Internet Appendix to: "Scale Economies, Bargaining Power, and Investment Performance: Evidence from Pension Plans"

A Equilibrium Solution in Generalized GP Model

This appendix introduces our three generalizations to the equilibrium asset pricing model with delegated management proposed by Gârleanu and Pedersen (2018)

A.1 Equilibrium Asset Management Fee

In this section, we derive the equilibrium active manager fee in a market with a single risky asset where plans can choose between not acquiring information or either acquiring information and managing assets internally or hiring a potentially informed external manager. The management fee is determined through Nash bargaining between the plan and the manager. The bargaining outcome depends on each party's utility from agreement and disagreement (i.e., their outside options). The manager's marginal gain from agreement is the fee f_i. To solve the Nash bargaining problem, we maximize the product of each party's surplus from agreement. Since the plan can potentially choose among three alternative management styles, we consider several outside options.

- **Outside option: another external active manager**. The welfare gain is given by $W_i + u_i^A - c_i^{EA} - f_i - (W_i + u_i^A - 2c_i^{EA} - f_i^*)$. Further, the Nash bargaining problem is given by

$$\max_{f_i}(c_i^{EA} + f_i^* - f_i)f_i.$$

  After imposing the same bargaining fee, we conclude $f_i^* = f_i = c_i^{EA}$.

- **Outside option: external passive management**. In this case the outside option is to go for a passive manager with payoff $W_i + u_i^P - c_i^{EP} - c_i^{EA}$ (since the plan searches again for another manager). The plan's gain from agreement is $W_i + u_i^A - c_i^{EA} - f_i - (W_i + u_i^P - c_i^{EP} - c_i^{EA}) = u_i^A - u_i^P + c_i^{EP} - f_i$. The Nash bargaining solution yields:

$$f_i = \frac{u_i^A - u_i^P + c_i^{EP}}{2}.$$

- **Outside option: internal active management**. The welfare gain is given by $W_i + u_i^A - c_i^{EA} - f_i - (W_i + u_i^A - c_i^{IA} - k - c_i^{EA})$, The Nash bargaining solution is given by

$$f_i = \frac{c_i^{IA} + k}{2}.$$

- **Outside option: internal passive management**. The welfare gain is given by $W_i + u_i^A - c_i^{EA} - f_i - (W_i + u_i^P - c_i^{IP} - c_i^{EA}) = u_i^A - u_i^P + c_i^{IP} - f_i$. The Nash bargaining solution is given by

$$f_i = \frac{u_i^A - u_i^P + c_i^{IP}}{2}.$$

Combining these four cases, we obtain

$$f_i = \min\left\{c_i^{EA}, \frac{1}{2}(u_i^A - u_i^P + c_i^{EP}), \frac{1}{2}(c_i^{IA} + k), \frac{1}{2}(u_i^A - u_i^P + c_i^{IP})\right\}.$$

In equilibrium we never observe $f_i = \frac{1}{2}(u_i^A - u_i^P + c_i^{EP})$ since that implies $c_i^{EA} \geq \frac{1}{2}(u_i^A - u_i^P + c_i^{EP})$. The gain from EA versus EP management is then given by

$$\begin{aligned} & u_i^A - c_i^{EA} - \frac{1}{2}(u_i^A - u_i^P + c_i^{EP}) - (u_i^P - c_i^{EP}) \\ &= \frac{1}{2}(u_i^A - u_i^P + c_i^{EP}) - c_i^{EA} < 0. \end{aligned}$$

Using similar reasoning we can also exclude $f_i = \frac{1}{2}(c_i^{IA} + k)$, since

$$\begin{aligned} & u_i^A - c_i^{EA} - \frac{1}{2}(c_i^{IA} + k) - [u_i^A - c_i^{IA} - k] \\ &= \frac{1}{2}(c_i^{IA} + k) - c_i^{EA} < 0. \end{aligned}$$

Finally, we can also exclude $f_i = \frac{1}{2}(u_i^A - u_i^P + c_i^{IP})$, since

$$\begin{aligned} & u_i^A - c_i^{EA} - \frac{1}{2}(u_i^A - u_i^P + c_i^{IP}) - [u_i^P - c_i^{IP}] \\ & = \frac{1}{2}\left(u_i^A - u_i^P + c_i^{EP}\right) - c_i^{EA} < 0. \end{aligned}$$

As a result, the equilibrium manager fee is given by

$$\boxed{f_i = c_i^{EA}.} \tag{A.1}$$

A.2 Passive and Active Asset Managers

We now derive the manager's condition to become active in equilibrium. Let $\bar{I}$ be the total number of plans, I^{EA} the number of external active plans, and I^{EP} the number of external passive plans. A passive manager expects $I^{EP}/(\bar{M} - M^A) + N/\bar{M}$ investors. We assume that the passive managers only collect fees from noise allocators, while rational plans pay no extra fee because they know the manager is passive (hence uninformed). An active manager expects $I^{EA}/M^A + N/\bar{M}$ investors and collects fees from both groups. Therefore, a manager becomes active if

$$\boxed{\frac{\bar{I}}{M^A}\mathbb{E}\left[c_i^{EA}\mathbb{1}\left(u_i^{EA} = \max\left\{u_i^{IP}, u_i^{IA}, u_i^{EP}, u_i^{EA}\right\}\right)\right] \geq k,} \tag{A.2}$$

where $u_i^{\cdot}$ denotes the certainty equivalent of each management style net of cost. In an interior equilibrium, the inequality becomes an equality.

A.3 Equilibrium

Finally, we solve for the asset market equilibrium. Before doing so, it is helpful to recall the definition of market inefficiency proposed by BP which plays an important role in the analysis:

$$\eta \equiv \frac{1}{2}\log\left(\frac{\mathrm{Var}(v|p)}{\mathrm{Var}(v|p,\zeta)}\right) = \gamma_i(u_i^A - u_i^P). \tag{A.3}$$

Part of the equilibrium consists of pinning down η. To this extent, let $I^A \equiv \{i : u_i^{IA} \text{ or } u_i^{EA} \geq \max(u_i^{IP}, u_i^{EP})\}$; the set of active investors. Conjecture that the price takes the form

$$p = \theta_0 + \theta_\zeta((\zeta - \bar{v}) - \theta_q(q - \bar{q})).$$

In Appendix A.7, we then show that

$$\boxed{\eta = -\frac{1}{2}\log\left(1 - \frac{\sigma_q^2\sigma_\varepsilon^2}{\tau^2 + \sigma_q^2\sigma_\varepsilon^2}\frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2}\right)}, \tag{A.4}$$

where

$$\boxed{\tau = \underbrace{\bar{I}\mathbb{E}\left[\frac{1}{\gamma_i}\mathbb{1}\left(i \in I^A\right)\right]}_{\text{Active investors}} + \underbrace{N\frac{M^A}{\bar{M}}\mathbb{E}\left[\frac{1}{\gamma_n}\right]}_{\text{Active noise allocators}}} \tag{A.5}$$

is the average risk-tolerance of all active agents in the economy. The endogenous variables are (f_i, M^A, η, τ), which can be solved for by (A.1), (A.2), (A.4) and (A.5).

Having solved for the equilibrium, we calibrate the model parameters as follows: $\bar{I} = 6,000$ (number of investors), $N = 300$ (number of noise allocators), $\bar{M} = 1,000$ (number of managers), the mean AUM is 40 billion, $\beta^{\mathrm{P}} = 0.75$ (power law coefficient for passive management), $\beta^{\mathrm{A}} = 0.9$ (power law coefficient for active management), $\gamma = \frac{3}{\text{mean AUM}}$ (CARA coefficient), $FC = 20$ million (fixed cost for internal management), $k = 15.3$ million (active management cost), $\bar{v} = (\bar{I} + N) \times$ mean AUM (total asset value), $\sigma_v = 0.2\bar{v}$ (volatility of asset value), $\sigma_\varepsilon = 0.3\bar{v}$ (volatility of error term), $\bar{q} = 1$ (expected supply, normalized to one), $\sigma_q = 0.03$ (volatility of supply).

A.4 Proof of Proposition 4.1

Part 1 is proved in the main text. To show part 2 and 3, assume that the returns to scale for passive and active management are the same: $\beta^{\mathrm{P}} = \beta^{\mathrm{A}} = \beta > 0$. To observe all 4 management styles in equilibrium, a necessary condition is the existence of $W_{EA}, W_{EP}, W_{IA} > 0$ such that

$$W_{EA} < \left(\frac{FC + k}{2\bar{c}^{EA} - \bar{c}^{IA}}\right)^{\frac{1}{\beta}}, \qquad W_{EA} < \left(\frac{\eta/\gamma}{2\bar{c}^{EA} - \bar{c}^{EP}}\right)^{1/\beta}, \quad W_{EA} < \left(\frac{\eta/\gamma + FC}{2\bar{c}^{EA} - \bar{c}^{IP}}\right)^{1/\beta} \tag{A.6}$$

$$W_{EP} > \left(\frac{\eta/\gamma - FC - k}{\bar{c}^{IA} - \bar{c}^{EP}}\right)^{1/\beta}, \quad W_{EP} < \left(\frac{FC}{\bar{c}^{EP} - \bar{c}^{IP}}\right)^{1/\beta} \tag{A.7}$$

$$W_{IA} < \left(\frac{\eta/\gamma - k}{\bar{c}^{IA} - \bar{c}^{IP}}\right)^{1/\beta}. \tag{A.8}$$

These conditions are derived by equating the certainty equivalent net of cost for each management style, and using that $\eta/\gamma > 0$ represents the certainty equivalent difference between passive and active management (recall (A.3)). If there are no differences in fixed cost, we can assume $FC = 0$, and the second term in (A.7) is not well-defined, which proves part 2.

For part 3, assume that $FC > 0$. For (A.6) to be well defined, we need $2\bar{c}^{EA} > \bar{c}^{IA}, 2\bar{c}^{EA} >$

$\bar{c}^{EP}, 2\bar{c}^{EA} > \bar{c}^{IP}$.[1] Additional restrictions are necessary to observe all management styles in equilibrium. We split the argument in several cases:

- $\eta/\gamma > FC + k$. For (A.7) and (A.8) to be well defined, we need $\bar{c}^{IA} > \bar{c}^{EP}, \bar{c}^{EP} > \bar{c}^{IP}$ and $\bar{c}^{IA} > \bar{c}^{IP}$. But this is clearly inconsistent, so an equilibrium with all 4 management styles cannot exist.

- $\eta/\gamma < k$. To have (A.7) and (A.8) well defined it must be that $\bar{c}^{IA} < \bar{c}^{EP}$ and $\bar{c}^{IA} < \bar{c}^{IP}$. In this case the first inequality in (A.7) and the inequality in (A.8) flip sign:

$$W_{EP} < \left(\frac{\eta/\gamma - FC - k}{\bar{c}^{IA} - \bar{c}^{EP}}\right)^{1/\beta}, \quad W_{IA} > \left(\frac{\eta/\gamma - k}{\bar{c}^{IA} - \bar{c}^{IP}}\right)^{1/\beta}. \tag{A.9}$$

  Now an equilibrium with all 4 management styles is possible. But the only equilibrium consistent with (A.6), (A.7), and (A.9) is $EA \to EP \to IP \to IA$, where the order is from small to large plans.

- $k < \eta/\gamma < FC + k$. In this case only the first inequality in (A.7) flips sign. It is again possible to have all 4 management styles. The only possible equilibrium in such case is $EA \to EP \to IA \to IP$.

Finally, to show part 4, we simply note that in the calibration of Figure 2a, $2\bar{c}^{EA} < \bar{c}^{EP}$, hence the second condition in (A.6) cannot hold.

