QUOTATION 1

AUTHOR, *SOURCE*

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AUTHOR

CONNOR ET AL.

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Dedicated to my family and friends.

Introduction

Chapter 1 Arithmetic on a Map Grid

Geologists and geophysicists make and use maps. These maps must have a common coordinate system. Otherwise, it is very difficult to use information plotted on different maps. In the past, a common coordinate system for most maps was not easy to achieve. During the 1990's, the advent of widely available global positioning system (GPS) receivers made it possible to achieve commonality and cross-referencing among coordinate systems. This common coordinate system vastly improved the quality and usefullness of maps. The downside, if there is one, is that we have to be much more careful about the way we gather and use data that are destined to be displayed on maps.

Core ideas presented in this chapter are:

- The Universal Transverse Mercator grid is used to make calculation on maps in a common reference frame.
- Calculations on maps often involve intervals – the distances between objects like measurement points on the ground or pixels on a satellite image.
- Latitude and longitude, the shape of the earth and map datums

The UTM Grid

Grid systems are widely used by geoscientists to simplify distance calculations on maps. Most maps are Cartesian grids. The y-axis is oriented N–S and y-coordinates are usually referred to as the Northing, or sometimes as distance north. The x-axis is oriented E–W and x coordinates are referred to as the Easting or distance east. Usually the units of these grids are meters.

The Universal Transverse Mercator (UTM) grid is the most important map grid system in use today. In this grid system, the

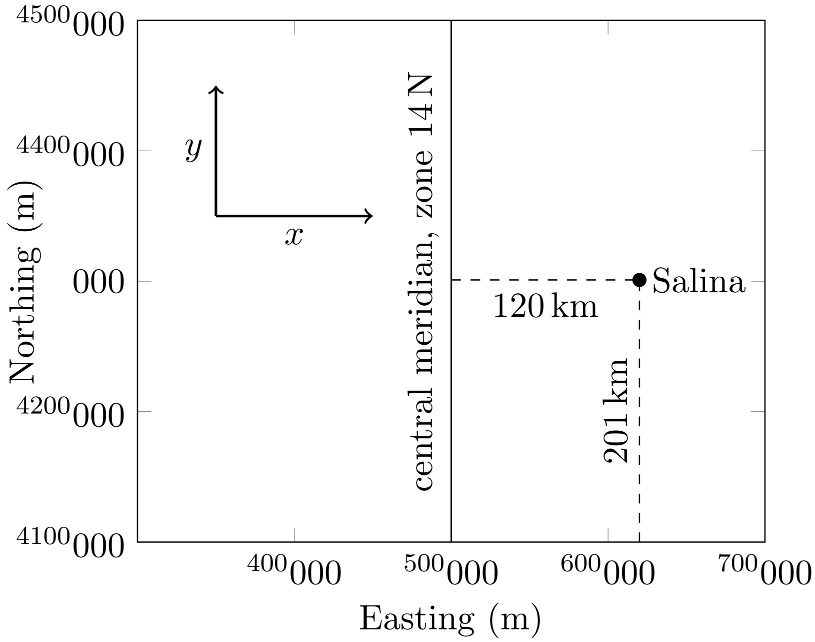


Figure 1.1: An example of the UTM grid. The town of Salina, Kansas (USA) is located at approximately 4301000 N, 620000 E, zone 14 N – or about 120 km East of the zone 14 N central meridian. The familiar *xy* - axes of the Cartesian coordinate system are shown for reference.

coordinates of a point on the surface of the Earth are given in terms of the *Northing* coordinate and *Easting* coordinate. The Northing for the UTM grid in the northern hemisphere is given in meters north of the equator. On a UTM map grid, one can think of the Northing coordinate as the distance North along the y-axis of the map. Negative numbers are avoided at all costs in the UTM grid. So, in the southern hemisphere, the Northing is given in meters south of the equator (a negative number) + 10,000,000 m. For example, the Northing of a point located 2000 km north of the equator is 2,000,000 N. The Northing of a point located 2000 km south of the equator is 8,000,000 N. In both the northern and southern hemispheres, UTM Northing is a positive number that increases as one walks north and decreases as one walks south.

The x-axis is perpendicular to the y-axis at the origin (central meridian) of the coordinate system. Walking East, in a positive x-direction on common graphs, increases the value of the Easting coordinate on the UTM grid; walking West decreases the value of the Easting coordinate.

Because the Earth is roughly spherical in shape, it is impossible to describe it with a Cartesian grid without distorting the grid. Consequently, the UTM grid system breaks up the Earth's surface into 60 zones to minimize this distortion. Each UTM zone has its own central meridian – a line that runs N–S through the center of the zone.

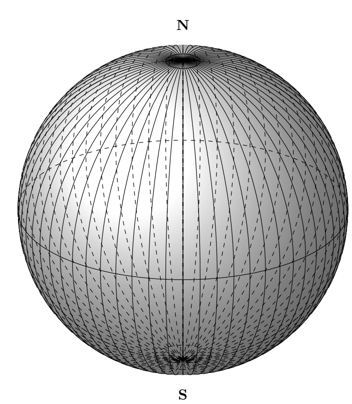


Figure 1.2: A globe showing the central meridians for UTM zones.

Again, negative numbers are avoided in the UTM grid. Consequently the value of the Easting along the central meridian (the origin of the x-axis) of any zone is 500,000 E, rather than 0 E. West of the central meridian Easting coordinates are less than 500,000 E; east of the central Easting coordinates are more than 500,000 E. There are enough zones so that, using this scheme, no negative Easting coordinates occur on Earth.

Easting and Northing coordinates in the UTM grid system are not unique. Points in different zones can have the same UTM coordinates because each zone has its own central meridian. Also, points in the southern hemisphere can have the same Northing coordinate as points in the northern hemisphere. In order to fully specify a map location the Easting, Northing and zone must be reported. These zones are uniquely numbered 1–60. It is also necessary to specify if the coordinate in the zone is in the northern hemisphere (N), or the southern hemisphere (S).

For example, the UTM coordinate:

```
4482963N    626371E    zone 10, N
```

corresponds to a location near Lassen Volcano, California (USA). The UTM coordinate:

```
4482963N    626371E    zone 17, N
```

corresponds to a location east of the city of Pittsburgh, Pennylvannia (USA). Both points are located approximately 4483 km North of the equator. Both points are located approximately 1264 km East of the central meridian in their respective zones.

Exercise 1.1 A college campus is in zone 17 N. Within this zone the Geosciences building on the college campus has the approximately UTM coordinates: 3104700 N, 360830 E. Suppose a GPS gives the coordinate of a point in the MLK Plaza (MLK1) on the same college campus to be 3104941 N, 360912 E, zone 17 N.

(a) Is MLK1 East or West of the Geosciences building?

(b) Is MLK1 North or South of the Geosciences building?

(c) How far north of the coordinate of the SCA building is MLK1?

(d) How far East of the coordinate of the SCA building is MLK1?

(e) What is the coordinate of a point 100 m west of the MLK Plaza point (MLK2)?

(f) Draw a map that shows the three coordinates (SCA building, MLK1 and MLK2) on a UTM grid.

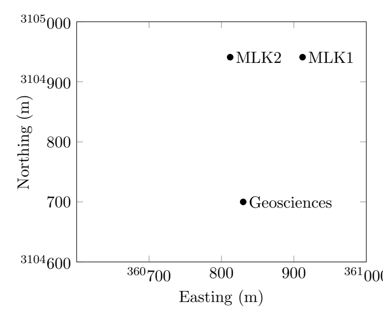


Figure 1.3: A map of three UTM coordinates

Exercise 1.2 What is the UTM coordinate of a point located in zone 17, 100 m west of the central meridian and 130 m south of the equator?

Solution 1.2 The value of the central meridian is 500000 E, so the easting of the UTM coordinate is $500000 - 100 = 499900$ E. The value of the northing at the equator is 10000000 N with respect to the southern hemisphere, so the northing of the UTM coordinate is $10000000 - 130 = 9999870$ N
The UTM coordinate is: 9999870 N, 499900 E, zone 17 S

Exercise 1.3 An MS student conducted a ground penetrating radar survey on pyroclastic deposits near Puluagua volcano, Ecuador. The deposits, and her survey line, crossed the equator. This created an offset in the northing coordinates along her line at the equator. To solve this problem, she created a "local" grid using the UTM coordinates of the southern hemisphere even for points located along the survey line in the northern hemisphere. That is, in her local grid, a point located 100 m north of the equator would have the northing coordinate 10000100 N, which is not an allowable UTM coordinate but is a fine value for the local grid. Unfortunately, her GPS reports locations in UTM coordinates, not in her local grid coordinates. The GPS coordinates of the beginning and end points along the survey line, in real UTM coordinates, are:

```
Point 1:    9999564 N    785342 E    zone 17, S
Point 2:    0000265 N    785358 E    zone 17, N
```

Convert to these coordinates to the MS student's local grid.

Solution 1.3 Point 1 is the same: 9999564 N, 785342 E zone 17, S. The northing of Point 2 must be converted to the local grid: 10000265 N, 785358 E zone 17, S

Grids, Intervals and Distances

OFTEN GEOSCIENTISTS want to make measurements along a line at even intervals, or on a map grid at even intervals. Example measurements made at even intervals might be the dip of a bed, the depth to the water table, or the variation in the Earth's gravity.

A SCHEMA: Beware that the measured interval is one less than the number of counts, unless the first count is zero. If you have N equally spaced stakes in the ground, there are $N - 1$ intervals between the stakes.

Exercise 1.4 Suppose you want to place stakes between MLK1 and MLK2 on a 10 m interval. The first stake is at MLK1, the second stake is 10 m west of MLK1, the last stake is at MLK2. How many stakes do you need?

Exercise 1.5 Suppose you only have 10 stakes, and want them spaced evenly with the first stake at MLK1 and the last stake at MLK2. What is the distance between the stakes?

Solution 1.5

$$(10-1)\ \text{spaces} \times x \frac{\text{m}}{\text{space}} = 100\ \text{m}$$

$$x = \frac{100\ \text{m}}{(10-1)\ \text{spaces}} \approx 11.11\ \text{m}$$

Exercise 1.6 Suppose you want to create a 100×100 m grid of stakes, on a 10 m interval. How many stakes do you need?

Exercise 1.7 Suppose you only want to use 49 stakes to create a 100×100 m grid with an even interval between stakes in each direction. What is the interval?

Solution 1.7 *49 stakes* $= \left[\frac{\text{total distance}}{\text{interval}} + 1\right]^2 = \left[\frac{100}{x} + 1\right]^2$
$\sqrt{49} = \frac{100}{x} + 1$
$x = \frac{100}{(\sqrt{49}-1)} \approx 16.67\,\text{m}$

Exercise 1.8 A satellite maps topography along a 2400 km-long transect in a N–S direction near the East Coast of North America. The satellite data are processed to provide one elevation measurement every 30 m of horizontal distance along the transect.

(a) How many elevation measurements are in the transect?

(b) If the UTM coordinate of the southernmost point in the satellite transect is 3200104 N, 500000 E, zone 17 N, what is the coordinate of the northernmost point along the transect?

Solution 1.8

(a) number measurements $= \frac{\text{total distance (meters)}}{\text{interval}\,(\frac{\text{meters}}{\text{measurement}})} +$ 1 measurement $= \frac{2400 \times 1000}{30} + 1 =$ 80001 measurements.

(b) For the Northing:
$3200104\text{ m} + (2400\text{ km} \times \frac{1000\text{ m}}{1\text{ km}}) =$ 5600104 N
The Easting is constant, so the coordinate is: 5600104 N, 500000 E, zone 17 N

Exercise 1.9 Your advisor asks you to measure the thickness of a limestone bed in 20 locations in the bed's outcrop area. A quick look at the geologic map indicates the unit crops out intermittently in an area approximately 1500 km $\times$ 500 km. Develop a strategy for collecting the thickness measurements that will convince your advisor that your thickness measurements represent the limestone bed thickness across the entire area.

Exercise 1.10 Your team is tasked to monitor groundwater contamination from a landfill along an E-W line extending from UTM coordinates 3202344 N, 345023 E to 3202344 N, 345381 E, zone 17N. In the absence of any additional information, you figure the wells should be equally spaced. Your total budget is $265,000 USD and you figure it will cost about $50,000 per well. Provide the UTM coordinates of the well locations.

Exercise 1.11 A digital elevation model (DEM) is a map grid of elevation measurements. Usually, one axis of the grid runs N–S and the other axis runs E–W. If the southwesternmost corner of the DEM is located at 3200104 N, 500000 E, zone 14 N, the spacing between elevation measurements in the grid is 30 m in both directions, and there are a total of 200 measurements in each direction, what is the UTM coordinate of the NE corner of the DEM?

Solution 1.11 For the Northing coordinate: $3200104 + ((200-1) \times 30) =$ 3206074 N. For the Easting coordinate: $500000 + ((200-1) \times 30) = 505970$ E

RECALL THAT THE DISTANCE FORMULA IS:

$$Distance = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{1.1}$$

where the coordinate of one point is x_1, y_1 and the coordinate of the second point is x_2, y_2

Exercise 1.12 Find the distance between the Geosciences building location and the MLK1 point.

```
Geosciences:     3104700 N    360830 E    zone 17, N
       MLK1:     3104941 N    360912 E    zone 17, N
```

Shape of the Earth

ORIGINALLY THE METER was intended to be defined in terms of the distance from the equator to the pole. Specifically, it was thought this distance is 10,000 kilometers. It turns out this is not exactly right, because the Earth is not quite spherical, and because at the time the meter was defined the distance between pole and equator was not precisely known. Nevertheless, it's close.

Exercise 1.13 Assuming that the distance from the equator to the north pole is 10,000 kilometers and that the Earth is a perfect sphere:

(a) How many meters is it from the south pole to the equator?

(b) Calculate the radius of the Earth (recall that the circumference of a circle is $2\pi r$, where r is the radius).

(c) Since there are 60 UTM zones, what is the maximum width of a zone at the equator (in meters)?

(d) Since the central meridian of a UTM zone is 500000 E, what are the maximum and minimum UTM Easting coordinates you would expect to find in any zone at the equator?

(e) The depth to the outer core is about 2900 km. What is the circumference of the Earth's outer core?

A SPHERE HAS CONSTANT RADIUS AND THE EARTH DOES NOT. The equatorial radius of the Earth is approximately 6378.137 km, the polar radius of the Earth is approximately 6356.752 km.

Consequently, the figure of the Earth is more precisely approximated by an oblate ellipsoid of revolution, often simply referred to as the reference ellipsoid. The planet is flattened along its axis of rotation, its equatorial bulge a result of rotation. The Earth is wider around its waistline (the equator) than around the poles. The amount of flattening of the ellipsoid can be expressed in terms of equatorial and polar radii:

$$f = \frac{R_{eq} - R_{po}}{R_{eq}} \tag{1.2}$$

where R_{eq} is the radius of the Earth at the equator and R_{po} is the radius at the pole.

The magnitude of flattening, f , is now very well determined from observing artificial satellite orbits around the Earth, and is near the value f = 1/298. Newton first discussed the flattening of the Earth, and estimated a value of 1/230. The radius of the Earth at a given latitude is given by

$$R_E = R_{eq}(1 - f sin^2\phi) \tag{1.3}$$

where: R_E is the Earth's radius at latitude ϕ, R_{eq} is the radius of the Earth at the equator, approximately 6378137 m, and f is the flattening.

