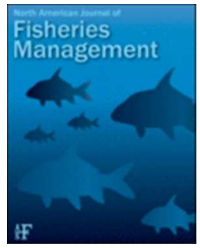

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Management Brief:

Quantile Regression Estimates of Body Weight for Walleye

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<A>Abstract

Quantile regression is a method of estimating fish weight at length for alternate portions of a probability distribution that has not received much attention in fisheries literature. Previous research has demonstrated that quantile regression can provide estimates of 75th quantile weight at length without bias, which is advantageous compared to previously-defined standard weight equations derived from linear or quadratic regression methods. The goal of this study was to demonstrate the utility of quantile regression as a tool to assess fish condition without bias using walleye *Sander vitreus* as a case study. Predicted weights of walleye derived from the 25th, 50th, and 75th percentiles for six distinct size categories were between the reference data set and independent data sets were used to demonstrate how populations may be compared at different management targets. A reference data set included 33,589 individual walleye weight-length values from 102 populations was used to create reference 75th quantile regressions and predictions of weight at 10-mm length classes and wider length categories. Randomly selected populations from the states of Georgia and South Dakota were used as independent data sets to account for the natural regional variability in walleye. Overall, this study demonstrates the relative ease with which quantile regression may be used to compare fish body condition between populations without bias.

<A>Introduction

Standard weight (W_s) equations and the concept of relative weight (W_r) were developed so that fisheries managers would have a “quick, inexpensive, and useful way of obtaining and interpreting fishery data for management purposes” (Wege and Anderson 1978). However, many W_s equations exhibit length-related biases, hampering evaluations of fish body condition which influences their ability to accurately assess fish body condition at all lengths (Gerow et al. 2005; Ranney et al. 2010). Additionally, there are a number of statistical concerns regarding estimating W_s equations, estimating the length-related biases associated with W_s equations (e.g., Gerow 2011; Ranney et al. 2011), and interpreting W_r (Brendan et al. 2003; Pope and Kruse 2007). Despite these concerns, the use of W_r as an evaluation tool has expanded beyond the scope envisioned by the originators (Cade et al. 2008; Ranney et al. 2011). Standard weight equations and W_r have been used to test whether body condition differs among or within fish populations and parametric tests are frequently used to compare W_r data (Murphy et al. 1990; Hyatt and Hubert 2001; Brendan et al. 2003). However, Cade et al. (2008) suggested that other means of comparing fisheries populations may provide a higher level of statistical rigor and be more relevant to the questions being asked. One tool to better compare changes in weight-length relationships is quantile regression (Cade and Noon 2003; Cade et al. 2008).

Quantile regression is a method to estimate different quantiles of a response variable distribution (Koenker and Bassett 1978; Cade and Noon 2003). In most regression applications, the mean response variable is estimated as a function of the predictor variable (Cade and Noon 2003). For example, the linear weight-as-a-function-of-length model

$$\log_{10}(W) = a + b \cdot \log_{10}(TL)$$

is an estimate of the mean response of $\log_{10}$-transformed weight (W) as a function of total $\log_{10}$-transformed length (TL). Quantile regression uses individual quantiles of the response variable (i.e., W) as a function of the predictor variable (i.e., TL). Thus, for a given quantile τ, $Q_W(\tau|TL)$ is the τ^{th} quantile Q as a function of TL (Cade et al. 2008). Quantile regression allows for estimation of all quantiles of W as a function of TL from τ = 0.01-0.99 (Cade et al. 2008).

Length-biased W_s equations may lead to low biased estimates of W_r at the upper length ranges for many species. Indeed, Ranney et al. (2010 and 2011) found that W_s equations developed for walleye *Sander vitreus* using either the regression-line percentile (RLP) technique or the empirical percentile (EmP) method were biased low. Regressing the 75th quantile of walleye weight as a function of length can generate unbiased predictions of walleye weight at the third quartile of weight in the third quartile (Cade et al. 2008). Using direct estimates of the 75th quantile of fish weight may provide a more direct means of comparing fish populations or individual fish to a reference population (Cade et al. 2008). However, little attention has been given to this method in the fisheries literature to date.

The goal of this study was demonstrate the utility of quantile regression as a tool to assess fish condition without bias using walleye as a case study. Predicted weights of walleye derived from the 25th, 50th, and 75th quantiles for six distinct size categories between the reference data set and independent data sets demonstrate how populations may be compared at different management targets. Walleye were considered an ideal candidate species for this study based on previous analyses (see Ranney et al. 2010, 2011), availability of data from a large geographic area, and their recreational and economic importance (Aiken 2011).