A.5 Generalization to Public and Private Assets

In this section, we generalize the GP model further by incorporating a second asset whose parameters are set to represent those of a private asset class. Assume that the signals for both assets are given by

$$\boldsymbol{\zeta} = \boldsymbol{v} + \boldsymbol{\varepsilon},$$

where $\boldsymbol{v}$ and $\boldsymbol{\varepsilon}$ are assumed to be independent and normally distributed with diagonal covariance matrices:

$$\boldsymbol{v} \sim N\left(\begin{bmatrix}\bar{v}_1\\ \bar{v}_2\end{bmatrix}, \begin{bmatrix}\sigma^2_{v_1} & 0\\ 0 & \sigma^2_{v_2}\end{bmatrix}\right), \quad \boldsymbol{\varepsilon} \sim N\left(\begin{bmatrix}0\\ 0\end{bmatrix}, \begin{bmatrix}\sigma^2_{\varepsilon_1} & 0\\ 0 & \sigma^2_{\varepsilon_2}\end{bmatrix}\right).$$

We refer to asset one as the *public asset* and asset two as the *private asset*. In this context, it is natural to assume $\sigma_{v_1} < \sigma_{v_2}$ and $\sigma_{\varepsilon_1} < \sigma_{\varepsilon_2}$, i.e., the private asset has a higher volatility as well as

[1] Notice that these conditions are violated under the model calibration in Section A.3.

higher uncertainty about its underlying value. Since the investor can acquire information about asset one and/or asset two, there are four possible certainty equivalents

$$
\begin{aligned}
u^P(W_i) &= -\frac{1}{\gamma_i}\log\left[\mathbb{E}\left[\max_{\boldsymbol{x}^P}\mathbb{E}\left(e^{-\gamma_i(W_i+\boldsymbol{x}^{P\prime}(\boldsymbol{v}-\boldsymbol{p}))}|\boldsymbol{p}\right)\right]\right]\\
u^{AP}(W_i) &= -\frac{1}{\gamma_i}\log\left[\mathbb{E}\left[\max_{\boldsymbol{x}^{AP}}\mathbb{E}\left(e^{-\gamma_i(W_i+\boldsymbol{x}^{AP\prime}(\boldsymbol{v}-\boldsymbol{p}))}|\zeta_1,\boldsymbol{p}\right)\right]\right]\\
u^{PA}(W_i) &= -\frac{1}{\gamma_i}\log\left[\mathbb{E}\left[\max_{\boldsymbol{x}^{PA}}\mathbb{E}\left(e^{-\gamma_i(W_i+\boldsymbol{x}^{PA\prime}(\boldsymbol{v}-\boldsymbol{p}))}|\zeta_2,\boldsymbol{p}\right)\right]\right]\\
u^{A}(W_i) &= -\frac{1}{\gamma_i}\log\left[\mathbb{E}\left[\max_{\boldsymbol{x}^{A}}\mathbb{E}\left(e^{-\gamma_i(W_i+\boldsymbol{x}^{A\prime}(\boldsymbol{v}-\boldsymbol{p}))}|\boldsymbol{\zeta},\boldsymbol{p}\right)\right]\right].
\end{aligned}
$$

Our empirical evidence suggests that it is rare for plans to manage alternative assets passively. Thus, we only consider the EA and IA options for the second asset class.[2] The portfolio solution with two assets is again of the mean-variance type

$$\boldsymbol{x} = \frac{1}{\gamma_i}\boldsymbol{\Sigma}_v^{-1}\left(\boldsymbol{\mu}-\boldsymbol{p}\right).$$

Because we assume $v_1 \perp\!\!\!\perp v_2$, $\boldsymbol{\Sigma}_v^{-1}$ is diagonal and the asset shares only depend on the price of their own asset. The conjectured prices take the form

$$
\begin{aligned}
p_1 &= \theta_1 + \theta_{1,\zeta}\left[(\zeta_1-\bar{v}_1) - \theta_{1,q}\left(q_1-\bar{q}_1\right)\right]\\
p_2 &= \theta_2 + \theta_{2,\zeta}\left[(\zeta_2-\bar{v}_2) - \theta_{2,q}\left(q_2-\bar{q}_2\right)\right].
\end{aligned}
$$

Following the calculations in GP, we define $\widehat{p}_1 = v_1 - \bar{v}_1 + \varepsilon_1 - \theta_{1,q}(q_1-\bar{q}_1)$. Then we obtain the

[2] It is only for some sub-asset classes such as REITS and some commodity funds where plans have the option to manage alternative assets passively. The assumption that only active management is observed in equilibrium can be incorporated in the model by choosing the parameters such that passive management leads to a big loss in utility (alpha).

conditional means and variances of each asset value

$$
\begin{aligned}
\mathbb{E}\left(v_1|p_1,p_2,\zeta_2\right) = \mathbb{E}\left(v_1|p_1\right) &= \bar{v}_1 + \frac{\sigma_{1,v}^2}{\sigma_{1,v}^2+\sigma_{1,\varepsilon}^2+\theta_{1,q}^2\sigma_{1,q}^2}\widehat{p}_1 \\
\mathbb{E}\left(v_1|p_1,\zeta_1,p_2,\zeta_2\right) = \mathbb{E}\left(v_1|p_1,\zeta_1\right) &= \bar{v}_1 + \frac{\sigma_{1,v}^2}{\sigma_{1,v}^2+\sigma_{1,\varepsilon}^2}(s_1-\bar{v}_1) \\
\mathrm{Var}(v_1|p_1,p_2,\zeta_2) = \mathrm{Var}(v_1|p_1) &= \frac{\sigma_{1,v}^2\left(\sigma_{1,\varepsilon}^2+\theta_{1,q}^2\sigma_{1,q}^2\right)}{\sigma_{1,v}^2+\sigma_{1,\varepsilon}^2+\theta_{1,q}^2\sigma_{1,q}^2} \\
\mathrm{Var}(v_1|p_1,\zeta_1,p_2,\zeta_2) = \mathrm{Var}(v_1|p_1,\zeta_1) &= \frac{\sigma_{1,v}^2\sigma_{1,\varepsilon}^2}{\sigma_{1,v}^2+\sigma_{1,\varepsilon}^2} \\
\mathbb{E}\left(v_2|p_1,\zeta_1,p_2,\zeta_2\right) = \mathbb{E}\left(v_2|p_2,\zeta_2\right) &= \bar{v}_2 + \frac{\sigma_{2,v}^2}{\sigma_{2,v}^2+\sigma_{2,\varepsilon}^2}(s_2-\bar{v}_2) \\
\mathrm{Var}(v_2|p_1,\zeta_1,p_2,\zeta_2) = \mathrm{Var}(v_2|p_2,\zeta_2) &= \frac{\sigma_{2,v}^2\sigma_{2,\varepsilon}^2}{\sigma_{2,v}^2+\sigma_{2,\varepsilon}^2}.
\end{aligned}
$$

In each line, the first equality follows from the independence assumption. It is now straightforward to solve for the unknown coefficients, $\{\theta_{z,0},\theta_{z,\zeta},\theta_{z,q}\}$ for $z\in\{1,2\}$, as in Appendix A.7.

The market clearing condition (see Equation (A.12) below) for the public asset remains unaffected. However, for the private asset, there are no passive investors. We also assume there are no noise allocators, so that in equilibrium all managers are active, i.e. the manager's indifference condition (A.2) becomes a strict inequality:

$$
\frac{I_2^{EA}}{M_2^A}f_{2,i} \geq k_2 \iff \frac{\bar{I}}{M_2^A}\mathbb{E}\left[c_{2,i}^{EA}\mathbb{1}\left(u_{2,i}^{EA}=\max\left\{u_{2,i}^{IP},u_{2,i}^{IA},u_{2,i}^{EP},u_{2,i}^{EA}\right\}\right)\right] > k.
$$

Furthermore, we assume that the plan cannot pull out from investing in the alternative asset, so that the equilibrium bargaining fee is still given by $f_{2,i}=c_{2,i}^{EA}$. The market clearing condition becomes

$$
q_2 = \sum_{i\in I_2^A} x_{2,i}^A,
$$

where I_2^A is the set of active investors in the alternative asset class. Having solved for the equilibrium, we calibrate the model parameters as in the public asset class, with the exception of: $N=0$ (number of noise allocators), $\bar{M}=400$ (number of managers), $\beta^{\mathrm{A}}=0.95$ (power law coefficient for active management), $FC=240$ million (fixed cost for internal management), $k=61.2$ million (active management cost), $\bar{v}=0.2\bar{v}_{\text{public}}$ (total asset value), $\sigma_v=0.4\bar{v}$ (volatility of asset value), $\sigma_\varepsilon=0.5\bar{v}$ (volatility of error term).

A.6 Return Performance

This section analyzes a plan's return performance. We first consider a plan's unconditional return performance in the public asset class. The expected gross return per unit of wealth of passive and active plans is given by

$$R_i^P = \frac{1}{W_i}\mathbb{E}\left[\frac{\mathbb{E}\left(v_1|p_1\right)-p_1}{\gamma_i \operatorname{Var}(v_1|p_1)}\left(v_1-p_1\right)\right] = \frac{1}{\gamma_i^{RRA}}\mathbb{E}\left[(SR_i^P)^2\right]$$
$$R_i^A = \frac{1}{W_i}\mathbb{E}\left[\frac{\mathbb{E}\left(v_1|p_1,\zeta_1\right)-p_1}{\gamma_i \operatorname{Var}(v_1|p_1,\zeta_1)}\left(v_1-p_1\right)\right] = \frac{1}{\gamma_i^{RRA}}\mathbb{E}\left[(SR_i^A)^2\right],$$

where $\gamma_i^{RRA} = \gamma_i W_i$ denotes the coefficient of relative risk aversion. Gârleanu and Pedersen (2018, Proposition 7) show that $\mathbb{E}\left[(SR_i^A)^2\right] > \mathbb{E}\left[(SR_i^P)^2\right]$, and hence $R_i^A > R_i^P$.

We now consider the gross return performance of a plan conditional on its size

$$\mathbb{E}\left(R|W_i,\gamma_i^{RRA}\right) = \mathrm{P}(\text{Active}|W_i,\gamma_i^{RRA})R_i^A + (1-\mathrm{P}(\text{Active}|W_i,\gamma_i^{RRA}))R_i^P. \tag{A.10}$$

The probability of being active conditional on size is given by

$$\begin{aligned}
&\mathrm{P}(\text{Active}|W_i,\gamma_i^{RRA})\\
&= \mathrm{P}(\text{Active}|W_i,\gamma_i^{RRA},\text{Rational allocator})\,\mathrm{P}(\text{Rational allocator}|W_i,\gamma_i^{RRA})\\
&+ \mathrm{P}(\text{Active}|W_i,\gamma_i^{RRA},\text{Noise allocator})\,\mathrm{P}(\text{Noise allocator}|W_i,\gamma_i^{RRA}).
\end{aligned} \tag{A.11}$$

The second term in this expression equals $\frac{M^A}{M}\frac{N}{I+N}$, which is independent of a plan's size. The first term is not monotonic as a function of size in our equilibrium, as the medium large plans opt for active management, but the largest plan employ passive management (see Figure 2 in the main paper). On the other hand, if we also condition the gross return (A.10) on being passive/active, then (trivially) the expected return is independent of size and strictly increasing from passive to active management.

The net return performance per unit of wealth is defined by

$$\mathbb{E}\left[R|W_i,\gamma_i^{RRA},c_i\right] - c_i(W_i)/W_i,$$

where c_i is one of the cost functions in (4.1)–(4.2). For W_i large enough, plans adopt internal passive management. Because $c_i(W_i)/W_i = O(W_i^{\beta^{\mathrm{P}}-1})$, we obtain that net returns are eventually increasing in

AUM. If we also condition the expected net return on being passive/active, then the net expected return is strictly increasing in size. The size effect on net-of-fee returns is more pronounced in the alternative asset class. In the calibrated model we find that only large plans use internal active management, whereas smaller plans use external active management (see Figure 2c in the main paper). Therefore, expected net returns are strictly increasing as a function of AUM in the private asset class.

A.7 Market Clearing with Heterogeneous Plans

This section shows how to clear the market in case of a single risky asset with heterogeneous plans. The final result is essentially contained in GP, but we include it to make the paper self contained.