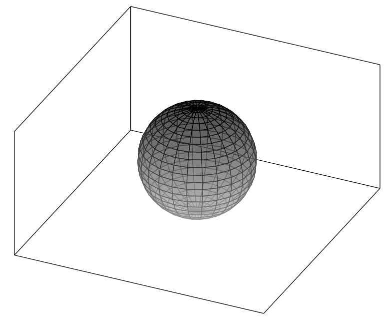


Figure 1.4: Plotted using a flattening of 1/298, it is clear that the flattening of the Earth is very subtle!

Exercise 1.14 Based on height above sealevel, Mt. Everest (8,848 m above sealevel, about 28 degrees N) is the highest place on Earth. Cotopaxi volcano (5,897 m above sealevel, about 1 degree N) is also a very high mountain. What if we measured height with respect to the center of the Earth? Which is taller: Cotopaxi or Everest?

Solution 1.14 Mt Everest:
$6378.137(1 - (1/298.257)\sin^2 28) + 8.848 = 6382.272\,\text{km}$
Cotopaxi volcano:
$6378.137(1 - (1/298.257)\sin^2 1) + 5.897 = 6384.034\,\text{km}$
The summit of Cotopaxi is further from the center of the Earth than is the summit of Mt. Everest.

Exercise 1.15 Finding map distances on a sphere?

Departures of the figure of the Earth from the reference ellipsoid occur primarily because of large scale density differences within Earth. These density differences cause undulations in the shape of Earth. So a further surface can be defined that accounts for these regional undulations. This surface is called the geoid. The two main characteristics of the geoid are:

- Earth's gravity field is perpendicular to the geoid everywhere - making the geoid an equipotential surface.
- The geoid coincides with the theoretical position of the surface of Earth's oceans at rest.

The geoid could be any equipotential surface. On Earth, it is simply convenient to refer the geoid to mean sea-level. The elevation of the geoid commonly deviates from the reference ellipsoid by up to 40 m. In some locations, such as off the coast of India, the geoid differs from the ellipsoid by more than 100 m. This means that if you sail on a ship from Madagascar to Bombay, you travel through a trough 100 m deep with respect to the reference ellipsoid. Actual topography varies with respect to both the ellipsoid and the geoid. Usually, elevation is referenced to the geoid, as in *meters above sea-level*, but it is possible to reference topography to the reference ellipsoid.

Latitude, Longitude, and Map Datums

Any geographic location on the surface of Earth can be expressed as a latitude and longitude. Unlike the UTM grid system, each point at the Earth's surface has a unique latitude and longitude. While elevation is usually referenced to the geoid, latitude and longitude are referenced to the ellipsoid. Two things are needed to express a position on the surface of Earth in terms of latitude and longitude:

- An origin for the coordinate system
- An equation for the reference ellipsoid

The origin is agreed upon by convention. Zero latitude corresponds to the equator (the plane orthogonal to the axis of Earth's rotation). Values of latitude vary from 0° N at the equator to 90° N at the north pole and 90° S at the south pole. Often, especially in many computer programs used by geoscientists, latitude is represented as positive in the northern hemisphere and negative in the southern hemisphere. That is, latitude varies from $+90°$ at the north pole to $-90°$ at the south pole. Longitude is more arbitrary. Zero longitude is designated as the meridian of the astronomical observatory in Greenwich, England. Longitude increases to the East. [1] The eastern hemisphere extends to a longitude of 180° E. Three different ways are used to represent longitude in the western hemisphere. The simplest approach, and most rarely used, is to simply let longitude values increase eastward until reaching 360° as the meridian at Greenwich, England, is approached from the west. In this system, longitude is uniquely expressed as any number, $0° - 360°$. Alternatively, longitude in the western hemisphere can be assigned a negative number, in which case longitude varies from $-180°$ to $+180°$. The most common means of expressing longitude is to express longitude using E and W to designate eastern and western hemisphere, respectively. In this system, longitude varies from 180° W to 180° E. Many computational errors have been made by forgetting which system is being used to designate longitude.

[1] Longitude counts upward to the East like on a familiar xy -plot with positive values increasing to the right as one faces North and looks down.

Various equations for the reference ellipsoid are in use. These equations differ in the average radius of the Earth chosen and the eccentricity (or flattening) of the ellipse. In the context of map coordinates, these different reference ellipsoids are referred to as different map datums. Three common map datums for North America are described in Table 1.1.

NAD27 refers to the North American Datum of 1927; NAD83 refers to the North American Datum of 1983; WGS refers to the

Ellipsoid (Datum)	R_{eq} (m)	$1/f$
Clarke 1866 (NAD27)	6378206.4	294.9786982
WGS84	6378137.0	298.257223563
GRS 1980 (NAD83)	6378137.0	298.257222101

Table 1.1: Common map datums used in North America and their parameters

World Geodetic System; GRS refers to Geodetic Reference Systems. These are the most common ellipsoids (sometimes called spheroids) and map datums that you are likely to encounter, but there are very many more. Prior to the mid-1980s, map datums and reference ellipsoids were derived from regional surveys. These ellipsoids were optimized, naturally enough to fit the figure of Earth in the regional area of interest. So, for example, the Clarke 1866 ellipsoid was derived for North America and its datum referenced to a location in central Kansas (meaning that the fit is optimal at this location). Departures from the geoid using Clarke 1866 NAD27 are minimized for North America, but are quite large for other parts of the globe. In the mid-1980s a global best-fit ellipsoid could be calculated using satellite data. The datum for this ellipsoid is Earth-centered, in other words, the center of Earth is essentially the datum's *origin*, and departure of the ellipsoid from the geoid is minimized globally.

The WGS84 datum is now commonly used, worldwide. Nevertheless, a large number of US maps in print were constructed using the Clarke 1866 ellipsoid and the NAD27 datum. In other parts of the world, other local ellipsoids and datums were used. [2] The difference between WGS84 and NAD27 is large in a state like Florida, that is far from the NAD27 origin. In Florida, the difference between NAD27 and WGS84 is about one second of latitude.

[2] *A given location at the surface of the Earth will have a different latitude and longitude depending on the map datum used.*

All this has two practical results for people who gather and use map data. First, the ellipsoid and datum used must be reported when map coordinates are given. Otherwise the data cannot be associated to a single specific location on the surface of Earth. Second, GPS receivers are very often used to determine position. Most GPS receivers will report position (*e.g.*, latitude and longitude) using one of any number of datums. If these data are to be plotted on a map, the datum used by the GPS receiver must match the datum used to create the map, otherwise the location of the data will be incorrectly plotted on the map. It is a very simple procedure to note the map datum. Surprisingly this information often is not noted, resulting in errors that can be very difficult to identify and fix later.

LATITUDE AND LONGITUDE can be expressed in decimal degrees (for example, MLK1: 28.0626° N, 82.4153° W, WGS84) or in degrees, minutes and seconds. To convert 28.0626° N to degrees and minutes, express the latitude with a whole number and a remainder. The remainder is multiplied by 60, because there are 60 minutes in 1 degree.

$$28.0626^\circ\ \mathrm{N} = 28^\circ + 0.0626^\circ \times \frac{60\ \text{minutes}}{\text{degree}} = 28^\circ\ 3.756' \tag{1.4}$$

To convert to degrees, minutes, and seconds the minutes must be treated as a whole number and remainder that is multiplied by 60, because there are 60 seconds in one minute.

$$28^\circ\ 3.756'\ \mathrm{N} = 28^\circ + 3' + 0.756' \times \frac{60\ \text{second}}{\text{minute}} = 28^\circ 3' 45.36''\ \mathrm{N} \tag{1.5}$$

Exercise 1.16 Convert the longitude 82.4153 W to degrees minutes seconds

Exercise 1.17 Convert latitude $87^\circ\ 47'\ 19''$ N into decimal degrees.

Exercise 1.18 Consider this set of map coordinates, given in UTM (zone 17) and in Latitude and Longitude:

Point A:	0000000 N	388719 E	0° N	82 ° W
Point B:	1105470 N	388736 E	1° N	82 ° W
Point C:	7208827 N	452845 E	65° N	82 ° W
Point D:	7320272 N	454615 E	66° N	82 ° W

(a) Points A and B are 1° of latitude apart. How far north is point B of point A in meters?

(b) Points C and D are also 1° of latitude apart. How far north is point D of point C in meters?

(c) Why is there a difference in the length of 1° of latitude between these pairs of points?

(d) All of the points are at the same longitude. Why does the UTM Easting coordinate change as a function of latitude?

Converting from Latitude/Longitude to UTM coordinates and back again

GIVEN THE DIFFERENCES IN MAP DATUMS AND IN COORDINATE SYSTEMS, it is essential for geologists and geophysicists to easily convert coordinates from one datum to another, or from one grid to another. Fortunately there are tools available for doing this conversion. One of the most reliable tools was developed by staff at the US

Geological Survey. This conversion tool is the **Proj.4 Cartographic Projections Library**. [3] Once installed on a computer, Proj.4 is useful for solving coordinate system conversion issues. Many tools have been developed for use on the Web, with graphical user interfaces (GUI), to make it easy to use Proj.4 or simialr software for map coordinate conversion. In the following, some simple map coordinate conversions using Proj.4 commands are described.

[3] Check out the Proj.4 website: `http://proj4.org/` for current versions and downloads. It is freely available.

For example, suppose a colleague specifies the latitude and longitude of 82° W, 27° N (NAD27 datum) for a work site near Tampa, Florida. The *cs2cs* program, part of the Proj.4 package, can be run to convert this coordinate to the NAD83 datum:

type:

```
cs2cs +proj=latlong +datum=NAD27 +to +proj=latlong +datum=NAD83 -f "%0.6f"
```

then type:

```
-82 27
```

Output of the *cs2cs* code using these parameters will be:

```
-81.999809     27.000336
```

That is, 82° W, 27° N (NAD27 datum) converts to 81.999809° W, 27.000336° N (NAD83 datum). Proj.4 can also convert from latitude/longitude to the UTM grid system using the program, *proj*:

```
proj +proj=utm +datum=WGS84 +zone=12 -f "%.0f"
```

then type:

```
111d17'55"W 38d34'N
```

Note that the datum (in this case WGS84) and UTM zone (in this case zone 12) must be specified. Output of *proj* is:

```
473985    4268733
```

corresponding to 473985 E, and 4268733 N in the UTM grid for zone 12 (WGS84 datum). Similarly, to convert UTM coordinates to Latitude and Longitude, one can use:

type:

```
proj -I +proj=utm +ellips=WGS84 +zone=12  -f "%.6f"
```

then type:

```
473985 4268733
```

this outputs:

```
-111.298620    38.566665
```

which corresponds to 111.298620° W, 38.566665° N (WGS84 datum). The **Proj.4 Cartographic Projections Library** contains vast resources for making such conversions, and can easily be applied to whole data files. Note, the *cs2cs* and *proj* programs can understand numbers in decimal degree format and in degrees/minutes/seconds format.

Exercise 1.19 Use *proj* or other reliable conversion software to determine the UTM coordinate of 28.0626° N, 82.4153° W, using the WGS 84 datum and the Clarke1866 datum. Compare the coordinates.

Solution 1.19
WGS84: 3104945.1 N, 360916.7 E
Clarke1866: 3104773.2 N, 360914 E
That is a difference of about 172 m in Northing coordinate and about 3 m in Easting coordinate.

Map Projections

A MAP PROJECTION transfers the coordinates of points located on the reference ellipsoid to a flat surface – a map. A huge number of map projections are in frequent use and essentially an infinite number can be derived. John Snyder [4] discusses the mathematical basis of various map projections in detail. He broadly states that a map projection can be chosen on the bases of:

[4] J. P. Snyder. *Map projections–A working manual*, volume 1395 of *U.S. Geological Survey Special Publication*. US Geological Survey, Reston, Va., 1987

- *Area.* Many types of map projection are equal area. A given area on one part of the map corresponds to an actual area on the surface of the Earth. This ratio is constant across the entire map. This is done at some expense - scale and angles are distorted on equal area map projections.

- *Shape.* Many map projections preserve shape and angle. These map projections are often called conformal

- *Scale.* No map projection preserves scale. That is, scale is not the same from one part of the map to another.

For large scale maps showing small areas, the differences between projections are fairly trivial. For large areas – like continents or ocean basin, shown on small scale maps, the differences are obvious and important to take into account. The key thing to remember about map projections when using maps is that depending on the maps projection the scale across the map changes, angles measured off the map can change, and areas in on part of the map may be differnt from areas seen on another part of the map. These features of map projections adversely affect map arithmetic!

Chapter 2 Venn Diagrams and Directed Graphs

Venn diagrams are used for advanced counting. Venn diagrams represent information about groups of data graphically. It may sound funny to discuss “advanced" counting, but almost all of the concepts introduced here are heavily used in programming computers to solve geological problems. Computer science, generally, greatly relies on advanced counting. Even if you never write a geoscience computer code yourself, you will be more successful as a geoscientist if you can converse with people who do write computer programs.

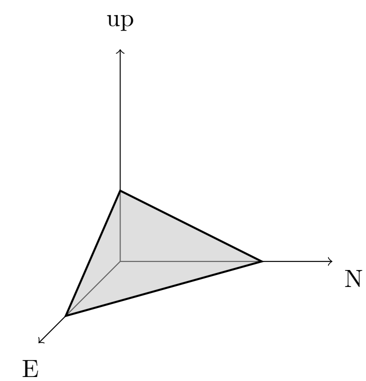


Figure 1.5: If a bedding plane is considered to be a geometric plane, its orientation in space can be measured. This plane has a strike and dip of 315°, 30°.

Consider a Venn diagram illustrating the skills of students during the first week of their geology field course (Figure 1.6). A complex set of skills is required to make a geologic map. Some of those skills are: (*a*) identifying the geology in the field, such as identifying sedimentary beds and the bedding planes that separate different sedimentary beds, (*b*) finding your location on the map so you can draw the geology you observe on the map, (*c*) making measurements, such as measuring the strike and dip of a sedimentary beds with a pocket transit. In order to make the map, all three skills (and more) are likely required. During the first week of field camp, of course, not everyone has yet acquired these skills. Consider the following Venn diagram that illustrates the distribution of students in the course by their skills.

