Music, Machine And Mathematics

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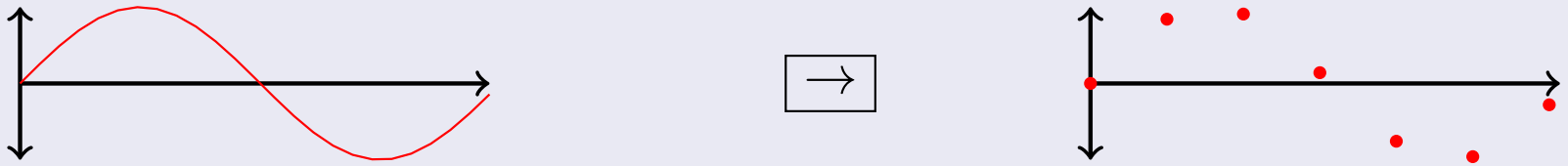
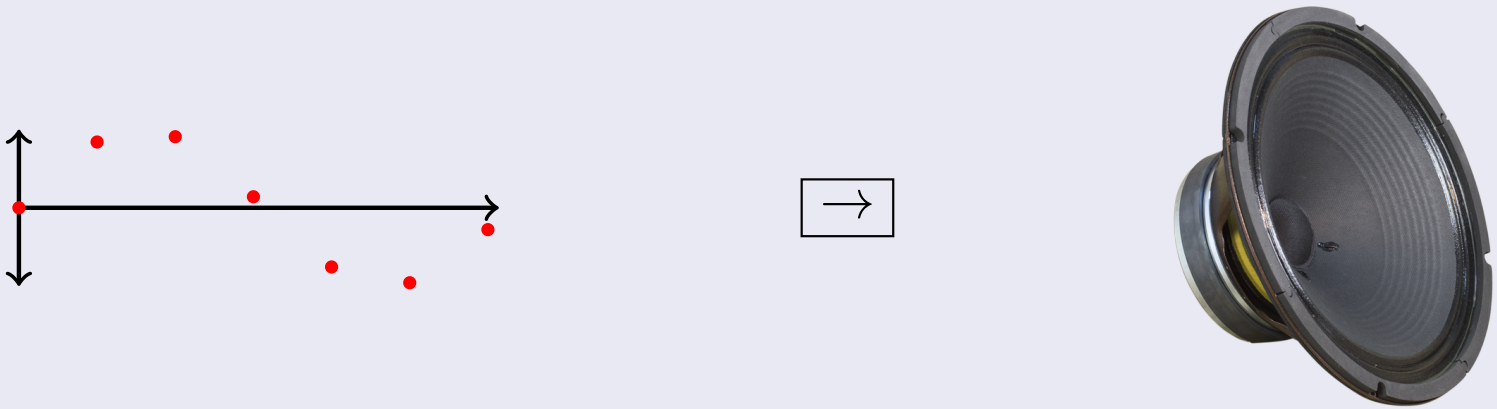
Overview

Computer Generation of Sound

What is Sound on a computer?



Then to a speaker



How to choose the sampling rate

Sampling rate = discretization

That is to say, the sampling rate has the same issues as discretization choices do in numerical mathematics. How can the necessary information be captured most efficiently?

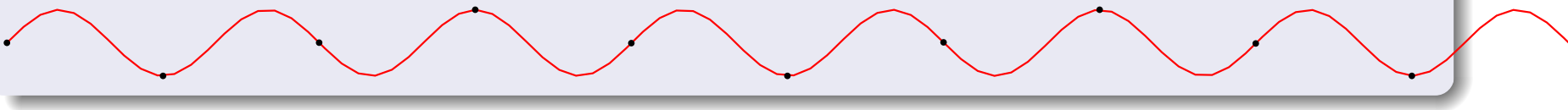
CDs

CDs use 44.1 kHz sampling. Why?

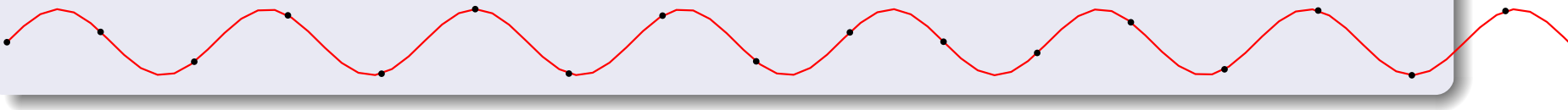
Capturing a pressure wave

Sampling at 1.75 of the period

Sampling at 0.75 of the period



Sampling at 0.45 of the period



Nyquist

Nyquist-Shannon Theorem

If a function Q is composed of continuous periodic waves (i.e. a nice fourier series) and has a highest frequency component in hertz f_{max} then the data of a sampling rate of at least $2f_{max}$ can be used to exactly determing Q.

Pd Examples

How Can 2 Sound Waves Interact?

First, what happens when two sound waves meet?

They add.

