PUBH 6717 – Decision Analysis for Health Care

Practice Problem Set 11: (Solutions)
Markov models

Question 1: The Canadian province of Ontario is considering a province-wide ban on indoor tanning as a means of preventing skin cancer, with a focus on young women. The Ontario Ministry of Health has just finished a large observational study on tanning behaviors and skin cancer incidence among women in Ontario to inform their decision.

In their surveillance study, they found that skin cancer risks differ substantially for individuals who visit tanning salons regularly ("regular tanners") and those who do not ("non-tanners"). The annual risk of skin cancer was 4% for regular tanners vs. 0.5% for non-tanners. (Assume that skin cancer risk depends only on current tanning behavior and not on tanning history).

The Ministry of Health also studied tanning behaviors. They estimated the annual probability of a non-tanner becoming a regular tanner, by age, among women. This probability begins increasing around age 12, peaks at age 24 and then begins to decrease. They also estimated the rate of a regular tanner becoming a non-tanner. This probability is low until age 30, after which it increases with age. These data are summarized in the Excel file "Ontario Indoor Tanning Parameters.xlsx".

Skin cancer resolves within one year of diagnosis, with 7% of cases resulting in death. Those who recover following a skin cancer diagnosis nearly all quit tanning, though they re-initiate indoor tanning at the same rate as their peers who have not experienced skin cancer.

The Ontario Ministry of Health is considering an indoor tanning ban for those 18 years of age and younger (reducing the rate of tanning initiation to zero among this age group). They are also considering a full indoor tanning ban, which would reduce the rate of tanning initiation to zero for everyone in Ontario. They would like to evaluate the impact that each of these policies would have over the lifetime of a cohort of 10-year-old girls. The Ministry of Health does not wish to discount outcomes.

(a) Draw a Markov model diagram of tanning behavior and skin cancer that the Ontario Ministry of Health could use to evaluate their tanning policies in terms of the desired outcomes. Use a yearly time step. Include the following states: "Non-tanners", "Regular tanners", "Skin cancer", "Dead from skin cancer", and "Dead from other causes". Label all transition probabilities and indicate which are changing over time.

(b) Which states are absorbing states? Which states are tunnel states?

Question 1 continued . . .

The Excel file "Ontario Indoor Tanning Parameters.xlsx" contains the age-specific (non-skin-cancer) mortality and age-specific probabilities of initiating and discontinuing indoor tanning for girls in Ontario. Using these data and the template provided on the "Markov model template" sheet, construct a Markov model of skin cancer incidence and death for 10 year-old girls in Ontario by filling in the cells highlighted in orange with the appropriate values. Assume that no girls are regular tanners at age 10. With current levels of tanning initiation/discontinuation, you should find that the remaining life-expectancy of a 10-year-old girl in Ontario is 70.56 years.

Using your Markov model, evaluate the following outcomes for each of the three strategies (do nothing, ban indoor tanning for those 18 years and younger, ban indoor tanning for everyone):

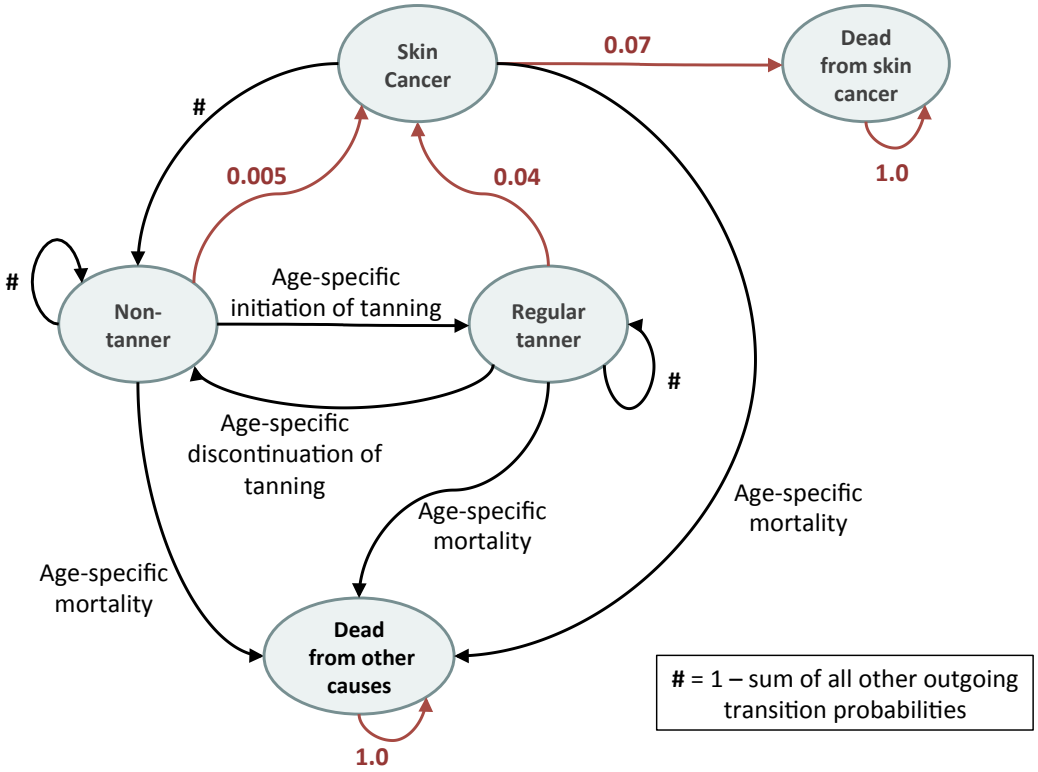
(c) 5pts: What is the remaining life-expectancy of a 10-year-old girl under each strategy?