Three students (let’s call them Chuck, Paul, and Rocco) have none of the required skills to make the map. Let’s define these students as members of set D. We say that Chuck, Paul and Rocco are elements of set D. We can write this relationship using notation:

$$D = \{Chuck\,, Paul\,, Rocco\} \tag{1.6}$$

In discussing sets in the context of computer programming languages like Python, PERL or C, sets are referred to as arrays, lists, or hashes. A mathematician would say there are three elements in Set D, or the cardinality of D is 3. This is written:

$$|D| = 3 \tag{1.7}$$

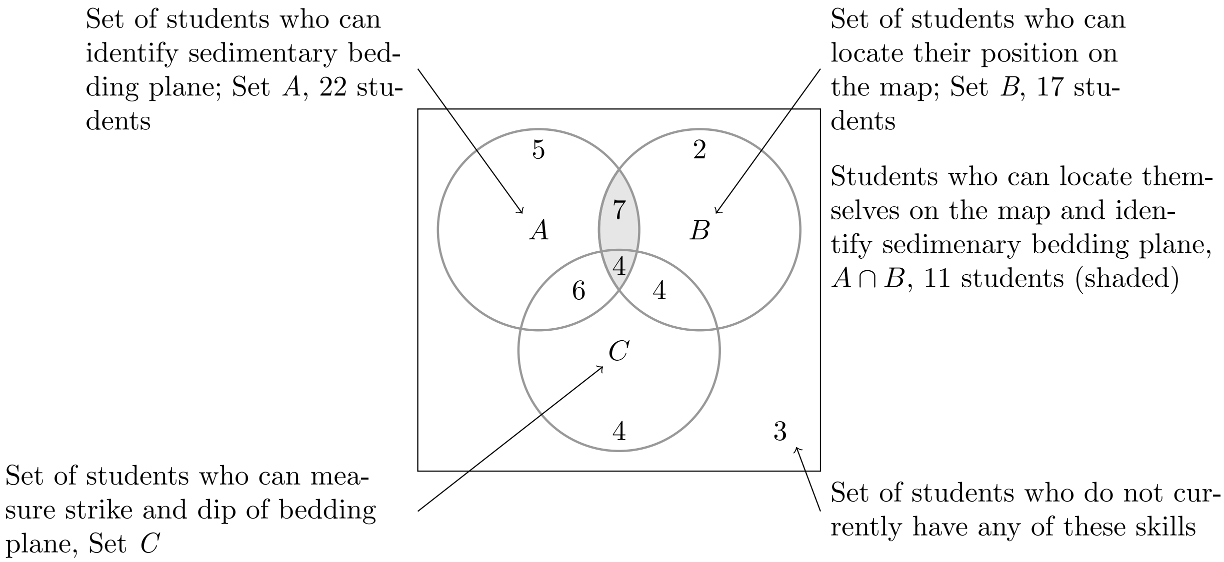


Figure 1.6: A Venn diagram revealing the relative mapping talents of students at the outset of their field course.

A computer scientist would say there are three elements in the array D. Or the "length" of D is 3. We can also indicate that Chuck is an element of Set D:

$$Chuck \in D \tag{1.8}$$

or that set D contains Chuck and Rocco:

$$D \ni \{Chuck, Rocco\} \tag{1.9}$$

This becomes very useful if we are going to quantify relationships based on sets. For example

$$\forall x \in D : P(x) \tag{1.10}$$

says that for all elements in D (Chuck, Paul, and Rocco) a proposition, $P(x)$, is true. In this case the proposition might be that they need to work on their mapping skills. Literally, $\forall$ can be read "for all" and so is called the universal qualifier. Similarly, the expression:

$$\exists x \in D : P(x) \tag{1.11}$$

means that there exists within set A at least one element for which the propositional function is true. For example, one might state that at least one of the members of set D has not taken the course prerequisites.

Exercise 1.20 Based on the Venn diagram (Figure 1.6):

(a) How many students are enrolled in this field course?

(b) How many students know how to measure a strike and dip?

(c) Suppose a student in this group has the ability to identify sedimentary beds and measure their strike an dip. Unfortunately, this student has not yet learned to locate themselves on a topographic map. If instructed to team with one other student, and not duplicate skills, who should this student team with? Answer the question by redrawing the Venn diagram with the appropriate area shaded.

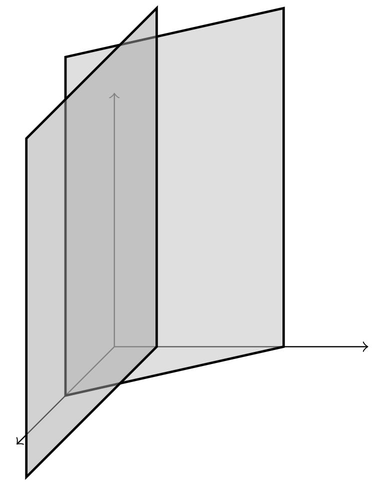


Figure 1.7: Some minerals have cleavage planes along which these minerals tend to break. The silicate mineral amphibole has two cleavage planes, making angles of 120° and 60°.

Exercise 1.21 Set A is the silicate minerals, set B is all minerals with cleavage.

(a) Draw a Venn diagram showing the relationship between Set A and Set B.

(b) List two minerals that exist for each bounded area on your Venn diagram.

(c) All silicate minerals that we know of occur within the crust of the Earth. Write a proposition using $\forall$ that expresses this concept.

(d) Some silicate minerals have conchoidal fracture. Write a proposition using $\exists$ that expresses this concept.

Exercise 1.22 The Smithsonian Institution has created the premier database of Holocene volcanic activity. [5] Using the Smithsonian online volcano database, search for the number of Holocene volcanoes that have human populations between 10,000 and 100,000 people living within 30 km of the volcano. How many calderas are there in the database, and how many of these calderas have human populations between 10,000 and 100,000 people living within 30 km? Make a Venn diagram showing your results.

[5] See the Smithsonian volcano database website: http://volcano.si.edu

Exercise 1.23 The United States Geological Survey (USGS) has worked with Association of American State Geologists (AASG) to create the National Geologic Map Database. [6] Search the map catalog for the total number of maps published by the USGS/AASG for the state of Alaska. Determine how many of these maps are bedrock geology maps. How many of the maps have scale 1:500,000? How many of the bedrock geology maps have scale 1:500,000? Make a Venn diagram of your results.

[6] https://ngmdb.usgs.gov/ngmdb/ngmdb_home.html

Exercise 1.24 The Paleobiology Database is a huge catalog of fossil occurrences. [7] Explore this database. Of the total number of Mesozoic fossil occurrences in the database, how many are Triassic in age? How many are Mollusca? How many are Triassic Mollusca? Plot your results on a Venn diagram.

[7] https://paleobiodb.org/#/

ADDITIONAL SET OPERATIONS are described using a notion for complementary sets, union of sets and intersections of sets. A univrsal set of cardinal numbers between 1 and 10, inclusive, can be defined as:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}. \tag{1.12}$$

Within the set U, define two more sets:

$$A = \{2, 3, 4, 5, 6, 7\}. \tag{1.13}$$

$$B = \{3, 4, 5, 6, 7, 8\}. \tag{1.14}$$

The complement of A consists of all elements in the unversal set, U, that are not included in A. The complement is written:

$$A^C = \{1, 8, 9, 10\}. \tag{1.15}$$

Even though $\{3, 4, 5, 6, 7\} \in B$, $\{3, 4, 5, 6, 7\}$ is also in A, so $\{3, 4, 5, 6, 7\}$ is not in the complement of A. The union of sets A and B is:

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}, \tag{1.16}$$

and the intersection is:

$$A \cap B = \{3, 4, 5, 6, 7\}, \tag{1.17}$$

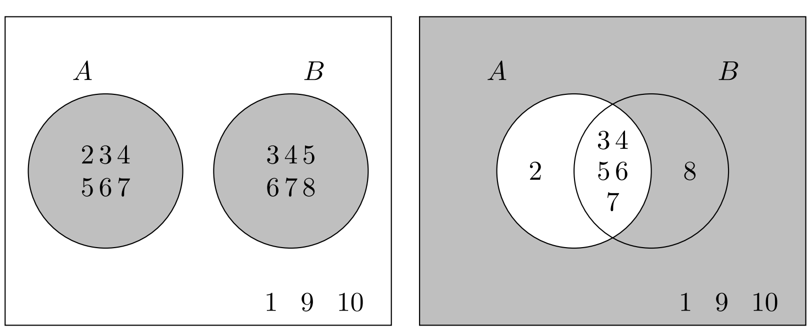


Figure 1.8: (*left*) Two sets, A and B each consist of some of the cardinal numbers between 1 and 10. (*right*) The complement of set A is A^C, shown as the shaded area.

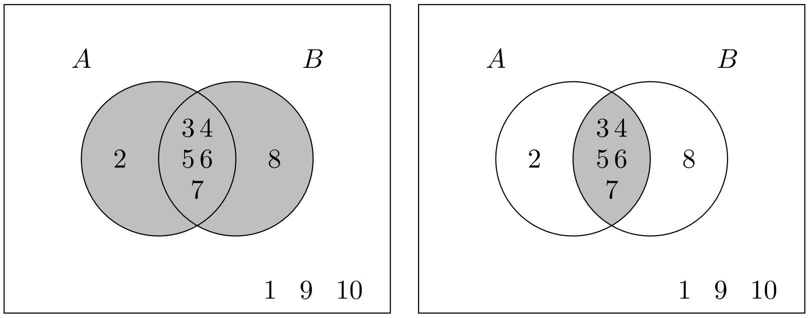


Figure 1.9: (*left*) The union of A and B shaded gray, $A \cup B$. (*right*) The intersection of A and B shaded gray, $A \cap B$.

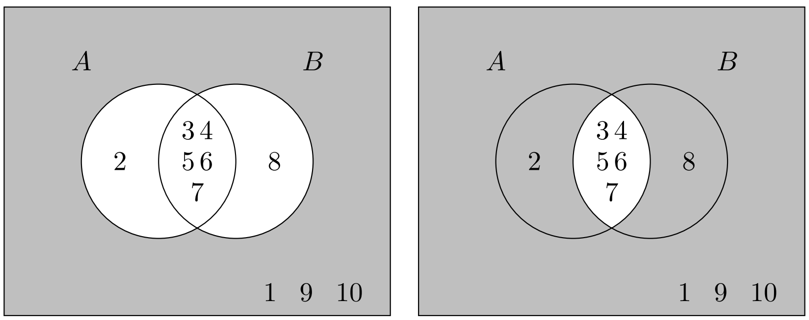


Figure 1.10: (*left*) The complement of the union of A and B shaded gray, $(A \cup B)^C$. (*right*) The complement of the intersection of A and B shaded gray, $(A \cap B)^C$.

Exercise 1.25 Databases are often used to compare and contrast the elements in sets. Consider a set of major silicate minerals we define as: feldspar[1], mica [2], amphibole [3], quartz [4], pyroxene [5], olivine [6]. garnet [7], kyanite [8]. The numbers in brackets are referred to as an index, key, or ID for each element in the set. Using these indicies, the set of major silicate minerals in granite is:

$$A = \{1, 2, 3, 4\} \tag{1.18}$$

and the set of major silicate minerals in diorite is:

$$B = \{1, 2, 3, 5, 6\} \tag{1.19}$$

List the major silicate minerals by their indicies in the sets:

(a) $A \cap B^C$ (that is, list the minerals of the compound set)

(b) $A^C \cap B$

(c) $A^C \cap B^C$

(d) $A^C \cup B^C$

Exercise 1.26 The difference between two sets is defined, for example, as the elements in A that are not included in B, and is written $A - B$. For example, in the above sets of major silicate minerals in granite and diorite, $A - B = \{4\}$, quartz. Find the sets:

(a) $B - A$

(b) $(A - B) \cup (B - A)$

(c) $(A \cup B) - (A \cap B)$

Directed Graphs

A graph is another way to visualize information. You are no doubt experienced using graphs that plot one variable against another, for example, plotting a parabola or line. Now consider a different kind of graph – one that deals with sets. In graph theory, a graph is a way to visualize a set of data in which the elements of the set are linked to one another in some way. For example, there may be a set of geological formations in a mapping area. Some of these formations are linked because they have a stratigraphic relationship (one overlies the other) or have some other relationship (e.g., a structural relationship in which one unit is faulted against the other). In this type of graph, elements of the set are visualized as nodes (often called vertices) and the relationship between elements is drawn as a line (often called an edge) that links vertices that have some relationship.

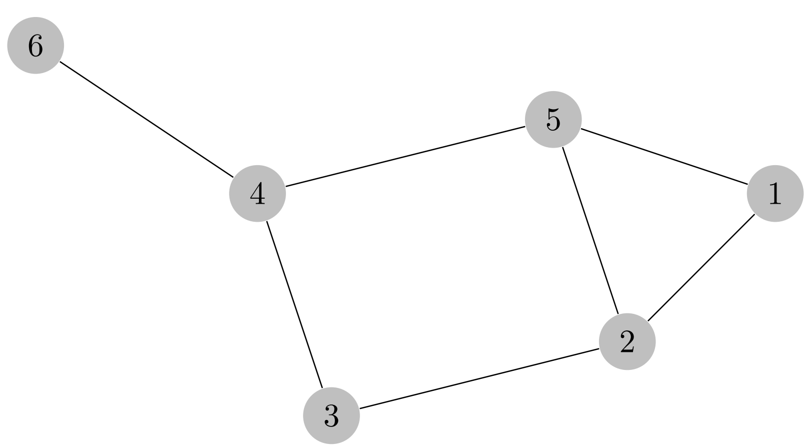


Figure 1.11: A simple graph showing nodes (shaded circles) and connections (lines).

The nodes of this graph form a set, $\{1, 2, 3, 4, 5, 6\}$, and additional information is provided by the edges (lines) between some nodes. If the numbers in this example index lithologic units on a map, the edges might represent contact relationships among these units. The edge, in this case, does not provide information about the nature of the geologic contact (conformity, unconformity, fault, etc.).

In a directed graph, the edges are represented by arrows that indicate flow, or some other directional relationship, between one vertex and another. Consider the following example of lithologic units in an outcrop.

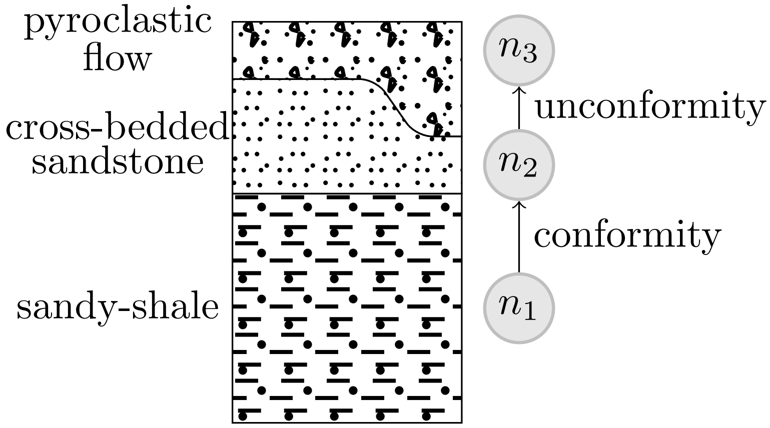


Figure 1.12: A stratigraphic section shows three lithologic units representing different rock types. The directed graph shows this statigraphic section as three nodes, representing the three rock types, and labelled edges, which show the stratigraphic relationship and the type of geologic contact.

The sandy-shale, $\{n_1\}$, is the geologic unit lowest in the stratigraphic section. The sandy-shale is overlain by the cross-bedded sandstone, $\{n_2\}$, so on the graph there is an edge between $\{n_1\}$ and $\{n_2\}$. We can infer based on the stratigraphic section that $\{n_2\}$ is younger than $\{n_1\}$ so the graph is directed, pointing toward the younger, stratigraphically higher unit. Similarly, there is a contact between $\{n_2\}$ and the overlying pyroclastic flow deposit, $\{n_3\}$. This relationship is also shown by an edge on the directed graph, with an arrow indicating that the pyroclastic flow deposit is stratigraphically higher, and younger, than the cross-bedded sandstone unit. Additional information is placed on the graph by labeling the edges to show the nature of the geologic contact. The graph indicates a conformable relationship between $\{n_1\}$ and $\{n_2\}$, and an unconformity between $\{n_2\}$ and $\{n_3\}$.

