

# Flavour Symmetry Embedded - GLoBES

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ABSTRACT: Abstract...

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## 1 Introduction

The discovery of neutrino oscillations points out the fact that neutrinos have mass, and provides evidence beyond the Standard Model (BSM). This phenomenon is successfully described by a theoretical framework with the help of three neutrino mixing angles ( $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ), two mass-square splittings ( $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$ ), and one Dirac CP phase ( $\delta$ ) [1–4]. Thanks to the great efforts in the past two decades, we almost have a complete understanding of such a neutrino oscillation framework. More data in the neutrino oscillation experiments is needed to determine the sign of  $\Delta m_{31}^2$ , to measure the value of  $\sin \theta_{23}$ , to discover the potential CP violation in the leptonic sector and even to constrain the size of  $\delta$  [4]. For these purposes, the on-going long baseline experiments (LBLs), such as the NuMI Off-axis  $\nu_e$  Appearance experiment (NO $\nu$ A) [5] and the Tokai-to-Kamioka experiment (T2K) [6], can answer these questions with the statistical significance  $\gtrsim 3\sigma$  in most of the parameter space. Based on the analysis with their data, the normal mass ordering ( $\Delta m_{31}^2 > 0$ ), the higher  $\theta_{23}$  octant ( $\theta_{23} > 45^\circ$ ), and  $\delta \sim 270^\circ$  are preferred so far [4]. The future LBLs, Deep Underground Neutrino Experiment (DUNE) [7], Tokai to Hyper-Kamiokande (T2HK) [8], and the medium baseline reactor experiment, the Jiangmen Underground Neutrino Observatory (JUNO) [9, 10] will further complete our knowledge of neutrino oscillations.

Flavour symmetry models are used to explain the origin of the neutrino mixing, and to predict the value of oscillation parameters (some of useful review articles are [11–17]). These models are motivated by some interesting features, *such as*  $\theta_{12} \sim 33^\circ$ , and  $\theta_{23} \sim 45^\circ$ .

Before the discovery of non-zero  $\theta_{13}$  measurement by Daya Bay experiment [18], the ‘tri-bi-maximal’ neutrino mixing (TBM) ansatz, which was proposed in 2002 by Harrison, Perkins, and Scott [19], fitted with the experimental data in a good agreement:

$$U_{\text{TBM}} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}.$$

With the fact that  $\theta_{13} \approx 8^\circ$ , several ways to obtain such non-zero value of  $\theta_{13}$  are proposed. One of popular proposals is to correct the tri-bi-maximal neutrino mixing such that

$$\sin \theta_{12} = (1 + s)/\sqrt{3}, \quad \sin \theta_{13} = r/\sqrt{2}, \quad \text{and} \quad \sin \theta_{23} = (1 + a)/\sqrt{2}.$$

The neutrino mixing ansatz can be realised by high-energy symmetries  $G_f$ . The symmetry of discrete groups  $G_f$ , preserved at the high energy but slightly broken at the lower energy, predicts the neutrino mixing, mass-square splittings, and the CP violation phase (Dirac and Majorana phases), with reduced degrees of freedom. The symmetries need to be broken at the low energy. Otherwise, the flavour of leptons cannot be distinguished. There are several approaches for the symmetry breaking, including the direct, indirect, and semi approaches. Direct approach preserves the residual symmetries of  $G_f$  in the charged-lepton or neutrino sector. On the other hand, there is no residual symmetry preserved in neither charged-lepton nor neutrino sector in the indirect approach. In the semi approach, the charged-lepton and neutrino sectors preserve different residual symmetries, respectively. This symmetry is broken by extending the Higgs sector or introducing the flavons. To achieve the  $\delta$  prediction, many models are based on a discrete family symmetry  $G_f$  together with a non-commuting CP symmetry  $H_{\text{CP}}$ . Broken in different approaches, the symmetry  $G_f \otimes H_{\text{CP}}$  can predict different patterns for the neutrino mixing. For example, in the semi-direct approach,  $S_4$  is preserved in the leading order, and leads the bimaximal (MB) or tri-bimaximal (TB) neutrino mixing, while the higher order terms bring the correction to the neutrino mixing.