For ease of notation, we abbreviate the variances of the value for the passive and active plans by

$$V^P := \mathrm{Var}(v|p)$$
$$V^A := \mathrm{Var}(v|p,\zeta).$$

Let x_i^P and x_i^A be the portfolio solutions for the passive and active plans, respectively. Due to the CARA utility assumption, the portfolio solution will always be of the mean-variance type. To clear the market, we equate supply and demand

$$q = \sum_{i\in I^A} x_i^A + \sum_{i\in \bar{I}\setminus I^A} x_i^P + \underbrace{\sum_n \frac{M^A}{\bar{M}} x_n^A + \left(1-\frac{M}{\bar{M}}\right) x_n^P}_{\text{Noise allocators}}, \tag{A.12}$$

Further expanding the right hand side yields

$$\begin{aligned}
&\left(\frac{-\theta_\zeta\left(-\left(q-\bar{q}\right)\theta_q-\bar{v}+\zeta\right)+\frac{\sigma_v^2(\zeta-\bar{v})}{\sigma_v^2+\sigma_\epsilon^2}+\bar{v}-\theta_0}{V^A}\right)\sum_{i\in I^A}\frac{1}{\gamma_i}\\
&\quad+\left(\frac{-\theta_\zeta\left(-\left(q-\bar{q}\right)\theta_q-\bar{v}+\zeta\right)+\frac{\sigma_v^2(-(q-\bar{q})\theta_q-\bar{v}+\zeta)}{\theta_q^2\sigma_q^2+\sigma_v^2+\sigma_\epsilon^2}+\bar{v}-\theta_0}{V^P}\right)\sum_{i\in \bar{I}\setminus I^A}\frac{1}{\gamma_i}\\
&\quad+\frac{M^A}{\bar{M}}\left(\frac{-\theta_\zeta\left(-\left(q-\bar{q}\right)\theta_q-\bar{v}+\zeta\right)+\frac{\sigma_v^2(\zeta-\bar{v})}{\sigma_v^2+\sigma_\epsilon^2}+\bar{v}-\theta_0}{V^A}\right)\sum_{n=1}^N\frac{1}{\gamma_n}\\
&\quad+\left(1-\frac{M^A}{\bar{M}}\right)\left(\frac{-\theta_\zeta\left(-\left(q-\bar{q}\right)\theta_q-\bar{v}+\zeta\right)+\frac{\sigma_v^2(-(q-\bar{q})\theta_q-\bar{v}+\zeta)}{\theta_q^2\sigma_q^2+\sigma_v^2+\sigma_\epsilon^2}+\bar{v}-\theta_0}{V^P}\right)\sum_{n=1}^N\frac{1}{\gamma_n}.
\end{aligned}$$

The expression above can be written as $c_0 + c_\zeta\zeta + c_q q$. Collecting coefficients and using market clearing,

we can solve for $c_q + \theta_q c_\zeta = 1$, which renders the solution

$$\theta_q = \frac{V^A\left(\sigma_v^2 + \sigma_\epsilon^2\right)}{\sigma_v^2\left(\sum_{i\in I^A}\frac{1}{\gamma_i} + \frac{M^A}{M}\sum_n \frac{1}{\gamma_n}\right)} = \frac{\sigma_\varepsilon^2}{\left(\sum_{i\in I^A}\frac{1}{\gamma_i} + \frac{M^A}{M}\sum_n \frac{1}{\gamma_n}\right)}$$

$$V^A = \frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}$$

$$V^P = \frac{\sigma_v^2(\sigma_\varepsilon^2 + \theta_q^2\sigma_q^2)}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2\sigma_q^2}.$$

Let $\tau = \sum_{i\in I^A}\frac{1}{\gamma_i} + \frac{M^A}{M}\sum_n \frac{1}{\gamma_n}$. Now we can obtain an expression for the market inefficiency. First note that

$$1 - \frac{V^A}{V^P} = \frac{\sigma_q^2\sigma_\varepsilon^2}{\tau^2 + \sigma_q^2\sigma_\varepsilon^2}\frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2}.$$

Therefore

$$\eta = -\frac{1}{2}\log\left(\frac{V^A}{V^P}\right) = -\frac{1}{2}\log\left(1 - \frac{\sigma_q^2\sigma_\varepsilon^2}{\tau^2 + \sigma_q^2\sigma_\varepsilon^2}\frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2}\right).$$

B CEM Data and Asset Allocation

This appendix discusses details of the CEM data collected from annual surveys sent out to a large sample of international pension plans. To participate in the survey (and to receive its results), CEM requires plans to report data on asset returns and costs by sub-asset class. Each of these sub-asset classes are further split into active/passive and internal/external management styles. CEM classifies internally managed holdings and returns as internal if the buy/sell decision is made within the pension fund organization. In addition, plans are asked to report policy returns, benchmarks, policy weights, and the number of external mandates—all within each sub-asset class—a unique feature of the CEM database. Other questions in the survey pertain to governance, operations and support costs as well as information such as the number of active plan members, the type of investments being offered and the percentage of the plan's liabilities due to retirees. Only a small number of variables are constructed by CEM themselves, such as a plan's asset volatility, which is computed using CEM's internal model.

A benefit of the CEM database is that there are no systematic biases in reporting related to performance. After consultation with CEM, it appears that plans' decision to report in a specific year is unrelated to their investment performance.[3] This conclusion is also reached in a study by Bauer, Cremers

[3] An important incentive for plans to participate in the CEM survey is to compare their performance and fees, as well

and Frehen (2010). However, most of the pension funds that provide data to CEM are typically larger in size, compared to the average pension plan. Our data show that the average plan size in the United States in 2019 was approximately $25 billion, and the maximum AUM recorded was $376 billion, which included 10 sponsors with over $100 billion in AUM. Notably, the eight largest U.S. sponsors in our data are among the top 10 largest DB plans nationwide.

According to our database, U.S. domiciled DB plans held a total of $3.79 trillion in AUM in 2019, compared to a total AUM of $8.1 trillion in aggregate across all U.S. DB plans (Investment Company Institute, 2021, p. 177). Of the total AUM, public plans contributed $2.53 trillion, while private plans contributed $1.27 trillion. Hence, our sample covers approximately 38% of the total AUM in the U.S. public sector, which amounted to $6.68 trillion in 2019. Moving outside the U.S., our coverage of AUM includes $1.64 trillion in Canada, $3.69 trillion in Europe (including the UK), and $1.2 trillion in the rest of the world.[4]

B.1 Asset Class Frequency and Geographic Coverage

To track pension plans' asset allocation, CEM groups each plan's holdings data into six major asset classes, namely stocks, fixed income, hedge funds and multi-asset, private equity, private debt, and real assets. These broad asset classes are further divided into sub-asset classes, as described in Section B.2 below.

Panel A in Table A.1 reports the total number of plans in the survey as well as the number of plans reporting holdings within each asset class for each year.[5] The total number of plans participating in the survey ranges from 123 in 1991 to 450 in 2012 and ends at 310 in 2019 (see Panel A).

The most frequently held asset classes are, by far, stocks and fixed income, followed by real assets and private equity, hedge funds, and private debt - the latter being distinctly less common than the other asset classes. Prior to 2000, it was uncommon for plans to hold private debt or hedge fund investments, but these asset classes have been increasingly embraced by plans during the latter years in our sample particularly after 2010, in the case of private debt.

Table A.2 shows the coverage for all countries in our database at two points in time, 2009 and 2019. In 2009, plans domiciled in countries such as Australia, South Korea, Sweden, New Zealand, France, the UK, and Denmark are included. Plans domiciled in China, Saudi Arabia, Switzerland, Germany, the

as asset allocations, with those of other pension plans.

[4]CEM provides exchange rates for all countries and years, which allows us to convert foreign currency denominated AUM to U.S. dollar AUM.

[5]From our discussions with CEM, a plan always reports its holdings and returns for every asset class.

Emirates or South Africa show up in the survey at some point during our sample. Still, the Netherlands, Canada, Norway, and the U.S. account for approximately 80% of total AUM throughout our sample.

B.2 Asset and Sub-asset Classes

The CEM database contains information about cost, returns and allocation at the sub-asset class level. In this section we provide details about each of these individual sub-asset classes.

B.2.1 Stocks

- U.S. Stocks: U.S. small, mid and large cap stocks. This category also includes U.S. 130/30 type investment strategies.
- Europe: Stock investments in the following countries: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and United Kingdom.
- Asia-Pacific: Stock investments in Australia, Hong Kong, Japan, New Zealand and Singapore.
- EAFE: Mandates invested primarily in Europe, Australasia, and the Far East (EAFE). Countries in this category include Australia, Austria, Belgium, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland and the United Kingdom
- Emerging: Emerging markets and any other countries not explicitly listed in the above categories.
- ACWI x U.S.: MSCI All Country World Index excluding the United States.
- Global: Mandates invested on a global basis.

B.2.2 Fixed Income

- U.S. fixed income: Mainly U.S. Treasury notes or U.S. mortgage backed securities.
- Long Bonds: Dedicated strategies where a manager has a mandate to invest in long bonds. Typically these bonds are due to mature between 10 and 30 years in the future.
- High Yield Bonds: Bonds issued by entities that do not meet the criteria for receiving investment-grade ratings from a major credit rating agency High yield mandates are included in this category as well.

- Bundled LDI: External mandates which blend fixed income and derivatives to generate returns aimed at hedging plan liabilities.
- Cash & Equivalents: Cash managed as a separate asset class, including cash underlying derivative positions.

B.2.3 Hedge Funds & Multi-asset

- Hedge Funds: Funded absolute return strategies, i.e. strategies that are equity market neutral.
- Funded Global TAA: Fully funded long-only segregated asset pool dedicated to tactical asset allocation.
- Risk parity: Portfolios aiming to distribute the overall portfolio risk evenly across various asset classes within a diversified portfolio. The portfolio is diversified while meeting return expectations through the use of leverage.

B.2.4 Private Equity

- Venture Capital.
- LBO and Energy partnerships.
- Other private equity: Unlisted equity investments in turnarounds, start-ups, mezzanine, distressed financing and energy partnerships.
- Diversified: All private equity investments if the plan does not distinguish between the above categories.

B.2.5 Private Credit

- Direct lending, non-traded loans, leveraged loans, distressed bank loan/debt products, mezzanine and other private debt or private credit arrangements.
- Mortgages: Direct mortgages, not including mortgage-backed securities. Mortgage-backed securities are treated as fixed income.

B.2.6 Real Assets

- Commodities: Actual physical investments in commodities (crude oil, sugar, copper etc.), commodity funds or products that may invest in an index like the S&P GSCI. Derivative exposures that are fully backed by cash (not just the margin requirement) are also included in this category.
- REITs: Real estate investment trust (REIT) is a type of entity that possesses and often manages income-generating real estate properties. These properties can encompass various forms of commercial real estate, including office buildings, apartment complexes, warehouses, shopping centers, hotels, and more.
- Real Estate: Direct real estate holdings, segregated real estate holdings, and more. nternal real estate management refers to in-house staff making decisions to buy or sell individual properties. Any other approach is considered an external real estate holding. This category also includes joint ventures.
- Infrastructure: Local distribution networks for utilities like electricity, water, and gas, as well as specific transportation assets like toll roads, airports, bridges, and tunnels. Internal infrastructure management indicates that in-house personnel are responsible for deciding when to acquire or divest these assets.
- Other Real Assets: Investments in real assets other than the classes described above.

B.3 Evolution in Asset Allocation

Figure A.1 shows the proportion of investments allocated to each of the six major asset classes for U.S. (top panel) and non-U.S. (bottom panel) DB plans. For U.S. plans, the average plan allocation to public equities (stocks) varies between 50 and 60% from the beginning of our sample (1991) until the Global Financial Crisis (2007-2008), after which it drops below 50% of portfolio holdings. These plans increasingly allocate to alternative asset classes by the end of our sample (2019)—from less than 8% in 1991 to almost 28% in 2019. Non-U.S. plans show a similar pattern of asset allocation over time, albeit with lower levels of stock investments.