Directed graphs can be expressed as sets. The set of geologic units observed in this stratigraphic section is $N = \{n_1, n_2, n_3\}$. The set of geologic contacts observed in the outcrop is $E = \{(n_1, n_2), (n_2, n_3)\}$. The types of contacts can also be defined:

$$\varphi(n_1, n_2) = \rightarrow\rightarrow \textit{ (or conformity)} \tag{1.20}$$

$$\varphi(n_2, n_3) = \sim\sim \textit{ (or unconformity)} \tag{1.21}$$

A path through the directed graph that shows all of these relationships:

$$W = (n_1, \rightarrow\rightarrow, n_2, \sim\sim, n_3) \tag{1.22}$$

In these equations, symbols are introduced to represent the conformity, $\rightarrow\rightarrow$ and the unconformity, $\sim\sim$.

We now have three representations of the stratigraphic section: (*i*) a sketch, which has meaning to geologists who can interpret the relationships; (*ii*) a directed graph that forces clearer labeling of the geologic features of interest in the outcrop; and (*iii*) mathematical representations of the sets, N and E, and the path, W, through the directed graph. These alternative representations of the same outcrop are useful for a few reasons:

- A geological sketch of the stratigraphic section allows one to introduce creative notation and detail on the sketch that highlight improtant features of the outcrop and section.

- Sketching a directed graph for a stratigraphic section, map or outcrop forces a systematic approach to describing all of the units and their contacts, and so reduces the chances of making mistakes by omitting observations.

- It becomes practical to do more advanced counting and classification. For example, the set of conformable contacts in this example is $C = \{(n_1, n_2)\}$. Geologists often consider packages or groups of geologic units, like the set C.

- In order to put geologic information into a database and to use the database, it is necessary to encode the outcrop observations in this way.

Now consider a nearby outcrop, in which the erosion of the channel containing the pyroclastic flow deposit is deeper.

This case is more complicated because there is also a geologic contact between $\{n_1\}$ and $\{n_3\}$. So there are the same number of vertices (geologic units), but more edges (contacts):

$$N = \{n_1, n_2, n_3\} \tag{1.23}$$

$$E = \{(n_1, n_2), (n_2, n_3), (n_1, n_3)\} \tag{1.24}$$

There are two paths on the directed graph and a full description of the outcrop shows both paths:

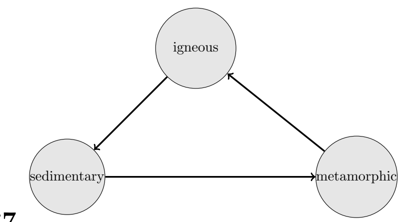
$$W_1 = (n_1, \rightarrow\rightarrow, n_2, \sim\sim, n_3) \tag{1.25}$$

$$W_2 = (n_1, \sim\sim, n_3) \tag{1.26}$$

Try sketching the form of this outcrop.

Exercise 1.27 In the game rock-paper-sissors, rock breaks sissors, sissors cuts paper, and paper smothers rock. The rock-paper sissors game can be expressed as a directed graph. In the geologist's version of this game,mid-crustal metamorphic rocks are partially melted to form igneous rocks, igneous rocks erode at the surface to become sedimentary rocks, sedimentary rocks change to metamorphic rocks under the high temperature and pressure conditions of burial. This cycle represents part of the rock cycle. Diagram this part of the rock cycle as a directed graph.

Exercise 1.28 Describe the hydrologic cycle as a directed graph.



Solution 1.27

Exercise 1.29 The following geologic section represents a zone of contact metamorphism around a granite intrusion. Create a directed graph that summarizes the outcrop relationships shown in the section. Be sure that zones, such as high grade hornfels, only appear once on the directed graph, even though some of these units appear on the section in more than one location (indicated by like shading).

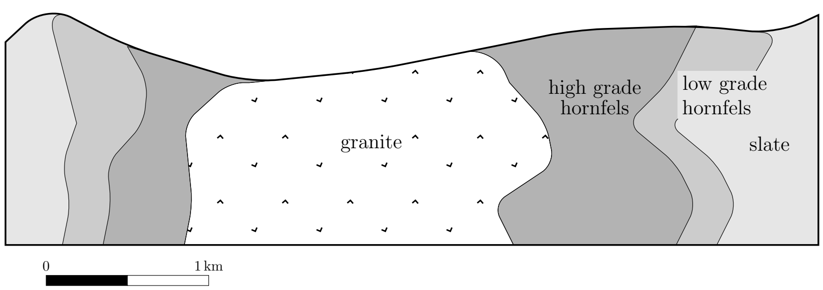


Figure 1.13: A geologic sketch showing the contact metamorphic zones between a granite intrusion and the slate the granite intruded.

Exercise 1.30 This block diagram (Figure 1.14) shows sedimentary formations in different patterns and shading. Puffy clouds indicate the Earth's surface. The diagram is meant to provide a 3D perspective on the orientation (strike and dip) of the sedimentary formations in the subsurface. Create a directed graph showing the relationships between these sedimentary formations.

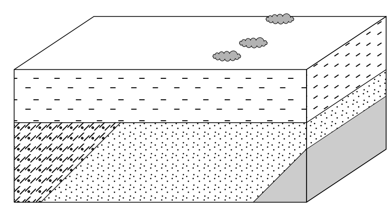


Figure 1.14: Three-dimensional perspective on the distribution of sedimentary formations in the subsurface. The different patterns and shades illustrate different sedimentary rock types.

Solution 1.30 The first step in solving this problem is to develop a nomenclature to describe the sedimentary formations. Let's call the gray shaded unit (perhaps a mudstone) n_1; the stippled unit (perhaps a sandstone) n_2, the unit with the wavy pattern n_3 and the unit shown by the dashed pattern n_4.

The second step involves identification of geologic contacts between formations using the block diagram. The block diagram shows there are geologic contacts between n_1 and n_2, between n_2 and n_3 and so on. Using our nomenclature, the full set of contacts is:

$$E = \{(n_1, n_2), (n_2, n_3), (n_2, n_4), (n_3, n_4)\} \tag{1.27}$$

The third step involves making the directed graph, which shows the units $n_1 - n_4$ as nodes, and connects nodes with arrows showing stratigraphic relationships. We can also add text to describe the nature of the geologic contact (conformity or unconformity).

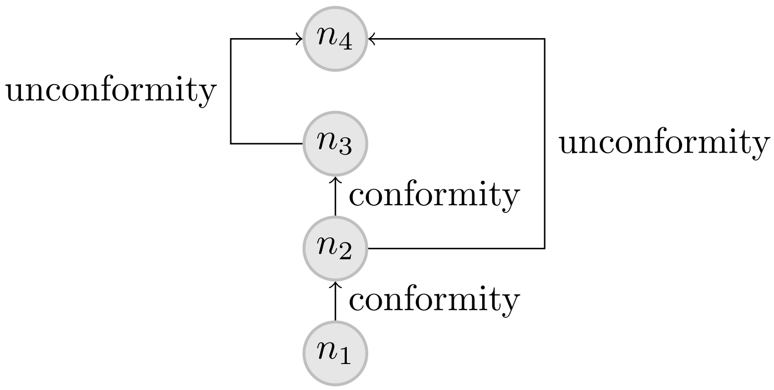


Figure 1.15: A directed graph for the block diagram shown in Figure 1.14.

Notice that the number of arrows on the directed graph is equal to the number of element pairs in set E.

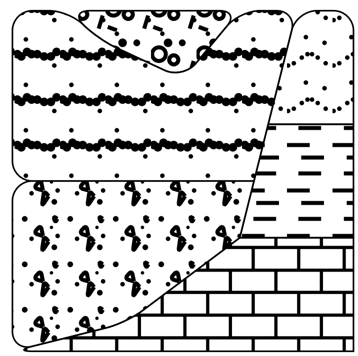


Figure 1.16: A geologic sketch of a buttress unconformity.

Exercise 1.31 The sketch (Figure 1.16) illustrates the stratigraphic section and relationships for several map units shown in cross section. One the right, the stratigraphically lowest unit is a limestone (brick pattern); above that is a shale (dash pattern); and above that is a ripply sandstone (wavy stippled pattern). To the left the units are volcanic. The lowest unit is a pyroclastic flow, followed by a bedded tephra fallout sequence. A lahar deposit fills the channel. The contact between the volcanic units to the left and the older sedimentary units to the right is usually referred to as a buttress unconformity and suggests the volcanic units inundated an older valley that was formed by erosion of the older sedimentary rocks. Draw a directed graph for geologic relationships shown on this sketch.

Exercise 1.32 Faults juxtapose different formations and lithologies (rock-types). The displacement of rocks by faults can also be summarized using directed graphs. Make a directed graph illustrating all of the contact relationships shown in this sketch of the cross section of a normal fault (Figure 1.17).

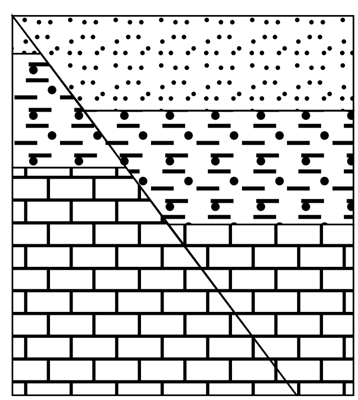


Figure 1.17: A geologic sketch of a normal fault displacing three sedimentary units.

Chapter 3 Quantities and Unit Conversion

GEOSCIENTISTS MEASURE THINGS. How long is the fault? What is the diameter of a foraminifer? What is the concentration of CO_2 in the Earth's atmosphere? What is the mass of tin in the deposit? What is the uplift rate of a mountain range? This emphasis on measurement means geoscientists must be proficient with the tools of measurement – from surveying tools and GPS to microscopy and mass spectrometers. So it is vital to understand quantities, units, and unit conversions.

Exercise 1.33 Since a quantity is a number times a unit, sketch a graph of $y = ax$. Label the slope on your graph and describe the graph.

Exercise 1.34 Paleontologists measure the sizes of fossils. For example, the length of a fossil shark's tooth might be measured with a micrometer. Comparison of the lengths of various sharks' teeth might reveal something about shark speciation and environment. Suppose measurements are obtained of the length of sharks' teeth in both millimeters (mm) and inches (in.).

(a) Sketch a graph with length (in) plotted on the x-axis (the abscissa) and length (mm) plotted on the y-axis (the ordinate). Describe the graph.

(b) Which is larger: an inch or a millimeter?

(c) How is the difference between an inch and a millimeter represented on your graph?

Solution 1.34

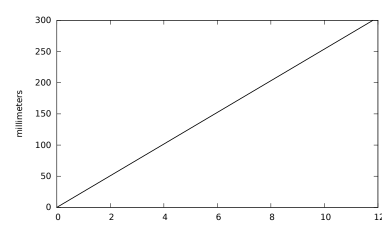


Figure 1.18: The graph is a straightline through the origin. From the slope of the graph, it is easy to deduce that the inch is longer than the millimeter. The positive slope > 1 indicates that millimeters increases at a faster rate than inches, so inches are the larger unit.

Exercise 1.35 An ounce is a unit of weight equal to one sixteenth of a pound or 16 drams or 28.349 grams (reported to five significant figures). There are 1000 grams in one kilogram.

(a) Which weighs more, one dram or one gram?

(b) Which weighs more, one pound or one kilogram?

Exercise 1.36 Given a constant value for a quantity (say 1 km length), consider the change in number with change in unit in the equation $Q = nu$. That is, if the constant distance is measured in tenths, of kilometers, fifths of kilometers, or half kilometers, what is the change in the number required to keep the quantity constant?

(a) To solve this, simply graph $Q = nu$, with u on the x-axis and n on the y-axis. The curve you plot is called hyperbolic, because n is equal to a constant divided by a variable. What happens to the number, n, when u is very big; very small?

(b) Now consider two quantities, Q_1 and Q_2, where $Q_2 > Q_1$. Plot the two equations, $Q_1 = nu$ and $Q_2 = nu$ on the same graph. What is the relationship between the two curves? Do they ever cross?

(c) Using either graph, as the unit of measure doubles (e.g., u changes from 1 to 2, what happens to the number, n?

Solution 1.36

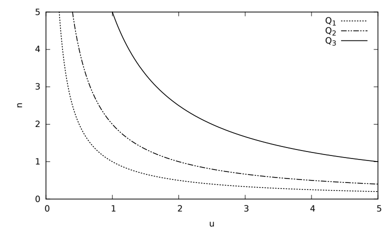


Figure 1.19: If $Q_3 > Q_2 > Q_1$, then Q_1 is alway above Q_2, which is always above Q_3. The curves never cross. For any of the curves, as u doubles, n halves.

Unit conversions with a single dimension

Exercise 1.37 One of your fossil shark's teeth is 4.2 inches long. What is it's length in microns, millimeters, centimeters, and meters?

Solution 1.37
$4.2 \text{ inches} \times \frac{2.54\,\text{cm}}{1\,\text{inch}} = 10.668\,\text{cm}$
$10.668\,\text{cm} \times \frac{1\,\text{m}}{100\,\text{cm}} = 0.10668\,\text{m}$
$10.668\,\text{cm} \times \frac{10\,\text{mm}}{1\,\text{cm}} = 106.68\,\text{m}$
$10.668\,\text{cm} \times \frac{10^4\,\mu}{1\,\text{cm}} = 1.0668 \times 10^5\,\mu$

Exercise 1.38 The average distance from the Sun to Jupiter is 5.2 AU (astronomical unit). The distance from the Sun to Earth is defined as 1 AU; there are 14,959,781 km in 1 AU. How far is Jupiter from the Sun in kilometers?

Exercise 1.39 Given the average concentration of CO_2 in the Earth's atmosphere is currently 405 ppm (parts per million), what is the percentage of CO_2 in the atmosphere?

Solution 1.39 $\frac{405\,\text{ppm}}{10^6} = 4.05 \times 10^{-4} = 0.0405 \times 10^{-2} = 0.0405\%$

Exercise 1.40 If the CO_2 concentration in the Earth's atmosphere rises by 45%, what will be the new concentration in ppm?

Solution 1.40 405 ppm $\times 0.45 +$ 405 ppm = 587.25 ppm

Exercise 1.41 Suppose you measure 0.015 ppm MTBE (Methyltert-butyl ether) in a sample of drinking water. MTBE is used to increase the octane rating of gasoline and sometimes contaminates aquifers. According to the U.S. Environmental Protection Agency, humans can taste MTBE in drinking water at a concentration of 20 ppb. Are you likely to be able to taste MTBE in your sample?

Exercise 1.42 Geochemists routinely estimate the concentration of elements, and simple compounds of elements in rocks using instruments like mass spectrometers. A mass spectrometer literally measures the mass of a spectrum of components (molecules) in a sample. A geochemical analysis reports the abundance of the "major element" K_2O in a rock sample to be 2.5%. In the same sample, the concentration of the "trace element" Sr is reported to be 300 ppm. This is a typical Sr concentration found in dacites, extrusive igneous rocks that are enriched in incompatible trace elements like Sr. Which is more abundant in this rock sample, K_2O or Sr?

