



## Quantile Regression Estimates of Body Weight at Length for Walleye

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Management Brief:  
Quantile Regression Estimates of Body Weight at Length in Walleye

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## Abstract

Quantile regression is a method of estimating fish weight at length for alternate portions of a probability distribution that has not received much attention in fisheries literature. Previous research has demonstrated that quantile regression can provide estimates of any quantile of weight at length without bias (including the 75<sup>th</sup> quantile which was often the focus of standard weight [ $W_s$ ] equations) which is advantageous compared to previously-defined standard weight equations derived from linear or quadratic regression methods. The goal of this study was to demonstrate the utility of quantile regression as a tool to assess fish weight at length at various portions of the probability distribution without bias using walleye *Sander vitreus* as a case study. Quantile regression models at the 75<sup>th</sup> quantile were developed for three randomly selected walleye populations from Georgia and South Dakota and compared to a large ( $n = 33,589$ ) reference population. Bootstrap resampling procedures indicated that only one population from the state of Georgia had an intercept and slope similar to the reference population. For the one population that had similar intercept and slope to the reference population, predictions of weight at various lengths still fell below the 95% confidence intervals for predicted weights of the reference population, suggesting that slight differences in intercepts and slopes in allometric relationships can result in predicted weights that still differ at some lengths. Predicted weights of walleye derived from the 10<sup>th</sup>, 25<sup>th</sup>, 50<sup>th</sup>, 75<sup>th</sup>, and 90<sup>th</sup> quantiles were used to demonstrate how individuals and populations may be compared at different management targets. Overall, this study demonstrates the relative ease with which quantile regression may be used to compare fish body condition between populations without bias.

## Introduction

Standard weight ( $W_s$ ) equations and the concept of relative weight ( $W_r$ ) were developed so that fisheries managers would have a “quick, inexpensive, and useful way of obtaining and interpreting fishery data for management purposes” (Wege and Anderson 1978). However, many  $W_s$  equations exhibit length-related biases, hampering evaluations of fish body condition which influences their ability to accurately assess fish body condition at all lengths (Gerow et al. 2005; Ranney et al. 2010; Ranney et al. 2011). Additionally, there are a number of statistical concerns regarding estimating  $W_s$  equations, estimating the length-related biases associated with  $W_s$  equations (e.g., Gerow 2011; Ranney et al. 2011), and interpreting  $W_r$  (Brendan et al. 2003; Pope and Kruse 2007). Despite these concerns, the use of  $W_r$  as an evaluation tool has expanded beyond the scope envisioned by the originators (Cade et al. 2008; Ranney et al. 2011). Standard weight equations and  $W_r$  have been used to test whether body condition differs among or within fish populations and parametric tests are frequently used to compare  $W_r$  data (Murphy et al. 1990; Hyatt and Hubert 2001; Brendan et al. 2003). However, Cade et al. (2008) suggested that other means of comparing fisheries populations may provide a higher level of statistical rigor and be more relevant to the questions being asked. One tool to better compare changes in weight-length relationships is quantile regression (Cade and Noon 2003; Cade et al. 2008; Cade et al. 2011; Crane et al. 2015; Crane and Farrell 2017).

Quantile regression is a method to estimate different quantiles of a response variable distribution (Koenker and Bassett 1978; Cade and Noon 2003). In most regression applications, the mean response variable is estimated as a function of the predictor variable (Cade and Noon 2003). For example, the linear weight-as-a-function-of-length model, common in fisheries applications,

$$\log_{10}(W) = a + b \cdot \log_{10}(TL)$$

is an estimate of the mean response of  $\log_{10}$ -transformed weight ( $W$ ) as a function of  $\log_{10}$ -transformed total length ( $TL$ ). Quantile regression estimates use individual quantiles of the response variable (i.e.,  $W$ ) as a function of the predictor variable (i.e.,  $TL$ ). Thus, for a given quantile  $\tau$ ,  $Q_W(\tau|TL)$  is the  $\tau^{\text{th}}$  quantile  $Q$  as a function of  $TL$  (Cade et al. 2008). Quantile regression allows for estimation of all quantiles of  $W$  as a function of  $TL$  from  $\tau = 0.01$ - $0.99$  (Cade et al. 2008).

Length-biased  $W_s$  equations may lead to low biased estimates of  $W_r$  at the upper length ranges for many species. Indeed, Ranney et al. (2010 and 2011) found that  $W_s$  equations developed for walleye *Sander vitreus* using either the regression-line percentile (RLP) technique (Murphy et al. 1990) or the empirical percentile (EmP) method (Gerow et al. 2005) were biased low. Regressing weight as a function of length and estimating the 75<sup>th</sup> quantile of weight for this functional relationship can generate unbiased predictions of walleye weight at the 75th quantile of weight (Cade et al. 2008). Using direct estimates from any quantile of fish weight—from 0.01 to 0.99—would provide a more statistically-valid means of comparing fish populations or individual fish to a reference population (Cade et al. 2008). However, little attention has been given to this method in the fisheries literature to date.

