

Flavour Symmetry Embedded - GLoBES

Jian Tang,^a Tse-Chun Wang^a

^aSchool of Physics, Sun Yat-Sen University, Guangzhou 510275, China

E-mail: tangjian5@mail.sysu.edu.cn, wangzejun@mail.sysu.edu.cn

ABSTRACT: Abstract...

Contents

1	Introduction	1
2	Overview FASE-GloBES	2
2.1	Download and Installation	4
2.2	Compile and Run	4
2.3	Model setting	4
2.4	Prior setting	5
3	Examples	5
3.1	Setup for a model	5
3.2	Constraint of model parameters	7
3.3	Model testing	8
4	Conclusion	8

1 Introduction

intro flavour symmetry, and summarise some phenon works

The discovery of neutrino oscillations points out the fact that neutrinos have mass, and provides evidence beyond the Standard Model (BSM). This phenomenon is successfully described by a theoretical framework with the help of three neutrino mixing angles (θ_{12} , θ_{13} , θ_{23}), two mass-square splittings (Δm_{21}^2 , Δm_{31}^2), and one Dirac CP phase (δ) [1–4]. Thanks to the great efforts in the past two decades, we almost have a complete understanding of such a neutrino oscillation framework. More data in the neutrino oscillation experiments is needed to determine the sign of Δm_{31}^2 , to measure the value of $\sin \theta_{23}$, to discover the potential CP violation in the leptonic sector and even to constrain the size of δ [4]. For these purposes, the on-going long baseline experiments (LBLs), such as the NuMI Off-axis ν_e Appearance experiment (NO ν A) [5] and the Tokai-to-Kamioka experiment (T2K) [6], can answer these questions with the statistical significance $\gtrsim 3\sigma$ in most of the parameter space. Based on the analysis with their data, the normal mass ordering ($\Delta m_{31}^2 > 0$), the higher θ_{23} octant ($\theta_{23} > 45^\circ$), and $\delta \sim 270^\circ$ are preferred so far [4]. The future LBLs, Deep Underground Neutrino Experiment (DUNE) [7], Tokai to Hyper-Kamiokande (T2HK) [8], and the medium baseline reactor experiment, the Jiangmen Underground Neutrino Observatory (JUNO) [9, 10] will further complete our knowledge of neutrino oscillations.

Flavour symmetry models are used to explain the origin of the neutrino mixing, and to predict the value of oscillation parameters (some of useful review articles are [11–17]). These models are motivated by some interesting features, *such as* $\theta_{12} \sim 33^\circ$, and $\theta_{23} \sim 45^\circ$.

Before the discovery of non-zero θ_{13} measurement by Daya Bay experiment [18], the ‘tri-bi-maximal’ neutrino mixing (TBM) ansatz, which was proposed in 2002 by Harrison, Perkins, and Scott [19], fitted with the experimental data in a good agreement:

$$U_{\text{TBM}} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}.$$

With the fact that $\theta_{13} \approx 8^\circ$, several ways to obtain such non-zero value of θ_{13} are proposed. One of popular proposals is to correct the tri-bi-maximal neutrino mixing such that

$$\sin \theta_{12} = (1 + s)/\sqrt{3}, \quad \sin \theta_{13} = r/\sqrt{2}, \quad \text{and} \quad \sin \theta_{23} = (1 + a)/\sqrt{2}.$$

The neutrino mixing ansatz can be realised by high-energy symmetries G_f . The symmetry of discrete groups G_f , preserved at the high energy but slightly broken at the lower energy, predicts the neutrino mixing, mass-square splittings, and the CP violation phase (Dirac and Majorana phases), with reduced degrees of freedom. The symmetries need to be broken at the low energy. Otherwise, the flavour of leptons cannot be distinguished. There are several approaches for the symmetry breaking, including the direct, indirect, and semi approaches. Direct approach preserves the residual symmetries of G_f in the charged-lepton or neutrino sector. On the other hand, there is no residual symmetry preserved in neither charged-lepton nor neutrino sector in the indirect approach. In the semi approach, the charged-lepton and neutrino sectors preserve different residual symmetries, respectively. This symmetry is broken by extending the Higgs sector or introducing the flavons. To achieve the δ prediction, many models are based on a discrete family symmetry G_f together with a non-commuting CP symmetry H_{CP} . Broken in different approaches, the symmetry $G_f \otimes H_{\text{CP}}$ can predict different patterns for the neutrino mixing. For example, in the semi-direct approach, S_4 is preserved in the leading order, and leads the bimaximal (MB) or tri-bimaximal (TB) neutrino mixing, while the higher order terms bring the correction to the neutrino mixing.