Solution 1.42
$\frac{300\,\text{ppm}}{10^6} = 3 \times 10^{-4} = 0.03 \times 10^{-2} = 0.03\%$, or there is about 83× more K_2O in a rock sample than Sr.

Exercise 1.43 A geologist measures the declination and inclination of the Earth's magnetic field in degrees, but must know the declination and inclination of the magnetic field in radians in order to use an equation to find the horizontal and vertical components of the magnetic field. If the measured declination is 15° and the inclination is 61°, what are the declination and inclination in radians?

Solution 1.43
declination: $15° \times \frac{\pi}{180°} = 0.26\,\text{rad}$
inclination: $61° \times \frac{\pi}{180°} = 1.1\,\text{rad}$

Exercise 1.44 In the U.S. and some other parts of the world, distances are often reported in inches (in), feet (ft), yards, and miles. Given: 1 in = 2.54 cm, 1 ft = 12 inches, 1 yard = 3 ft, 1 mile = 5280 ft,

(a) How many inches are in one meter?

(b) How many meters are in 25 yards?

(c) How many kilometers are in five miles?

(d) Sometimes it is assumed that 1.6 km = 1 mile to simplify conversions. What is the percentage error in using this value, instead of the exact value (1 in = 2.54 cm) in converting from miles to kilometers?

Solution 1.44
$1\,\text{mile} \times \frac{5280\,\text{feet}}{1\,\text{mile}} \times \frac{12\,\text{inch}}{1\,\text{foot}} \times \frac{2.54\,\text{cm}}{1\,\text{inch}} \times \frac{1\,\text{m}}{100\,\text{cm}} \times \frac{1\,\text{km}}{1000\,\text{m}} = 1.609344\,\text{km}$ exactly. The percentage error is the difference assumed-exact divided by exact, or

$$\frac{1.6 - 1.609344}{1.609344} \times 100\% = 0.580609242\% \tag{1.28}$$

Exercise 1.45 A laser rangefinder is an instrument that sends out a pulse of light and senses the time required for the light to reach an object, like an outcrop, reflect off the object and return to the laser. Suppose the flight time for one measurement is 210 nanoseconds. Light travels 0.299792458 meters in one nanosecond.

(a) How far does the light pulse travel?

(b) Often the speed of light is approximated as 1 foot per nanosecond. How many feet does the light pulse travel using this approximation and what is the percentage error compared to the more accurate speed given in meters per second?

Exercise 1.46 A trucking company reports a fuel spill of 40,000 gallons within a watershed. How many liters of fuel were spilled? Note that 1 quart ≈ 0.94635 liters.

Solution 1.46
$40000 \text{ gallons} \times \frac{4\,\text{quarts}}{1\,\text{gallon}} \times \frac{0.94635\,\text{lt}}{1\,\text{quart}} = 151416\,\text{lt}$
or, since the spill is reported with one significant figure: 200000 lt.

Unit conversions in higher dimensions

Exercise 1.47 Which is larger, a 40–m^2 apartment in Tokyo, or a 500-square-foot apartment in New York?

Solution 1.47
$500 \text{ square feet} \times \frac{12\times 12\,\text{sq in}}{1\,\text{square ft}} \times \frac{2.54\times 2.54\,\text{square cm}}{1\,\text{square inch}} \times \frac{1\,\text{square m}}{100\times 100\,\text{square cm}} = 46.45152\,\text{m}^2$
The NYC apartment is larger.

Exercise 1.48 A potential client asks a geologist to submit a bid for making a geologic map of a 10-square-mile area. How big is the mapping area in square kilometers?

Exercise 1.49 A stratigrapher determines the volume of an oil shale unit is $1.3 \times 10^9\,\text{m}^3$. What is the volume in cubic kilometers?

Solution 1.49
$1.3 \times 10^9\,\text{m}^3 \frac{1\,\text{km}^3}{1000\times 1000\times 1000\,\text{m}^3} = 1.3\,\text{km}^3$

Exercise 1.50 An instrument remotely senses the sulfur dioxide flux from a volcano to be 525 metric tonnes per day. What is the flux in $\text{kg}\,\text{s}^{-1}$? Note: one metric tonne is 1,000 kg. Note: the term metric tonne is redundant since a tonne, by definition is metric, as opposed to a ton, which by definition is not metric. Since ths confuses a lot of people, metric tonne is used to be absolutely clear (and redundant)!

Solution 1.50
$525 \text{ metric tonnes per day} \times \frac{1000\,\text{kg}}{1\,\text{metric tonne}} \times \frac{1\,\text{day}}{24\,\text{hr}} \times \frac{1\,\text{hr}}{3600\,\text{s}} = 6.08\,\text{kg}\,\text{s}^{-1}$

Exercise 1.51 By definition, 1 milliliter occupies a volume that is 1 cm on a side if represented as a cube. This volume of water weighs 1 g.

(a) How many liters of water are in a cubic volume that is 0.5 m on a side?

(b) How much does this cube weigh (in kg)?

Exercise 1.52 One acre-foot is a common measure of volume of surface or groundwater in the U.S. One square mile is 640 acres. If one inch of rain falls on a watershed that measures 1200 square miles, what is the volume of this rainwater in acre-feet?

Solution 1.52
$1200 \text{ square miles} \times 640 \text{ acres/mile} \times 1/12 = 64000 \text{ acre feet}$

Exercise 1.53 Hectare is the SI unit of land area. One hectare is 10^4 m^2. In the U.S., land area is usually reported in acres. Which is larger, a 120-hectare (120 ha) vineyard in France or a 300-acre apple orchard in Washington?

Solution 1.53
$$300 \text{ acre} \times \frac{1\,\text{sq mile}}{640\,\text{acre}} \times \frac{5280\times 5280\,\text{square feet}}{1\,\text{square mile}} \times \frac{12\times 12\,\text{sq in}}{1\,\text{square ft}} \times \frac{2.54\times 2.54\,\text{square cm}}{1\,\text{square inch}} \times \frac{1\,\text{square m}}{100\times 100\,\text{square cm}} \approx 1,214,056\,\text{m}^2$$
and

120 ha $\times \frac{10^4\,\mathrm{m}^2}{1\mathrm{ha}} = 1,200,000\,\mathrm{m}^2$
The apple orchard is larger.

Dimensional analysis

DIMENSIONAL ANALYSIS IS AN EXTREMELY USEFUL TECHNIQUE FOR STUDYING EQUATIONS, like those commonly found in geology publications. In dimensional analysis, an equation is expressed in terms of the fundamental units of length (L), mass (M), temperature (Q) and time (T). Even if an equation appears to be exceedingly complex, it is usually quick work to express the equation using these fundamental units. Doing so helps one understand the relationship between physical quantities used in the equation. Dimensional analysis is also a quick way to check if equations are correct, since the fundamental units on the right hand of the equation must be the same as the fundamental units on the left hand of the equation if the equation is physically correct.

Exercise 1.54 Newton's second law of motion, $F = ma$, relates force to mass and acceleration.

(a) Express Newton's second law of motion in SI units

(b) Express Newton's second law of motion in fundamental units (L, M, T).

Exercise 1.55 Lithostatic pressure is the pressure at some depth due to the weight of overlying rocks. The equation for lithostatic pressure is:

$$P = \rho g h \tag{1.29}$$

where P is pressure, ρ is density, g is gravitational acceleration, and h is depth.

(a) Express this equation for lithostatic pressure in SI units

(b) Express this equation for lithostatic pressure in fundamental units (L, M, T).

(c) Given Newton's second law of motion and the equation for lithostatic pressure, both expressed in fundamental units, how is pressure related to force?

Solution 1.54
In SI units: $\frac{\mathrm{kg\,m}}{\mathrm{s}^2} = \mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}$
In Fundamental units: $\frac{ML}{T^2} = M\frac{L}{T^2}$

Exercise 1.56 Youngs modulus is used in geology to describe the relationship between force applied to a body and its deformation. Rocks behave in a linear-elastic manner when small forces are applied. "Elastic" means that rocks will deform when a force is applied, then return to their original shape when the force is removed. "Linear" means that there is a linear relationship between the force applied and the amount of deformation. Young's modulus is the ratio of the stress (force applied per unit area) to the strain (deformation in response to the force):

$$E = \frac{F/A}{\Delta L/L_o} \tag{1.30}$$

where F is the force, A is the area to which the force is applied, L_o is the original length of the object, and ΔL is the change in length of the object. As an example, think of a long cylindrical core of rock drilled out of an outcrop. If a weight the core is extended by pulling it, the change in length of the core of rock compared to force applied is Young's modulus. If the force is so high that the core breaks, it is not possible to measure ΔL, so it is not possible to determine Young's modulus.

(a) Write the equation for Young's modulus is SI units

(b) Write the equation for Young's modulus in fundamental units (L, M, T).

Exercise 1.57 Volcanoes deform when new magma rises upward beneath the volcano. The new volume of magma causes a space problem, and this is accommodated by "inflating" the volcano like a balloon. The vertical deformation (upward movement) of the surface of the volcano in response to intrusion of magma can be estimated using the Mogi model:

$$U_z = \frac{3a^3 \Delta P d}{4G(d^2 + r^2)^{1.5}} \tag{1.31}$$

where U_z is the vertical deformation in response to a spherical intrusion of radius a, depth d. ΔP is the excess pressure of the intrusion, r is the distance from the intrusion to the point where U_z is estimated. G is a modulus, like Young's modulus, but is called the shear modulus. Express this equation in fundamental units (L, M, T).

Solution 1.56
$\frac{M}{LT^2} = \frac{ML}{T^2}\frac{1}{L^2}\frac{L}{L}$
which means that the units of Young's modulus are the same as the units of pressure (Pa).

Exercise 1.58 Mean heat flow on continental crust is about 61 $\mathrm{mW/m^2}$. The unit mW is a milliwatt. A Watt is a unit of power (energy per second; Joule per second). A Joule has the SI units $\mathrm{kg\,m^2\,s^{-2}}$ (Think of the equation for kinetic energy, $KE = 1/2mv^2$ and you can see it has units of Joules. Express the units of heat flow in fundamental units (L, M, T).

Exercise 1.59 Energy is absorbed, emitted, and reflected by nearly all natural objects. Some of this energy is transferred in the visible part of the spectrum, that is, at wavelengths we can perceive with our eyes. Some of the energy is transferred at wavelengths we cannot see, either in the ultraviolet or infrared parts of the spectrum. Geologists routinely measure the light emitted or reflected by rocks, vegetation and other features across this energy spectrum. These "remote sensing" measurements allow geologists to classify and study natural features remotely – from airplanes, satellites or telescopes. In remote sensing, the Stephan-Boltzmann law is used to estimate the temperature of objects from the spectrum of light they emit. The law states that the total energy radiated by a blackbody per unit surface area is proportional to the fourth power of the black body's surface temperature. That is, it is possible to estimate the surface temperature of an object, like the Sun or a rock, from the spectrum of energy that the object emits. The Stephan-Boltzmann law is:

$$j^* = \sigma T^4 \tag{1.32}$$

where j^* is the power per unit area ($\mathrm{W\,m^{-2}}$), T is temperature (in Kelvin) and σ is Stephan-Boltzmann's constant.

(a) What are the SI units of σ, Stephan-Boltzmann's constant?

(b) Write j^* in fundamental units (L, T, Q, M).

(c) Write the entire Stephan-Boltzmann law in fundamental units (L, T, Q, M)

Solution 1.58
$\mathrm{mW/m^2} = \mathrm{kg\,m^2\,s^{-3}}\ \mathrm{m^{-2}}$
$\frac{M}{T^3}$

Exercise 1.60 Lots of natural processes involve diffusion. Diffusion can be expressed in one-dimension as:

$$\frac{\partial C}{\partial t} = \alpha \frac{\partial^2 C}{\partial x^2} \tag{1.33}$$

This is a partial differential equation, in which C is concentration (such as the concentration of a pollutant in groundwater, or the concentration of sediment in a stream), t is time and x is distance. The ∂ symbol refers to "change in", like the symbol Δ, and so has no units. In words, the equation says that the change in concentration with respect to time is equal to a proportionality constant, α, times the rate of change in concentration with respect to distance, x. Although partial differential equations can be complicated to solve, it is easy to check their dimensionality. In checking the dimensionality, just ignore the ∂, or "change in" symbol. Taking it one step at a time

$$\frac{\partial C}{\partial t} = \frac{\text{kg}}{\text{m}^3}\frac{1}{\text{s}} = \frac{M}{L^3 T} \tag{1.34}$$

and

$$\frac{\partial^2 C}{\partial x^2} = \frac{kg}{m^3}\frac{1}{m^2} = \frac{M}{L^5} \tag{1.35}$$

What are the SI and fundamental units of the proportionality constant α, which is called the diffusivity, and also called the diffusion coefficient?

Solution 1.60
$\frac{\text{kg}}{\text{m}^3}\frac{1}{\text{s}} = \alpha\frac{\text{kg}}{\text{m}^3}\frac{1}{\text{m}^2}$
$\alpha = \frac{\text{m}^2}{\text{s}} = \frac{L^2}{T}$
or
$\frac{M}{L^3 T} = \frac{L^2}{T}\frac{M}{L^5}$

Exercise 1.61 In heat transfer, the thermal diffusivity is:

$$\alpha = \frac{k}{\rho c_p} \tag{1.36}$$

where k is the bulk thermal conductivity (W/(mK)), ρ is density (kg/m^3), and c_p is heat capacity (J/(kg K)). In general, rocks have low thermal diffusivity (they are good insulators. For example, for sandstone, $\alpha \approx 1 \times 10^{-6}\,\text{m}^2\,\text{s}^{-1}$. Write the equation for thermal diffusivity in fundamental units (M, L, T, Q).

Exercise 1.62 The diffusion of heat through a rock in 3D can be described using the following equation, as long as the thermal diffusivity is uniform throughout the rock:

$$\frac{\partial u}{\partial t} = \alpha\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \tag{1.37}$$

where u is the temperature in the rock at a point, x, y, z and at time, t. Write this equation in its fundamental units.

Exercise 1.63 Geoscientists often model groundwater flow using computer programs that calculate results of partial differential equations for specific model geometries. For example, one common problem is to estimate the change in height of the groundwater table (or hydraulic head) in response to pumping of groundwater from a well. the program MODFLOW, written by scientists at the US Geological Survey, solves this an related problems. The partial differential equation describing this problem is:

$$\frac{\partial h}{\partial t} = \alpha\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right] - G \tag{1.38}$$

where h is the hydraulic head at a point, x, y, z and at time, t. The units of h are meters, since it represents the height of water above a datum. The weight of this water produces pressure gradients and causes flow, or change in h with time. As in other diffusion problems,

α is the diffusivity. G is the sink term, representing groundwater lost by pumping the well. What are the units of G?

Solution 1.63

$\frac{L}{T} = \frac{L^2}{T}\left[\frac{L}{L^2} + \frac{L}{L^2} + \frac{L}{L^2}\right] - G$

$\frac{L}{T} = \frac{L}{T} + \frac{L}{T} + \frac{L}{T} - \frac{L}{T}$

In order for the units to balance, G must have the same units as $\frac{\partial h}{\partial t}$ and so represents the change in hydraulic head with time due to groundwater pumping.