<A>Methods

I used walleye data from Ranney et al. (2010, 2011) to generate first, second, and third quartile (τ) regression models, corresponding to the 25th, 50th, and 75th quantiles (τ = 0.25, 0.50, and 0.75). These values are appropriate benchmarks of comparison for population weight data. Estimating the 75th quantile of weight as a function of length allows for a comparison of a given fish to the species-wide 'above average' body weight; the 50th quantile allows for comparison of fish to a species-wide "average" body weight; the 25th quantile allows for comparison of fish to a species-wide "poor" body weight. I filtered these data prior to analyses with the filtering methods described in Ranney et al. (2010) because data quality can influence parameter estimation, model fit, and predictive ability (Belsley et al. 1980).

I used the quantile regression package "quantreg" (Koenker 2017) in R version 3.3.3 (R Development Core Team 2017) to conduct the linear quantile regressions of $\log_{10}$-transformed weight (g) as a function of $\log_{10}$-transformed length (mm) for $\tau = 0.25$, 0.50, and 0.75. I pooled data from 102 populations ($N = 33{,}589$ walleye) and assigned equal weights to all populations regardless of contributed sample size (Cade et al. 2008). I refer to these data and the quantile regressions from this dataset as the "reference" dataset and regressions.

I solicited additional walleye weight and length data from state fisheries management agencies in Georgia and South Dakota to evaluate length-related bias between the quantile regression equation derived from the reference data set compared to independent data. I conducted linear quantile regressions as described above on three populations of walleye randomly selected from each state ($N = 313$, 795, and 199 for the GA 1, GA 2, and GA 3 populations, respectively; $N = 392$, 280, and 140 for the SD 1, SD 2, and SD 3 populations, respectively). I refer to these data and the quantile regressions from these datasets as the "state population" datasets and regressions.

To compare slope (β_1) and intercept (β_0) values from state populations to the reference population, I bootstrapped the estimates for the 75th quantile linear slope and intercept parameters 10,000 times for the reference data set and each state population data set. I used $\log_{10}$-transformed weight and length data to develop all linear regressions. For each of the 10,000 quantile regressions for the reference and individual state population data, I randomly selected

100 individual weight-length pairs with replacement. I compared population-level mean 75th quantile regression coefficients and their bootstrapped 95% confidence intervals to the 75th quantile regression coefficients from the reference data (Pope and Kruse 2007). If the bootstrapped 95% confidence intervals for slope for the reference population did not overlap the slope point estimates for state populations, I used the equation

$$(RS_1 - RS_2) \pm t_{(1-\alpha/2;df)} \cdot \sqrt{SE^2{}_1 + SE^2{}_2},$$

where RS_1 is the first regression slope, RS_2 is the second regression slope, SE_1 is the standard error of the first slope, and SE_2 is the standard error of the second slope (Pope and Kruse 2007) to compare slope point estimates. If the interval around the difference in slopes contains zero, the difference between the two estimated coefficients is statistically nonsignificant (Pope and Kruse 2007).

To demonstrate how populations may be compared at different management targets, I developed quantile regressions at $\tau = 0.25$, 0.50, and 0.75 and predicted weight values at each τ for 10-mm length classes from 150 mm to 750 mm. I also categorized the walleye data sets (reference and state populations) according to length categories established by Gabelhouse (1984). These length categories correspond to substock (SS), stock-quality (S-Q), quality-preferred (Q-P), preferred-memorable (P-M), memorable-trophy (M-T), and greater than trophy (>T). For each length category, I estimated the first, second, and third quartile (i.e., 25th, 50th, and 75th percentile) values of weight using the Blom method (Blom 1958; Gerow 2009), an unbiased estimator of quantiles (Gerow 2009).

<A>Results

Estimates of the slope derived from the 75th quantile calculated from the reference data set varied between 3.173 and 3.38 and intercept values varied between -5.976 and -5.430 (Table 1). The lowest slope value in the state population data was in the SD 2 population and the largest slope was found in the GA 1 population; the lowest intercept was in the GA 1 population and the largest was in the SD 2 (Table 1).