Currently, some works discuss on how the future experiments can be used for testing these flavour symmetry models in the phenomenological point of view, *e.g.* [20–23]. A large number of these works are based on the c-library – **General Long Baseline Experiment Simulator (GLOBES)** [24, 25], which is a convenient simulation tool to simulate neutrino oscillation experiments via the Abstract Experiment Definition Language (AEDL). Some AEDL files for experiments are also available on **GLOBES** website. In addition, the working group of DUNE experiment also releases their AEDL files [26]. **GLOBES** can be taken as one of popular and useful tools in the community of neutrino oscillation physics. However, it has not yet to be extended for the purpose of analysing flavour symmetry models.

In this work, we will present our simulation tool **Flavour Symmetry Embedded - GLOBES (FaSE-GLOBES)** that is a simulation code based on GLOBES library and specific for studying the flavour symmetry.

2 Overview FaSE-GLOBES

The FLAVOR Symmetry Embedded (FASE) is a supplemental tool for General Long Baseline Experiment Simulator (GLOBES) in order to analysis how a leptonic flavour sym-

metry model can be tested in neutrino oscillation experiments. FASE is written in the c/c++ language, and consist with three codes **FASE_GLoBES.c**, **model-input_diag.c** and **model-input.c**. The user defines the model in to **model-input_diag.c** or **model-input.c**, while **FASE_GLoBES.c** does not need to be touched.

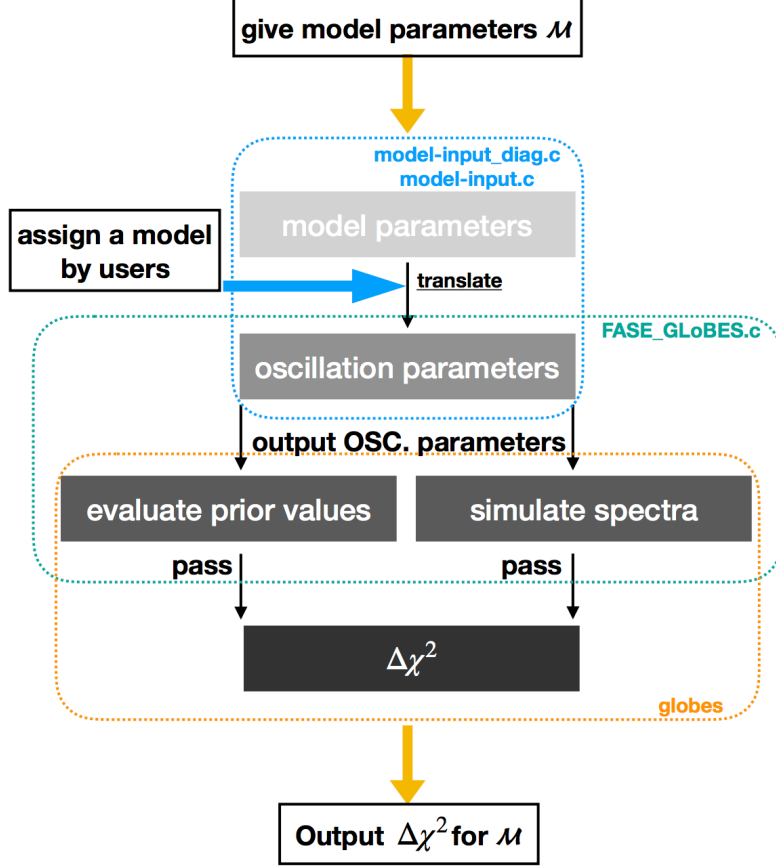


Figure 1. A scheme to correlate the model parameters with standard neutrino oscillation parameters. The error propagation is implemented in the simulation code up to the spectra analysis.

