means of

298 directly evaluating fisheries weight at length for alternate portions of a probability distribution

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299   (Cade and Noon 2003). Quantile regression procedures are available in common data analysis

300   software: XLStat for Excel, PROC QUANTREG in SAS, and the "statsmodels" library in

301   Python. It is my hope that quantile regression will become a common means of evaluating

302   various distributions of fish weight at length and will help fisheries scientists and managers

303   compare weight-at-length of disparate populations without the length biases of W_s and the ad-hoc

304   methods used to analyze W_r data.

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377   \<A\> Supplements

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379   A repository that includes the data and R code used in this manuscript is available at

380   https://github.com/stevenranney/waeQuantiles.

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389   Table 1. Estimates of intercept and slope values from bootstrapped replicates of quantile

390   regression with $\tau = 0.75$ for seven different data sets of walleye. Bootstrap replicates

391   were estimated by resampling with replacement 5,000 $\log_{10}$-weight and $\log_{10}$-length pairs

392   1,000 times.

Population	Intercept (β_0)			Slope (β_1)		
	2.5%	Estimate	97.5%	2.5%	Estimate	97.5%
Reference	-5.717	-5.702	-5.687	3.271	3.277	3.282
GA1	-5.954	-5.713	-5.471	3.186	3.275	3.364
GA2	-5.551	-5.451	-5.352	3.135	3.173	3.212
GA3	-5.678	-5.508	-5.339	3.135	3.201	3.266
SD1	-5.497	-5.425	-5.353	3.140	3.169	3.197
SD2	-5.356	-5.174	-4.992	3.000	3.070	3.139
SD3	-5.692	-5.603	-5.514	3.186	3.223	3.259

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406   Table 2. Differences and 95% confidence intervals around the differences in intercepts and slope

407   of each state population from the reference population. *P*-value is from a test of the null

408   hypothesis that the intercept and slope from the quantile regression where $\tau = 0.75$ for

409   each state population is equal the reference population intercept and slope. Asterisk

410   indicates populations with significantly different intercept and slope from the reference

411   population.

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Population	Intercept (β_0)				Slope (β_1)			
	2.5%	Estimate	97.5%	*p*-value	2.5%	Estimate	97.5%	*p*-value
GA1	-0.2484	-0.0107	0.2271	0.9299	-0.0898	-0.0019	0.0861	0.9668
GA2*	0.1486	0.2507	0.3528	< 0.0001	-0.1423	-0.1032	-0.064	< 0.0001
GA3*	0.0062	0.194	0.3819	0.0429	-0.1483	-0.0757	-0.0032	0.0407
SD1*	0.2024	0.2772	0.352	< 0.0001	-0.1375	-0.108	-0.0784	< 0.0001
SD2*	0.3549	0.5281	0.7014	< 0.0001	-0.273	-0.207	-0.141	< 0.0001
SD3*	0.0129	0.0991	0.1853	0.0242	-0.089	-0.054	-0.019	0.0025

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425   Table 3. Weight (g) estimates of walleye *Sander vitreus* at 10-mm length class for τ = 0.10, 0.25,

426   0.50, 0.75, and 0.90 of the reference data set. Estimates of weight were derived from

427   linear quantile regressions of $\log_{10}$-weight as a function of $\log_{10}$-length.