Bootstrapped 95% confidence intervals of 75th quantile slope and intercept for the reference dataset overlapped the 75th quantile estimates of slope and intercept for two of three Georgia populations and only one South Dakota populations (Table 1; Figure 1). The intervals around the difference in slopes between the reference population and GA 2, SD 1, and SD 2 did not contain zero (Table 2), suggesting that the 75th quantile slope from the GA 2, SD1, and SD 2 populations is significantly lower than the slope of the reference population.

Estimates of weight for each $\tau = 0.25$, 0.50, and 0.75 at each 10-mm length class ranged from 23.8, 25.5, and 27.5 g at the 155-mm length class to 4494.7, 4870.6, and 5288.1 g at the 745-mm length class (Table 3). Estimated quartile values across length categories varied widely (Table 4), particularly when state populations are compared to the reference population. In the SS and M-T length categories for some state populations, there was only one individual in each length category. Because of this, for some state populations, only the first quartile could be estimated (Table 4). Third quartile Blom-method estimates of fish weight in the Q-P length category for all state populations were greater than 3rd quartile Blom-method estimates of fish weights from the reference population except for $\tau = 0.25$ in the SD 1 population. Conversely, in the P-M length category, none of the estimated quartiles for any of the state populations were greater than the reference population (Table 4). There were no individuals greater than trophy length in any of the state population data.

<A>Discussion

Quantile regression of fisheries weight-length data is a means to compare individual fish weights and population regression data without the biases and statistical limitations inherent in W_s equations and W_r values (Cade et al 2008). While W_r is an appropriate tool for evaluating how a management action may impact fish condition, quantile regression of appropriately large reference datasets provide fisheries managers and researchers a means by which conspecific fish populations and individuals can be compared with statistical rigor (Cade et al. 2008; Ranney et al. 2011). Though W_r and W_s certainly have a use in fisheries science, the biases inherent in W_s equations (see Ranney et al. 2010 and 2011 for a discussion) do not provide the statistical validity with which to compare individuals and populations. Quantile regression provides the capability to compare unbiased estimates of quartiles (Gerow 2009) with the Blom method (Blom 1958).

With unbiased estimates of 75th quantile weight for 10-mm length categories, a fisheries manager (or angler) need only compare the length and weight of an individual fish to the reference table. For example, if a 580 mm walleye weighs 2,100 g, the fisheries manager can compare those values to the 585 mm length class in Table 2. Thus, a fish that weighs 2,100 g and is 580 mm long is somewhere between the 50th and 75th quantile estimate of weight for the reference population. Further, any additional quantile weights between $\tau = 0.01$ and 0.99 may be readily calculated and included in a tabular form as a quick reference for fish body condition to identify potential areas for growth bottlenecks. For example, a fisheries manager who may be responsible for the GA 1 population can determine that the 75th quantile of fish in the Q-P length category weigh nearly 100 g more than the 75th quantile estimate of weight for the species-wide reference populations (Table 4). However, in the P-M length category, walleye from the same population fall well below the species-wide standards at all estimated quantiles (Table 3). This could be an indication that prey availability at length categories beyond the Q-P category may be limiting walleye growth. Similarly, the manager responsible for the SD 1 population can see that the population of walleye in the Q-P length category is tracking well with the species-wide reference populations.

Though I have refrained from referring to quantile regressions as estimates of 'condition,' I did select the 25th, 50th, and 75th quantiles shown here for a reason. Historically, W_s equations were estimated at the 75th percentile and if the ratio of individual fish weight to W_s times 100 was greater than 100, then that individual is assumed to be in good or 'above average' condition (Wege and Anderson 1978; Neumann et al. 2012). Here, because quantile regression is estimating the response of $\log_{10}$-transformed fish weight as a function of $\log_{10}$-transformed fish length at the quantile specified, we can use phrases like "above average," "average", and "poor" weight for $\tau = 0.25$, 0.5, and 0.75 because that is one interpretation of what those quantiles represent. Other quantiles and more-refined definitions could be provided to identify benchmarks for comparison in other species or for different management priorities. For example, the 75th percentile was initially used by Wege and Anderson (1978) to estimate game fishes in 'good' condition. Lower values of τ for non-game or threatened or endangered fishes may be a more appropriate benchmark for comparison.