The FASE-GLoBES is FASE working with GLoBES as shown in Fig. 1, in which three parts are shown: 1. **the parameter translation** (the blue box), 2. **giving oscillation-parameter values** (the green box), and 3. **the χ^2 -value calculation** (the orange box). The idea behind this flow chart Fig. 1 is that given a set of value for model parameter, the corresponding values for oscillation parameters are obtained by a translation, which is assigned by the user in **model-input_diag.c** or **model-input.c**. And then, through **FASE_GLoBES.c**, these oscillation-parameter values are passed in to **GLoBES** library to simulate the event spectrum for evaluating the χ^2 value.

API functions to use FASE are listed:

1. `MODEL_init(N_{para})`,
2. `FASE_glb_probability_matrix`,

3. `FASE_glb_set_oscillation_parameters`,
4. `FASE_glb_get_oscillation_parameters`,
5. `FASE_prior_OSC`,
6. `FASE_prior_model`.

The first one is to initialise FASE with the parameter N_{para} the number of input parameters. The next three functions need to be included to replace the default GLoBES probability engine by the one that can read the output from `model-input__diag.c` or `model-input.c`, as follows

```
glbRegisterProbabilityEngine(6,
&FASE_glb_probability_matrix,
&FASE_glb_set_oscillation_parameters,
&FASE_glb_get_oscillation_parameters,
NULL);
```

The final two functions are prior functions. Once the user gives the prior in oscillation (model) parameters, the user needs to call `FASE_prior_OSC` (`FASE_prior_model`) as follows.

```
glbRegisterPriorFunction(FASE_prior_OSC, NULL, NULL, NULL);
or
glbRegisterPriorFunction(FASE_prior_model, NULL, NULL, NULL);
```

In the following, we give more details about using FASE-GLoBES. In Sec. 2.1, we will give the instruction to download and install GLoBES and FASE-GLoBES. Then, in Sec. 2.2, we will introduce how to compile and run the code. More complicated is setting the model, and the way to do this will be described in Sec. 2.3. Finally, we will discuss how to set up the prior in Sec. 2.4.

2.1 Download and Installation

download and install GLoBES and FASE-GLoBES

[will fill in after the code ready on Github]

2.2 Compile and Run

[will fill in after the code ready on Github]

To initial **FASE-GLoBES**, we need to include the following code

```
MODEL_init( $N_{para}$ );
```

where N_{para} is the number of model parameters.

2.3 Model setting

FASE includes three c++ files: `model-input__diag.c`, `model-input.c`, and `FASE_GLoBES.c`. As shown in the previous paragraph, these files are for different service. In `model-`

input_diag.c and **model-input.c**, the user can assign the relation between the oscillation and model parameter sets. The difference between these two code is that in **model-input.c**, the user give the relation between these two parameter sets directly, while in **model-input.c**, the mass matrix, which is provided by the user, can be diagonalised to obtain the corresponding oscillation-parameter values.

For different methods to include the model, we use different **c++** file. Adopting the mass matrix expressed by model parameters, we assign this model in **model-input_diag.c**. In function **MtoS**, we give the mass matrix in the 3×3 complex matrix **Mass_Matrix[]**. Passing the complex matrix **Mass_Matrix[]** into the function **STAN_OSC**, we will get the corresponding oscillation parameters in an array **OSC_PARAMS**, components of which are the value for θ_{12} , θ_{13} , θ_{23} , δ , Δm_{21}^2 , and Δm_{31}^2 . The expression of **Mass_Matrix[]** is given by the user. For the first four components, values are given in the unit of **rad**, while the other two are in **eV²**. We can also assign the relation between oscillation and model parameters in **model-input.c**. The relations are given in **MtoS** as well.

2.4 Prior setting

The **c++** file **FASE_GLoBES.c** then will call these oscillation-parameter values and pass these to simulate the event spectrum. The χ^2 value from priors is also given in **FASE_GLoBES.c**. And these functions are given by the user.