Chapter 4 Estimates, significant figures, and error

Often geoscientists are called upon to make estimates. How fast can we pump groundwater before the well goes dry? How much gold is in those hills? If the retaining wall for our containment pond breaks, what will be the maximum arsenic concentration downstream? If the volcano erupts, how big will the eruption be? It's important to answer such questions accurately (make an estimate that is close to the true value) and it is equally important – some would say more important – to provide an estimate of the precision of your answer. One simple way to express precision is to give a range, and be certain your answer is within this range. There are much more sophisticated approaches to estimating confidence, using probability, confidence intervals, and the like, but for the moment stick with the idea that you want to report a best-estimate and a range. In this geoscience game (which some geoscience consultants gamble quite a lot to play) there is a big penalty if the true value falls outside your estimated range. There is also a penalty if your range is so broad that your estimate is useless, or nearly useless, or not as useful as the next consultant's estimate. The best answer is a narrow range that you are certain encompasses the true value.

In the following, give a best estimate, range and justification for your estimate and range. To get a handle on estimates and ranges, try these problems without looking up the answer first. Be courageous – with time and experience your ability to estimate will improve dramatically if you practice.

Exercise 1.64 The tallest mountain in North America is Mt. Denali. How high is it (in meters above sea-level)?

Solution 1.64

You might have no idea how tall Denali is, but you can estimate. You might realize that it is taller than other mountains you know in North America. many mountains are higher than 4,000 m, so choose that as a lower bound. It is unlikely that Denali is as high as the tallest Himalyan peaks, which are over 8,000 m. So the range might be 4,000 – 8,000 masl. If we do not have additional information, we might choose the midpoint as the best estimate, 6000 m. What is the "real" elevation of Mt. Denali? Look it up!

Exercise 1.65 What is the Earth's radius at any latitude or longitude (km)?

Exercise 1.66 How deep is the deepest point in the Gulf of Mexico?

Exercise 1.67 What is the average geothermal gradient beneath Tampa, Florida, in the upper 5 km of the crust?

Exercise 1.68 How thick is the Earth's lithosphere directly beneath Tampa, Florida?

Exercise 1.69 What is the diameter of Barringer Meteor Crater (Az)?

Exercise 1.70 How many days will elapse before another earthquake causes fatalities?

This last question is an example of the kind of question that geological "experts" are frequently asked by the public. Obviously one cannot know the true value, but one can provide an informed estimate of the range. Try making this estimate (including the range!) using information on fatal earthquakes available on-line. Then monitor the news for fatal earthquakes and evaluate your solution.

Significant figures

WHENEVER A NUMBER IS REPORTED IN THE GEOSCIENCES (SAY THE MEASURED LENGTH OF A TRILOBITE) AN UNCERTAINTY SHOULD BE REPORTED ABOUT THE NUMBER. Sometimes the uncertainty is informally reported using significant figures. For example, if the length is reported to be 6.34 in., then it is implied that the length measurement is precise to approximately $1/100^{th}$ of one inch. When most geoscientists see a number reported to the second decimal place they will assume the measurement is good to the second decimal place. That is, if the length of this particular trilobite was re-measured more precisely to the third decimal place the length might be 6.338 in. or 6.344 in., but the length should not be 6.353 in., given the significant figures used in the original measurement. This is the information implied in the use of significant figures.

Often we estimate in the geosciences because we have imprecise information. How thick is the aquifer? What is the diameter of the asteroid? How far is the outcrop up the trail? Usually people answer these types of questions with estimates. If one reports the thickness of an aquifer is estimated to be 100 m thick (that is, using one significant figure), his or her colleagues can conclude that the aquifer actually might be 125 m thick or 79 m thick, but not 250 m thick. When we use such estimates in equations (e.g., what is the volume of the aquifer?) we must know how well we know the calculated value, given the imprecision in our estimated values.

Vacher [8] lays out a detailed explanation of the use and abuse of significant figures. Read his paper. From that paper, a summary of rules about significant figures:

[8] H. L. Vacher. Computational geology 1–significant figures! *Journal of Geoscience Education*, 46(3):292–295, 1998

- When you multiply or divide, round off to the same number of figures as in the factor with the least number of figures.
- When you add or subtract, it is the position of the digits that is important, not how many there are. Imagine that the various terms that are added or subtracted together are in a vertical column with the decimal points all lined up. Spot the term that goes least far to the right. Round off to that position.
- If you have a calculation that involves multiple steps, keep additional digits through the intermediate results and round off at the end. If you need to report one of the intermediate results, round off to the appropriate number of significant figures when you report it, but go back and pick up the discarded figures when you proceed further with the calculation.

Exercise 1.71 A geologist needs a trench dug across a fault in order to determine slip on the fault (see Marshak chapter 10, especially Figure 10.38). The trench is made with a backhoe, and the backhoe operator charges by the volume of material removed. The final dimensions of a trench are measured to be 26.5 m long, 2.5 m wide and 2.0 m deep. What is the volume of the trench?

Solution 1.71
$26.5\,\text{m} \times 2.5\,\text{m} \times 2.0\,\text{m} = 132.5\,\text{m}^3$ but there are only two significant figures, so the correct answer is $130\,\text{m}^3$

Exercise 1.72 A geologist is asked to determine the density of a rock in units of g/cc. The rock sample is weighed on a scale that reports 166.038 g. The sample is then placed in a graduated cylinder and is found to displace 83 cc water. What is the density of the sample?

Exercise 1.73 A student estimates the height of an outcrop to be 10 m, and the thickness of a bed in the outcrop to be $1/5^{th}$ of the total outcrop height. What is the thickness of the bed?

Exercise 1.74 You measure the thickness of a bed in an outcrop to be 2.23 m. You estimate that the outcrop is $1/5^{th}$ of the total outcrop height. What is your estimate of the height of the outcrop? (there is one (1) significant figure in the answer because the fraction has one significant figure).

Exercise 1.75 A geologist measures a set of beds that comprise a formation. The measured bed thicknesses are 1.01 m., 5.3 m, 23 cm, 12.2 m, 64 cm. What is the total thickness of the formation?

Exercise 1.76 Arc-Info (a GIS tool) is used to estimate the area of a lava flow to be 1,234,234 m^2. From the map, the thickness of the lava flow is estimated to be 5 m. What is the volume of the lava flow?

Exercise 1.77 There are exactly 2.54 cm in 1 in. Using a micrometer, a geologist measures the length of a fossil trilobite to be 6.344 in. (a very precise measurement). What is the length of the fossil trilobite in cm?

Exercise 1.78 uppose two water wells penetrate an aquifer. These two wells are 110 m apart, and aligned in the direction of local groundwater flow. The potentiometric surface is 87.53 m above sea-level (masl) in on well, and 87.01 masl in the other well. If the hydraulic conductivity is 4.3 m per day, and the effective porosity is 0.21, what is the average velocity with which a tracer in the groundwater would move from one well to the other? To solve this, use Darcy's Law for velocity:

$$v = -\frac{K}{n_e}\frac{\Delta h}{\Delta s} \tag{1.39}$$

where v is the average tracer velocity, n_e is the effective porosity, Δh id the difference in elevation of the potentiometric surface between the two wells (the hydraulic head), and Δs is the distance between the two wells, in the direction of groundwater flow.

Exercise 1.79 The radius of the Earth is given as 6378 km. What is the volume of the Earth in cubic kilometers? Assume the Earth is a sphere and the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.

Solution 1.75
The total is: $1.01 + 5.3 + 0.23 + 12.2 + 0.64 =$19.38 m
Rounding this to the proper decimals gives: 19.4 m
note that the position of the digits is important, rather than the fact that some measurements are only reported with two significant figures.

Solution 1.77
6.344 in. $\times \frac{2.54\,\text{cm}}{1\,\text{in.}} = 16.11376\,\text{cm}$
There are four significant figures in 6.344 in., so the correct answer is 16.11 cm
Note that the conversion factor is exact, so there are not three but four significant figures.

Solution 1.79
$V = \frac{4}{3}\pi 6378^3 = 1.086781293 \times 10^{12}\,\text{km}^3$
The radius is reported with four significant figures, so the correct answer is $1.087 \times 10^{12}\,\text{km}^3$
Note that 4 and 3 are constants, and so have no effect on the significant figures. Similarly, π is an irrational number, and a constant, and has no effect on the significant figures as long as π is approximately using more four or more digits.

Ranges, midpoints and the median

ANOTHER WAY TO REPORT UNCERTAINTY IS BY REPORTING THE UNCERTAINTY DIRECTLY. For example, if the length if a trilobite is measured by 10 geologists, we might find the length to be 6.36 in. ± 0.04 in. In this example, the number 6.36 represents the most likely value of the length measurement. The uncertainty (± 0.04 in.) indicates something about the dispersion of measurements.

The standard way to report uncertainty in measurements (measurement error) is to use the standard deviation of the distribution of measurements. The standard deviation is a statistical characterization of the range of measurements. Literally, 68% of the measurements fall within one standard deviation. In this method, the most likely value is usually the estimated mean, and the uncertainty might be one or two standard deviations from the mean.

If the reported uncertainty number reflects the total range of measurements rather than the standard deviation, the uncertainty is called the maximum error. The most likely value in this case in the midpoint of the range, and the uncertainty is the range of measurements. Frequently, geologists report a range of measurements (e.g., the length of the trilobite is 6.34 – 6.38 in.). In this case, the uncertainty is the maximum error.

Exercise 1.80 Ten geologists measure the length of a single trilobite, reporting varying precision. Their measurements are 6.344 in., 6.38 in., 6.353 in., 6.375 in., 6.342 in., 6.365 in., 6.34 in., 6.349 in., 6.35 in., 6.3456 in.

(a) What is the maximum range of reported values?

(b) Give the length of the trilobite with midpoint and maximum error.

Solution 1.80
minimum: 6.34 in. maximum: 6.38 in. range: 6.34 in. – 6.38 in.
6.36 in $\pm$ 0.02 in.
where 6.36 in. is the midpoint in the range.

The midpoint of the range is not a particularly good measure of the most likely value. Instead, the median is often used. For a very large group of measurements, we expect the median to be the number that divides the group in half. 50% of the measurements will have values greater than the median, 50% will have values less than the median. The way to find the median is to sort the data in ascending or descending order, and pick the middle value. In the case where there are an even number of measurements (10 in the case of the trilobite measurements described previously) the median is the average of the middle two measurements.

Exercise 1.81 For the ten trilobite measurements:

(a) Find the median.

(b) Give the length of the trilobite with median and maximum error.

(c) Interpret the differences you observe between the median and the midpoint. Why are these estimates of the most likely value different?

Solution 1.81
The median is less than the midpoint because there are more short measurements than long measurements. The range and midpoint are not sensitive to outliers (measurements that are very different from most).

Exercise 1.82 One way to look at the distribution of all the data is with a histogram.

(a) Neatly plot a histogram of the trilobite length measurements, using a bin size of 0.01 in.

(b) Label the midpoint and the median on the histogram.

(c) Explain the difference between the midpoint and the median for these data using the histogram.

Exercise 1.83 Notice that each of the trilobite measurements is $1/10^{th}$ of the total dataset, or 10%. One way to think of this distribution is that 100% of the measurements are $\geq$ 6.34 in., 90% of the measurements are $\geq$ 6.344 in, ..., 10% of the measurements are $\geq$ 6.38 in.

(a) Make a plot with the measured trilobite lengths on the x-axis and the percentage of measurements equal to or greater than the corresponding length on the y-axis.

(b) Find the median trilobite length directly from this plot.

(c) This type of graph is often called a survival plot. For example, for the set of all cigar smokers, the x-axis might be the age of death and the y-axis might be the percentage of smokers who live to the age of death or longer. What is the meaning of your survivor plot of trilobite lengths?

(d) Based on your plot, if 20 measurements were made instead of 10, what would you expect the longest measured length to be (you have to estimate from your graph using a range)?

Exercise 1.84 Suppose a geologist gathers samples of a Precambrian granite pluton (see Marshak, chapter 6) and makes three radiometric age determinations on these samples. The three age determinations are 624 $\pm$ 34 Ma, 631 $\pm$ 24 Ma and 691 $\pm$ 23 Ma.

(a) Should she select the median age or the midpoint as the best estimate of the age of the granite pluton?

(b) Should she use the average of the age determinations of the granite age or the median?

Solution 1.84

She should select the median age over the midpoint: $631 \pm 24\,\text{Ma}$. The three age determinations suggest that the midpoint ($621 + \frac{691-624}{2}$ = 654.5 Ma) is skewed by the much older age. The average age is: $AVG = \frac{624+631+691}{3} = 648\,\text{Ma}$, which is close to the midpoint in this case. The median is more likely representative of the true age than the average (or mean) in this case because one number is much different (older age) than the other two.

Propagation of Error

Notice in the previous problem about radiometric age determinations for granites each age determination ($624 \pm 34\,\text{Ma}$, $631 \pm 24\,\text{Ma}$ and $691 \pm 23\,\text{Ma}$) has uncertainty. When we do calculations with these numbers, we must propagate the uncertainty through the analysis. There are rules for error propagation and these mathematical rules can become very complicated. If the absolute errors (data ranges) are used, some simplified rules for error propagation apply:

1. For addition and subtraction of values with errors, errors add. For example:

$$(x \pm \Delta x) + (y \pm \Delta y) = (x + y) \pm (\Delta x + \Delta y) \tag{1.40}$$

$$(x \pm \Delta x) - (y \pm \Delta y) = (x - y) \pm (\Delta x + \Delta y) \tag{1.41}$$

$$\frac{[(x \pm \Delta x) + (y \pm \Delta y)]}{2} = \frac{1}{2}(x + y) \pm \frac{1}{2}(\Delta x + \Delta y) \tag{1.42}$$

2. For multiplication or division of values with errors, errors add. For example:

$$(x \pm \Delta x) \times (y \pm \Delta y) = (xy) \pm xy \left[\frac{\Delta x}{x} + \frac{\Delta y}{y}\right] \tag{1.43}$$

$$\frac{(x \pm \Delta x)}{(y \pm \Delta y)} = (x/y) \pm x/y \left[\frac{\Delta x}{x} + \frac{\Delta y}{y}\right] \tag{1.44}$$

$$\frac{[(x \pm \Delta x) \times (y \pm \Delta y)]}{a} = \frac{x}{ay} \pm \frac{x}{ay} \left[\frac{\Delta x}{x} + \frac{\Delta y}{y}\right] \tag{1.45}$$

Using these somewhat simplified rules, answer the following questions.

Exercise 1.85 The addition of errors (sometimes called the accumulation of error) is extremely important in map surveying. Suppose three stakes are laid out in a line for a geophysical survey. The distance between stake 1 and stake 2 is 100 m ± 0.25 m and the distance between stake 2 and stake 3 is 85 m ± 0.33 m, where absolute errors are reported. What is the distance, with uncertainty, between stakes 1 and 3?

Solution 1.85
The errors add in this case:

$$(100 \pm 0.25) + (85 \pm 0.33) = 185 \pm (0.25 + 0.33) = 185 \pm 0.5 \quad (1.46)$$

Exercise 1.86 Suppose the manufacturer of a laser rangefinder reports the instrument has an absolute error of 0.1 m for distances $< 500\,\mathrm{m}$. A geoscientist makes 8 shots (each $< 500\,\mathrm{m}$) along a transect across his map area, and figures the total distance traversed to be 2990 m. What is the uncertainty in this estimate, given the absolute error reported by the manufacturer?