The goal of this study was demonstrate the utility of quantile regression as a tool to assess fish weight at length without bias using walleye as a case study. Direct comparison of population-specific quantile regression lines—based on well-established statistical theory—show how managers can directly compare fish populations to each other or to a reference population. Predicted weights of walleye derived from  $\tau = 0.10, 0.25, 0.50, 0.75$ , and  $0.90$  across 10 millimeter (mm) length classes demonstrate how populations may be compared at different

management targets. Walleye were considered an ideal candidate species for this study based on previous analyses (see Ranney et al. 2010, 2011), availability of data from a large geographic area, and their recreational and economic importance (Aiken 2011).

## Methods

I used walleye data from Ranney et al. (2010, 2011) to generate regression models. I filtered these data prior to analyses with the filtering methods described in Ranney et al. (2010) because data quality can influence parameter estimation, model fit, and predictive ability (Belsley et al. 1980). I refer to these data and any quantile regressions from this dataset as the “reference” dataset and regressions. I solicited additional walleye weight and length data from state fisheries management agencies in Georgia and South Dakota. I randomly selected from each state three walleye populations. Sample size was not a consideration when randomly selecting populations. I refer to these data and the quantile regressions from these datasets as the “state” datasets and regressions.

I used the linear quantile regression function `rq()` in the quantile regression package “quantreg” (Koenker 2017) in R version 3.3.3 (R Development Core Team 2017) to regress  $\log_{10}W$  as a function of  $\log_{10}TL$  to estimate 75<sup>th</sup> quantile intercepts ( $\beta_0$ ) and slopes ( $\beta_1$ ) from the reference and state population data. I used  $\tau = 0.75$  because this is similar to the established value used in  $W_s$  equations (though  $W_s$  is not a true estimate weight at length at  $\tau = 0.75$ ) and could be considered an estimator of fish with “above average” body weight. Previous quantile regression analyses (i.e., Cade et al. 2008; Cade et al. 2011) have shown that a single linear model can provide estimates for separate population-level  $\beta_0$  and  $\beta_1$  when a categorical factor (i.e., reference

and each state population identifier) and its interaction with the continuous predictor variable (i.e.,  $\log_{10}TL$ ) are included in the same model. I used the model

$$Q_{\log_{10}W}(\tau | \log_{10} TL, I_j) = \beta_{0j}(\tau)I_j + \beta_{1j}(\tau)I_j \log_{10} TL$$

where  $\beta_{0j}$  and  $\beta_{1j}$  are the intercepts and slopes for each reference and state population and  $I_j$  is the population identifier. I estimated the standard error of  $\beta_{0j}$  and  $\beta_{1j}$  by resampling with replacement 5,000  $\log_{10}W$ - $\log_{10}TL$  pairs 1,000 times. I calculated 95% confidence intervals around each  $\beta_{0j}$  and  $\beta_{1j}$  with the  $t$  statistic = 1.960.

I used a reparametrized version of the linear model described above to estimate which state populations had different  $\beta_0$  and  $\beta_1$  from the reference population. To do this, I set the reference population as the base level then set the contrasts to be among the rest of the categorical factors (population identifier and its interaction with the continuous predictor variable). That model then becomes

$$Q_{\log_{10}W}(\tau | \log_{10} TL, I_j) = \log_{10} \beta_0(\tau) + \beta_1(\tau) \log_{10} TL + \beta_{0j}(\tau)I_j + \beta_{1j}(\tau)I_j \log_{10} TL$$

where  $\beta_0$  and  $\beta_1$  are intercepts and slope for the reference population and  $\beta_{0j}$  and  $\beta_{1j}$  then become the proportionate differences in intercept and slope, respectively, for each state population  $I_j$  from the reference population. Similar to estimating the standard error of the  $\beta_0$  and  $\beta_1$  for each population-level model, I estimated the standard error of the differences in  $\beta_{0j}$  and  $\beta_{1j}$  from the reference population  $\beta_0$  and  $\beta_1$  with resampling. I resampled 5,000  $\log_{10}W$ - $\log_{10}TL$  pairs 1,000 times and calculated the 95% confidence intervals around the differences in  $\beta_0$  and  $\beta_1$  with the  $t$  statistic = 1.960. Alpha for all statistical tests was set to 0.05.

I compared quantiles of weight at specific lengths from each state population to the reference population by regressing  $\log_{10}W$  as a function of  $\log_{10}TL$  with  $\tau = 0.05, 0.10, \dots, 0.95$  in 0.05 increments for each state populations. I then estimated weight and 95% confidence

138 intervals of weight for the midpoints of the length categories for walleye [midpoints: substock =  
139 125 mm; stock-quality = 315 mm; quality-preferred = 445; preferred-memorable = 570 mm;  
140 memorable-trophy = 695 mm (Gabelhouse 1984)].