3 Examples

3.1 Setup for a model

We now take the littlest seesaw model in the tri-direct approach as an example. In this model, the atmospheric and solar flavon vacuum alignments are $\langle \phi_{\text{atm}} \rangle \propto (1, \omega^2, \omega)^T$ and $\langle \phi_{\text{sol}} \rangle \propto (1, x, x)^T$, where stands for a cube root of unity and the parameter x is real because of the imposed CP symmetry. Under this model, the light left-handed Majorana neutrino mass matrix is given by

$$m_\nu = m_a \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + e^{i\eta} m_s \begin{pmatrix} 1 & x & x \\ x & x^2 & x^2 \\ x & x^2 & x^2 \end{pmatrix}, \quad (3.1)$$

where x , η , m_a , and the ratio $r \equiv m_s/m_a$ are four parameters and will be constrained by experimental data. We note that from Eq. (3.1), $m_1 = 0$ and the normal mass ordering are imposed, and will need to be imposed in **FASE-GLoBES**. We will introduce the way to do it in Sec. 2.4.

```
int MtoS(double OSC_PARAMS[6], double M_para[])
{
double x=M_para[0];
double eta=M_para[1];
double r=M_para[2];
double ma=M_para[3];
```

Table 1. A summary of the relation between oscillation parameters and TDLS model parameters [?]. Two requirements are imposed by TDLS: the smallest mass state $m_1 = 0$ and the normal mass ordering. The sign of $\sin \delta$ depends on the sign of $x \cos \psi$: “+” (“−”) is for $x \cos \psi > 0$ (< 0).

model parameters	x, η, r, m_a
combinations of model parameters	$y = \frac{5x^2+2x+2}{2(x^2+x+1)}(m_a + e^{i\eta}m_s),$ $z = -\frac{\sqrt{5x^2+2x+2}}{2(x^2+x+1)}[(x+2)m_a - x(2x+1)e^{i\eta}m_s],$ $w = \frac{1}{2(x^2+x+1)}[(x+2)^2m_a + x^2(2x+1)^2e^{i\eta}m_s],$ $\sin \psi = \frac{\Im(y^*z+wz^*)}{ y^*z+wz^* }, \quad \cos \psi = \frac{\Re(y^*z+wz^*)}{ y^*z+wz^* }.$ $\sin 2\theta = \frac{2 y^*z+wz^* }{\sqrt{(w ^2- y ^2)^2+4 y^*z+wz^* ^2}},$ $\cos 2\theta = \frac{ w ^2- y ^2}{\sqrt{(w ^2- y ^2)^2+4 y^*z+wz^* ^2}}.$
oscillation parameters	$\Delta m_{21}^2 = m_2^2 = \frac{1}{2} \left[y ^2 + w ^2 + 2 z ^2 - \frac{ w ^2- y ^2}{\cos \theta} \right],$ $\Delta m_{31}^2 = m_3^2 = \frac{1}{2} \left[y ^2 + w ^2 + 2 z ^2 + \frac{ w ^2- y ^2}{\cos \theta} \right],$ $\sin^2 \theta_{12} = 1 - \frac{3x^2}{3x^2+2(x^2+x+1)\cos^2 \theta},$ $\sin^2 \theta_{13} = \frac{2(x^2+x+1)\sin^2 \theta}{5x^2+2x+2},$ $\sin^2 \theta_{23} = \frac{1}{2} + \frac{x\sqrt{3(5x^2+2x+2)}\sin 2\theta \sin \psi}{2[3x^2+2(x^2+x+1)\cos^2 \theta]},$ $\cos \delta = \frac{\cot 2\theta_{23}[3x^2-(4x^2+x+1)\cos^2 \theta_{13}]}{\sqrt{3} x \sin \theta_{13}\sqrt{(5x^2+2x+2)\cos^2 \theta_{13}-3x^2}},$ $\sin \delta = \pm \csc 2\theta_{23} \sqrt{1 + \frac{(x^2+x+1)^2 \cot^2 \theta_{13} \cos^2 2\theta_{23}}{3x^2[3x^2 \tan^2 \theta_{13} - 2(x^2+x+1)]}}.$

```
double ms=ma*r;
```

```
double complex Mass_Matrix[] = {ma+ms*(cos(eta) + I*sin(eta)), ma*(cos(6.6666e-1*M_PI)
+ I*sin(6.6666e-1*M_PI))+x*ms*(cos(eta) + I*sin(eta)), ma*(cos(6.6666e-1*M_PI)
+ I*sin(6.6666e-1*M_PI))*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))+x*ms*(cos(eta)
+ I*sin(eta)), ma*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))+x*ms*(cos(eta)
+ I*sin(eta)), ma*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))*(cos(6.6666e-1*M_PI)
+ I*sin(6.6666e-1*M_PI))+x*x*ms*(cos(eta) + I*sin(eta)), ma+x*x*ms*(cos(eta) +
I*sin(eta)), ma*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))*(cos(6.6666e-1*M_PI)
+ I*sin(6.6666e-1*M_PI))+x*ms*(cos(eta) + I*sin(eta)), ma+x*x*ms*(cos(eta) + I*sin(eta)),
ma*(cos(6.6666e-1*M_PI) + I*sin(6.6666e-1*M_PI))+x*x*ms*(cos(eta) + I*sin(eta))};
```

```
STAN_OSC(Mass_Matrix,OSC_PARAMS);
```

```
return 0;
}
```

```
double MtoS(double osc_para[6], double M_para[])
{
/*example: tri-direct*/
```

```

double x_in = M_para[0];
double eta_in = M_para[1];
double r_in = M_para[2];
double ma_in = M_para[3];

osc_para[0]=TDth12(x_in,eta_in,r_in, ma_in);
osc_para[1]=TDth13(x_in,eta_in,r_in, ma_in);
osc_para[2]=TDth23(x_in,eta_in,r_in, ma_in);
osc_para[3]=TDdCP(x_in,eta_in,r_in, ma_in);
osc_para[4]=TDdm21(x_in,eta_in,r_in, ma_in);
osc_para[5]=TDdm31(x_in,eta_in,r_in, ma_in);

return 0;
}