Comparisons of population-specific regression parameters to the reference data set suggest that half of the state populations are not significantly different from the reference population (Pope and Kruse 2007). The t distribution is used to calculate the interval around the differences in slopes when confidence intervals around a slope estimate from one population do not overlap the slope estimate from another population (Pope and Kruse 2007). However, the *t* distribution is a probability distribution used when estimating parameters for normal distributions (Zar 1999). A cursory glance at the distribution from which the slopes were drawn for the state population data used here suggest that those distributions are not normal, thus negating the assumption of normality when constructing confidence intervals around the regression coefficients. Interestingly, the residuals from 75th quantile regression models followed no discernable pattern beyond the fact that close to 75% of the residuals were less than zero. This pattern was expected, given that the quantile regression model where $\tau = 0.75$ is estimating the 75th quantile of $\log_{10}$-transformed weight as a function of $\log_{10}$-transformed length. Further research into the statistical properties of the distributions of 75th quantile regression slopes and intercepts of fisheries $\log_{10}$-transformed weight-length data is warranted. One potential method of comparing slope estimates for two populations from non-normal distributions estimated from 75th quantile regressions is to compare the distributions with a two-sample Kolmogorow-Smirnov test (Sokal and Rohlf 1981).

Quantile regressions of fisheries $\log_{10}$-transformed weight-length data provide a statistically valid means of comparing 75th quantile estimates of weight. While I do not advocate eliminating W_s and W_r as a means of evaluating fish condition, quantile regressions provide a simple and tractable means of directly evaluating fisheries weight at length for alternate portions of a probability distribution (Cade and Noon 2003). Quantile regression is relatively new to the fisheries profession and has yet to be used as a means to evaluate fish body-weight growth patterns. It is my hope that quantile regression will become a common means of evaluating various distributions of fish weight at length and will help fisheries managers and scientists compare weight-at-length of disparate populations without the length biases of W_s and W_r.

<A>Acknowledgements

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Table 1. Estimates of intercept and slope values from 10,000 bootstrap replicates of third quartile (75^{th} percentile) quantile regression of $\log_{10}$-transformed weight as a function of $\log_{10}$-transformed length for seven different data sets of walleye. The reference data set contains 33,589 individuals from 102 populations. Population sizes for the remaining Georgia (GA) and South Dakota (SD) populations are N=313, 795, and 199 for the Georgia populations, respectively, and N = 392, 280, and 140 for the South Dakota populations, respectively. Bootstrap replicates were estimated by resampling with replacement 100 $\log_{10}$-transformed weight-length pairs and conducting third quantile regressions on those resampled weight-length data. Intercept and slope values were estimated from those 10,000 bootstrapped replicates.

Population	Intercept (β_0)			Slope (β_1)		
	2.5%	mean	97.5%	2.5%	mean	97.5%
Reference	-5.9760	-5.7072	-5.4307	3.1731	3.2786	3.3817
GA 1	-6.2369	-5.7305	-5.4038	3.1595	3.2808	3.4688
GA 2	-5.7012	-5.4394	-5.1300	3.0498	3.1685	3.2709
GA 3	-5.7972	-5.5427	-5.3364	3.1334	3.2138	3.3127
SD 1	-5.5704	-5.4269	-5.2743	3.1106	3.1697	3.2267
SD 2	-5.4793	-5.2000	-4.9472	2.9821	3.0794	3.1860
SD 3	-5.6950	-5.5948	-5.4959	3.1808	3.21982	3.2623

Table 2. Intervals around the differences in slopes (β_1) between the reference population and three state populations. The intervals here do not contain zero, suggesting that the slope of the GA 2, SD 1, and SD 2 populations are significantly lower than the slope for the reference population.

Population	Lower interval	Upper interval
GA 2	0.1085	0.1115

SD 1	0.1076	0.1099
SD 2	0.1976	0.2006

Table 3. Weight (g) estimates of walleye *Sander vitreus* at 10-mm length class for the first, second, and third quartile (i.e., 25^{th}, 50^{th}, and 75^{th} percentile) of the reference data set. Estimates of weight were derived from non-linear third quartile (75^{th} percentile) regressions of weight as a function of length.