The math of 2 wave friends

Wave_1 & Wave_2

Suppose $\text{Wave}_1 = A\cos(wt + k)$ and $\text{Wave}_2 = A\cos(w't + k')$. Then,

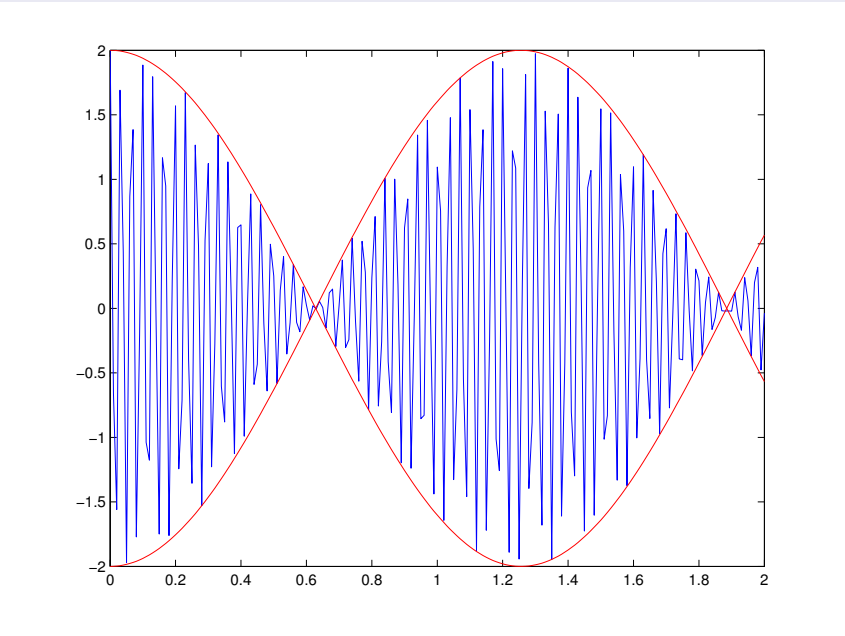
$$\begin{aligned} W_1 + W_2 =& A\cos(wt + k) + A\cos(w't + k') \\ =& 2A\cos(\frac{(w + w')t + (k + k')}{2})\cos(\frac{(w - w')t + (k - k')}{2}) \end{aligned}$$

Aural Beating

The math of the given example

Suppose $\text{Wave}_1 = \cos(440t)$ and $\text{Wave}_2 = \cos(435t)$. Then,

$$\begin{aligned} W_1 + W_2 &= \cos(440t) + \cos(435t) \\ &= 2\cos\left(\frac{875t}{2}\right)\cos\left(\frac{5t}{2}\right) \end{aligned}$$



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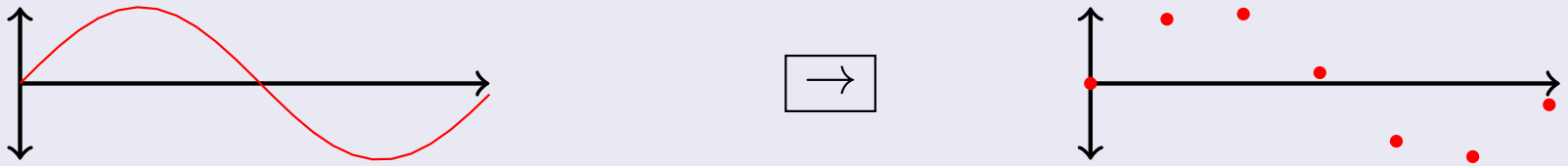