```

The functions TDth12, TDth13, TDth23, TDdCP, TDdm21, and TDdm31 are included for obtaining the value of θ_{12} , θ_{13} , θ_{23} , δ , Δm_{21}^2 , and Δm_{31}^2 according to Table 1, respectively.

3.2 Constraint of model parameters

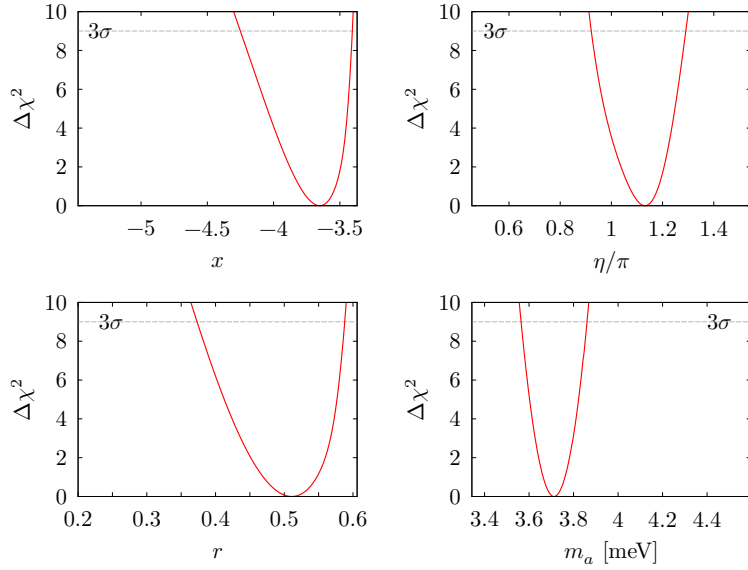


Figure 2. The $\Delta\chi^2$ value against each model parameters for MOMENT. True values for the model parameters are used $(x, \eta, r, M_a) = (-3.65, 1.13\pi, 0.511, 3.71 \text{ meV})$. The range shown here is according to the 3σ uncertainty with NuFit4.0 results: $-5.475 < x < -3.37$ (red band), $0.455 < \eta/\pi < 1.545$ (dark grey band), $0.204 < r < 0.606$ (blue band), $3.343 < m_a/\text{meV} < 4.597$ (yellow band).