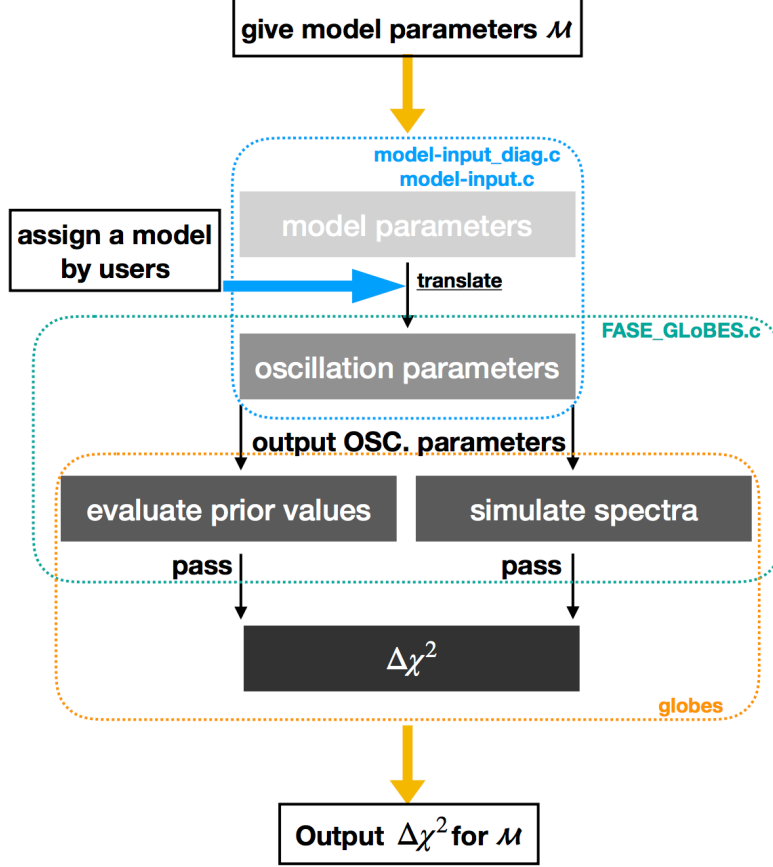
Currently, some works discuss on how the future experiments can be used for testing these flavour symmetry models in the phenomenological point of view, *e.g.* [20–23]. A large number of these works are based on the c-library – **General Long Baseline Experiment Simulator (GLOBES)** [24, 25], which is a convenient simulation tool to simulate neutrino oscillation experiments via the Abstract Experiment Definition Language (AEDL). Some AEDL files for experiments are also available on **GLOBES** website. In addition, the working group of DUNE experiment also releases their AEDL files [26]. **GLOBES** can be taken as one of popular and useful tools in the community of neutrino oscillation physics. However, it has not yet to be extended for the purpose of analysing flavour symmetry models.

In this work, we will present our simulation tool **Flavour Symmetry Embedded - GLOBES (FaSE-GLOBES)** that is a simulation code based on GLOBES library and specific for studying the flavour symmetry.

## 2 Overview FASE-GLOBES

**FLA**vor **S**ymmetry **E**mbodied (**FASE**) is a supplemental tool for General Long Baseline Experiment Simulator (**GLOBES**) in order to analysis how a leptonic flavour sym-

metry model can be tested in neutrino oscillation experiments. **FASE** is written in the `c/c++` language, and consist with three codes **FASE\_GLoBES.c**, **model-input\_diag.c** and **model-input.c**. The user defines the model in to **model-input\_diag.c** or **model-input.c**, while **FASE\_GLoBES.c** does not need to be touched.



**Figure 1.** A scheme to correlate the model parameters with standard neutrino oscillation parameters. The error propagation is implemented in the simulation code up to the spectra analysis.

The **FASE-GLoBES** is **FASE** working with **GLoBES** as shown in Fig. 1, in which three parts are shown: 1. **the parameter translation** (the blue box), 2. **giving oscillation-parameter values** (the green box), and 3. **the  $\chi^2$ -value calculation** (the orange box). The idea behind this flow chart Fig. 1 is that given a set of value for model parameter, the corresponding values for oscillation parameters are obtained by a translation, which is assigned by the user in **model-input\_diag.c** or **model-input.c**. And then, through **FASE\_GLoBES.c**, these oscillation-parameter values are passed in to **GLoBES** library to simulate the event spectrum for evaluating the  $\chi^2$  value.