Solution 1.86
Since the errors add, and the absolute error is thought to be the same for each measurement, the total error is:

$$8 \times 0.1\,\mathrm{m} = 0.8\,\mathrm{m} \quad (1.47)$$

Of course, there are likely measurement errors as well! Also, it is possible that errors will cancel so the true error might be less than the maximum error. This factor can be accommodated using more complicated approaches to the error propagation (using partial derivatives).

Exercise 1.87 A GPS manufacture reports absolute errors for horizontal position on their handheld GPS unit to be ±5 m, and absolute error on the vertical position to be ±15 m (usually the uncertainty in GPS is approximately 3 × larger for the vertical measurement because of the geometry of GPS satellites visible to the GPS receiver). With this GPS unit, an Environmental Science tech gathers three GPS coordinates for stakes along a profile line. The coordinates are:

stake 1	360342 E	3100457 N	123 masl
stake 2	360354 E	3100854 N	135 masl
stake 3	360367 E	3101073 N	152 masl

(a) What is the difference in elevation, with uncertainty reported, between stakes 1 and 3?

(b) How far north is stake 3 of stake 1, with uncertainty reported?

Exercise 1.88 The lithostatic pressure at the base of the crust (see Figure 2.16 in Marshak) is calculated as:

$$P = \rho g h \quad (1.48)$$

where P is pressure, ρ is average density of the crust, g is gravitational acceleration ($9.8\,\mathrm{m\,s^{-2}}$), and h is the depth to the Moho. If the thickness of the crust is estimated to be 30–35 km and the average density is estimated to be 2600-2700 $\mathrm{kg\,m^{-3}}$, what is the lithostatic pressure at the Moho, with uncertainty?

Solution 1.88
Use the midpoints for the ranges:

$$P = 2650 \times 9.8 \times 32500 = 844\,\mathrm{MPa} \quad (1.49)$$

$$\Delta P = \rho g h \left[\frac{\Delta h}{h} + \frac{\Delta \rho}{\rho} \right] = 844\,\text{MPa} \left[\frac{2.5}{32.5} + \frac{50}{2650} \right] = 80.8\,\text{MPa} \quad (1.50)$$

The lithostatic pressure at the Moho for these estimates is: 844 ± 80.8 MPa. Note that g is treated as a constant with no uncertainty.

Exercise 1.89 A group of geophysics students decide to repeat Eratosthene's experiment to estimate of the radius of the Earth (See Marshak, page 18). They determine their two measurement locations to be 800 km apart with two GPS units (each with an absolute uncertainty of ± 5 m horizontal assuming a spherical Earth, giving an uncertainty in the distance between them of ± 14 m. Repeating the experiment (Figure 1.6 in Marshak), they determine the angle cast by the Sun's rays to be $7.0^\circ - 8.0^\circ$. What is their estimate of the circumference of the Earth, given the ranges of uncertainties in their distance and angle measurements?

Exercise 1.90 A group of student geologists visit an outcrop along the coastline and take strike and dip measurements. The range of dip measurements made by the group is $26^\circ - 34^\circ$. The group estimates the distance to a nearby hill to be 1–1.2 km in the up-dip direction. Assuming the dip of the bed remains constant over this distance, how high up the hill would they expect to find an outcrop of the dipping bed? Calculate the height and uncertainty, realizing that the height h of the bed above their current position is the distance, x, times the tangent of the dip, or:

$$h = x \tan(dip) \quad (1.51)$$

Solution 1.90 The height of the outcrop on the hill is:
$h = x \tan(dip) = 1.1\,\text{km} \times \tan(30^\circ) = 630\,\text{m}$

The uncertainty in height resulting from the given ranges of angle and distance measurements is:
$\Delta h = x \tan(dip) \left[\frac{0.1}{1.1} + \frac{\tan 4^\circ}{\tan 30^\circ} \right] = 630 \times (0.9 + 0.12) = 133\,\text{m}$

Given the range of estimates the group would expect to find the bed outcropping in the hill at 630 ± 132 m above the elevation of the coastline.

Chapter 5 Proportion and percentage

If we say y is proportional to x, we are implying a specific mathematical relationship: $y = ax$. f we say y is inversely proportional to x, we are implying another mathematical relationship: $y = \frac{a}{x}$.

Exercise 1.91 Consider an equation in which y is directly proportional to x: $y = 2x$ over the range $x = 0$ to $x = 10$.

(a) Plot the graph of y vs. x

(b) Now assume y is inversely proportional to x: $y = 2/x$, over the range $x = 0$ to $x = 10$. Plot this graph.

(c) Explain why the second plot is of a curve and the first plot is of a straight line.

Exercise 1.92 n the equation for lithostatic pressure, $P = \rho g h$, where P is lithostatic pressure, ρ is rock density, g is gravity, and h is depth, sketch the relationship between P and h for constant ρ and constant g. What type of relationship is this?

Exercise 1.93 Sediment transport on beaches is related to the action of waves in the swash zone. The run-up of a wave onto the beach (the maximum distance water travels up the beach slope as the wave crashes) is directly proportional to the height of the wave measured off-shore. To a first approximation, the run-up, r, is equal to the wave height, h. This relationship is observed to hold for wave heights from several centimeters to several meters. Graph the relationship between r and h.

Exercise 1.94 Some volcanic eruptions are pulsatory in the sense that the volcano erupts magma in explosions that occur at relatively regular intervals. These explosions may be a few seconds apart to several days apart. From observations, it is found that the viscosity of the magma, η (Pa s) is directly proportional to the median of the time elapsed between pulsatory eruptions, α (s). The median of

Solution 1.92

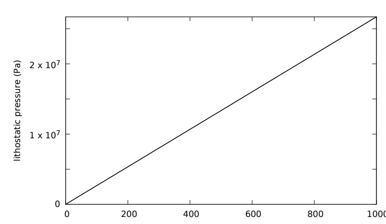


Figure 1.20: Lithostatic ressure is directly proportional to depth, so is a straightline on the plot intersecting the origin.

the time elapsed between pulsatory eruptions is called the repose interval. The observed relationship is that, to one order of magnitude, $\eta = 100 \times \alpha$.

(a) What are the units of the proportionality constant (100) in this equation?

(b) Rewrite the equation so that α has units of hours instead of seconds, making sure to recalculate the proportionality constant.

(c) Graph the proportional relationship between magma viscosity and repose interval over several orders of magnitude, from $\alpha = 10^1$ s to $\alpha = 10^8$ s.

Exercise 1.95 Tracers are used to determine the velocity with which groundwater moves, or contaminants in groundwater move, through a geologic formation. The velocity of a tracer is directly proportional to the discharge per unit area and inversely proportional to the porosity of the geologic formation.

$$v = \frac{q}{\phi} \tag{1.52}$$

where v is the velocity of the tracer ($\mathrm{m\,s^{-1}}$), q is the discharge per unit area ($\mathrm{m\,s^{-1}}$), and ϕ is the fractional porosity. The fractional porosity theoretically varies from near zero for geologic formations with very low porosity to near one for extremely porous formations, but in actual rocks rarely is $\phi > 0.3$. The discharge per unit area is a function of the pressure gradient in the geologic formation that drives groundwater flow, the permeability of the formation, and the viscosity of water, as described by Darcy's Law.

(a) Make a graph showing two curves, the change of tracer velocity as a function of porosity for $q = 0.1\,\mathrm{m\,s^{-1}}$ and $q = 0.2\,\mathrm{m\,s^{-1}}$.

(b) Based on the graph, describe how tracer velocity changes as a function of porosity.

(c) How does a small uncertainty in porosity affect your estimate of tracer velocity compared to a small uncertainty in discharge per unit area? Compare large and small fractional porosity.

Exercise 1.96 Ground penetrating radar (GPR) is a geophysical technique used to investigate the shallow subsurface, to depths of up to about 10 m. GPR is able to resolve geologic bedding due to changes in the velocity of the radar wave associated with changes in rock properties and because radar waves are reflected back toward the surface by some geologic contacts. In a practical sense, the thinnest geologic bed that can be observed by GPR is proportional to the average velocity of the radar wave in the subsurface, and inversely proportional to the frequency of the radar wave that the GPR system transmits. The minimal thickness of a bed that can be resolved is, roughly:

$$t_m = \frac{1}{2}\frac{v}{f} \qquad (1.53)$$

where t_m is the minimum thickness of an observable bed (m), v is the average velocity of the radar wave ($\mathrm{m\,s^{-1}}$), and f is the frequency of the radar wave ($\mathrm{s^{-1}}$). Typical GPS systems can transmit waves at frequencies of 100–900 MHz, where 1 MHz is $1 \times 10^6\,\mathrm{s^{-1}}$.

(a) Given the velocity of a radar wave in a formation is 0.1 $\mathrm{m\,ns^{-1}}$, what are the minimum thicknesses of observable beds for waves transmitted at frequencies 100–900 MHz? Show your answer as a graph.

(b) Given a GPR system transmitting radar waves at 200 MHz, what are the minimum thicknesses of observable beds for waves transmitted at average velocities of 0.0 – 0.2 $\mathrm{m\,ns^{-1}}$? Show your answer as a graph.

(c) Describe, in words, the factors that govern the minimum observable thickness of a bed with GPR. Describe a geological problem that might be solved by transmitting radar waves at higher, as opposed to lower frequencies.

Exercise 1.97 Gravitational potential is related to the potential energy that exists about an object due to its mass. For example, the Earth is a massive object that creates gravitational potential. An object near the Earth, such as a person, has potential energy due to the mass of the Earth. Gravitational potential is inversely proportional to distance above the Earth's surface. An object at higher elevation has greater gravitational potential than an object at lower elevation. It requires less work to fall off a cliff than to climb up the cliff, so gravitational potential is higher at the top of the cliff than at its base. The gravitational potential is:

$$U = -\frac{GM}{r} \qquad (1.54)$$

where U is gravitational potential – the potential energy per unit mass ($\mathrm{kg\,m^2\,s^{-2}} \cdot \mathrm{kg^{-1}} = \mathrm{m^2\,s^{-2}}$), G is the gravitational constant, M is the mass of the Earth (kg), and r is distance above the center of mass of the Earth (m). This equation applies to the gravitational potential from the surface of the Earth upward.

(a) What are the SI units of the gravitational constant of proportionality, G?

(b) Sketch a graph of the change in gravitational potential with distance above the Earth's surface. Since G and M are constant, just assume $GM = 1$ to get to know the shape of the graph.

(c) What is the value of U at great distance ($r \to \infty$) from the surface of the Earth (regardless of the exact values of G and M)?

(d) Where is U minimum (regardless of the exact values of G and M)?

Exercise 1.98 When groundwater is pumped from a well, water flows through the surrounding rock toward the well at a velocity that is proportional to the volume of water pumped from the well as a function of time, V. Consider a well that is drilled into a thick homogeneous aquifer and only draws water through its base (the sides of the water well are sealed). To a first approximation, one can think of an imaginary sphere, or radius, r, with the base of the well at its center. The velocity of water through the sphere, v, is everywhere the same on the surface of this imaginary sphere.

(a) Write a mathematical expression, that shows the volume of water pumped from the well as a function of time ($\mathrm{m^3\,s^{-1}}$) is proportional to the velocity of the water moving through the surface of the sphere ($\mathrm{m\,s^{-1}}$). Recall that the surface area of a sphere is $A = 4\pi r^2$.

(b) If the pumping rate at the well doubles, what happens to the velocity of water in the aquifer measured at the surface of the sphere?

(c) Notice that in your mathematical expression, if you have formulated it correctly (!), the volume of water pumped from the well as a function of time is *not* directly proportional to r but is directly proportional to r^2. For a given pumping rate, sketch a graph of the change in velocity, v as a function of r.

Map scale problems and equality ratios

Exercise 1.99 In Mexico, most maps are printed at a scale of 1:50000. How far on the ground will you walk to move a distance of 1.5 cm on the map?

Solution 1.99
$\frac{1}{50{,}000} = \frac{1.5\,\mathrm{cm}}{x}$
cross multiply to obtain: $x =$ $75{,}000\,\mathrm{cm} = 750\,\mathrm{m}$

Exercise 1.100 A geoscientist needs to print her map at a scale that will fit easily on a journal page. The width of her map area is 150 m and she needs to print the map to fit a space no wider than 6 in. What is the best scale to print her map?

Exercise 1.101 A geologist plans to make a map that will cover as much of a 30 cm × 20 cm mylar sheet as possible. The mapping area measures 1300 m by 1900 m. Find a map scale that will maximize the use of the mylar sheet and that fits the entire mapping area on this single sheet.

Exercise 1.102 A geologist makes a photocopy of his base-map. The original scale of the base-map is 1:24,000. With the photocopy machine, he selects the zoom feature to increase the size of the printed map by 250%. Assuming the photocopy is exactly 250% larger, what should be the new map scale (note that on a photocopy machine or in most software, when you "zoom" by 100%, the size does not change)?

Exercise 1.103 Geology students decide to represent Phanerozoic time using a roll of toilet paper that is reported to have 1000 sheets. Taking the Phanerozoic to start 541 Ma and the Holocene to start 12,000 yBP, how many sheets of toilet paper represent the duration of the Holocene?

Exercise 1.104 A photomicrograph shows a zircon crystal in granite. On the photomicrograph, the apparent length of the zircon is 3.5 cm. The magnification of the photomicrograph is reported to be 2500×. What is the actual length of the zircon crystal? (Note that this is a scale question. 1:1000 is the same as a magnification of 1000×).

Exercise 1.105 Silicate melts and the igneous rocks that form from them consist mainly of a group of oxides. In a "whole rock" analysis, the weight of common oxides is measured in a laboratory using a mass spectrometer. Often, the sum of the weights of common oxides analyzed does not equal the total weight of the analyzed sample, because some elements or compounds in the rock go unanalyzed in a whole rock analysis, and because of analytical error. Consider this whole rock analysis for a tholeiitic basalt that erupted at a mid-ocean ridge.

Solution 1.100

The best ratio should be $\frac{6\,\text{in.}}{150\,\text{m}} = \frac{1}{x}$
Convert 6 in. to meters: $6\,\text{in.} \times \frac{2.54\,\text{cm}}{1\,\text{in.}} \times \frac{1\,\text{m}}{100\,\text{cm}} = 0.1524\,\text{m}$
$\frac{1}{x} = \frac{0.1524\,\text{m}}{150\,\text{m}}$
$x \approx 984\,\text{m}$, so it is appropriate to print the map at a scale of 1:1000, and it will occupy slightly less than the 6 in. maximum width, and yet be printed at a convenient scale.

Solution 1.102

$\frac{2.5}{24000} = \frac{1}{x}$
x = 9600; the new map scale is 1:9600

Oxide	wt percent	normalized wt. percent
SiO2	48.77	
TiO2	1.15	
Al2O3	15.90	
Fe2O3	1.33	
FeO	8.62	
MnO	0.17	
MgO	9.67	
CaO	11.16	
Na2O	2.43	
K2O	0.08	
P2O5	0.09	
H2O	0.03	
total		100

(a) Complete the table by normalizing the weight percent oxides so they total to 100%. Note that this is a proprtion problem – namely for SiO_2:

$$\frac{48.77\,\text{wt}\,\%}{99.4\,\text{wt}\,\%} = \frac{x}{100\,\text{normalized}\,\text{wt}\,\%} \tag{1.55}$$

(b) Notice that Fe is reported twice in the table, once as ferric iron (valence 3+) and once as ferrous iron (valence 2+). What weight percent of the total iron oxide (FeO + Fe_2O_3)in the analysis is ferric iron (Fe_2O_3)?