141 To demonstrate how individual fish may be compared at different management targets, I  
142 used the reference dataset to create linear quantile regressions of  $\log_{10}W$  as a function of  $\log_{10}TL$   
143 at  $\tau = 0.10, 0.25, 0.50, 0.75$ , and  $0.90$  and predicted total weight values of each  $\tau$  at 10-mm  
144 length classes from 155 mm to 745 mm. I pooled data from 102 populations ( $N = 33,597$   
145 walleye) and assigned equal weights to all populations regardless of contributed sample size  
146 (Cade et al. 2008). These values of  $\tau$  are appropriate benchmarks of comparison for population  
147 weight data. Estimating the 90<sup>th</sup> quantile of weight as a function of length corresponds to a fish  
148 of “excellent” body weight; the 75<sup>th</sup> quantile allows for a comparison of fish to the reference  
149 population “above average” body weight; the 50<sup>th</sup> quantile allows for comparison of fish to a  
150 reference population “average” body weight; the 25<sup>th</sup> quantile allows for comparison of fish to a  
151 reference population “below average” body weight; the 10<sup>th</sup> quantile allows for comparison of  
152 fish to a reference population “poor” body weight. I exponentiated the predicted values of  
153  $\log_{10}W$  to convert weight back to g [e.g.,  $10^{(\log_{10}W)}$ ].

154

## 155 <A>Results

156 The filtered reference data set included 102 populations and 33,597 individual  
157 observations of weight and length. The three randomly sampled populations from each state  
158 contained  $N = 313, 795$ , and  $199$  for the GA1, GA2, and GA3 populations, respectively, and  $N =$   
159  $392, 280$ , and  $140$  for the SD1, SD2, and SD3 populations, respectively. Individuals from  
160 populations in Georgia were collected with fall gillnetting and late-winter electrofishing.



Individuals from South Dakota populations were collected with summer gillnets. Collection of individuals in the reference dataset was of unknown methods.

The 75<sup>th</sup> quantile regression model  $\beta_0$  and  $\beta_1$  estimates for the reference dataset were -5.702 and 3.277, respectively (Table 1). The 95% confidence interval for the reference data set was  $\beta_0 = -5.717$  to  $-5.687$  and  $\beta_1 = 3.271$  to  $3.282$ . The lowest  $\beta_0$  was in the GA 1 population ( $-5.713$ ) and the largest was in the SD 2 population ( $-5.174$ ; Table 1). The lowest  $\beta_1$  value in the state population data was in the SD 2 population ( $3.070$ ) and the largest in the GA 1 population ( $3.275$ ; Table 1).

Bootstrapped 95% confidence intervals of 75<sup>th</sup> quantile  $\beta_0$  and  $\beta_1$  for the reference dataset overlapped the 75<sup>th</sup> quantile estimates of slope and intercept for only one of three Georgia populations and none of the South Dakota populations (Table 2). Estimates and 95% confidence intervals of weight at total length equal to the midpoints of length categories across all quantiles from  $\tau = 0.05, \dots, 0.95$  by 0.05 increments overlapped considerably for all three populations from Georgia and the reference population, especially at  $TL = 125$  mm (Figure 1, Panel A). Overlap in the confidence intervals for all Georgia populations and the reference population is evident across the remaining four weight-at-total-length estimates, though becomes less pronounced at the upper length ranges for GA2. There was less overlap in the confidence interval bands for the South Dakota populations at lower lengths ( $TL = 125, 315$ , and  $445$  mm) than at higher lengths ( $TL = 570$  and  $695$  mm; Figure 1, Panel B). Estimates of weight at  $\tau = 0.10, 0.25, 0.50, 0.75$ , and  $0.90$  from the reference population for each 10-mm length class ranged from 23.6, 25.3, 27.3, 29.8, and 32.1 g at the 155 mm length class to 4,107.1, 4,385.1, 4,730.0, 5,114.9, and 5,532.0 g at the 745 mm length class (Table 3).

## 184 &lt;A&gt;Discussion

185           Quantile regression of fisheries weight-length data is a means to compare individual fish  
186 weights and population regression data without the biases and statistical limitations inherent in  
187  $W_s$  equations and  $W_r$  values (Cade et al 2008). Quantile regression of a reference dataset(s)  
188 provide fisheries scientists and managers a means by which conspecific fish populations and  
189 individuals can be compared with statistical rigor (Cade et al. 2008; Ranney et al. 2011).  
190 Fisheries scientists and managers may consider that  $W_r$  and  $W_s$  still have a use in fisheries  
191 science; however, the biases inherent in  $W_s$  equations (see Ranney et al. 2010 and 2011 for a  
192 detailed discussion) do not provide the statistical validity with which to compare individuals and  
193 populations. Even when comparing the results of local management actions (e.g., prey  
194 augmentation; Cade et al. 2008), quantile regression was demonstrated as superior to  $W_r$  and  
195 there was no need to have a  $W_s$  equation from a large reference population. Those working with  
196 weight-length data should think long and hard to consider whether  $W_r$  is relevant to the questions  
197 they are asking. Linear quantile regression provides the capability to compare unbiased estimates  
198 of quantiles and is rooted in standard linear model procedures rather than the ad-hoc methods in  
199 use today to compare  $W_r$  values.