10-mm length class	Percentile		
	25th	50th	75th
155	23.8	25.5	27.5
165	29.4	31.4	33.9
175	35.8	38.2	41.3
185	43.0	46.0	49.8
195	51.3	54.9	59.3
205	60.6	64.9	70.2
215	71.1	76.1	82.3
225	82.7	88.6	95.8
235	95.6	102.5	110.9
245	109.9	117.8	127.5
255	125.6	134.7	145.8
265	142.8	153.2	165.8
275	161.6	173.4	187.7
285	182.0	195.5	211.6
295	204.2	219.4	237.5
305	228.3	245.3	265.5
315	254.2	273.2	295.8
325	282.1	303.4	328.5
335	312.2	335.7	363.6
345	344.4	370.5	401.2
355	378.8	407.6	441.5
365	415.6	447.3	484.6
375	454.8	489.7	530.5
385	496.6	534.8	579.4
395	541.0	582.7	631.3
405	588.0	633.5	686.5
415	637.9	687.4	744.9
425	690.6	744.4	806.8
435	746.4	804.7	872.2
445	805.2	868.3	941.1
455	867.2	935.3	1013.9
465	932.4	1005.9	1090.5
475	1001.0	1080.1	1171.0
485	1073.1	1158.1	1255.7

495	1148.7	1240.0	1344.5
505	1228.0	1325.8	1437.7
515	1311.0	1415.7	1535.3
525	1397.9	1509.9	1637.4
535	1488.8	1608.3	1744.3
545	1583.7	1711.1	1855.9
555	1682.8	1818.4	1972.4
565	1786.1	1930.4	2094.0
575	1893.8	2047.1	2220.8
585	2005.9	2168.7	2352.8
595	2122.6	2295.3	2490.3
605	2244.1	2427.0	2633.2
615	2370.2	2563.8	2781.9
625	2501.3	2706.0	2936.3
635	2637.4	2853.6	3096.7
645	2778.5	3006.8	3263.1
655	2924.9	3165.7	3435.6
665	3076.6	3330.3	3614.5
675	3233.7	3500.9	3799.8
685	3396.4	3677.5	3991.7
695	3564.6	3860.3	4190.2
705	3738.7	4049.3	4395.6
715	3918.6	4244.7	4608.0
725	4104.5	4446.7	4827.4
735	4296.5	4655.3	5054.1
745	4494.7	4870.6	5288.1

Table 4. Predicted values of first, second, and third quartile (i.e., $\tau = 0.25$, 0.5, and 0.75) weight by length category for a reference population of walleye *Sander vitreus* and six state populations, three from Georgia (GA) and three from South Dakota (SD). Quartile values were estimated by the Blom method (Gerow 2009). Length categories are substock (SS), stock-quality (S-Q), quality-preferred (Q-P), preferred-memorable (P-M), and memorable-trophy (M-T) and correspond to those from Gabelhouse (1984). There were no individuals greater than trophy length (>T) in any of the seven reference or state population datasets. Cells with "-" values represent populations in which there was only one individual in that length category and only the first quartile could be estimated.

Pop	SS			S-Q			Q-P			P-M			M-T		
	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
Ref	**60**	**85**	**115**	**220**	**331**	**430**	**660**	**850**	**1050**	**1560**	**1871**	**2268**	**3041**	**3405**	**3859**
GA 1	146	-	-	200	348	424	793	966	1152	1491	1789	2100	2795	3163	3582
GA 2	74	90	102	193	251	315	781	884	996	1419	1700	2000	2840	3600	3985
GA 3	110	-	-	183	226	254	778	968	1149	1420	1699	2045	2500	-	-
SD 1	19	22	26	277	324	383	632	852	1064	1425	1546	1834	3207	-	-
SD 2	122	-	-	165	333	446	722	889	1053	1434	1522	1641	3012	-	-
SD 3	19	23	30	270	308	394	853	1030	1238	1399	1598	1897	2720	2829	3135

Figure Legends

Figure 1. Estimates of slope and intercept values from 10,000 bootstrap replicates of third quartile (75th percentile) quantile regression for seven different data sets. The 'reference' slope and intercept values (crossed circle) were estimated from 33,589 individuals from 102 equally-weighted populations. Sample size for the Georgia populations (open shapes) were N = 313, 795, and 199 (GA 1, GA 2, and GA 3, respectively), and N = 392, 280, and 140 for the South Dakota populations (filled shapes; SD 1, SD 2, and SD 3, respectively). Each of the 10,000 linear quantile regressions was estimated from 100 randomly selected weight-length pairs, with replacement. Weight and length values were $\log_{10}$-transformed to linearize the data. The GA 2 population is almost directly underneath the SD 1 population.