(c) Recall that one mole is 6.022×10^{23} of something. One mole of SiO_2 is 6.022×10^{23} molecules of SiO_2, and consists of 1 mole of Si atoms and 2 moles of O atoms. The molecular weight of Fe (regardless of valence) is 55.845 g mol^{-1}. The molecular weight of oxygen is 16 g mol^{-1}. Using the normalized weight percents for the tholeiite sample, what mole percent of total Fe oxides in the tholeiite sample is ferric oxide (Fe_2O_3)?

Solution 1.105

Based on the normalized weights, 1.34 g (Fe_2O_3) / 159.69 $\frac{g}{mol}$ = 0.0084 mol

based on the normalized weights, 8.67 g (Fe_2O_3) / 71.845 $\frac{g}{mol}$ = 0.12 mol

$\frac{0.0084}{0.0084+0.12} \times 100 = 6.5$ mol % (Fe_2O_3) in the sample

Looking back, this answer makes sense because 1 mol (Fe_2O_3) weights roughly twice as much as 1 mol FeO.

Proportions of three things – ternary diagrams

TERNARY DIAGRAMS ARE GRAPHS THAT ARE USED TO PLOT THE RATIOS OF THREE VARIABLES, such as the proportions of three constituents in a mixture. Ternary diagrams are usually plotted as equilateral triangles. Usually before plotting, the three components shown on ternary plots are normalized to sum to 100%. Ternary plots are frequently used in the geosciences to classify rocks, mineral an soils. For example, Shepard's diagram is a ternary plot that is used to classify marine clastic sediments based on the relative abundance (by weight) of clay, silt and sand.

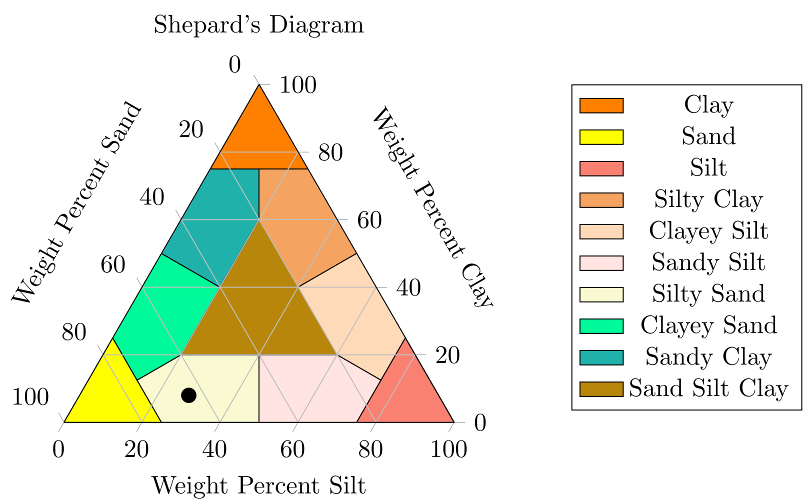


Figure 1.21: A ternary diagram plots the proprtion of three things in a sample. Here, the ternary diagram is called Shepard's diagram and plots the proportion of clay, silt and sand-sized particles in a sediment sample. The color-code shows sample names, based on the proportion of these three particle size groups.

Component	normalized wt. percent
clay	8
silt	28
sand	64
total	100

A grain-size analysis is required to determine the relative contributions of clay (0.98 – 3.9 μm diameter particles), silt (3.9 – 62.5 μm diameter particles) and sand (62.5 μm – 1 mm diameter particles) by weight. Because there is likely other material in the sample, such as biogenic limestone, insoluble salts, and/or fine gravel, the normalized weight showing the relative contributions of the three components only must be calculated before the samples can be classified using Shepard's diagram. That is, the clay, sand and silt components must add to 100% and any other compnents left out of the normalization. The sample shown by the black solid circle on the above ternary diagram consists of the following proportions of clay, silt and sand shown by weight percent in the Table at right.

This sample is classified as a silty sand. The classification, in turn, is used to describe the depositional environment and provenance of clastic marine sediments.

Exercise 1.106 The Maryland Geological Survey undertook a large study of the clastic sediments of Chesapeake Bay. The report was published by the U.S. Geological Survey as Water-Resources Investigations Report 03–4123. Hundreds of samples of Chesapeake Bay sediments were dredged as part of their study. Three samples are shown in this table:

Component	Sample 1 (g)	normalized wt. percent	Sample 2 (g)	normalized wt. percent	Sample 3 (g)	normalized wt. percent
clay	249.09		56.35		78.602	
silt	183.54		102.45		172.92	
sand	4.37		353.45		141.48	
other	15.67	0	23.01	0	19.03	0
total		100		100		100

(a) Complete the table by normalizing filling in the normalized weight column for each sample.

(b) Plot the three samples on the ternary diagram.

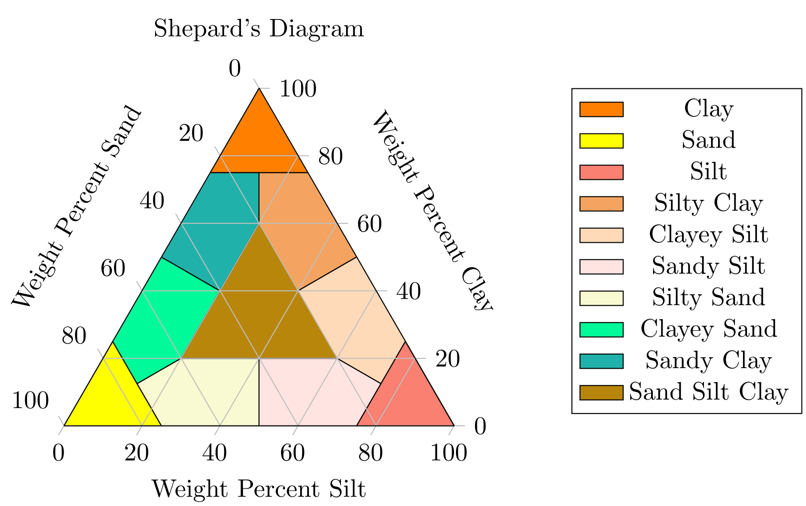


Figure 1.22: Plot the proportions of sand, silt and clay for the three samples on this diagram.

(c) Classify the three samples by name.

Exercise 1.107 Ternary diagrams are extremely important in geochemistry. Geochemists classify igneous rocks using ternary diagrams and use them to explain processes like solid solution in mineral chemistry. The ternary diagram of FeO, Fe_2O_3, and TiO_2 explains much of the mineralogy of magnetic minerals.

(a) Note where these minerals plot on the FeO, Fe_2O_3, and TiO_2 ternary diagram: wüstite (FeO), rutile (TiO_2), hematite (Fe_2O_3). Plot and label these minerals on the ternary diagram: magnetite (Fe_3O_4 = FeO + Fe_2O_3), ilmenite ($FeTiO_3$), ulvospinel (Fe_2TiO_4), and pseudo-brookite (Fe_2TiO_5).

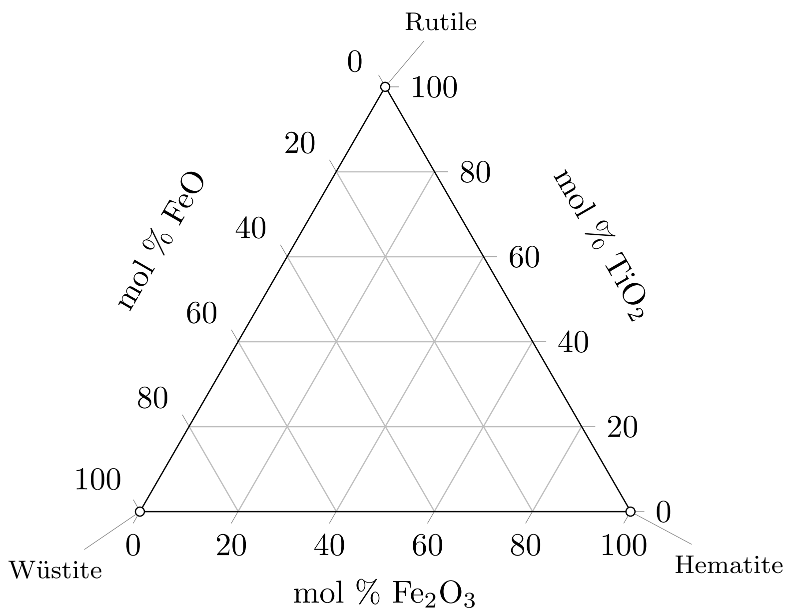


Figure 1.23: A ternary diagram for magnetic minerals. Note that ilmenite is FeO + TiO2; psuedobrookite is Fe2O3 + TiO2; ulvospinel is FeO+FeO+TiO2

(b) Titanomagnetic is a mineral that makes a complete solid solution between the end-member mineral ilmenite and magnetite on the ternary diagram. The solid solution is a straight line between these end member minerals, and is formed by substituting Fe^{2+} and Ti^{4+} for Fe^{3+} in magnetite. Plot the solid solution line for titanomagnetite on the ternary diagram. Give reasonable mole percents FeO and Fe_2O_3 for titanomagnetite that includes 20% mol TiO_2.

(c) Oxidation of minerals involves increasing the proportion of oxygen in a mineral's formula. Consider adding 1/2 mol oxygen to 1 mole ulvospinel. What is the resulting mineral? Plot a line on your ternary diagram that shows the addition of oxygen to ulvospinel, from zero additional O_2 to 1/2 mol O_2. Indicate the direction of increasing oxidation by adding an arrowhead to the line.

(d) Plot on the ternary diagram a tholeiite (basalt) sample containing 1.34 wt % Fe_2O_3, 8.56 wt % FeO, and 1.16 wt % TiO_2

Exercise 1.108 In 2015, total U.S. consumption of coal was approximately 800 million short tons, total U. S. natural gas consumption was approximately 30 trillon cubic feet, and about 7 billon barrels of petroleum. Create a ternary diagram that shows the proportion of these hydrocarbons consumed in the U. S. in 2015 by volume. Hint: convert the units to a common volume, such as cubic meters.

Exercise 1.109 In 2015, total U.S. consumption of coal was approximately 800 million short tons, total U. S. natural gas consumption was approximately 30 trillon cubic feet, and about 7 billon barrels of petroleum. Create a ternary diagram that shows the proportion of these hydrocarbons consumed in the U. S. in 2015 by volume. Hint: convert the units to a common volume, such as cubic meters.

Exercise 1.110 Natural waters are classified by relative cation and anion abundance in the water. The major cation ternary diagram compares the relative abundances of calcium (Ca^{2+}), magnesium (Mg^{2+}) and sodium + potassium (Na^{+} + K^{+}). The anion ternary diagram usually compares the relative abundance of chlorine (Cl^{-}) sulfate (SO_4^{-2}) and bicarbonate (HCO_3^{-}). In fact these two ternary diagrams are used together in a Piper diagram to classify water types. Plot the following water samples on cation and anion ternary plots.

Sample #	Ca^{2+} (mg/l)	Mg^{2+} (mg/l)	Na^{+} (mg/l)	K^{+} (mg/l)	HCO_3^{-} (mg/l)	Cl^{-} (mg/l)	SO_4^{-2} (mg/l)
1	104	6.8	7.0	2.8	280	14	25
2	82	5.2	6.8	3.6	195	13	35
3	3.6	1.2	6.8	1.9	13	5.2	9.5

Samples 1 and 2 come from a karst aquifer and sample 3 comes from a sandstone aquifer.

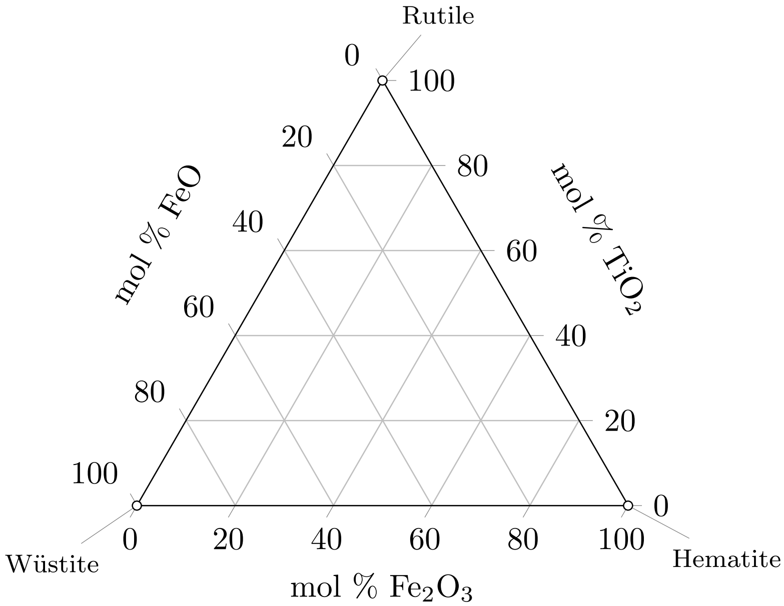


Figure 1.24: A ternary diagram for magnetic minerals. Note that ilmenite is FeO + TiO2; psuedobrookite is Fe2O3 + TiO2; ulvospinel is FeO+FeO+TiO2

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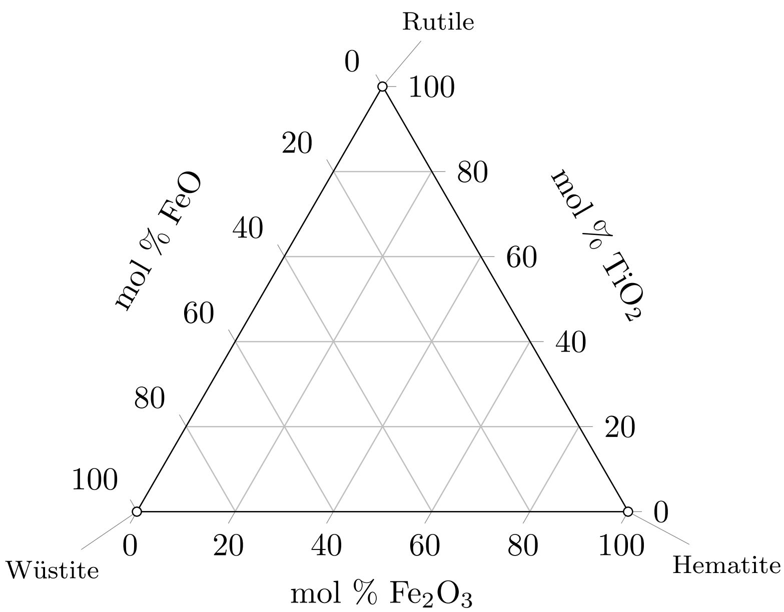

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Bibliography

J. P. Snyder. *Map projections–A working manual*, volume 1395 of *U.S. Geological Survey Special Publication*. US Geological Survey, Reston, Va., 1987.

H. L. Vacher. Computational geology 1–significant figures! *Journal of Geoscience Education*, 46(3):292–295, 1998.