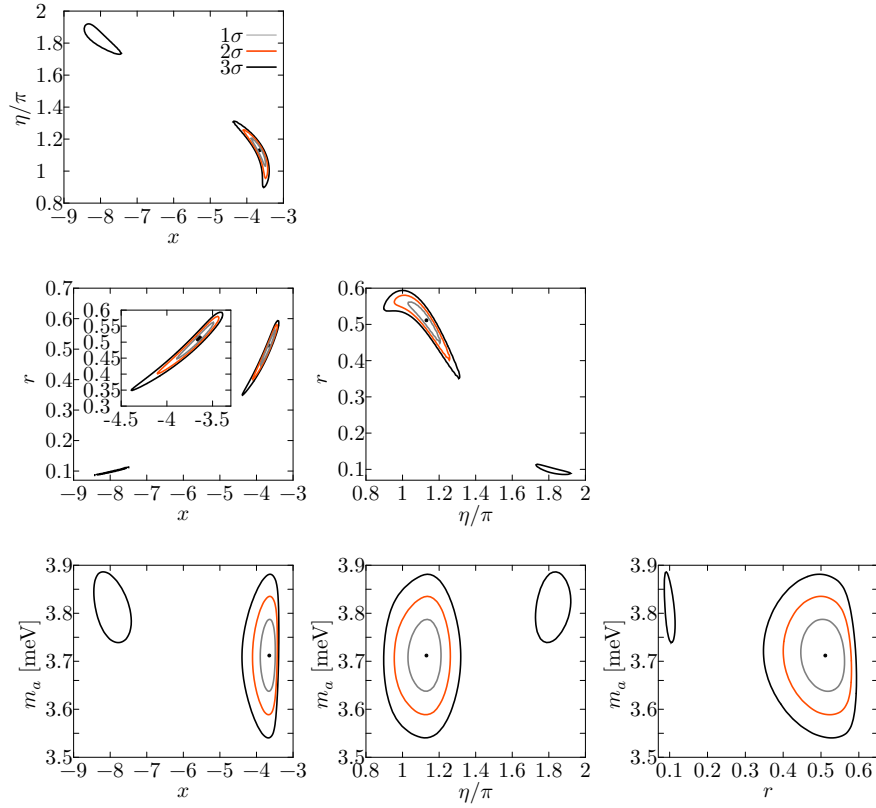


Figure 3. Precision measurements of any two model parameters at 3σ confidence level in the framework of three neutrino oscillations taking uncertainties of the current global fit results, for MOMENT, at 1σ (gray), 2σ (orange), 3σ (black). True values for the model parameters are used $(x, \eta, r, M_a) = (-3.65, 1.13\pi, 0.511, 3.71 \text{ meV})$.

3.3 Model testing

4 Conclusion

References

- [1] B. Pontecorvo, *Neutrino Experiments and the Problem of Conservation of Leptonic Charge*, *Sov. Phys. JETP* **26** (1968) 984.
- [2] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870.
- [3] B. Pontecorvo, *Inverse beta processes and nonconservation of lepton charge*, *Sov. Phys. JETP* **7** (1958) 172.
- [4] I. Esteban, M. C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni and T. Schwetz, *Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of θ_{23} , δ_{CP} , and the mass ordering*, *JHEP* **01** (2019) 106 [[1811.05487](#)].
- [5] NOvA collaboration, *The NOvA Technical Design Report*, .

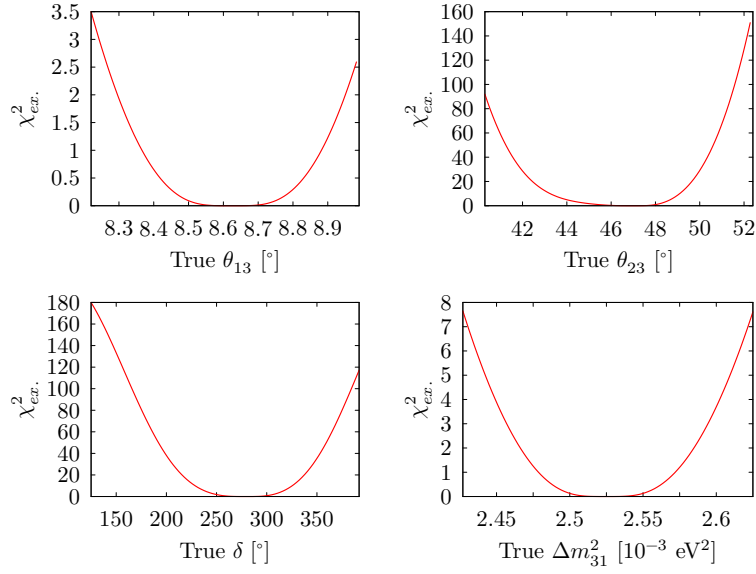


Figure 4. The $\chi^2_{ex.}$ value for tri-direct littlest seesaw model for θ_{13} , θ_{23} , δ and Δm^2_{31} . The range for each parameter is taken according to the 3σ uncertainty in NuFit4.0 results.