Even larger shifts have taken place during our sample period in the sub-asset classes that comprise the six main asset classes. Figure A.2a shows that the allocation of U.S. plans to broad-based U.S. stock strategies (U.S. Broad/All) is 86% at the beginning of our sample, dropping to 18% by 2019. In

turn, these U.S. plans allocated more to international stock strategies, such as ACWI ex U.S., EAFE, emerging markets, and global (12% in 1991 versus 58% in 2019), and allocated more to specialized market capitalization strategies such as small, medium and large cap stocks. Trends in fixed-income investments show a similar movement from broad-based to more specialized mandates. Allocations between alternative sub-asset classes exhibit a trend toward greater allocations to hedge funds (Figure A.2c), LBOs (Figure A.2d), private credit (Figure A.2e), and natural resources and infrastructure (Figure A.2f).

Figure A.3 shows the time-series evolution in total AUM by asset class, aggregated across all U.S. (left panel) and non-U.S. (right panel) plans in our sample. We note a marked shift towards greater coverage of plans outside the U.S. during our sample.

U.S. plans' investments in equities rise steadily from around $252 billion at the beginning of the sample to $1.5 trillion at the end. This increase reflects the cumulative effect of high equity returns during our sample along with increased inflows to equity investments for existing plans and the increased number of plans included in the CEM survey. Fixed income investments rise from $182 billion in 1991 to more than $1.24 trillion in 2019. The remaining asset classes all start at low levels in the early sample but rise steadily, ending at levels that exceed $300 billion in 2019 for real assets and private equity and just below $250 billion for hedge funds and multi-asset.

For non-U.S. plans, a very different allocation pattern emerges with stocks and fixed income holdings following almost identical paths, both ending near $1.7 trillion in 2019. Among the alternative asset classes, real assets are relatively more important for non-U.S. plans than for their U.S. counterparts, although the ranking at the end of the sample is the same as that for U.S. plans.

Supplementing Figure A.3, Table A.3 reports the time-series evolution in asset allocation for U.S. and non-U.S. plans during our sample. For U.S. plans (Panel A), stock holdings account for a little over half (55%) of total asset values in the early sample, peaking at a share of 63% in 1999, before declining to 40% in 2019. Fixed income holdings account for 35-40% of overall portfolio values in the 1990s, before falling to a range of 25-30% between 1999 and 2007 and retaining a fairly steady portfolio weight averaging 31% from 2008 to 2019.

Hedge fund and multi-asset holdings rise from roughly 1% in 2004 to more than 4% in 2009. In the last five years of the sample, plans hold around 6% of their assets in hedge funds. Allocations to private equity start out around 2% at the beginning of the sample, rise to 4% in 2000, before doubling to 8% in 2010 and remaining in the 8-9% range for the rest of our sample. Private debt holdings account for less

than 0.1% of AUM prior to 2003 but rise modestly to end up at 2% in 2019. Finally, real assets hover around 4% during the nineties, rise to around 7% over the next decade and end up at 10% in the last years in our sample.

For non-U.S. plans (Panel B), we observe similar patterns. At the beginning of the sample, the vast majority of plan assets is allocated to stocks and fixed income. In contrast to the U.S. sample, however, fixed income takes up most of the investments (58%), followed by stock holdings (36%) in the early part of the sample. Over time, alternative asset classes become more prominent, with hedge fund and multi-asset holdings accounting for 3.9% of total assets at the end of our sample. Private equity (6.5%) and real assets (14.7%) in particular also comprise a significant portion of total assets. At the end of our sample, stock holdings are the major source of non-US plans' asset allocation (41.1%), closely followed by fixed income (31.9%).

Figure A.4 shows the investment shares of sub-asset classes for non-U.S. plans. For stocks, we see an increase in the allocation to "Global" and "Emerging Market" equities. This is also true for "Global" fixed income allocations. In private equity, we see increased portfolio weights on limited buyouts venture capital. Since the Global Financial Crisis, we also see an increase in the allocation to private credit in lieu of mortgages. In the real asset class, allocations to infrastructure increase whereas there is a divestment from real estate.

In summary, stocks and fixed income account for more than 90% of the total value of pension plans' asset holdings in the early nineties. This share has declined to about 70% at the end of our sample, with real assets, private equity and hedge fund investments accounting for most of the increased allocation to alternative asset classes. While stocks and bonds thus remain by far the most important asset classes, alternative assets are clearly gaining significant ground, having nearly tripled their share of pension plans' portfolios from roughly 10% to close to 30% during our 29-year sample.

B.4 Asset Management Mandate

In each sub-asset class, the AUM of a sponsor are managed according to their *asset management mandate* (or style). CEM provides information at the sub-asset class level about the following management mandates:

- Internally managed: the buy-sell decisions for the underlying assets (e.g., individual stocks, bonds, property) are made within the organization. This also includes wholly-owned subsidiaries.
- Externally managed: the buy-sell decision for the underlying assets are made by third-party entities,

such as money managers.

- Passively managed (or indexed): designed to either replicate broad capital market benchmarks (e.g., the S&P 500 for U.S. stocks) or dedicated to matching liability requirements.
- Actively managed: assets given to an external manager to manage according to a set of objectives and constraints.
- Limited partnerships: investments in funds with a predetermined lifespan, where assets are sold, and invested capital is returned upon reaching the investment horizon.
- Co-investments: minority investments directly made into an operational company in conjunction with a financial sponsor or another private equity investor, typically in the context of a leveraged buyout, recapitalization, or growth capital transaction.
- Fund of Funds: Investments in funds whose holdings consist primarily of other funds.

B.4.1 Empirical Evidence on Mandates

Table A.4 shows how small (bottom 30th percentile in AUM) and large (top 30th percentile in AUM) plans allocated sub-asset classes to the four management styles in 2009 and thus complements Table 1 in the main text that shows similar evidence for 2019. External active management is dominant for small plans, particularly in the private asset classes but also for most sub-asset classes in stocks and fixed income. For stocks and fixed income, some plans also use external passive management, particularly for ACWI ex U.S., Other, U.S. Broad stocks, and inflation indexed and long bonds. In contrast, large plans use internal allocation far more often than the smaller plans. This holds both among stock and fixed income investments and involves both internal active and internal passive management. Among the holdings in the private asset classes, internal active management plays an important role for the private equity "other" assets, mortgages (private debt), commodities, infrastructure, real estate and REIT investments.

In results not reported here, we find that, across all asset classes, non-U.S. plans manage a significantly higher portion of their investments internally compared to their U.S. peers. Differences are particularly large for fixed income, private debt, and real assets in which the proportion of internally managed assets for non-U.S. plans exceeds that of U.S. plans by more than 20%.

In some cases, plans use multiple investment management styles to allocate their holdings within a particular sub-asset class. For those plans that do not adopt a single management style for all of their

holdings in a particular sub-asset class, Table A.5 shows the allocation share to the six possible pairwise combinations of the four investment management styles. The table covers only the largest plans because this usage of multiple investment management styles within a single sub-asset class is extremely uncommon among smaller plans. For stock accounts, combinations of external active and external passive as well as combinations of external active and internal active management are the most widespread pairs, but external active combined with internal passive management is also not uncommon. Among fixed income investments as well as investments in the private asset classes, combinations of external active and internal active management is the most common pairing.

Table A.6 shows statistics on the number of sub-asset classes per plan/year that are internal and external actively managed. We find that external active management is more common than internal active management. Additionally, on average, a greater amount of AUM is allocated to internal active management in comparison to external active management. These trends are consistent across all asset classes, and lend support to the hypothesis that only big plans have the expertise and resources to set up internal teams. Furthermore, internal management tends to be utilized exclusively for a select few specialized sub-asset classes. Figure A.5 shows a bar chart of the number of sub-asset classes that are actively managed, either internally or externally by a specific plan. For stocks and fixed income, the number of sub-asset classes that are internally managed is always lower than the externally managed assets, with the exception of plans that invest in a single fixed income sub-asset class.

B.5 Asset Allocation Decisions

We examine plans' asset allocation decisions by modeling the weight of asset class A for plan i in year t ($\omega_{iAt} = \text{AUM}_{iAt}/\text{AUM}_{it}$), as follows:

$$\begin{aligned}\omega_{iAt} = c_A + \lambda_{At} + \beta_{1,A}\log(\text{AUM}_{it-1}) + \beta_{2,A}\text{Cost}_{iAt-1} \\ + \beta_{3,A}\text{Private}_i + \beta_{4,A}\text{nonUS}_i + \beta_{5,A}\text{LiabilityRetiree}_{it} + \epsilon_{iAt}.\end{aligned} \tag{B.1}$$

The list of regressors is similar to that adopted earlier with two exceptions. First, our cost variable (Cost_{iAt-1}) is now the lagged per-dollar cost for plan i in asset class A measured as a fraction of AUM and denoted in bps. Second, we also control for liability-related effects on asset allocation decisions by including $\texttt{LiabRetiree}_{it}$, the fraction of a plan's total liabilities owed to retirees. We do so because plans are likely to consider their liability structure when deciding how much to allocate to asset classes with different risk characteristics. For example, more mature plans may allocate a larger fraction of their

portfolio to fixed income. However, only fewer plans report data on liabilities, decreasing the sample size in regressions including `LiabRetiree` as a covariate. We estimate equation (B.1) using fractional regressions and nonparametric estimates.

Across estimation methods, we find consistent evidence that larger plans allocate less of their portfolios to stocks, fixed income, hedge funds, and private debt. Conversely, they allocate a significantly greater share of their investments toward private equity and real assets.[6] Investment management costs are important in asset allocation decisions, with higher costs leading to lower investments to shares to that specific asset class. Estimates on liabilities owed to retirees indicate that larger liabilities lead to lower investments to stocks (Table A.7).

B.5.1 Asset Allocation and Size: Fractional Regressions

We use fractional regressions (Papke and Wooldridge, 1996, 2008) to estimate the probability of allocating to specific asset classes, following the model described in (B.1). In our regressions we use lagged log size, lag cost, an indicator variable that takes the value of 1 for plans domiciled outside of the US, and an indicator variable that takes the value of 1 for private plans as covariates.

Figure A.6 shows the distribution of portfolio allocation to a given asset class as a share of total AUM.[7] Allocation to public asset classes nearly follows a normal distribution with a mean of 52 percent for equities and 36 percent for fixed income. The distribution of AUM share of private asset classes resembles a power law distribution. From a modeling perspective, the Cragg model with a hurdle at the zero lower bound clearly is an unsuitable tool to estimate the relationship between plan size and allocation decisions.

Table A.7 presents average partial effect estimates of lagged log size ($\log(\mathrm{AUM}_{\mathrm{i,t-1}})$), lagged per-dollar cost measured as a fraction of AUM denoted in bps (Cost_{it-1}), and liability owed to retirees ($\mathrm{LiabRetiree}_{it}$) (in panel B). Average partial effects indicate that for almost all asset classes an increase in size leads to lower allocations except for private equity and real assets. Strikingly, on average, a 100 percent increase in size leads to a 3 percent decrease in equity allocation, 1.5 percent decrease in hedge fund and multi asset allocation. For fixed income, private credit, and private equity a 100 percent increase in size leads to a less than 1 percent decrease in allocation. Average partial effect estimates on liabilities owed to retirees are highly significant and negative for stocks. A doubling in liabilities is associated with a 7 percent decline in equity allocation. Surprisingly, a doubling in liabilities is associated

[6] Our estimates are consistent with the findings reported in Dyck and Pomorski (2011).
[7] We define the share of share of total AUM allocated to a given asset class as: $\omega_{iAt} = \mathrm{AUM}_{iAt}/\mathrm{AUM}_{it}$.

with a 3.7 percent increase in allocation to hedge funds and multi assets. However, the positive APE estimate is mostly due to the bundling of hedge funds and funded TAA.