API functions to use **FASE** are listed:

1. `MODEL_init( $N_{para}$ )`,
2. `FASE_glb_probability_matrix`,

3. `FASE_glb_set_oscillation_parameters,`
4. `FASE_glb_get_oscillation_parameters,`
5. `FASE_prior_OSC,`
6. `FASE_prior_model.`

The first one is to initialise FASE with the parameter  $N_{para}$  the number of input parameters. The next three functions need to be included to replace the default GLoBES probability engine by the one that can read the output from `model-input__diag.c` or `model-input.c`, as follows

```
glbRegisterProbabilityEngine(6,
&FASE_glb_probability_matrix,
&FASE_glb_set_oscillation_parameters,
&FASE_glb_get_oscillation_parameters,
NULL);
```

This probability engine can work with the oscillation or model parameter. It can be set by the user with the parameter `PARAMETER`. If `PARAMETER=STAN` (`PARAMETER=MODEL`) the probability engine works with oscillation (model) parameters. The final two functions are prior functions. Once the user gives the prior in oscillation (model) parameters, the user needs to call `FASE_prior_OSC` (`FASE_prior_model`) as follows.

```
glbRegisterPriorFunction(FASE_prior_OSC,NULL,NULL,NULL);
or
glbRegisterPriorFunction(FASE_prior_model,NULL,NULL,NULL);
```

We note that except for setting the probability engine and the prior function, the other parts in the main code should follow with the GLoBES manual.

In the following, we give more details about using **FASE-GLoBES**. In Sec. 2.1, we will give the instruction to download and install **GLoBES** and **FASE**. Then, in Sec. 2.2, we will introduce how to compile and run the code. More complicated is setting the model, and the way to do this will be described in Sec. 2.3. Finally, we will discuss how to set up the prior in Sec. 2.4.

## 2.1 Download and Installation

### download and install GLoBES and FASE-GLoBES

[will fill in after the code ready on Github]

## 2.2 Compile and Run

As FASE works based on GLoBES, the user should use the **GLoBES** Makefile, but include the binary file for FASE. To do so, we include the script in the Makefile,

```
{makefile_execution}: my_program.o FASE_GLoBES.o model-input_diag.o
gcc my_program.o FASE_GLoBES.o model-input_diag.o -o my_executable $(LD_FLAGS)
```

```
$(local_LDFLAGS)
```

or

```
{makefile_execution}: my_program.o FASE_GLoBES.o model-input.o
```

```
gcc my_program.o FASE_GLoBES.o model-input.o -o my_executable $(LDFLAGS) $(local_LDFLAGS)
```

where {compile\_execusion} is the makefile commend for the program my\_program.o and the execution my\_executable is the output. After giving this script in the **GLoBES Makefile**, the user needs to makefile the program my\_program on the terminal.

```
makefile {makefile_execution}
```

And, the user can execute the program by typing the following commend on the terminal.

```
./my_executable
```

The user can also compile the program without the **GLoBES Makefile** with the following script.

```
gcc my_program.o FASE_GLoBES.o model-input_diag.o -lglobes -LGLB_DIR/lib/ -o my_executable
```

or

```
gcc my_program.o FASE_GLoBES.o model-input.o -lglobes -LGLB_DIR/lib/ -o my_executable
```

And, the execution is also in the same way: ./my\_executable.

Finally we need to initialize **FASE** by giving  $N_{para}$  which is the number of model parameters. To do so, we need to include the following script in the main code.

```
MODEL_init( $N_{para}$ );
```

### 2.3 Model setting

FASE includes three c++ files: **model-input\_diag.c**, **model-input.c**, and **FASE\_GLoBES.c**.

As shown in the previous paragraph, these files are for different service. In **model-input\_diag.c** and **model-input.c**, the user can assign the relation between the oscillation and model parameter sets. The difference between these two code is that in **model-input.c**, the user give the relation between these two parameter sets directly, while in **model-input.c**, the mass matrix, which is provided by the user, can be diagonalised to obtain the corresponding oscillation-parameter values.

For different methods to include the model, we use different c++ file. Adopting the mass matrix expressed by model parameters, we assign this model in **model-input\_diag.c**. In function **MtoS**, we give the mass matrix in the  $3 \times 3$  complex matrix **Mass\_Matrix[]**. Passing the complex matrix **Mass\_Matrix[]** into the function **STAN\_OSC**, we will get the corresponding oscillation parameters in an array **OSC\_PARAMS**, components of which are the value for  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ , and  $\Delta m_{31}^2$ . The expression of **Mass\_Matrix[]** is given by the user. For the first four components, values are given in the unit of **rad**, while the other two are in **eV<sup>2</sup>**. We can also assign the relation between oscillation and model parameters in **model-input.c**. The relations are given in **MtoS** as well.