- [6] T2K collaboration, *The T2K Experiment*, *Nucl. Instrum. Meth.* **A659** (2011) 106 [[1106.1238](#)].
- [7] DUNE collaboration, *Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE)*, [1512.06148](#).
- [8] HYPER-KAMIOKANDE WORKING GROUP collaboration, *A Long Baseline Neutrino Oscillation Experiment Using J-PARC Neutrino Beam and Hyper-Kamiokande*, 2014, [1412.4673](#).
- [9] JUNO collaboration, *JUNO Conceptual Design Report*, [1508.07166](#).
- [10] JUNO collaboration, *Neutrino Physics with JUNO*, *J. Phys.* **G43** (2016) 030401 [[1507.05613](#)].
- [11] G. Altarelli and F. Feruglio, *Discrete Flavor Symmetries and Models of Neutrino Mixing*, *Rev. Mod. Phys.* **82** (2010) 2701 [[1002.0211](#)].
- [12] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada and M. Tanimoto, *Non-Abelian Discrete Symmetries in Particle Physics*, *Prog. Theor. Phys. Suppl.* **183** (2010) 1 [[1003.3552](#)].
- [13] S. F. King and C. Luhn, *Neutrino Mass and Mixing with Discrete Symmetry*, *Rept. Prog. Phys.* **76** (2013) 056201 [[1301.1340](#)].
- [14] S. F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, *Neutrino Mass and Mixing: from Theory to Experiment*, *New J. Phys.* **16** (2014) 045018 [[1402.4271](#)].
- [15] S. F. King, *Models of Neutrino Mass, Mixing and CP Violation*, *J. Phys.* **G42** (2015) 123001 [[1510.02091](#)].
- [16] S. F. King, *Neutrino Mixing: from experiment to theory*, *Nucl. Part. Phys. Proc.* **265-266** (2015) 288.