Figure A.7 plots the conditional mean of allocation in percent of total AUM to different asset classes for different plan sizes. For public asset classes (stocks and fixed income) there appears to be a negative linear monotonic relationship between allocation and plan size. However, this relationship appears to be non-linear for private asset classes and positive for private equity and real assets.

To test more formally whether the relations in Figure A.7 are monotonic, we use the monotonic relation test of Patton and Timmermann (2010). The monotonic relation is specified to be either positive ("+") or negative ("–") as specified for the different asset classes in Table A.9. Under the null hypothesis, there is no positive (rep. negative) monotonic relation between plan size and allocation to a given asset class. Conversely, there is a monotonic relation between lagged plan size and allocation to a given asset class under the alternative. For example, if we specify a negative (decreasing) relation under the alternative, small p-values indicate that larger plans allocate a smaller amount of their investments to a given asset class.

To implement the test, each year (t) we sort plans by AUM, keeping only those plans that also report holdings the following year ($t+1$). We then form equal weighted quartile portfolios for the size-sorted plans going from the smallest to the largest plans. We conduct these tests only for those asset classes for which we have a sufficient number of observations, leading us to drop private debt. The results are reported in Table A.9. We find significant evidence of a monotonically decreasing relation between plan size and allocations to stocks and hedge funds and multi-asset mandates. Furthermore, the test also provides evidence for a monotonically increasing relation between plan size and allocations to private equity and real assets. Only for fixed income do we fail to reject the null of no monotonic relation between plan size and allocation.

C Cost Data

CEM collects detailed cost data at the sub-asset class level. In general, all costs—internal and external to the pension plan—related to management of plan assets are included in the survey.[8]

[8] For our empirical results, we proxy the plan's cost by average cost relative to AUM in a specific sub-asset class and year. This measure of cost also includes performance fees.

C.1 Cost Components

We list the various cost components that a plan reports to CEM below:

Internal investment costs

- Compensation, benefits and direct expenditures associated with the staff overseeing internal portfolios. If staff is responsible for multiple asset classes, the cost is split according to the estimated time allocation
- Consulting, research, legal, trading systems and other third party costs.
- General operating expenses, including rent, utilities, IT services, investment accounting, financial control, and human resources. These costs are also allocated based on usage.

External investment costs

- Base fees remitted to third-party managers including investment management fees, manager-of-manager fees, commitment fees and fees netted from returns.
- Performance fees paid to (third-party) managers.
- Costs associated with balanced mandates, proportionally allocated based on actual holdings.
- Compensation, benefits and direct expenses for staff members primarily responsible for selecting, monitoring, and overseeing external managers.
- Third-party investment management fees prior to any deductions for rebates. These rebates constitute the limited partners' portion of specific fee income realized by the partner in connection with the fund, such as fees related to break-ups, monitoring, and funding.

Limited partnership costs

- Unreturned Invested Capital: Contributed capital less contributed capital attributable to realized investments less the aggregate amount of write-downs, if any, with respect to unrealized investments. This is often the amount on which fees are based after the investment period ends.
- Percentage fee on unreturned invested capital (post investment period): Private equity management fees are typically paid as a percentage of the committed amount during the investment period and as a percentage of unreturned invested capital after the investment period ends.

- Rebate percentage: the limited partners' share of certain fee income realized by the General Partner in connection with the fund such as fees for break-up, monitoring and funding.

Other expenses

- Oversight of the fund, including expenses such as staff salaries, direct costs (e.g., travel, director fees, director's insurance, etc.), and unallocated overhead related to the supervision of fund assets.
- Trustee and custodial costs.
- Consulting costs for manager searches, scenario testing, system consulting, and internal or external costs for performance measurement.
- Legal fees related to the entire fund which includes, among others, fiduciary insurance and printing.
- Fund of Funds Costs: top-layer management fees levied by the fund-of-funds manager as the manager base fees. It also includes the expenses incurred in the underlying funds. In cases where this data is unavailable, CEM applies a standard default.

C.2 Trends in Investment Management Costs

Asset management costs play a key role in our analysis, so we examine how they have evolved over time. We measure the aggregate investment management cost in an asset class/year as the sum of AUM-based fees and performance-related fees (value-weighted across all plan sub-asset class allocations within that asset class during that year). Further, before reporting, we scale the median value of this cost by the "grand average" cost, which is value-weighted across all asset classes, plans, and years in our data.[9] This generates a new measure of average asset class cost/year, *scaled cost*, which is expressed as a percentage of this grand average cost. While this scaling does not show the actual cost level in bps/year, it allows us to interpret time trends in management costs as well as to compare relative costs across different asset classes and management mandates.[10]

First, consider investment management costs for stock holdings (Figures A.8a and A.8d). For passively managed accounts, median costs increase slightly over time from 5% to 8% of average costs when internally managed, but decline more significantly, from 18% to 9%, when externally managed. Hence,

[9] While we use the raw, highly granular data in our empirical analysis to follow, here, we use this transformation to preserve the confidentiality of granular asset class cost *levels*, as requested by CEM.

[10] For example, a *scaled cost* of 50% implies a median cost for an asset-class/year of half the average cost across all asset-classes and years. Appendix Table A.12 reports scaled costs by asset class and country-of-domicile for plans for selected years.

by the end of our sample, the cost of internal and external passive management converge, suggesting that passive management has increasingly become a "commoditized" investment management service. Active equity management costs are far higher, at 22% of average costs for internally managed accounts, and rising from 81% to 110% of average costs for externally managed accounts with no sign of convergence. Part of the reason for this non-convergence, as we will show, is that plans tend to internalize the active management of the least specialized, lower-cost sub-asset classes (e.g., broad cap U.S. stocks), while maintaining external managers for high-cost segments (e.g., emerging market stocks and small cap stocks) which require more specialized knowledge to manage actively.

The evolution in investment management costs for fixed income allocations is similar (Figures A.8b and A.8e): a downward trend in external passive management costs toward that of internal passive management reduces the spread from more than 10 bps/year at the beginning of our sample to 3 bps/year at the end. Similarly, external active management costs in fixed income hover around 50 bps/year versus 10 for internal active management, with no sign of convergence during our sample, again consistent with an increased specialization of external active managers to sub-asset classes. For alternative asset classes, spreads between internal and external management costs and passive versus active are generally larger in magnitude, but display similar convergence in internal versus external passive management costs (largely REITS and commodities), but not for external versus internal active management costs (Figures A.8c and A.8f).[11]

In summary, passively managed assets have become largely commoditized, resulting in lower costs, especially for large plans that use internal management at the end of our sample period (as shown in Figure 1). External active management fees have, in general, remained durably higher than internal active management costs, due to an increasing specialization of external active managers and the alpha benefits such specialization brings to pension plans—as Figure 1 shows, even large plans continue to allocate substantial assets to external active managers.

C.3 Variation in Costs by Investment Management Mandate

Investment management mandate is a key determinant of costs, but there is considerable heterogeneity in how much individual plans pay in fees. We present several figures that illustrate this heterogeneity. As in the main text, we scale all cost figures by the grand average cost, averaged across plans, asset classes and years. Hence, all cost are expressed in percentage units relative to the average cost in our

[11] Due to the smaller number of plans that invest in private asset classes, particularly early in our sample, we show results aggregated across the four individual private asset classes.

sample.[12]

We begin by presenting box plots in Figure A.9, with the median and interquartile range of 2019 plan-level costs for public asset classes and the four management mandates represented in our sample, scaled (to maintain proprietary data confidentiality required by CEM) by the grand-average cost, i.e., costs averaged across asset classes, across plans, and over time. For both stocks and fixed income, the cost ranges are low and narrow for passively managed accounts (IP and EP). Internal active (IA) management costs are a little higher, on average, than passive management fees and slightly more dispersed among stock and fixed income accounts. Median costs grow notably bigger, and cost ranges wider, for external actively (EA) managed accounts, which charge far higher fees than all other account types. We note (in unreported tests) that this holds across all sub-asset classes and throughout our sample.

Figure A.10 presents box-and-whisker plots displaying how total costs evolve over time for the active, passive, internal, and external management styles. Costs are aggregated across asset classes on a value-weighted basis. The scaled median internal management cost (top left panel) fluctuates around 14% of average costs with no discernible trend. Internal management costs are very homogeneous across plans. For example, the 95th percentile of scaled internal management costs is at most 62% of average costs.

In sharp contrast, scaled median external management costs are trending up starting at around 100% in 1999 to around 133% of average costs in 2019. Differences in external management costs across plans are also far higher than what we see for internal management costs with 95% bands ranging from 22% to nearly 450% of average costs towards the end of our sample.

Median passive management costs have declined modestly from around 18% of average costs in the early sample to close to 9% of average costs per year at the end. Differences in passive management costs across plans are also very modest, with the 95% bands ranging from 2% to 22% of average costs at the end of the sample.

In contrast, median active management costs rise from close to two-thirds of average costs in the early sample to about 150% of average costs at the end of the sample. The spread in active investment management costs is also very large, with the 95% confidence band going from close to zero to nearly 400% of average costs at the end of the sample.

The plots in Figure A.10 show management costs aggregated across different asset classes whose weights are shifting over time. To isolate the impact of shifts in the weights of individual asset classes, Figures A.11–A.12 plot investment management costs for individual asset classes segregated by internally

[12] We implement this scaling to preserve confidentiality of the cost levels. However, this transformation of costs still allows us to compare cost across different asset classes and years.

vs. externally managed assets and passively vs. actively managed assets. Hence, these plots show both the time-series variation and the degree of heterogeneity in management costs by asset class and management style.

We begin by examining equity investment cost. In most years, plans' passive, internal management costs for stocks amount to less than 7% of average costs, whereas internal active costs are somewhat higher, varying in the range 10-45% of average costs. In both cases, there is no discernible time-series trend in public market investment management costs. Fees for externally-managed stock portfolios (right panels in Figure A.11) are notably higher, with a (scaled) median annual cost that varies between 9% and 22% of average costs for passive management and active management fees between 56% and 130% of average costs. Overall, we find a far greater degree of variation in the costs of externally managed stock portfolios than for internally managed ones.

Figure A.12 shows similar plots for fixed income investments. For internal passively managed fixed income portfolios, median costs fluctuate between 4% and 9% of average costs, with three-quarters of plans paying less than 11% of average costs in most years and always less than 18%. The costs for actively managed internal portfolios are similar. The costs of externally managed fixed income portfolios fluctuate at a higher level, around 11% of average costs for passive portfolios, and 44% for actively managed portfolios. Again, a trend in these fees is notably absent with year-to-year variation more likely to reflect shifts in the composition of our sample of plans.

C.3.1 Variations at the Sub-asset Class Level

Figure A.13 provides further granularity by plotting median costs for the most important sub-asset classes in our sample. First consider management costs for U.S. Large and Small cap stocks. Median passive management costs are declining over time whereas active costs for internal and external management are steady around 67% of average costs for active large cap and 11% of average costs for internal management. Median external active management costs for small cap portfolios are around 130% of average costs versus 44% for internal active management.

Median costs for passively managed EAFE mandates converge to approximately 11% of average costs and we see a similar trend for passive management of Broad stocks whose median cost converges to around 7% of average costs. Median active management costs have maintained their gap between external and internal management of about 67% of average costs. Median internal management cost for EAFE is around 44% and 22% of average costs for Broad stocks. External active management costs for

EAFE mandates amount to 110% and 90% of average costs for Broad stocks.

Median internal passive costs for U.S. fixed income fluctuate between 2% and 4% of average costs in most years. External passive management costs start considerably higher but trend downward, converging towards internal passive management costs at the end of the sample. Median internal active management costs are around 11% of average costs versus 44% for external active management costs.

The last panels in Figure A.13 show that median internal passive management costs for Canadian fixed income have been fluctuating around 11% of average costs, and external passive management costs converged to 11% of average costs toward the end of our sample. Median internal active management costs is around 11% of average costs without any considerable trend; external active management start above 44% and decrease to approximately 38% of average costs.