## 2.4 Prior setting

Given a set of values for model parameters, **FASE\_GLoBES.c** will obtain the corresponding oscillation-parameter values from **model-input\_diga.c** or **model-input.c**, and will pass these values to simulate the event spectrum and to give the prior value. Two gaussian prior functions are provided in **FASE** – **FASE\_prior\_OSC** and **FASE\_prior\_model**. These two functions are for different purposes. If the user give the prior in oscillation (model) parameters, the user should register **FASE\_prior\_OSC** (**FASE\_prior\_model**) for the prior with the **GLoBES** function **glbRegisterPriorFunction**, as we introduced in the beginning of this section. The Gaussian prior is

$$\chi_{prior}^2 = \sum_i \frac{(\theta_i - \theta_i^c)^2}{\sigma_i^2}, \quad (2.1)$$

where  $\theta_i$  is one of parameters constrained by prior,  $\theta_i^c$  ( $\sigma_c$ ) is the central value (Gaussian width) of the prior for  $\theta_i$ . We note that  $\theta_i$  can be either model or oscillation parameter. But, oscillation and model parameters can be mixed up together in the using of prior.

The values of  $\theta_i^c$  and  $\sigma_i$  need to be given by the user through three arrays: **Central\_prior**, **UPPER\_prior**, and **LOWER\_prior**, in which there are six components. To treat asymmetry of width for the upper and lower Gaussian widths, we give values in two arrays **UPPER\_prior**, and **LOWER\_prior**, respectively. If the user gives the prior in model parameters, the order of each component follow with the setup of input of the probability engine. While the user gives the prior in oscillation parameters, the six components in order are  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ , and  $\Delta m_{31}^2$ . The first four parameter are in **rad**, and the final two are in **eV<sup>2</sup>**.

Finally, some restrictions are imposed by the studied flavour symmetry model. We set up these restrictions in the function **model\_restriction**, which is in **model-input\_diga.c** and **model-input.c**. In the function **model\_restriction**, the user needs to *return* 0 once the restriction is broken. For example, if the normal ordering is imposed, we give “ **if** (**DMS31**<0) { **return** 1;} ” in **model\_restriction**, where **DMS31** is for  $\Delta m_{31}^2$ . Then, when the restriction is broken, **model\_restriction** returns the value 1 to the prior function **FASE\_prior\_OSC** or **FASE\_prior\_model**. In the following, the prior function will give  $10^6$  for the  $\chi^2$  value, and it will be selected out when we study the statistically reasonable region in the parameter space.

## 3 Examples

### 3.1 Setup for a model

We now take the littlest seesaw model in the tri-direct approach as an example. In this model, the atmospheric and solar flavon vacuum alignments are  $\langle \phi_{\text{atm}} \rangle \propto (1, \omega^2, \omega)^T$  and  $\langle \phi_{\text{sol}} \rangle \propto (1, x, x)^T$ , where stands for a cube root of unity and the parameter  $x$  is real because of the imposed CP symmetry. Under this model, the light left-handed Majorana neutrino mass matrix is given by

$$m_\nu = m_a \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + e^{i\eta} m_s \begin{pmatrix} 1 & x & x \\ x & x^2 & x^2 \\ x & x^2 & x^2 \end{pmatrix}, \quad (3.1)$$

where  $x$ ,  $\eta$ ,  $m_a$ , and the ratio  $r \equiv m_s/m_a$  are four parameters and will be constrained by experimental data. We note that from Eq. (3.1),  $m_1 = 0$  and the normal mass ordering are imposed, and will need to be imposed in **FASE-GLoBES**. Therefore, the restrictions in this model are  $m_a > 0$  and  $r > 0$ .

**Table 1.** A summary of the relation between oscillation parameters and TDLS model parameters [?]. Two requirements are imposed by TDLS: the smallest mass state  $m_1 = 0$  and the normal mass ordering. The sign of  $\sin \delta$  depends on the sign of  $x \cos \psi$ : “+” (“−”) is for  $x \cos \psi > 0$  ( $< 0$ ).