(d) What is the lifetime risk of dying from skin cancer for a 10-year-old girl under each strategy?

(e) There are currently 150,000 10-year-old girls living in Ontario. What would be the total number of skin cancer cases averted over the lifetime of this cohort for each type of tanning ban?

Solutions

(a) Markov model diagram:
Constant probabilities are shown in red – all others change over time as the cohort ages.



(b) "Non-skin cancer death" and "Skin cancer death" are both absorbing states. "Skin cancer" is a tunnel state.

(c) Remaining life-expectancy.

Calculate the number of life years accrued each cycle (that is, the proportion of the cohort that is still alive each year) and sum:

Strategy	Remaining life-expectancy of a 10-year-old girl (in years)
Do Nothing	70.56
Ban for 18 years and younger	71.25
Ban for everyone	72.45

(d) Lifetime risk of dying from skin cancer.

There are two absorbing states for death, “Dead from skin cancer” and “Dead from other causes”. 100% of the cohort will be in one of these two states by the end of the (lifetime) time horizon – those that died of skin cancer will be in the “Dead from skin cancer” state, while everyone else will be in the “Dead from other causes” state.

Therefore, the lifetime risk of dying from skin cancer is just the proportion of the cohort in the “Dead from skin cancer” state at the end of the time horizon:

Strategy	Lifetime risk of dying from skin cancer of a 10-year-old girl
Do Nothing	5.9%
Ban for 18 years and younger	4.8%
Ban for everyone	2.6%

(e) Skin cancer cases averted.

Calculate the number of skin cancer cases each cycle (that is, the proportion of the cohort in the skin cancer state) and sum:

Strategy	Number of skin cancer cases	Skin cancer cases averted (compared to no ban)
Do Nothing	126,285	-
Ban for 18 years and younger	103,291	22,994
Ban for everyone	54,812	71,473

Question 2: Christmastitis is a health disorder that arises from a combination of holiday stress, inclement weather, and an overdose of holiday pop music. Though Christmastitis most often develops during the holiday season, its effects are irreversible and life-long. In fact, suffers of Christmastitis only have a remaining life-expectancy of 17.92 months (undiscounted) from the onset of their illness.

Once a person develops Christmastitis, they are generally in a chronic state of illness, with a 2% monthly risk of death and $50/month in healthcare costs. Luckily their quality of life is not impacted in this chronic state (utility of 1.0). However, Christmastitis sufferers are at risk (a 10% monthly probability) of developing an acute episode of Holiday Blues, characterized by low quality of life (a utility of 0.50), higher healthcare costs, and an even greater risk of death. The severity of a Holiday Blues episode varies over time. The first month of a Holiday Blues episode is the most severe, with a 30% risk of death, which decreases to a 20% risk of death in the second month and a 15% risk of death each month after that. A person experiencing a Holiday Blues episode incurs greater healthcare costs as well. The first month of an episode costs an average of $500 in healthcare costs, while each subsequent month incurs a cost of $100 on average. There is a probability each month that a person will recover from a Holiday Blues episode and return to the chronic Christmastitis state. This probability is 45% in the first month, 25% in the second month, and 15% each month after that. State-specific quantities for Christmastitis are summarized in Table 1.