Supplementing these figures, Table A.10 shows regressions of costs (in bps) by sub-asset class on dummies such as external, and active. Across all sub-asset classes, external investment management is significantly costlier than internal management and active management is costlier than passive management. External investment management is disproportionately costly in the private sub-asset classes and for specialized sub-asset classes such as emerging market stocks and bonds and high yield bonds.

Table A.10 does not control for plan size. To highlight the importance of plan size, Table A.11 presents regression results of the power law in investment costs at the sub-asset class level, as discussed in Equation (5.2) in the main text. The table shows that economies of scale are higher (lower β estimates) for passively managed EAFE and U.S. broad stocks, and for inflation-indexed bonds. For the alternative asset classes we find lower scales of economy for the cost of managing diversified private equity, real estate, and REITS. Consistent with these findings, Figure A.14 illustrates that large plans pay lower external management fees in nearly all sub-asset classes compared to small plans.

Figure A.15 highlights the size effect within the same product market for the 11 most populated products across all years, where "populated" refers to the number of plans investing in a given product (defined by year, sub-asset class, and mandate). In each case, the figure shows that the linear approximation fits the data well, and the estimated slope is consistently well below one.

C.3.2 Estimating Switching Costs: The Role of the Parallel Trends Assumption

In the main text, we also estimate the costs from switching management style. Here, we briefly elaborate on the underlying assumption that we use to estimate the effect on costs of a switch from external to internal management. To account for unobserved confounding variables such as bargaining power, we

rely on the following parallel trends assumption

$$\begin{aligned}&\mathbb{E}\left(\text{Cost}_{iat}(\text{Internal}_{iat}=0,\text{Internal}_{iat-1}=0)-\text{Cost}_{iat-1}|\text{Internal}_{iat}=1,\text{Internal}_{iat-1}=0,x_{iat}\right)\\=&\mathbb{E}\left(\text{Cost}_{iat}(\text{Internal}_{iat}=0,\text{Internal}_{iat-1}=0)-\text{Cost}_{iat-1}|\text{Internal}_{iat}=0,\text{Internal}_{iat-1}=0,x_{iat}\right).\end{aligned} \tag{C.1}$$

The parallel trends assumption (C.1) stipulates that the change in management cost is equal between the treatment group (plans switching from external to internal management) and the control group (plans continuing with external management) in the absence of treatment ($\text{Internal}_{iat}=0$), once we condition on x_{iat}. These observed variables allow us to control for differences in costs induced by plan size (captured by AUM_{iat}), active vs. passive management (captured by Active_{iat}), public vs. private plans (captured by Private_i), U.S. vs. non-U.S. plans (captured by nonUS_i), and sub-asset class heterogeneity. We include the latter as a control to impose that plans can only be matched within the same sub-asset class because of the large heterogeneity in costs (and potential confounding variables) across different sub-asset classes.[13]

C.4 Management Costs by Country of Domicile

Investment management costs depend not only on investment management style and asset class but also on country-of-domicile for the investment plan. To illustrate this, Table A.12 presents plans' mean cost per asset class by country-of-domicile in 2009 and 2019, again measured relative to the grand average cost figure. Across countries and at both points in time, management costs are lowest for fixed income balances, followed by stock portfolios. Private credit and real asset accounts fall in the middle in most countries with hedge funds and, particularly, private equity management costs being much higher.[14] The table also shows interesting geographical variation in costs, with surprisingly similar costs of managing stocks and fixed income assets in the U.S., Canada, and the Netherlands and relatively low costs of managing public assets in countries such as Australia and Sweden. The cost of managing private equity and hedge fund assets is quite similar across domiciles, while conversely we see bigger geographical differences in the cost of managing real assets, probably due to the very heterogeneous nature of this

[13] Note that the parallel trends assumption rules out potentially time-varying omitted confounding variables that affect both management costs and the choice of external management. Once the set of potentially confounding variables is specified, treatment and control units are matched based on their propensity score, which is a measure of how similar the plans are along the variables contained in x_{iat}. Finally, an estimate of the effect of internal management on cost is obtained by forming an average between treatment and control units in the matched set (see Imai, Kim and Wang (2021, Equation 18)). We use the `PanelMatch` package in `R` developed by these authors to carry out the estimation.

[14] As we show in the paper, these broad cost estimates conceal a lot of variation related to changes in investment management styles (active versus passive, external versus internal).

asset class.

D Returns, Benchmarks and Risk-Adjustments

D.1 Benchmarks

The CEM database contains a detailed list of returns, policy weights and return benchmarks, all available at the sub-asset class level. We state the definition of these variables below.

- Returns: Actual full-year returns for a specific sub-asset class. Returns are categorized as gross returns and net returns (net of cost).
- Policy weights: Weights that reflect plans' long-term policy, normal or target asset mix such as 60% stocks and 40% bonds. Policy weights add to 100% and are provided at year-end levels.
- Benchmarks: Broad investable capital market indexes (for example, the S&P500 for U.S. stocks) used to gauge asset class performance. If multiple benchmarks apply for an asset class, each benchmark is weighted accordingly (e.g., 60% S&P 500 and 40% Russell 3000). Our data sample contains a total of 15,101 different policy benchmarks, which also includes some esoteric benchmarks tailored to specialized investments such as the Dow Jones Brookfield Global Infrastructure Index or the KOSPI 3-year average return.
- Total policy return: Returns that track the policy mix and/or benchmark changes through the year.

D.2 Asset Class Return Performance

Table A.13 reports summary statistics for gross-of-fee returns grouped by asset class, averaged across all plan-years. Over our sample period, private equity holdings earned the highest mean return (15.9% per annum), followed by stock holdings (10.8%), real assets (8.4%), and private debt (7.7%). Hedge funds & multi assets (7.1%) and fixed income (6.9%) earned the lowest average sample returns. Volatility estimates, reported on the diagonal of Panel B in Table A.13, show that private equity is by far the most volatile asset class (21.6% per annum), followed by stocks (16.9%), real assets (12.7%), hedge funds (12.4%) and private debt (11.5%). Fixed income holdings, unsurprisingly, record the lowest volatility (7.4%).

While expected return performance is clearly an important driver of plans' asset allocation decisions, it is by no means the only explanation for the increased importance of alternative asset classes over our sample. The possibility of reducing portfolio-level return volatility by diversifying across asset classes has also been a key determinant of these decisions.

To better understand the extent to which pension plans gained from diversification across asset classes, Panel B in Table A.13 reports the average correlation across our six asset classes. Stock returns are positively correlated with returns on all other asset classes and have the lowest correlation with real assets (0.202) and fixed income (0.277) and the highest correlation with hedge funds and multi assets (0.858). Fixed income returns, on the other hand, are negatively correlated with returns on both real assets and private equity, though insignificantly so. The correlation between fixed income returns and returns on hedge funds and multi asset (0.548) or returns on private debt (0.598) is much stronger.

These correlation estimates are sufficiently low to imply clear diversification benefits from adding alternative asset classes to the plans' public asset holdings, with the possible exception of hedge funds and multi assets whose returns were highly correlated with both stock and fixed income returns during our sample.

D.3 Risk Adjustment Regressions

An alternative to studying policy-adjusted returns is to correct for plans' return exposures to a small set of the most important risk factors. With less than 30 annual return observations per asset class, we need to choose the risk factors judiciously, in many cases eliminating factors whose coefficient estimates are insignificant. In particular, we consider the following risk factors for the individual asset classes:

- Stocks: The Fama and French (1993) three factor model: market excess return (Market), small minus big (SMB), and high minus low book-to-market ratios (HML).
- Fixed Income: U.S. Aggregate Bond Index, U.S. Corporate Index, U.S. High Yield Index, Global Diversified Index, U.S. Long Treasury (1–3 years).
- Hedge Funds and multi asset: The seven factor model of Fung and Hsieh (2001) which includes the market excess return (Market), a bond trend, currency trend, commodity trend, size spread, bond market and credit spread factor.
- Private Equity, Private debt and Real assets: A subset of the seven factor model that includes the market excess return, size spread, and bond factor. For these asset classes we also include lags of

each factor to account for staleness in returns.

D.4 Construction of risk factors

We next describe the construction of the Fung and Hsieh (2001) risk factors that are used in Section D.7 of the main text. Three factors are obtained from Hsieh's website:[15] the Bond trend-following factor, the Currency trend-following factor and the commodity trend-following factor. In addition, we construct the following factors ourselves (following instructions on Hsieh's website):

- Equity Market Factor: Constructed using monthly S&P 500 returns.
- Size spread factor: Russel 2000 index monthly return - S&P500 monthly return.
- Bond Market Factor: Monthly changes in the 10-year treasury constant maturity yield (month end-to-month end).[16]
- Credit Spread Factor: Monthly changes in Moody's Baa yield less 10-year treasury constant maturity yield (month end-to-month end).[17]

All series are annualized from their underlying monthly values. Since the risk factors are most appropriate for U.S. plans, we only construct portfolios based on U.S. plans.[18]

D.5 Factor regression results

The "portfolio" columns in Table A.14 show estimates from regressing equal-weighted asset-class returns on the risk factors as in Equation (D.2) of the main text.[19] For the stock portfolio, the market excess return factor obtains a highly significant loading of 0.95 which is close to unity, both in an economic and statistical sense.[20] The size factor is also significant but the coefficient is an order of magnitude smaller than the market factor. The book-to-market factor is insignificant. Overall, these three factors generate an R^2 of 0.95, suggesting that most of the time-series variation in plans' (aggregate) stock returns is explained by the market factor. At -0.69%, the average plan alpha is negative but statistically insignificant.

[15] https://faculty.fuqua.duke.edu/~dah7/HFRFData.htm

[16] Available at the Federal Reserve Bank of St. Louis: https://fred.stlouisfed.org/series/DGS10

[17] Available at the Federal Reserve Bank of St. Louis: https://fred.stlouisfed.org/series/DBAA and https://fred.stlouisfed.org/series/DGS10

[18] The risk factor regressions for the alternative asset classes that use the Fung and Hsieh (2001) factors have fewer time series observations since the factor data only go back to 1994.

[19] These regressions use excess returns net of costs, but the results are nearly identical if instead we use gross excess returns.

[20] Betas on the market return significantly higher than unity would be consistent with plans applying leverage.

For the fixed income portfolios, the Bloomberg U.S. Aggregate Bond Index, a credit risk factor and a term structure variable all generate highly significant and positive estimates. The time-series R^2 (0.97) is even higher for the fixed income portfolio than for the stock portfolio (0.95). After adjusting for these risk factors, the average fixed income portfolio generates a positive and statistically significant alpha of 65 bps.

For hedge funds and multi asset mandates, the market, size spread factor and bond market factors obtain statistically significant coefficients which explain 92% of the time-series variation in average returns. For the private equity portfolio, the market equity excess return and its lagged value both obtain significant coefficients as does the concurrent bond market factor. These factors explain 74% of the variation in returns. Finally, risk factors explain a notably smaller fraction of the time-series variation for private credit and real assets with R^2 values of 0.40 and 0.44, respectively. For these asset classes, only the equity market return or its lag generate statistically significant coefficients, in both cases with values that are quite small (0.22 and 0.27, respectively). Average alpha estimates for the alternative asset classes tend to be greater in absolute terms, though only statistically significant for one of the four alternative asset classes.

The "pooled" columns in Table A.14 show estimates of the factor loadings using pooled panel regressions on individual plan-year observations. The risk factors retain even stronger statistical power over individual plans' returns in the pooled panel regressions. Unsurprisingly, however, the explanatory power of the risk factors over individual plans' returns is somewhat lower than for the aggregate regressions. This is to be expected because of idiosyncratic variation in individual plans' returns around their benchmarks due to active management.

These results suggest that traditional risk-adjustment methods work particularly well for the two most liquid asset classes (stocks and fixed income) as well as for hedge funds, but do a worse job at tracking performance in the most illiquid asset classes in our sample such as private equity, private debt, and real assets.