Total length (mm)	Quantile (τ)				
	0.10	0.25	0.50	0.75	0.90
155	23.6	25.3	27.3	29.8	32.1
165	28.9	31.0	33.6	36.6	39.5
175	35.1	37.7	40.7	44.4	47.8
185	42.1	45.2	48.9	53.3	57.4
195	50.1	53.7	58.1	63.3	68.2
205	59.0	63.3	68.5	74.6	80.4
215	69.1	74.0	80.1	87.2	94.0
225	80.2	86.0	92.9	101.2	109.1
235	92.5	99.1	107.2	116.7	125.8
245	106.1	113.7	122.9	133.7	144.2
255	121.0	129.7	140.2	152.5	164.4
265	137.3	147.1	159.0	172.9	186.6
275	155.1	166.1	179.6	195.3	210.6
285	174.4	186.8	201.9	219.5	236.8
295	195.4	209.2	226.1	245.8	265.2
305	218.0	233.4	252.2	274.1	295.8
315	242.4	259.5	280.4	304.7	328.8
325	268.6	287.6	310.7	337.6	364.3
335	296.8	317.7	343.2	372.8	402.4
345	326.9	349.9	378.0	410.5	443.1
355	359.1	384.3	415.2	450.8	486.6
365	393.4	421.0	454.8	493.8	533.1
375	430.0	460.1	497.0	539.5	582.5
385	468.8	501.7	541.8	588.1	635.0
395	510.1	545.7	589.4	639.6	690.7
405	553.8	592.4	639.8	694.2	749.7
415	600.0	641.8	693.1	752.0	812.1
425	648.9	694.0	749.5	813.0	878.0
435	700.4	749.1	808.9	877.4	947.6
445	754.7	807.2	871.6	945.2	1,021.0
455	812.0	868.3	937.5	1,016.6	1,098.1
465	872.1	932.6	1,006.9	1,091.7	1,179.3
475	935.3	1,000.1	1,079.7	1,170.5	1,264.5
485	1,001.6	1,070.9	1,156.1	1,253.2	1,353.9
495	1,071.1	1,145.1	1,236.2	1,339.9	1,447.6

505	1,143.9	1,222.9	1,320.1	1,430.6	1,545.7
515	1,220.1	1,304.2	1,407.8	1,525.6	1,648.4
525	1,299.7	1,389.3	1,499.6	1,624.8	1,755.7
535	1,382.9	1,478.1	1,595.4	1,728.4	1,867.8
545	1,469.7	1,570.8	1,695.4	1,836.5	1,984.7
555	1,560.2	1,667.4	1,799.6	1,949.3	2,106.6
565	1,654.5	1,768.2	1,908.2	2,066.7	2,233.7
575	1,752.8	1,873.0	2,021.4	2,189.0	2,366.0
585	1,855.0	1,982.1	2,139.1	2,316.3	2,503.6
595	1,961.3	2,095.6	2,261.4	2,448.5	2,646.7
605	2,071.8	2,213.5	2,388.6	2,586.0	2,795.4
615	2,186.5	2,336.0	2,520.6	2,728.7	2,949.8
625	2,305.6	2,463.1	2,657.7	2,876.8	3,110.0
635	2,429.1	2,594.9	2,799.8	3,030.3	3,276.1
645	2,557.1	2,731.5	2,947.1	3,189.5	3,448.4
655	2,689.8	2,873.1	3,099.8	3,354.4	3,626.8
665	2,827.1	3,019.7	3,257.8	3,525.2	3,811.6
675	2,969.3	3,171.4	3,421.4	3,701.9	4,002.8
685	3,116.4	3,328.3	3,590.6	3,884.6	4,200.5
695	3,268.5	3,490.5	3,765.5	4,073.5	4,405.0
705	3,425.7	3,658.2	3,946.3	4,268.8	4,616.3
715	3,588.0	3,831.4	4,133.0	4,470.4	4,834.5
725	3,755.6	4,010.2	4,325.8	4,678.5	5,059.8
735	3,928.6	4,194.8	4,524.7	4,893.3	5,292.2
745	4,107.1	4,385.1	4,730.0	5,114.9	5,532.0

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438   **Figure Legends**

439   Figure 1. Quantile regression (τ = 0.05, …, 0.95 by 0.05 increments) predictions and 95%

440   confidence intervals of weight for the midpoint of each length category [substock, stock-

441   quality, quality-preferred, preferred-memorable, memorable-trophy (Gabelhouse 1984)]

442   for the reference dataset and three populations of walleye from Georgia (A) and South