200           Estimates of weight at length from quantile regression can be used in place of  $W_r$  in  
201 nearly all instances in which  $W_r$  is subject to statistical analysis. Linear quantile regression is a  
202 more statistically rigorous tool that can be used to compare changes in weight-length  
203 relationships pre- and posttreatment or to compare the weight-length relationships of multiple  
204 populations (e.g., Cade et al. 2008; Cade et al. 2011; Crane et al. 2015; Crane and Farrell 2017).  
205 Populations that are monitored on a yearly basis (e.g., trophy fisheries or threatened and  
206 endangered populations) are also well-suited to quantile regression analysis.

Slight differences in  $\beta_0$  and  $\beta_1$  in allometric relationships can result in large differences in predictions of weight at various lengths. For example, though GA1 and the reference population had similar  $\beta_0$  and  $\beta_1$  (Table 1), the predictions of weight at the 75<sup>th</sup> quantile for the GA1 population at higher lengths (Figure 1, Panel A) fell outside the 95% confidence intervals for the reference dataset. Similarly, differences in  $\beta_0$  and  $\beta_1$  in allometric relationships does not imply that predicted weights will be different across length ranges. At TL = 315 (Figure 1, Panel B), the SD1 and SD2 populations had almost identical predictions of weight at the 75<sup>th</sup> quantile when compared to the reference population. This suggests that the joint effect of  $\beta_0$  and  $\beta_1$  is what matters most and that it is still difficult to make conclusive statements about how predictions (and populations) differ until actual predictions of weight have been compared with each other. I could have achieved better precision in the confidence intervals around the predictions of weight at length at various quantiles for both the state and reference populations had I resampled > 5,000 weight-length pairs and bootstrapped the regressions > 1,000 times. However, resampling pairs to the size of the entire dataset ( $n = 35,716$ ) and running 10,000 bootstrap regressions would have required significant additional computing time.

With unbiased estimates of any quantile of weight for 10-mm length categories, a fisheries manager (or angler) can compare the length and weight of an individual fish to the reference values in Table 3 to determine how that individual compares to the reference population. For example, if a 580 mm walleye weighs 2,100 g, the fisheries manager can compare those values to the 585 mm length class in Table 3. Thus, a fish that weighs 2,100 g and is 580 mm long is somewhere between the 25<sup>th</sup> and 50<sup>th</sup> quantile estimate of weight for the reference population. If interested, a fisheries manager could determine at which  $\tau$  that 580 mm fish weighing 2,100 g lies. Further, any additional quantile weights between  $\tau = 0.01$  and 0.99

and their confidence intervals may be readily calculated and included in tabular form as a quick reference for fish body weight to identify potential areas for growth bottlenecks. For example, a fisheries manager who may be responsible for the SD1 and SD2 populations can determine that the 75<sup>th</sup> quantile of fish at the S-Q midpoint (Figure 1, Panel B, TL = 315) are tracking well with the reference dataset. However, at the M-T midpoint (Figure 1, Panel B, TL = 695), walleye from those same populations fall well below the species-wide standards at all estimated quantiles except  $\tau = 0.15$  and below. This could be an indication that prey availability at length categories beyond the Q-P category may be limiting walleye growth. Similarly, the manager responsible for all Georgia populations can see that the populations of walleye at the Q-P midpoint (Figure 1, Panel A, TL = 445) are all tracking well with the species-wide reference populations. Table 3 could easily be converted to imperial measurements if so desired.

Though I have refrained from referring to quantile regressions predictions of weight as estimates of “condition,” I did select the quantiles shown in Table 3 for a reason. Historically,  $W_s$  equations were estimated at the 75<sup>th</sup> percentile (Wege and Anderson 1978) and if the ratio of individual fish weight to  $W_s \times 100$  was greater than 100, then that individual is assumed to be in good or ‘above average’ condition (Wege and Anderson 1978; Neumann et al. 2012). Here, because quantile regression is estimating the response of  $\log_{10}$ -transformed fish weight as a function of  $\log_{10}$ -transformed total length at the quantile specified, we can use phrases like “excellent”, “above average,” “average”, “below average”, and “poor” weight for  $\tau = 0.90, 0.75, 0.50, 0.25$ , and  $0.10$ , respectively, because that is one interpretation of what those quantiles represent. Other quantiles and more-refined definitions could be established to identify benchmarks for comparison in other species or for different management priorities. Lower values of  $\tau$  for non-game or threatened or endangered fishes may be a more appropriate benchmark for

comparison. State and regional fisheries scientists and managers can determine which quantile or set of quantiles best fits the question they are asking or the species they are investigating. The ease with which quantile models can be estimated and compared makes the need to set a “standard” quantile (or set of quantiles) superfluous.