D.6 Factor exposures in policy-adjusted returns

In Figure A.16, we show box and whisker plots of the policy-adjusted gross returns. These are roughly centered around zero for all asset classes. Our approach of risk-adjusting returns by subtracting the plan-specific policy benchmark returns (Figure A.17) can be criticized on the grounds that some plans could earn abnormal returns by deviating from their policy targets. To address this concern, we next

examine whether significant exposures to systematic risk factors remain after subtracting policy returns from plan returns.

To obtain meaningful estimates, we require that the plans have ten or more annual return observations, and we limit our analysis to stocks and fixed income. Moreover, we only include a single risk factor to reduce the number of parameters estimated for each asset class. Our single-factor regressions for individual plans' policy-adjusted returns thus take the form

$$\widetilde{r}_{iAt} = \alpha_{iA} + \beta'_{iA} F_{At} + \epsilon_{iAt}. \tag{D.1}$$

For stocks, we use the market excess return while for fixed income portfolios we use the Bloomberg U.S. aggregate bond Index excess return as the single risk factor. Table A.15 summarizes the results. Across 199 plans with the required number of observations on stock returns, the mean and median values of β_{iA} are -0.0003 and 0.003, respectively, while the mean and median values of α_{iA} are both 0.003. Interquartile ranges are quite narrow: -0.001 to 0.0082 (alpha estimates) and -0.023 to 0.0252 (beta estimates).

For fixed income, we find similar results: Across 203 plans with at least 10 annual observations, the mean and median estimate of α_{iA} is 0.003 and 0.002 (interquartile range of -0.0013 to 0.0064), respectively, while the mean and median estimate of β_{iA} is -0.004 and 0.002 (interquartile range of -0.0929 to 0.1038).

These results show that the policy-adjustment procedure succeeds in capturing the vast amount of systematic risk exposures in plans' returns and that the plans choose market risk exposures that are very close to those laid out in the policy benchmarks for both stock and fixed income holdings.

D.7 Risk-adjusted Return Performance

Policy returns constitute a natural benchmark against which to measure individual plans' return performance. However, it is more common to measure investment performance by adjusting for plans' exposure to a small set of risk factors. Such an approach is not feasible here, however, because most plans have short return records in the CEM database.

As an alternative to conducting plan-level risk adjustments, for each asset class, we form equal-weighted portfolios that comprise up to 29 years of annual plan-level returns. We refer to this equal-weighted aggregate return for asset class A in year t as $\bar{r}_{At}$ and use it to estimate the following risk-adjustment regression:

$$\bar{r}_{At} - r_{ft} = \alpha_A + \beta'_A \mathrm{F}_{At} + \epsilon_{At}, \tag{D.2}$$

where r_{ft} is the risk-free rate and F_{At} refers to the risk factors used for asset class A. We consider both the Fama and French (1993) three-factor model and the seven-factor model of Fung and Hsieh (2001) which includes the market excess return, a bond trend, currency trend, commodity trend, size spread, bond market and credit spread. The risk factor regressions provide a very good fit for stocks, fixed income, and hedge funds and a somewhat poorer fit for plans' returns in the three remaining alternative asset classes. Appendix D.4 provides further details.

Repeating the earlier analysis from Table 8 on the plan-year risk-adjusted returns, we find results that are broadly in line with those obtained for the policy-adjusted returns. The last four columns in Table 8 shows results for stocks, fixed income, alternative assets and the total portfolio. We find that the largest plans continue to produce significantly higher risk-adjusted returns on the alternative asset classes both on a gross and net of cost basis. Using risk-adjusted returns also leads to higher coefficient estimates on the size variable for fixed income and total portfolio returns.

D.8 Performance in Sub-Asset Classes

To help further pin down the relation between variables such as AUM and investment performance in different markets, we estimate panel regressions of policy-adjusted net returns on individual sub-asset classes. We only consider those sub-asset classes for which we have sufficiently many observations to obtain reasonably precise parameter estimates. Table A.16 presents results for a set of sub-asset classes chosen on the basis that they have at least 1,000 observations.

Our estimates show that a plan's log-AUM is significantly positively related to policy-adjusted net returns for the EAFE, U.S. Large Cap, Global, and Emerging categories but fails to be significantly related to stock investments such as U.S. small cap or ACWI ex U.S. Moreover, the economic effect of AUM can be quite large: the increases in average annual returns associated with moving from a plan in the 10th percentile to a plan in the 90th percentile of holdings in a given sub-asset class are 50 bps (EAFE), 54 bps (U.S. Large Cap), 67 bps (Global), and 62 bps (Emerging).

Examining the log-AUM coefficient estimates more closely, we see that they are bigger for the net return regressions than for gross returns for all sub asset classes with exception of U.S. Small caps. This suggests that the largest plans' better performance in these sub-asset classes, as compared to their smaller peers, is, at least in part, driven by their ability to reduce costs.

We also find a significant relation between log-AUM and policy-adjusted *net* returns for fixed income (Canada, Global, Inflation Indexed), hedge funds, private equity (Diversified private equity, Other) and real assets (REITs). The coefficients for Hedge funds, Diversified private equity and Other private equity are very large (0.46, 0.79 and 1.29 respectively), so that moving from a plan in the tenth percentile to a plan in the 90th percentile of the (2019) size distribution in these sub-asset classes is associated with increases in mean returns of 192 bps, 403 bps, and 537 bps, respectively. Once again, the coefficient estimates on log-AUM tend to be notably higher for net returns than for gross returns, consistent with bigger cost savings for the largest plans also in these sub-asset classes.

Although many of the estimates on the private plan dummy are quite large and positive, we only find two instances (U.S. Broad or All stocks and Diversified private equity) for which private plans appear to produce policy-adjusted net returns whose means are significantly different from those of public plans.

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Tables

Panel A: Total number of observations by asset class and year

	Stocks	Fixed Income	Hedge & multi ass.	Private Equity	Private Debt	Real Assets	Total Public	Total Private	Total
1991	122	122	17	69		100	33	90	123
1992	163	162	30	85	2	129	31	132	163
1993	216	216	34	112	2	160	55	164	219
1994	265	267	40	137	6	201	76	192	268
1995	294	297	49	139	9	223	96	201	297
1996	292	295	44	134	10	210	91	204	295
1997	271	272	32	130	10	201	95	177	272
1998	285	285	28	140	12	201	99	186	285
1999	305	305	25	144	16	207	114	191	305
2000	285	286	29	148	17	203	112	174	286
2001	294	294	42	154	18	201	117	177	294
2002	274	274	56	145	18	184	108	166	274
2003	278	278	68	149	21	191	108	170	278
2004	286	286	78	158	28	210	108	178	286
2005	298	299	91	157	37	218	116	183	299
2006	291	291	105	161	36	218	111	180	291
2007	356	358	150	213	43	266	123	235	358
2008	336	339	156	209	41	261	115	224	339
2009	336	337	157	215	36	257	115	222	337
2010	348	348	172	226	45	267	120	228	348
2011	375	376	206	252	56	314	115	262	377
2012	448	447	253	304	80	381	204	246	450
2013	445	445	265	308	97	381	201	247	448
2014	422	421	255	294	103	368	206	218	424
2015	361	362	209	267	109	315	148	215	363
2016	345	347	204	256	113	304	145	202	347
2017	349	352	200	260	144	312	154	198	352
2018	333	336	196	249	152	304	147	189	336
2019	307	310	176	236	159	282	136	174	310

Panel B: Total number of plan count by frequency of observation

# of obs	1	2	3	4	5	6	7	8	9	10
Plan Count	240	124	134	65	54	39	45	59	29	31
# of obs	11	12	13	14	15	16	17	18	19	20
Plan Count	21	24	29	22	22	15	16	17	16	17
# of obs	21	22	23	24	25	26	27	28	29	
Plan Count	16	7	12	18	14	9	15	12	17	

Table A.1: **Number of participants per asset class and year (Panel A) and by frequency of participation (Panel B)**. Panel A presents the total number of observations (plans) per asset class and year. Panel B presents the time series frequency of unique plans in the CEM database.

	2009				2019			
	Stocks	Fixed Income	Other assets	AUM (%)	Stocks	Fixed Income	Other assets	AUM (%)
U.S.	1132.77	801.62	501.33	53.31	1525.22	1236.51	1032.26	36.69
Canada	254.81	203.64	164.88	13.63	497.31	371.9	767.06	15.51
Australia	32.93	29.6	12.42	1.64	77.01	52.13	40.86	1.65
Belgium	0	0	0	0	0	0	0	0
China	0	0	0	0	101.86	48.7	108.67	2.5
Denmark	4.35	15.67	3.29	0.51	0	0	0	0
Emirates	0	0	0	0	0	0	0	0
Finland	30.68	53.16	21.85	2.31	60.18	55.5	50.08	1.6
France	16.02	14.83	1.49	0.71	0	0	0	0
Germany	0	0	0	0	0	0	0	0
Netherlands	146.67	243.74	132.29	11.43	447.34	615.54	349.73	14.27
NewZealand	4.1	5.5	4.24	0.3	17.99	6.51	7.18	0.31
Norway	195.7	146.78	0	7.49	913.32	384.64	50.57	12.98
OtherUSD	0.08	0.04	0.02	0	44.21	25.85	11.5	0.78
SaudiArabia	0	0	0	0	12.13	8.77	7.86	0.28
SouthAfrica	0	0	0	0	15.89	4.51	2.48	0.22
SouthKorea	51.52	199.14	10.81	5.72	228.25	304.15	69.26	5.79
Sweden	37.89	44.48	8.34	1.98	68.65	54.33	34.75	1.52
Switzerland	0	0	0	0	0	0	0	0
UK	29.03	8.1	7.26	0.97	223.84	207.7	177.85	5.89

Table A.2: **AUM allocation by asset class and country in 2009 and 2019**. This table shows total AUM allocated to Stocks, Fixed Income and Other assets in billions of USD for all countries in the CEM database. Other assets bundles the asset classes: Private Equity, Private Debt and Real Assets. AUM (%) denotes the share of total AUM per country, which is defined by $\text{Share}_{At} = \sum_i \text{AUM}_{iAt} / \sum_i \sum_A \text{AUM}_{iAt}$, where AUM_{iAt} denotes total AUM of plans in country i in asset class A in year t.