model parameters	$x, \eta, r, m_a$
combinations of model parameters	$y = \frac{5x^2+2x+2}{2(x^2+x+1)}(m_a + e^{i\eta}m_s),$ $z = -\frac{\sqrt{5x^2+2x+2}}{2(x^2+x+1)}[(x+2)m_a - x(2x+1)e^{i\eta}m_s],$ $w = \frac{1}{2(x^2+x+1)}[(x+2)^2m_a + x^2(2x+1)^2e^{i\eta}m_s],$ $\sin \psi = \frac{\Im(y^*z+wz^*)}{ y^*z+wz^* }, \quad \cos \psi = \frac{\Re(y^*z+wz^*)}{ y^*z+wz^* }.$ $\sin 2\theta = \frac{2 y^*z+wz^* }{\sqrt{( w ^2- y ^2)^2+4 y^*z+wz^* ^2}},$ $\cos 2\theta = \frac{ w ^2- y ^2}{\sqrt{( w ^2- y ^2)^2+4 y^*z+wz^* ^2}}.$
oscillation parameters	$\Delta m_{21}^2 = m_2^2 = \frac{1}{2} \left[  y ^2 +  w ^2 + 2 z ^2 - \frac{ w ^2- y ^2}{\cos \theta} \right],$ $\Delta m_{31}^2 = m_3^2 = \frac{1}{2} \left[  y ^2 +  w ^2 + 2 z ^2 + \frac{ w ^2- y ^2}{\cos \theta} \right],$ $\sin^2 \theta_{12} = 1 - \frac{3x^2}{3x^2+2(x^2+x+1)\cos^2 \theta},$ $\sin^2 \theta_{13} = \frac{2(x^2+x+1)\sin^2 \theta}{5x^2+2x+2},$ $\sin^2 \theta_{23} = \frac{1}{2} + \frac{x\sqrt{3(5x^2+2x+2)}\sin 2\theta \sin \psi}{2[3x^2+2(x^2+x+1)\cos^2 \theta]},$ $\cos \delta = \frac{\cot 2\theta_{23}[3x^2-(4x^2+x+1)\cos^2 \theta_{13}]}{\sqrt{3} x \sin \theta_{13}\sqrt{(5x^2+2x+2)\cos^2 \theta_{13}-3x^2}},$ $\sin \delta = \pm \csc 2\theta_{23} \sqrt{1 + \frac{(x^2+x+1)^2 \cot^2 \theta_{13} \cos^2 2\theta_{23}}{3x^2[3x^2 \tan^2 \theta_{13} - 2(x^2+x+1)]}}.$

```

int MtoS(double OSC_PARAMS[6], double M_para[])
{
double x=M_para[0];
double eta=M_para[1];
double r=M_para[2];
double ma=M_para[3];
double ms=ma*r;

double complex Mass_Matrix[] = {ma+ms*(cos(eta) + I*sin(eta)), ma*(cos(6.6666e-1*M_PI)
+ I*sin(6.6666e-1*M_PI))+x*ms*(cos(eta) + I*sin(eta)), ma*(cos(6.6666e-1*M_PI)
+ I*sin(6.6666e-1*M_PI))*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))+x*ms*(cos(eta)
+ I*sin(eta)), ma*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))+x*ms*(cos(eta)
+ I*sin(eta)), ma*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))*(cos(6.6666e-1*M_PI)
+ I*sin(6.6666e-1*M_PI))+x*x*ms*(cos(eta) + I*sin(eta)), ma+x*x*ms*(cos(eta) +
I*sin(eta)), ma*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))*(cos(6.6666e-1*M_PI)

```



```
+ I*sin(6.6666e-1*M_PI))+x*ms*(cos(eta) + I*sin(eta)), ma+x*x*ms*(cos(eta) + I*sin(eta)),
ma*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))+x*x*ms*(cos(eta) + I*sin(eta))};
```

```
STAN_OSC(Mass_Matrix,OSC_PARAMS);
```

```
return 0;
}

double MtoS(double osc_para[6], double M_para[])
{
/*example: tri-direct*/

double x_in = M_para[0];
double eta_in = M_para[1];
double r_in = M_para[2];
double ma_in = M_para[3];

osc_para[0]=TDth12(x_in,eta_in,r_in, ma_in);
osc_para[1]=TDth13(x_in,eta_in,r_in, ma_in);
osc_para[2]=TDth23(x_in,eta_in,r_in, ma_in);
osc_para[3]=TDdCP(x_in,eta_in,r_in, ma_in);
osc_para[4]=TDdm21(x_in,eta_in,r_in, ma_in);
osc_para[5]=TDdm31(x_in,eta_in,r_in, ma_in);

return 0;
}
```

The functions TDth12, TDth13, TDth23, TDdCP, TDdm21, and TDdm31 are included for obtaining the value of  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ , and  $\Delta m_{31}^2$  according to Table 1, respectively.