I have used a reference dataset that spans a large portion of the geographic range of walleye. Given that this paper is meant as an introduction to using quantile regression to estimate fish body weight at length, this seemed like a reasonable first step. This is not meant to suggest, however, that fisheries scientists and managers should always use such a robust data set to which they can compare their own populations. Indeed, though fisheries managers in Georgia may want to compare their populations to those of the upper Midwest United States periodically, it seems more likely that managers in one region would be most interested in making within-region comparisons. As a result, regional reference data sets could be established and used as reference populations, depending upon the comparison of interest. Fisheries scientists and managers could easily define a useful organization of geography and habitat (e.g., reservoirs, lakes, rivers, etc.) so that consistency within a region is maintained. Making the decision as to which regions and habitats should be grouped together is challenging; however, the well-developed statistical method by which those comparisons can be made should allow fisheries scientists and managers to spend more time considering the organization question rather than having to consider how to conduct that analysis.

The minimum sample size needed for a reference dataset has not been investigated. However, given that confidence interval estimates are made with the  $t$  statistic (Zar 1999), if sample sizes are low, then confidence intervals of any range will be less precise. The minimum number of populations that should be included in a reference dataset has also not been

investigated. Provided that the reference dataset contains enough populations to be representative of the species across the spatial extent of the comparison(s) being made, the number of populations in the reference dataset is likely of little concern because all populations are given equal weights, regardless of contributed sample size (Cade et al. 2008). If there is a relevant weighting scheme based on survey design, weights can be incorporated into the linear quantile regression model by implementing appropriate weighting arguments. Averaging quantile estimates across multiple populations—based on constructing appropriate contrasts in the linear model—is an alternative approach to having a single “reference” population (Cade et al. 2011).

Many state and provincial management agencies already have standard methods in place to collect fisheries weight-length data. In these instances, I would not presume to suggest alternative methods for “standardized” collections. For management agencies that do not have a standard method by which to collect fisheries length-weight data for a given species, I would recommend establishing protocols as soon as is reasonable (see Bonar et al. 2009 for guidelines). Recognizing that time of year may impact weight of an individual fish (e.g., pre-spawn vs. post-spawn), state reference datasets that are compiled from a standardized collection method would be consistent. For larger, regional reference datasets, the variability inherent across standardized collection methods from contributing management agencies (e.g., pre-spawn vs. late summer vs. under ice) would likely negate any biases that could be inherent in the data (e.g., heavier fish collected during pre-spawn, lighter fish collected in mid-winter). Additionally, resampling a dataset considered to be representative of the species across the spatial extent under consideration when estimating the  $\beta_0$  and  $\beta_1$  coefficients of the quantile regression models would serve to reduce the influence of collection method on estimates of  $\beta_0$ ,  $\beta_1$ , or the differences between the  $\beta_0$  and  $\beta_1$  of the reference population and any other populations being compared.

Frequency of reference dataset compilation should be determined by fisheries scientists and managers. Climate change may impact different geographic regions at different rates. As a result, defining the frequency with which reference datasets should be established is beyond the scope of this paper. Climate change could impact growth of fishes across geographic regions and latitudes and reference datasets should reflect those changes. Regardless of how climate change affects fish growth, the ease with which population comparisons can be made with quantile regression will allow fisheries scientists and managers more time to think about their research questions and management goals rather than the best way to conduct analyses of their data.

Quantile regressions of fisheries  $\log_{10}$ -transformed weight-length data provide a statistically valid means of comparing any quantile estimates of weight. While I do not advocate eliminating  $W_s$  and  $W_r$  as a means of evaluating fish condition in management-appropriate situations, quantile regression provides a simple, tractable, and statistically-valid means of directly evaluating fisheries weight at length for alternate portions of a probability distribution (Cade and Noon 2003). Quantile regression procedures are available in common data analysis software: XLStat for Excel, PROC QUANTREG in SAS, and the “statsmodels” library in Python. It is my hope that quantile regression will become a common means of evaluating various distributions of fish weight at length and will help fisheries scientists and managers compare weight-at-length of disparate populations without the length biases of  $W_s$  and the ad-hoc methods used to analyze  $W_r$  data.

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390 <A> Supplements

391 A repository that includes the data and R code used in this manuscript is available at

392 <https://github.com/stevenranney/waeQuantiles>.

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Table 1. Estimates of intercept and slope values from bootstrapped replicates of quantile regression with  $\tau = 0.75$  for seven different data sets of walleye. Bootstrap replicates were estimated by resampling with replacement 5,000  $\log_{10}$ -weight and  $\log_{10}$ -length pairs 1,000 times.