Two interventions were recently developed for patients with Christmastitis to treat or prevent Holiday Blues episodes. The first, a pharmaceutical treatment, increases the monthly probability of recovery to 45% when taken during a Holiday Blues episode. Unfortunately, due to diagnostic delays, a patient can only begin to receive the treatment after having symptoms for two months (e.g. at the beginning of the third month of an episode). Furthermore, 30% of Christmastitis suffers have a genetic variant that does not respond to treatment, meaning they incur the cost of treatment, but receive none of the benefit. The treatment costs $200 each month it is taken.

The second intervention is an implantable device that can quickly detect and resolve a Holiday Blues episode. The device prevents an episode of Holiday Blues from progressing beyond the first month (i.e. it increases the probability of recovery in the first month to 0.70). Implantation of the device costs $10,000 and has a 10% risk of surgical death. There are no monthly maintenance costs thereafter.

Table 1: State-specific quantities for Christmastitis disease progression.

(monthly quantities)	Chronic Chrismastitis	Holiday Blues – month 1	Holiday Blues – month 2	Holiday Blues – month 3+	Dead
Probability of death	0.02	0.30	0.20	0.15	N/A
Probability of recovery	N/A	0.45	0.25	0.15	N/A
Utility	1.0	0.5	0.5	0.5	0
Healthcare cost	$50	$500	$100	$100	$0

(a) Draw a state transition diagram of the natural history of Christmastitis (e.g. without intervention). Label all states and transition probabilities.

(b) Construct a Markov model of the natural history of Christmastitis in Excel. Calculate the distribution of a cohort of patients newly diagnosed with Christmastitis in monthly time steps for 180 months (15 years). Calculate the remaining life-expectancy of the cohort (in months), both undiscounted and discounted at 3% per year. You should get an undiscounted LE of 17.92 months and a discounted LE of 17.16 months.
    i) Submit a print-out of your state transition matrix.
    ii) Submit a print-out of the cohort distribution for months 0 to 24.

Note: Recall that to discount at a rate of r_{year} per year, the yearly discount factor is $1/(1+r_{year})^t$, where is t the number of years in the future. The monthly discounted factor is $1/(1+ r_{year})^{t/12}$ where t is the number of months in the future.

(c) Construct a decision tree that could be used for a cost-effectiveness analysis of the two interventions for Christmastitis. Clearly label all probabilities.

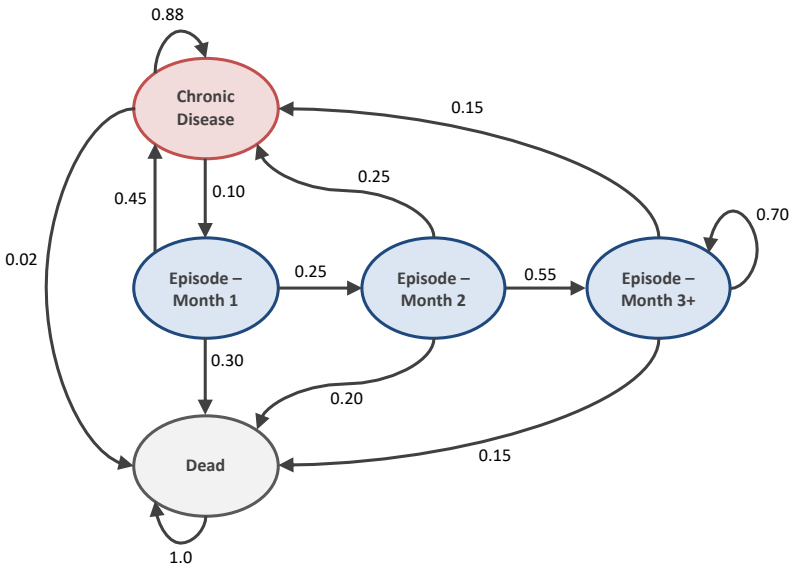
(d) Using your Markov model from part (b), calculate the remaining (discounted) quality-adjusted life-expectancy and healthcare costs needed for the terminal node values of your decision tree in part (c). Appropriately label the terminal nodes in your decision tree.

(e) Calculate the overall expected costs and benefits (in terms of QALE) of a patient newly diagnosed with Christmastitis under each of the three strategies (No Intervention, Pharmaceutical Treatment, and Implantable Device).