The restrictions  $m_a > 0$  and  $r > 0$  need to be setup in `model_restriction` as follows.

```
double model_restriction(double model [])
{
double x=model[0];
double eta=model[1];
double r=model[2];
double ma=model[3];

if(ma<0) {return 1;}
if(r<0) {return 1;}

return 0;
}
```

If there is no any restrictions, we simply *return* 0 in `model_restriction` as follows.

```
double model_restriction(double model []){ return 0;}
```

### 3.2 Initialise the code

In the beginning of main code, we need to initialise GLOBES and FASE, and include the considered experiment (here we consider MOMENT experiment) as follows.

```
glbInit(argv[0]);
glbInitExperiment("expMOMENT_FIX_FLUX_150KM_addATM_NC.glb",&glb_experiment_list[0],
&glb_num_of_exps);
MODEL_init(4);
```

Then we register the probability engine.

```
glb_init_probability_engine();
glbRegisterProbabilityEngine(6,
&FASE_glb_probability_matrix,
&FASE_glb_set_oscillation_parameters,
&FASE_glb_get_oscillation_parameters,
NULL);
```

In the following we define the model-parameter values, and set the true value in model parameters.

```
float degree = M_PI/180;
float x_true,eta_true,r_true,ma_true;
x_true=-3.65029; eta_true=1.13067*M_PI; r_true=0.511325; ma_true=3.71199e-3;
glb_params true_values = glbAllocParams();
glb_params test_values = glbAllocParams();
glb_params input_errors = glbAllocParams();
glb_params centers = glbAllocParams();
glbDefineParams(true_values,x_true,eta_true,r_true,ma_true,0,0);
glbSetDensityParams(true_values,1.0,GLB_ALL);
glbCopyParams(true_values,test_values);
PARAM=MODEL;
glbSetOscillationParameters(true_values);
glbSetRates();
```

We then finally set up the projection. The projection **free** is to set all four parameters free to find the minimal value of  $\chi^2$ , which project is used for studying the constraint on  $x$  and  $\eta$ .

```
glb_projection projection = glbAllocProjection();
glb_projection free = glbAllocProjection();
glbDefineProjection(free,GLB_FREE,GLB_FREE,GLB_FREE,GLB_FREE,GLB_FIXED,GLB_FIXED);
glbSetDensityProjectionFlag(free, GLB_FREE, GLB_ALL);
glbDefineProjection(projection,GLB_FIXED,GLB_FIXED,GLB_FREE,GLB_FREE,GLB_FIXED,GLB_FIXED);
```

```
glbSetDensityProjectionFlag(projection, GLB_FREE, GLB_ALL);
```

### 3.3 Prior setup

To set up the prior, we need to register the prior function. As mentioned, when we use the prior in oscillation and model parameters, we register `FASE_prior_OSC` and `FASE_prior_model`, respectively. Here we present an example setting up priors in oscillation parameters according to NUFIT4.0 Table 2.

**Table 2.** The best fit and  $3\sigma$  uncertainty, in the results of NuFit4.0 [4].

Parameter	$\theta_{12}/^\circ$	$\theta_{13}/^\circ$	$\theta_{23}/^\circ$	$\delta/^\circ$	$\Delta m_{21}^2/10^{-5}\text{eV}^2$	$\Delta m_{31}^2/10^{-3}\text{eV}^2$
best fit	33.82	8.61	49.6	215	7.39	2.525
$3\sigma$ Range	31.61 – 36.27	8.22 – 8.99	40.3 – 52.4	125 – 392	6.79 – 8.01	2.47 – 2.625

We therefore register `FASE_prior_OSC` for the prior function as follows.

```
glbRegisterPriorFunction(FASE_prior_OSC, NULL, NULL, NULL);
```