Population	Intercept ( $\beta_0$ )			Slope ( $\beta_1$ )		
	2.5%	Estimate	97.5%	2.5%	Estimate	97.5%
Reference	-5.717	-5.702	-5.687	3.271	3.277	3.282
GA1	-5.954	-5.713	-5.471	3.186	3.275	3.364
GA2	-5.551	-5.451	-5.352	3.135	3.173	3.212
GA3	-5.678	-5.508	-5.339	3.135	3.201	3.266
SD1	-5.497	-5.425	-5.353	3.140	3.169	3.197
SD2	-5.356	-5.174	-4.992	3.000	3.070	3.139
SD3	-5.692	-5.603	-5.514	3.186	3.223	3.259

Table 2. Differences and 95% confidence intervals around the differences in intercepts and slope of each state population from the reference population.  $P$ -value is from a test of the null hypothesis that the intercept and slope from the quantile regression where  $\tau = 0.75$  for each state population is equal the reference population intercept and slope. Asterisk indicates populations with significantly different intercept and slope from the reference population.

Population	Intercept ( $\beta_0$ )				Slope ( $\beta_1$ )			
	2.5%	Estimate	97.5%	$p$ -value	2.5%	Estimate	97.5%	$p$ -value
GA1	-0.2484	-0.0107	0.2271	0.9299	-0.0898	-0.0019	0.0861	0.9668
GA2*	0.1486	0.2507	0.3528	< 0.0001	-0.1423	-0.1032	-0.064	< 0.0001
GA3*	0.0062	0.194	0.3819	0.0429	-0.1483	-0.0757	-0.0032	0.0407
SD1*	0.2024	0.2772	0.352	< 0.0001	-0.1375	-0.108	-0.0784	< 0.0001
SD2*	0.3549	0.5281	0.7014	< 0.0001	-0.273	-0.207	-0.141	< 0.0001
SD3*	0.0129	0.0991	0.1853	0.0242	-0.089	-0.054	-0.019	0.0025

Table 3. Weight (g) estimates of walleye *Sander vitreus* at 10-mm length class for  $\tau = 0.10, 0.25, 0.50, 0.75$ , and  $0.90$  of the reference data set. Estimates of weight were derived from linear quantile regressions of  $\log_{10}$ -weight as a function of  $\log_{10}$ -length.

Total length (mm)	Quantile ( $\tau$ )				
	0.10	0.25	0.50	0.75	0.90
155	23.6	25.3	27.3	29.8	32.1
165	28.9	31.0	33.6	36.6	39.5
175	35.1	37.7	40.7	44.4	47.8
185	42.1	45.2	48.9	53.3	57.4
195	50.1	53.7	58.1	63.3	68.2
205	59.0	63.3	68.5	74.6	80.4
215	69.1	74.0	80.1	87.2	94.0
225	80.2	86.0	92.9	101.2	109.1
235	92.5	99.1	107.2	116.7	125.8
245	106.1	113.7	122.9	133.7	144.2
255	121.0	129.7	140.2	152.5	164.4
265	137.3	147.1	159.0	172.9	186.6
275	155.1	166.1	179.6	195.3	210.6
285	174.4	186.8	201.9	219.5	236.8
295	195.4	209.2	226.1	245.8	265.2
305	218.0	233.4	252.2	274.1	295.8
315	242.4	259.5	280.4	304.7	328.8
325	268.6	287.6	310.7	337.6	364.3
335	296.8	317.7	343.2	372.8	402.4
345	326.9	349.9	378.0	410.5	443.1
355	359.1	384.3	415.2	450.8	486.6
365	393.4	421.0	454.8	493.8	533.1
375	430.0	460.1	497.0	539.5	582.5
385	468.8	501.7	541.8	588.1	635.0
395	510.1	545.7	589.4	639.6	690.7
405	553.8	592.4	639.8	694.2	749.7
415	600.0	641.8	693.1	752.0	812.1
425	648.9	694.0	749.5	813.0	878.0
435	700.4	749.1	808.9	877.4	947.6
445	754.7	807.2	871.6	945.2	1,021.0
455	812.0	868.3	937.5	1,016.6	1,098.1
465	872.1	932.6	1,006.9	1,091.7	1,179.3
475	935.3	1,000.1	1,079.7	1,170.5	1,264.5
485	1,001.6	1,070.9	1,156.1	1,253.2	1,353.9
495	1,071.1	1,145.1	1,236.2	1,339.9	1,447.6