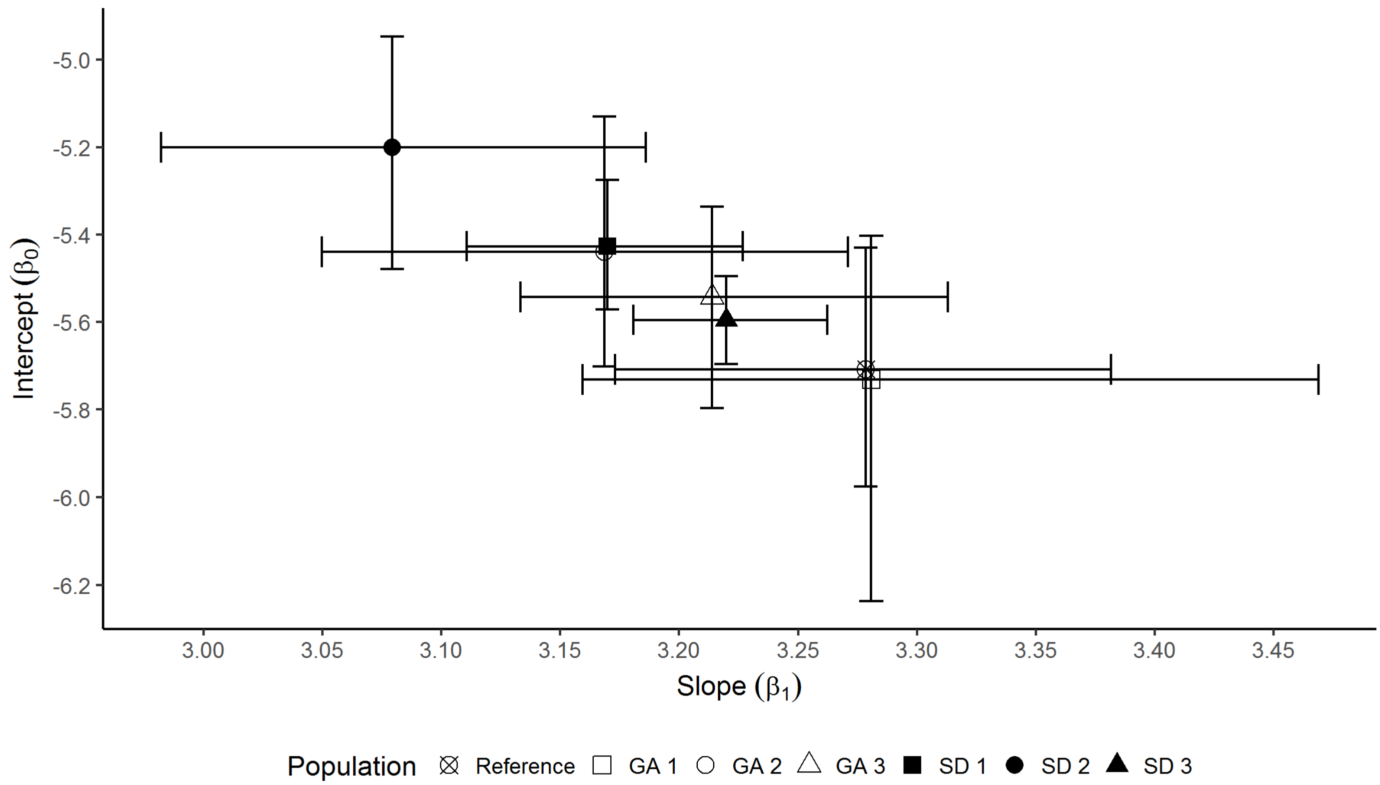
Intercept (β_0)
-5.0
-5.2
-5.4
-5.6
-5.8
-6.0
-6.2
Slope (β_1)
3.00
3.05
3.10
3.15
3.20
3.25
3.30
3.35
3.40
3.45
Population
Reference
GA 1
GA 2
GA 3
SD 1
SD 2
SD 3

Figure 1.

<A> Supplements

A repository that includes the data and R code used in this manuscript is available at https://github.com/stevenranney/waeQuantiles. Users can pull this repository and evaluate the code and data for themselves.

Additionally, I developed an R Shiny application to explore the distribution of bootstrapped quantile regression slope estimates in support of lines 208-211. That app can be viewed at https://stevenranney.shinyapps.io/slopeDistDemo/.