And, then we give the value for the central values, and upper and lower Gaussian widths as follows.

```
UPPER_prior[0]=36.27*degree; LOWER_prior[0]=31.61*degree; Central_prior[0]=33.82*degree;
UPPER_prior[1]=8.99*degree; LOWER_prior[1]=8.22*degree; Central_prior[1]=8.61*degree;
UPPER_prior[2]=52.4*degree; LOWER_prior[2]=40.3*degree; Central_prior[2]=49.6*degree;
UPPER_prior[3]=392*degree; LOWER_prior[3]=125*degree; Central_prior[3]=215*degree;
UPPER_prior[4]=8.01e-5; LOWER_prior[4]=6.79e-5; Central_prior[4]=7.39e-5;
UPPER_prior[5]=2.625e-3; LOWER_prior[5]=2.427e-3; Central_prior[5]=2.525e-3;
```

```
int i;
for (i=0;i<6;i++) {UPPER_prior[i]=fabs(UPPER_prior[i]-Central_prior[i])/3;
LOWER_prior[i]=fabs(LOWER_prior[i]-Central_prior[i])/3;}
for (i=0; i<6; i++) glbSetOscParams(centers,0,i);
glbSetDensityParams(centers,1.0,GLB_ALL); glbCopyParams(centers,input_errors);
glbSetCentralValues(centers); glbSetInputErrors(input_errors);
```

The parameter `degree` is defined as  $\pi/180$ .

### 3.4 Constraint of model parameters

To get the constraint of  $x$  and  $\eta$ , we obtain the minimum of  $\chi^2$  value,

```
glbSetProjection(free);
float res0=glbChiNP(true_values,NULL,GLB_ALL);
```

Then, we set two loops to get the  $\Delta\chi^2$  value for different  $x$  and  $\eta$

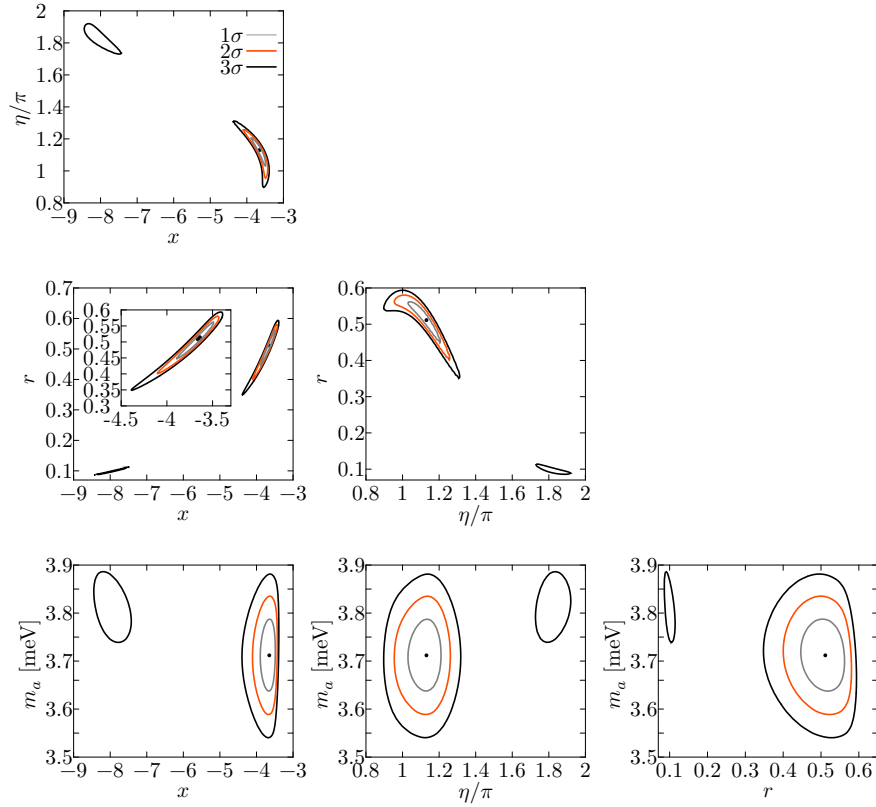
```
float x,eta,r,ma,dx,deta;
```

```

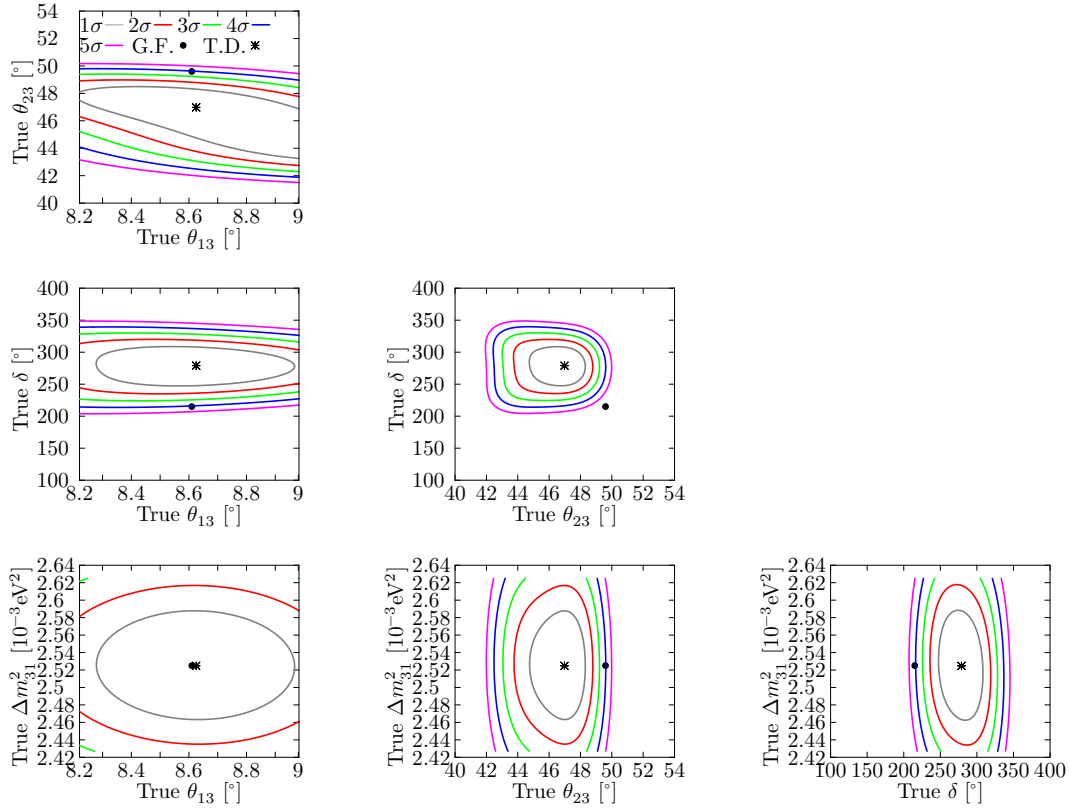
float lower_x,upper_x,lower_eta,upper_eta;
FILE* File=fopen("data/constraint_x_eta.dat", "w");
lower_x=-5; upper_x=-3; lower_eta=0.8*M_PI; upper_eta=1.5*M_PI;
dx=(upper_x-lower_x)/100; deta=(upper_eta-lower_eta)/100;
glbSetProjection(projection);
for (x=lower_x;x<=upper_x;x=x+dx){
for (eta=lower_eta;eta<=upper_eta;eta=eta+deta){
glbSetOscParams(test_values,x,0); glbSetOscParams(test_values,eta,1);
float res=glbChiNP(test_values,NULL,GLB_ALL);
fprintf(File,"%f %f %f \n",x,eta/M_PI,res-res0);
} fprintf(File,"\n");}