443   Dakota (B). Confidence bands around the Reference population are narrow and appear as
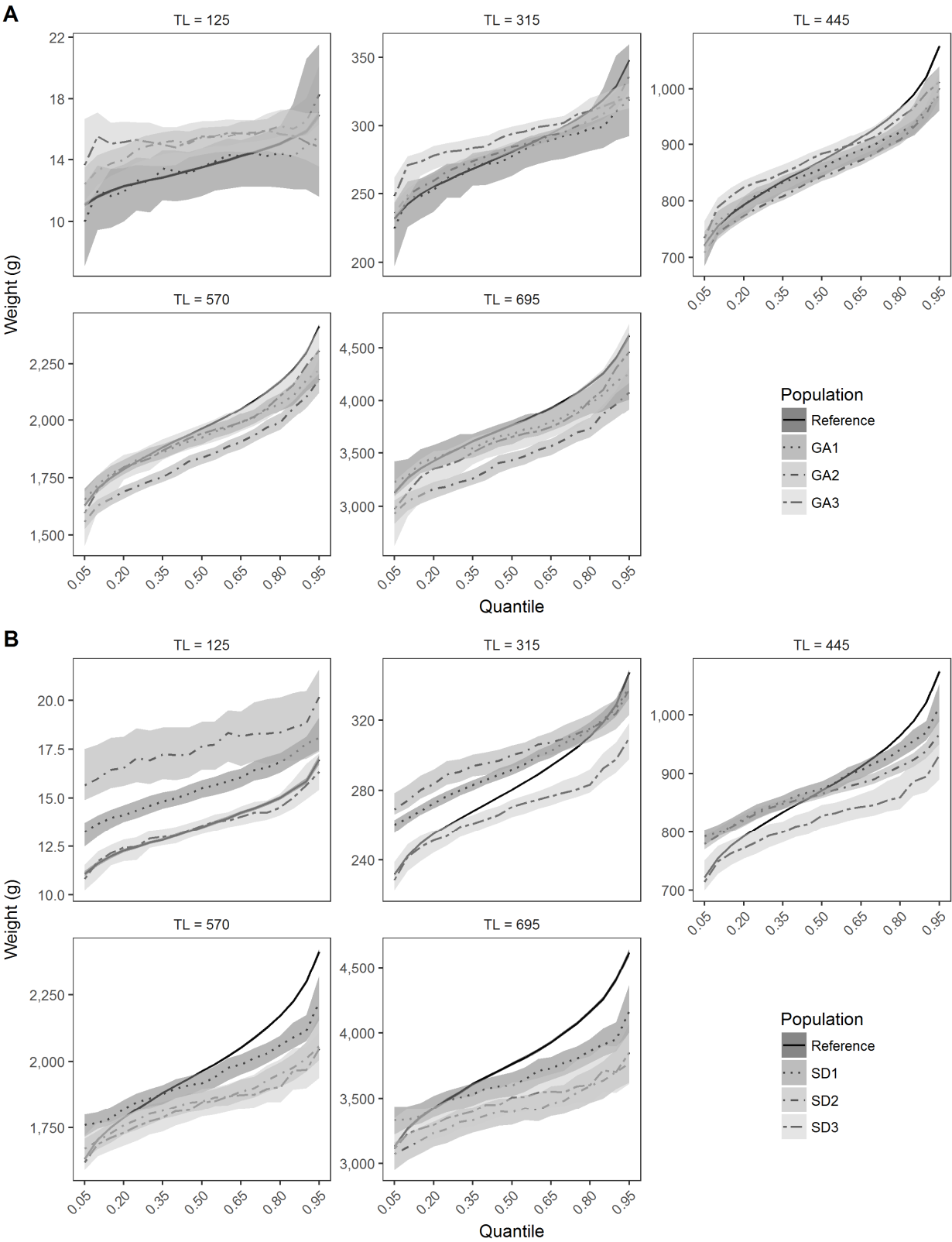
444   a solid line.

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451 Figure 1.