(f) Plot each strategy in terms of its costs and benefits. Draw the efficient frontier and list the incremental cost-effectiveness ratios for strategies on the frontier.
    i) Which strategies (if any) are dominated?
    ii) If you chose to invest in the Pharmaceutical Treatment strategy, what could you say about your cost-effectiveness threshold?

Solutions

(a) Markov diagram



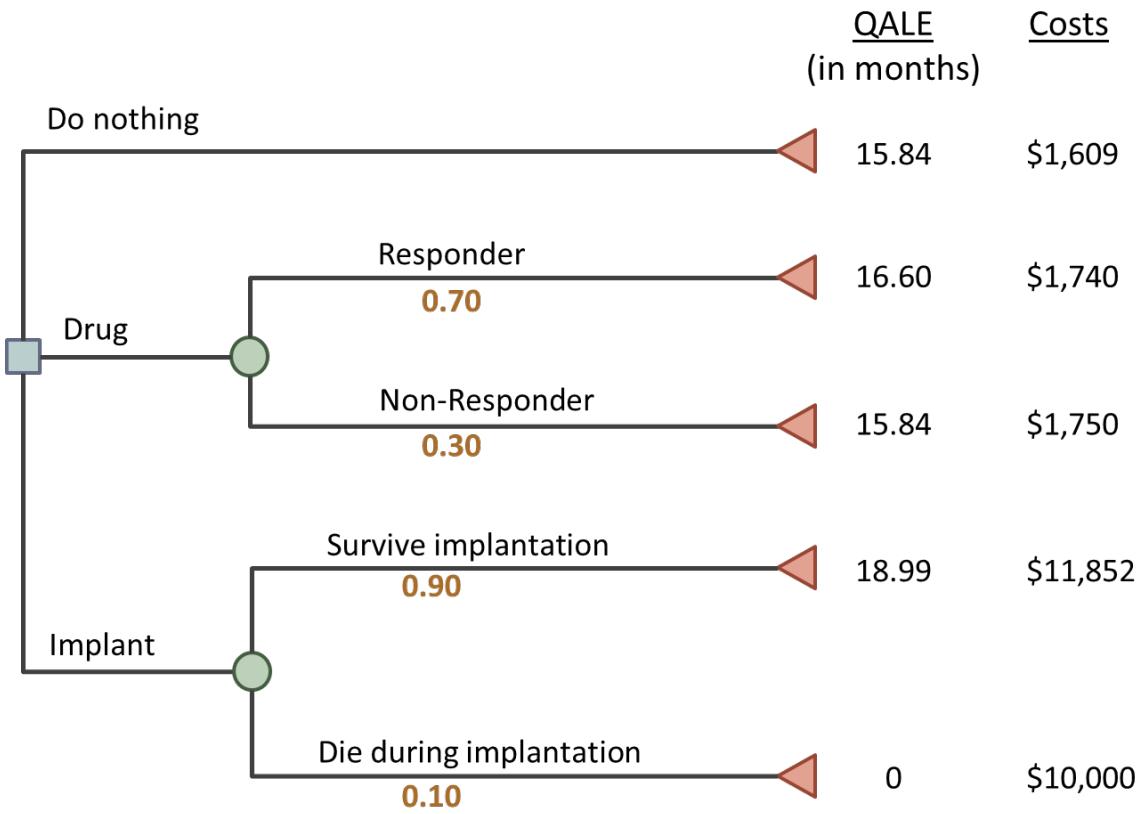
(b) State Transition matrix

	Chronic	Episode - month 1	Episode month 2	Episode month 3+	Dead
Chronic	0.88	0.1	0	0	0.02
Episode - month 1	0.45	0	0.25	0	0.3
Episode month 2	0.25	0	0	0.55	0.2
Episode month 3+	0.15	0	0	0.7	0.15
Dead	0	0	0	0	1

Cohort distribution:

Month	Chronic	Episode - month 1	Episode month 2	Episode month 3+	Dead
0	1	0	0	0	0
1	0.880	0.100	-	-	0.020
2	0.819	0.088	0.025	-	0.068
3	0.767	0.082	0.022	0.014	0.115
4	0.719	0.077	0.020	0.022	0.162
5	0.676	0.072	0.019	0.026	0.207
6	0.636	0.068	0.018	0.029	0.249
7	0.599	0.064	0.017	0.030	0.290
8	0.564	0.060	0.016	0.030	0.329
9	0.532	0.056	0.015	0.030	0.366
10	0.502	0.053	0.014	0.029	0.401
11	0.474	0.050	0.013	0.028	0.435
12	0.447	0.047	0.013	0.027	0.466
13	0.422	0.045	0.012	0.026	0.496
14	0.398	0.042	0.011	0.025	0.524
15	0.376	0.040	0.011	0.023	0.550
16	0.355	0.038	0.010	0.022	0.576
17	0.335	0.035	0.009	0.021	0.599
18	0.316	0.033	0.009	0.020	0.622
19	0.299	0.032	0.008	0.019	0.643
20	0.282	0.030	0.008	0.018	0.663
21	0.266	0.028	0.007	0.017	0.681
22	0.251	0.027	0.007	0.016	0.699
23	0.237	0.025	0.007	0.015	0.716
24	0.224	0.024	0.006	0.014	0.732

(c) Decision tree



(d) Remaining life expectancy for terminal nodes are labeled in the tree above. The key is to simulate each situation with the correct parameters in the Markov model.

For a responder to the pharmaceutical treatment, their probability of recovery from "Holiday Blues – Month 3+" is 0.45 and their cost of being in the "Holiday Blues – Month 3+" is $300 / month. For a non-responder to the treatment, all transition probabilities are unchanged, but their cost in the "Holiday Blues – Month 3+" state is increased to $300 / month. Note that it doesn't make much of a difference cost-wise!

For a survivor of the implant, their probability of recovery from "Holiday Blues – month 1" is now 0.70, meaning that, in fact, the probability of progression to month 2 is 0, effectively eliminating any long-term Holiday Blues episodes. Note that episodes can reoccur, which is why we use a Markov model rather than making it a chance node in the tree. Whether or not a patient survives, they incur a cost of $10,000 upfront.

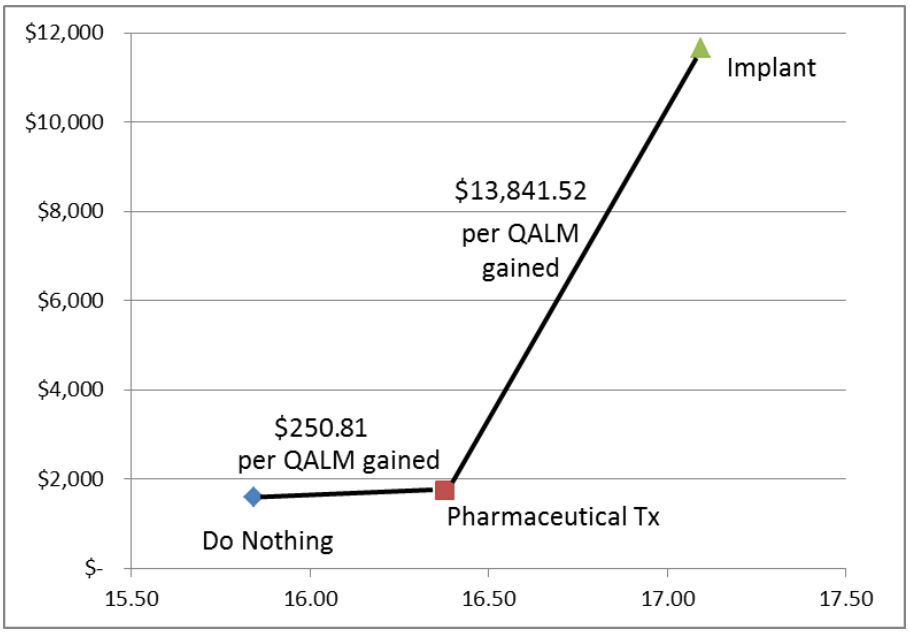
(e) Roll-back the tree. The expected costs and benefits of each strategy are:

Strategy	Cost	QALMs
Do Nothing	$1,609.38	15.84
Pharmaceutical	$1,743.02	16.38
Implant	$11,666.68	17.09

(f) No strategies are strongly or weakly dominated. Incremental cost-effectiveness ratios are shown below:

Strategy	Cost	QALMs	Inc. Cost	Inc. Effect	ICER
Do Nothing	$1,609.38	15.84	-	-	-
Pharmaceutical	$1,743.02	16.38	$133.65	0.53	$250.81
Implant	$11,666.68	17.09	$9,923.66	0.72	$13,841.52

Cost-effectiveness plot:



If an insurer would choose to cover the pharmaceutical intervention, it means that their cost-effectiveness threshold must be between $251 and $13,841 per quality-adjusted life-month.