```

We use the same code to obtain the constraint of any other combination of two parameters. Then, we can have the result as in Fig. 2.



**Figure 2.** Precision measurements of any two model parameters at  $3\sigma$  confidence level in the framework of three neutrino oscillations taking uncertainties of the current global fit results, for MOMENT, at  $1\sigma$  (gray),  $2\sigma$  (orange),  $3\sigma$  (black). True values for the model parameters are used  $(x, \eta, r, M_a) = (-3.65, 1.13\pi, 0.511, 3.71 \text{ meV})$ .



**Figure 3.** The 2-D exclusion contour for tri-direct littlest seesaw model on the plane of any two true standard parameters, from  $1\sigma$  to  $5\sigma$ . The range for each parameter is taken according to the  $3\sigma$  uncertainty in NuFit4.0 results. The black dot denotes the best fit of NuFit4.0 results  $((\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{21}^2, \Delta m_{31}^2) = (33.82^\circ, 8.61^\circ, 49.6^\circ, 215^\circ, 7.39 \times 10^{-5} \text{ eV}^2, 2.525 \times 10^{-3} \text{ eV}^2))$ , while the star is the prediction by the tri-direct littlest seesaw model with NuFit4.0 results  $((\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{21}^2, \Delta m_{31}^2) \sim (36.25^\circ, 8.63^\circ, 47^\circ, 279^\circ, 7.39 \times 10^{-5} \text{ eV}^2, 2.525 \times 10^{-3} \text{ eV}^2))$ .

### 3.5 Model testing

## 4 Conclusion

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