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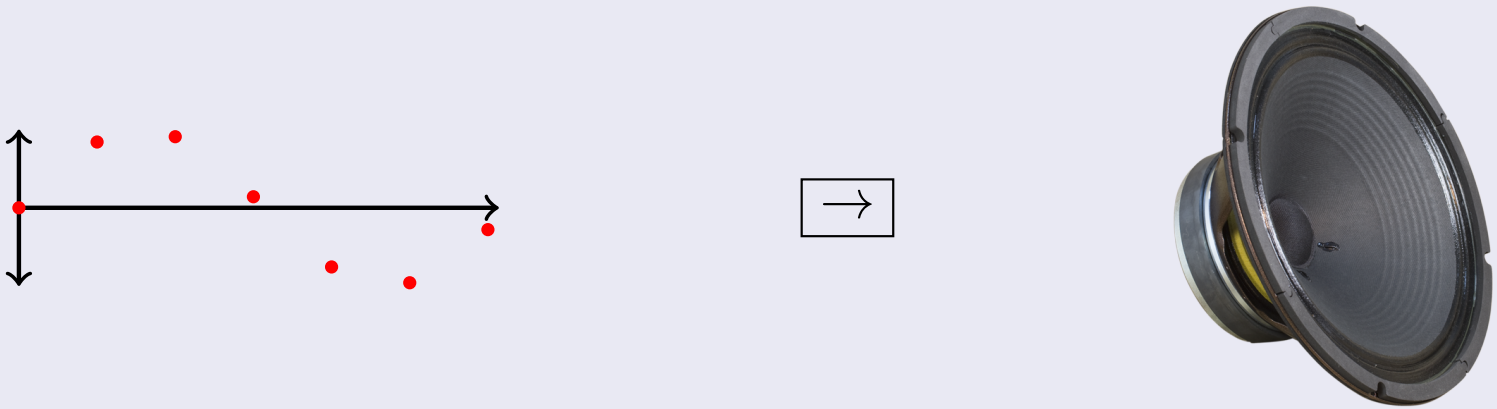
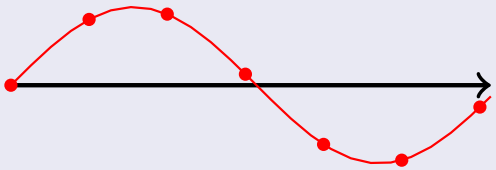
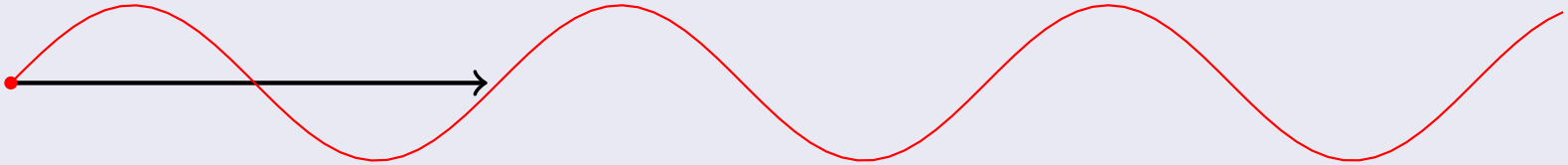


Sampling at Δt

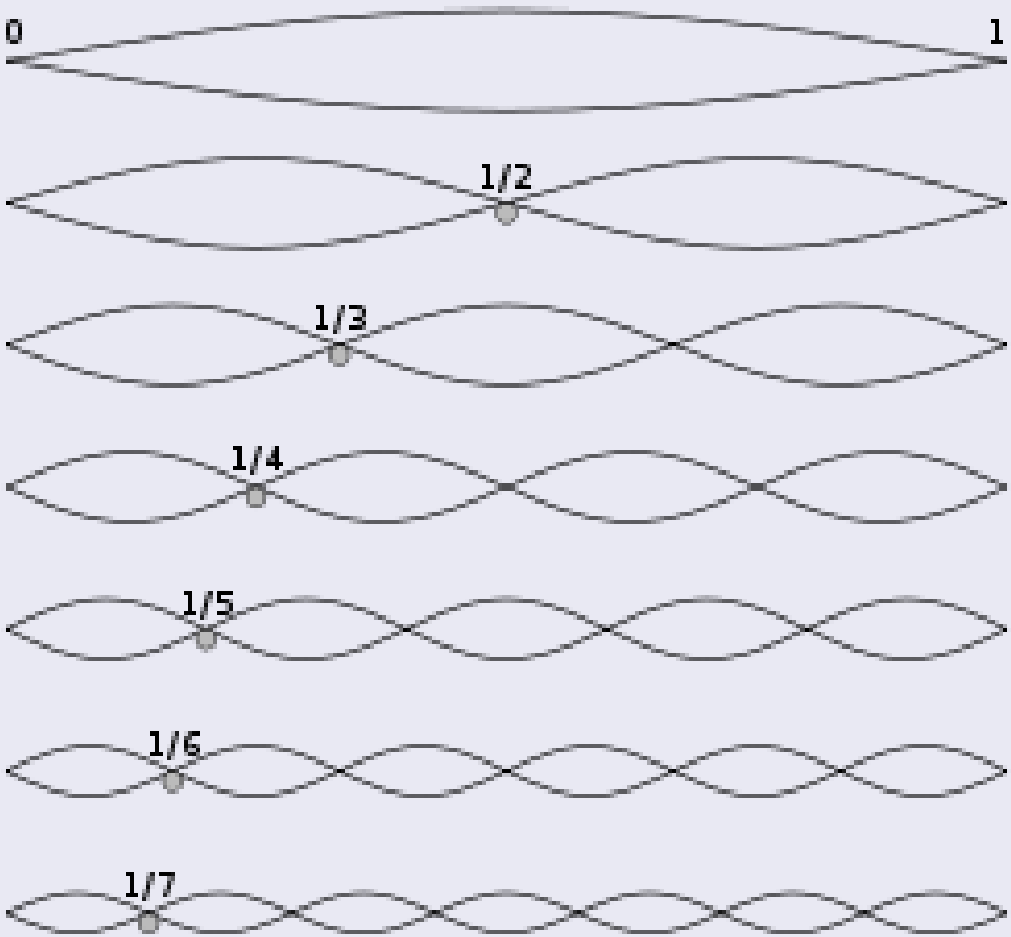


Sampling at Δt



Sounds from instruments

Fundamental and overtones



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