505	1,143.9	1,222.9	1,320.1	1,430.6	1,545.7
515	1,220.1	1,304.2	1,407.8	1,525.6	1,648.4
525	1,299.7	1,389.3	1,499.6	1,624.8	1,755.7
535	1,382.9	1,478.1	1,595.4	1,728.4	1,867.8
545	1,469.7	1,570.8	1,695.4	1,836.5	1,984.7
555	1,560.2	1,667.4	1,799.6	1,949.3	2,106.6
565	1,654.5	1,768.2	1,908.2	2,066.7	2,233.7
575	1,752.8	1,873.0	2,021.4	2,189.0	2,366.0
585	1,855.0	1,982.1	2,139.1	2,316.3	2,503.6
595	1,961.3	2,095.6	2,261.4	2,448.5	2,646.7
605	2,071.8	2,213.5	2,388.6	2,586.0	2,795.4
615	2,186.5	2,336.0	2,520.6	2,728.7	2,949.8
625	2,305.6	2,463.1	2,657.7	2,876.8	3,110.0
635	2,429.1	2,594.9	2,799.8	3,030.3	3,276.1
645	2,557.1	2,731.5	2,947.1	3,189.5	3,448.4
655	2,689.8	2,873.1	3,099.8	3,354.4	3,626.8
665	2,827.1	3,019.7	3,257.8	3,525.2	3,811.6
675	2,969.3	3,171.4	3,421.4	3,701.9	4,002.8
685	3,116.4	3,328.3	3,590.6	3,884.6	4,200.5
695	3,268.5	3,490.5	3,765.5	4,073.5	4,405.0
705	3,425.7	3,658.2	3,946.3	4,268.8	4,616.3
715	3,588.0	3,831.4	4,133.0	4,470.4	4,834.5
725	3,755.6	4,010.2	4,325.8	4,678.5	5,059.8
735	3,928.6	4,194.8	4,524.7	4,893.3	5,292.2
745	4,107.1	4,385.1	4,730.0	5,114.9	5,532.0

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**Figure Legends**

Figure 1. Quantile regression ( $\tau = 0.05, \dots, 0.95$  by 0.05 increments) predictions and 95% confidence intervals of weight for the midpoint of each length category [substock, stock-quality, quality-preferred, preferred-memorable, memorable-trophy (Gabelhouse 1984)] for the reference dataset and three populations of walleye from Georgia (A) and South Dakota (B). Confidence bands around the Reference population are narrow and appear as a solid line.