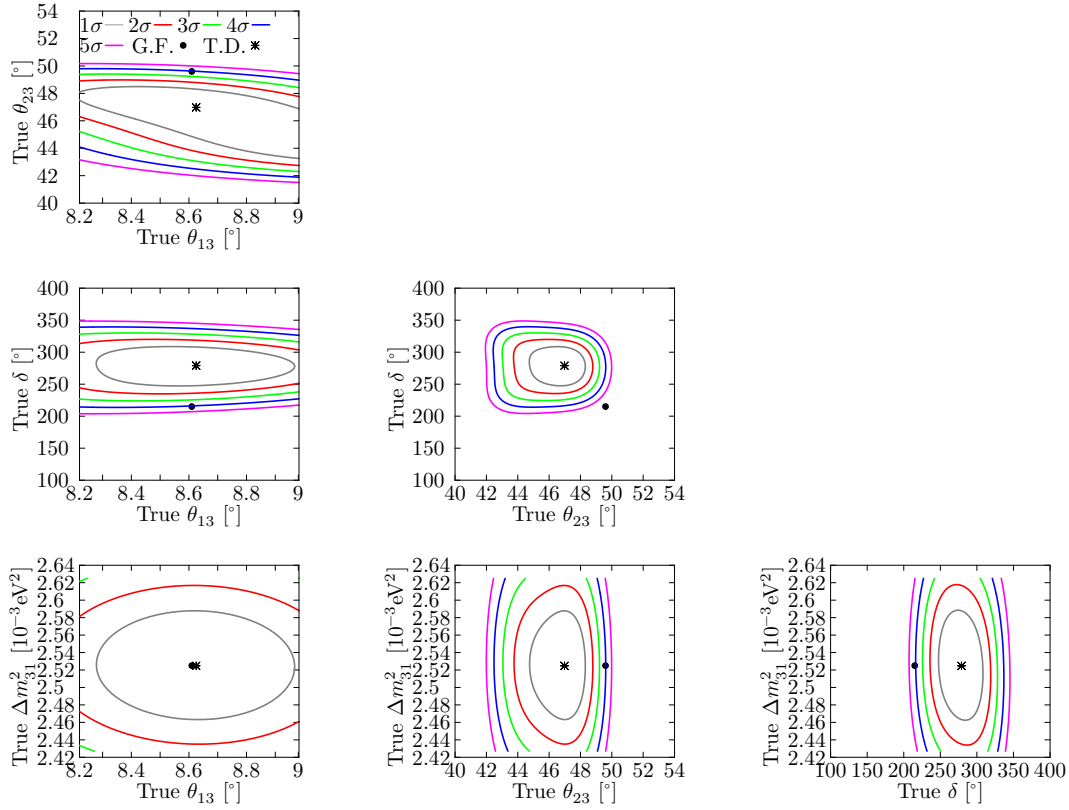


Figure 5. The 2-D exclusion contour for tri-direct littlest seesaw model on the plane of any two true standard parameters, from 1σ to 5σ . The range for each parameter is taken according to the 3σ uncertainty in NuFit4.0 results. The black dot denotes the best fit of NuFit4.0 results $((\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{21}^2, \Delta m_{31}^2) = (33.82^\circ, 8.61^\circ, 49.6^\circ, 215^\circ, 7.39 \times 10^{-5} \text{ eV}^2, 2.525 \times 10^{-3} \text{ eV}^2))$, while the star is the prediction by the tri-direct littlest seesaw model with NuFit4.0 results $((\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{21}^2, \Delta m_{31}^2) \sim (36.25^\circ, 8.63^\circ, 47^\circ, 279^\circ, 7.39 \times 10^{-5} \text{ eV}^2, 2.525 \times 10^{-3} \text{ eV}^2))$.

- [17] S. F. King, *Unified Models of Neutrinos, Flavour and CP Violation*, *Prog. Part. Nucl. Phys.* **94** (2017) 217 [[1701.04413](#)].
- [18] DAYA BAY collaboration, *Spectral measurement of electron antineutrino oscillation amplitude and frequency at Daya Bay*, *Phys. Rev. Lett.* **112** (2014) 061801 [[1310.6732](#)].
- [19] P. F. Harrison, D. H. Perkins and W. G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett. B* **530** (2002) 167 [[hep-ph/0202074](#)].
- [20] P. Ballett, S. F. King, S. Pascoli, N. W. Prouse and T. Wang, *Precision neutrino experiments vs the Littlest Seesaw*, *JHEP* **03** (2017) 110 [[1612.01999](#)].
- [21] S. S. Chatterjee, P. Pasquini and J. Valle, *Probing atmospheric mixing and leptonic CP violation in current and future long baseline oscillation experiments*, *Phys. Lett. B* **771** (2017) 524 [[1702.03160](#)].
- [22] G.-J. Ding, Y.-F. Li, J. Tang and T.-C. Wang, *Confronting Tri-direct CP-symmetry models to neutrino oscillation experiments*, *Phys. Rev. D* **100** (2019) 055022 [[1905.12939](#)].
- [23] J. Tang and T. Wang, *Study of a tri-direct littlest seesaw model at MOMENT*, *Nucl. Phys. B*

[952](#) (2020) 114915 [[1907.01371](#)].

- [24] P. Huber, M. Lindner and W. Winter, *Simulation of long-baseline neutrino oscillation experiments with GLoBES (General Long Baseline Experiment Simulator)*, *Comput. Phys. Commun.* **167** (2005) 195 [[hep-ph/0407333](#)].
- [25] P. Huber, J. Kopp, M. Lindner, M. Rolinec and W. Winter, *New features in the simulation of neutrino oscillation experiments with GLoBES 3.0: General Long Baseline Experiment Simulator*, *Comput. Phys. Commun.* **177** (2007) 432 [[hep-ph/0701187](#)].
- [26] DUNE collaboration, *Experiment Simulation Configurations Used in DUNE CDR*, [1606.09550](#).