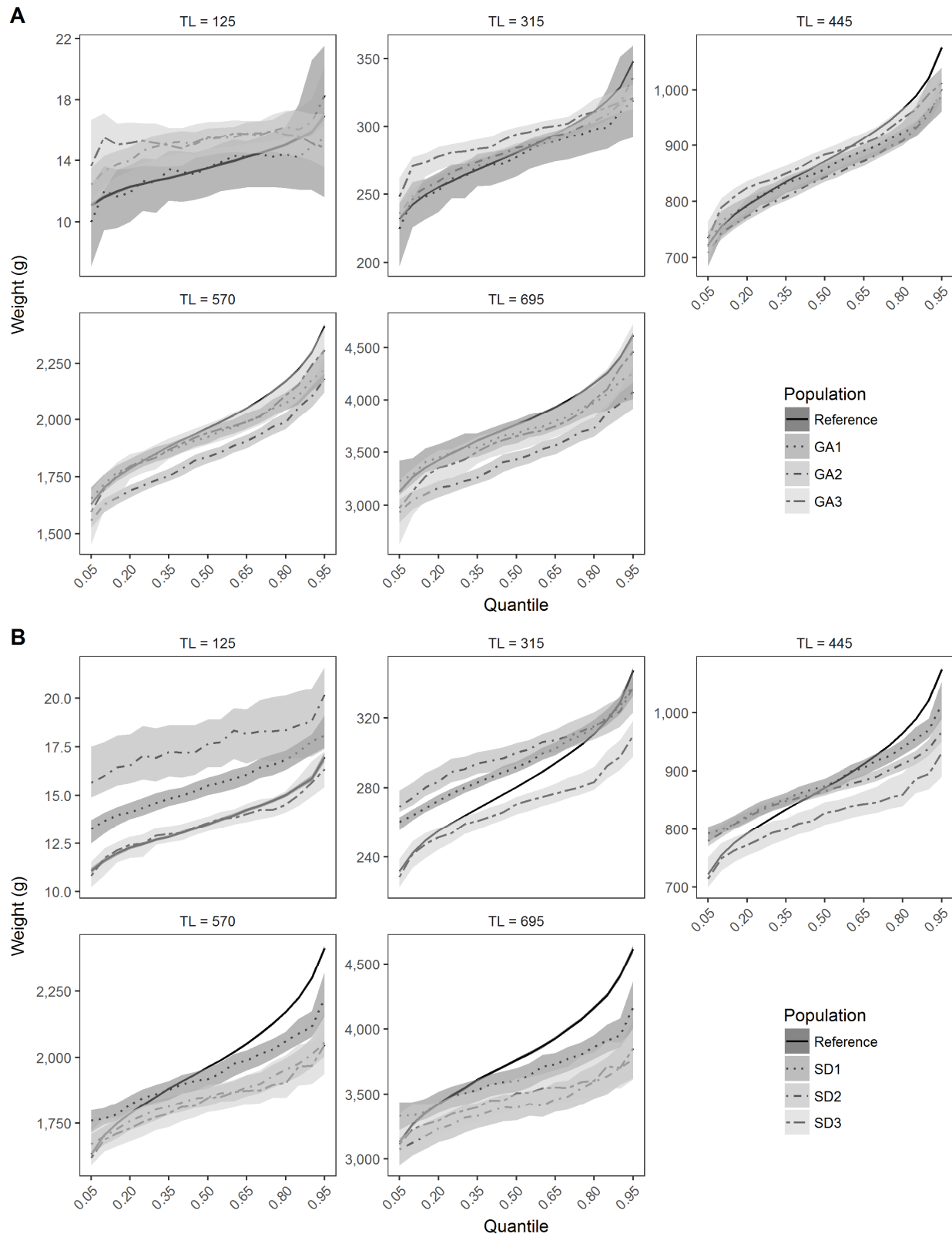
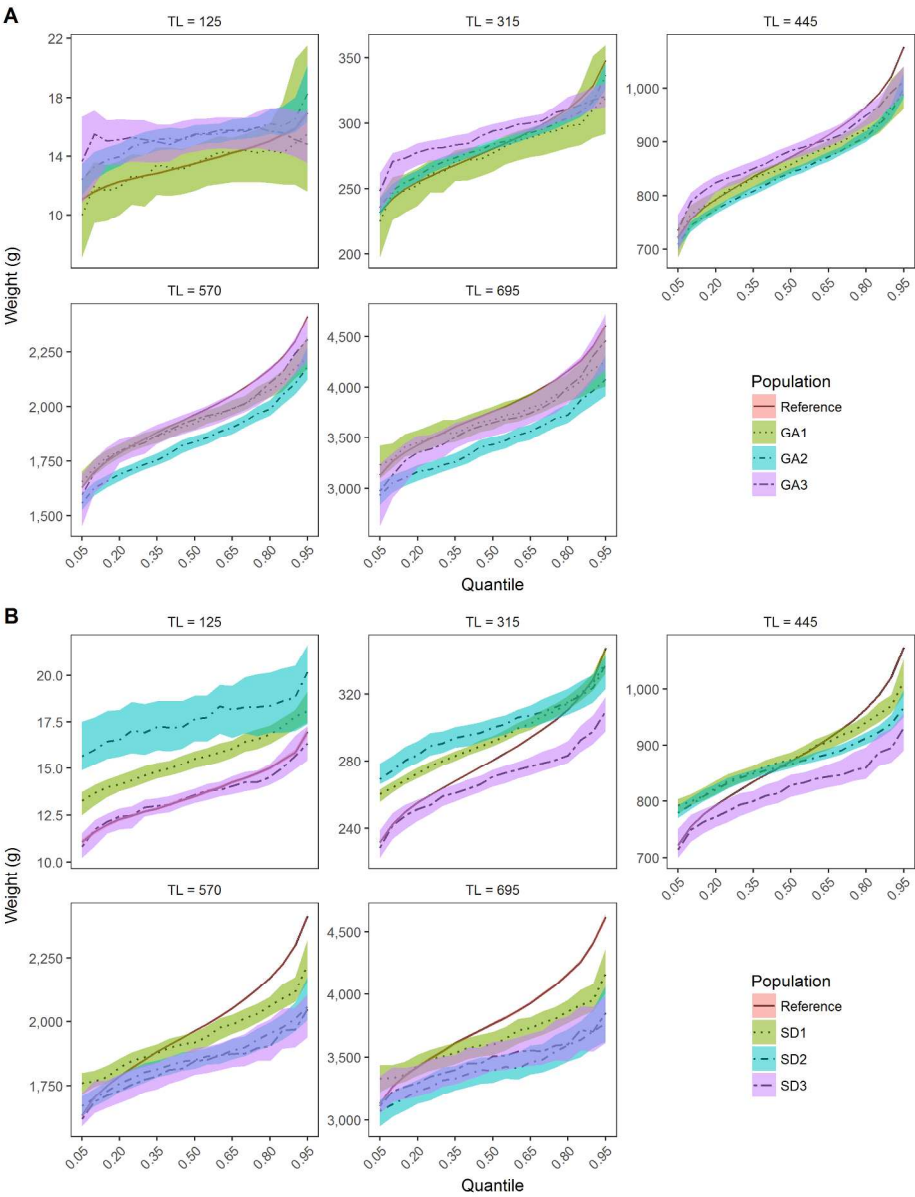


Figure 1.



This is a color version (\*.tiff) of Figure 1 embedded in the manuscript.

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