	Panel A: U.S. Plans						Panel B: Non-U.S. Plans					
	Stocks	Fixed Income	Hedge & multi ass.	Private Equity	Private Debt	Real Assets	Stocks	Fixed Income	Hedge & multi ass.	Private Equity	Private Debt	Real Assets
1991	53.63	38.76	0.73	1.91		4.98	35.81	57.64	0.43	1.14		4.97
1992	54.09	37.91	1.33	1.82	0.09	4.76	36.86	56.32	0.31	1.69	0.49	4.34
1993	55.83	35.72	1.72	2.54	0.07	4.12	39.85	52.71	0.29	2.25	0.82	4.08
1994	54.25	37.03	1.42	2.79	0.07	4.43	41.47	47.38	0.27	1.82	3.68	5.38
1995	53.80	37.48	1.76	2.46	0.05	4.44	45.47	44.68	0.37	0.72	3.25	5.51
1996	56.46	35.30	1.18	2.64	0.03	4.39	50.43	39.61	0.42	1.20	2.73	5.61
1997	58.61	33.34	1.04	2.76	0.01	4.24	52.15	38.53	0.60	1.73	1.75	5.26
1998	60.32	31.76	1.11	2.68	0.01	4.13	52.25	38.00	0.62	1.96	1.75	5.43
1999	63.06	29.59	0.83	2.78	0.01	3.73	50.72	37.43	0.56	2.38	2.89	6.02
2000	61.05	29.75	0.84	4.04	0.02	4.30	49.20	37.69	0.72	3.29	2.82	6.27
2001	59.96	30.48	0.71	4.02	0.03	4.80	50.10	37.72	0.45	2.93	2.08	6.72
2002	58.84	31.11	0.72	3.98	0.03	5.31	50.64	36.64	0.56	2.69	1.49	7.97
2003	60.19	29.80	1.03	3.96	0.07	4.95	49.42	38.72	0.65	2.16	1.80	7.27
2004	62.54	27.58	1.35	3.62	0.11	4.81	50.76	38.96	0.94	1.89	1.24	6.21
2005	61.94	26.78	1.83	3.86	0.49	5.10	49.14	40.93	0.88	1.67	0.82	6.56
2006	60.90	26.28	2.34	4.17	0.74	5.57	47.72	40.35	1.45	2.02	0.73	7.74
2007	56.64	28.24	3.05	5.00	0.74	6.32	47.54	39.21	1.76	2.48	0.87	8.13
2008	48.84	31.93	3.76	7.09	0.66	7.71	41.99	41.88	2.32	3.40	1.09	9.32
2009	46.51	32.91	4.59	7.90	0.66	7.43	38.67	45.62	2.16	3.49	0.97	9.08
2010	48.79	31.21	4.43	8.15	0.66	6.76	42.63	41.83	1.96	3.84	0.87	8.86
2011	46.38	31.16	5.02	9.06	0.65	7.73	40.31	41.54	2.50	4.18	0.80	10.68
2012	44.61	31.59	5.36	9.36	0.73	8.35	40.45	40.85	2.94	4.16	0.84	10.75
2013	46.58	29.57	5.61	8.78	0.96	8.50	43.34	37.83	3.26	3.94	0.81	10.83
2014	46.33	29.68	5.92	8.40	1.07	8.61	43.99	36.65	3.85	4.09	0.74	10.68
2015	45.21	29.47	6.13	8.48	1.29	9.42	43.09	36.00	4.12	4.30	0.89	11.60
2016	44.00	29.51	6.05	8.61	1.48	10.36	42.32	33.82	4.30	5.21	1.14	13.21
2017	44.17	30.43	5.75	8.09	1.62	9.94	43.41	33.10	3.79	5.08	1.38	13.24
2018	41.99	31.36	6.12	8.61	1.81	10.10	41.37	32.70	3.97	5.90	1.55	14.52
2019	40.20	32.59	5.86	8.93	2.15	10.27	41.08	31.86	3.85	6.52	1.97	14.72

Table A.3: **Aggregate Asset Allocation for U.S. (Panel A) and non-U.S. (Panel B) plans**. This table shows the share of total AUM dedicated to each of the six asset classes during each of the years in our sample: $\omega_{At} = \sum_i \text{AUM}_{iAt} / \sum_i \sum_A \text{AUM}_{iAt}$, where i indicates plans, t indicates year, A indicates the asset class, estimated separately for U.S. and non-U.S. plans.

	Small Plans (in %)				Large Plans (in %)			
Stocks	IP	EP	IA	EA	IP	EP	IA	EA
ACWI x. U.S.		39.93		60.07		16.78	7.26	75.96
EAFE		8.97		91.03	18.06	15.95	11.25	54.74
Emerging		27.82		72.18	10.76	4.82	12.57	71.85
Global		11.41		88.59	7.29	3.16	55.39	34.16
Other	7.79	92.21			16.94	0.76	39.98	42.32
U.S. Broad	0.89	45.84		53.27	24.48	30.76	12.74	32.02
U.S. Large Cap		29.26		70.74	31.95	18.50	13.92	35.62
U.S. Mid Cap								
U.S. Small Cap		25.04		74.96	13.19	14.65	7.73	64.43
Fixed Income								
Bundled LDI								
Cash			12.51	87.49			42.89	57.11
Convertibles								100.00
EAFE						13.05		86.95
Emerging				100.00			25.74	74.26
Global				100.00	1.60	0.51	77.56	20.33
High Yield				100.00	1.32	0.01	8.48	90.19
Inflation Indexed	39.55	42.23	1.71	16.51	30.87	5.74	35.43	27.97
Long Bonds	1.04	49.66	2.18	47.12	7.08	2.23	15.00	75.69
Other		2.52		97.48	80.74	0.23	8.93	10.10
U.S.		17.73		82.27	2.32	6.13	41.47	50.09
Hedge & multi ass.								
Funded TAA				100.00			1.08	98.92
Hedge Fund				100.00				100.00
Risk Parity								100.00
Private Equity								
Div. Private Eq.				100			7.57	92.43
LBO							0.76	99.24
Other							38.68	61.32
Venture Capital				100			0.08	99.92
Private Debt								
Mortgages				100			87.10	12.90
Credit							28.87	71.13
Real Assets								
Commodities				100.00	11.86	8.27	40.13	39.74
Infrastructure				100.00			64.60	35.40
Nat. Resource				100.00			15.76	84.24
Other			11.52	88.48			14.93	85.07
Real Estate			6.55	93.45			29.96	70.04
REIT				100.00	5.22	1.55	48.25	44.98

Table A.4: **Small and large plans' investment allocation by sub-asset class and management structure in 2009**. This table shows the share (in %) of AUM allocated to the four management mandates: Internal Passive (IP), External Passive (EP), Internal Active (IA), and External Active (EA) for the given sub-asset classes. The share is calculated as follows: $\omega_{ats} = \frac{\text{AUM}_{ats}}{\text{AUM}_{at}}$, where $\text{AUM}_{ats} = \sum_i \text{AUM}_{iats}$, and $\text{AUM}_{at} = \sum_s \sum_i \text{AUM}_{iats}$, where i denotes plan i, a indicates the sub-asset class, t denotes the year 2009, and s denotes one of the four mandates. The shares are calculated separately for small and large plans, defined by the bottom and top 30th percentile of AUM in 2009 respectively. For small and large plans, rows sum up to 100%.

Sub-Asset Class	EA&EP	EA&IA	EA&IP	IA&IP	EP&IP	EP&IA	EA	EP	IA	IP
Stocks										
ACWI X U.S.	47.62 (10)	4.76 (1)	14.29 (3)				28.57 (6)	4.76 (1)		
EAFE	35.71 (10)	10.71 (3)	7.14 (2)				35.71 (10)	7.14 (2)	3.57 (1)	
Emerging	31.15 (19)	8.20 (5)	6.56 (4)	1.64 (1)			42.62 (26)	4.92 (3)	3.28 (2)	1.64 (1)
Global	20.37 (11)	24.07 (13)	1.85 (1)	1.85 (1)			27.78 (15)	16.67 (9)	7.41 (4)	
Other		9.09 (2)					50.00 (11)	4.55 (1)	18.18 (4)	18.18 (4)
U.S. Broad	31.58 (12)	5.26 (2)	5.26 (2)		5.26 (2)		23.68 (9)	18.42 (7)	10.53 (4)	
U.S. Large Cap	23.08 (6)	7.69 (2)	15.38 (4)	7.69 (2)			15.38 (4)	19.23 (5)	3.85 (1)	7.69 (2)
U.S. Mid Cap			25.00 (2)				37.50 (3)	12.50 (1)	12.50 (1)	12.50 (1)
U.S. SmallCap	14.29 (4)	14.29 (4)	10.71 (3)	7.14 (2)			50.00 (14)			3.57 (1)
Fixed Income										
Bundled LDI				16.67 (1)			33.33 (2)	50.00 (3)		
Cash		16.44 (12)					39.73 (29)		43.84 (32)	
Emerging	4.88 (2)	24.39 (10)	4.88 (2)	4.88 (2)			51.22 (21)	7.32 (3)	2.44 (1)	
Global		11.11 (3)	3.70 (1)	7.41 (2)	3.70 (1)		29.63 (8)	7.41 (2)	37.04 (10)	
High Yield	2.27 (1)	20.45 (9)					75.00 (33)		2.27 (1)	
Inflation Index	9.76 (4)		2.44 (1)	2.44 (1)		4.88 (2)	9.76 (4)	12.20 (5)	34.15 (14)	24.39 (10)
Long	3.57 (1)	10.71 (3)					50.00 (14)	3.57 (1)	7.14 (2)	25.00 (7)
Other		9.38 (3)	3.13 (1)				62.50 (20)	6.25 (2)	15.63 (5)	3.13 (1)
U.S.	17.07 (7)	9.76 (4)	2.44 (1)	2.44 (1)		2.44 (1)	41.46 (17)	2.44 (1)	19.51 (8)	2.44 (1)
Hedge & Multi ass.										
Funded TAA		14.29 (3)					66.67 (14)		19.05 (4)	
Risk Parity		23.08 (3)					76.92 (10)			
Private Equity										
LBO		4.00 (1)					96.00 (24)			
Other		17.39 (4)					69.57 (16)		13.04 (3)	
VC		10.00 (3)					90.00 (27)			
Div. PE		20.99 (17)					79.01 (64)			
Private Debt										
Private Credit		22.81 (13)					73.68 (42)		3.51 (2)	
Real Assets										
Commodities	5.56 (1)	16.67 (3)	5.56 (1)	11.11 (2)			50.00 (9)		5.56 (1)	5.56 (1)
Infrastructure		36.67 (22)					60 (36)		3.33 (2)	
Nat. Resource		29.73 (11)					70.27 (26)			
Other		5.56 (1)					83.33 (15)		11.11 (2)	
Real Estate		36.67 (33)					58.89 (53)		4.44 (4)	
REITs	10.00 (3)	10.00 (3)	3.33 (1)				40.00 (12)	13.33 (4)	13.33 (4)	10.00 (3)

Table A.5: **Plans' relative allocation to multiple investment mandates**. This table shows the 2019 allocation share to different pairs of management mandates for large plans that utilize more than one management style within the same sub-asset class. Large plans belong to the top 30th percentile by AUM. The total number of plans are indicated in parentheses and rows sum to 100%.

		Stocks			Fixed income			Hedge & multi ass.		
Year	Style	Mode	Mean	Avg. AUM	Mode	Mean	Avg. AUM	Mode	Mean	Avg. AUM
1999	IA	1	1.70	2841.78	1	1.74	1998.31	1	1.00	911.38
	EA	3	2.76	727.69	2	2.00	558.10	1	1.00	662.12
2009	IA	1	2.20	3597.23	1	2.04	3008.48	1	1.00	69.89
	EA	3	3.40	816.36	2	2.51	911.88	1	1.46	728.44
2019	IA	1	2.40	8865.01	1	2.24	4686.10	1	1.07	2723.24
	EA	4	3.18	1552.97	2	2.95	1415.54	1	1.61	1487.49

		Private equity			Private credit			Real assets		
Year	Style	Mode	Mean	Avg. AUM	Mode	Mean	Avg. AUM	Mode	Mean	Avg. AUM
1999	IA	1	1.05	459.16	1	1.00	611.12	1	1.19	512.69
	EA	1	1.20	374.62	1	1.00	1751.71	1	1.18	400.05
2009	IA	1	1.09	733.78	1	1.00	2492.83	1	1.45	1605.56
	EA	1	1.72	688.94	1	1.00	323.78	1	1.96	532.30
2019	IA	1	1.29	2930.15	1	1.13	2581.10	1	1.79	4728.25
	EA	1	2.13	1270.12	1	1.38	606.23	2	2.96	836.72

Table A.6: **Frequency of internal and external active management.** This table shows the mode and the mean of how often each plan employs internal (IA) –and external active (EA) management for sub-asset classes in a given asset class for the years 1999, 2009 and 2019. The mode and the mean are calculated across plans within a given year and asset class. Avg. AUM denotes the average AUM (in millions U.S. dollar) allocated to IA or EA management within each asset class.

	Fractional Regressions: Average Partial Effect					
	Stocks	Fixed income	Hedge & multi ass.	Private equity	Private debt	Real assets
	Panel A: Equation (B.1) excluding `LiabilityRetired`					
$\log(\text{AUM}_{it-1})$	**-2.97**	**-0.30**	**-1.44**	**0.78**	**-0.16**	**0.93**
	(0.106)	(0.086)	(0.120)	(0.045)	(0.064)	(0.043)
Cost_{it-1}	**-0.09**	**-0.01**	**0.05**	**0.02**	0.00	**0.02**
	(0.012)	(0.006)	(0.008)	(0.004)	(0.005)	(0.004)
Obs.	7798	7796	2773	4710	1138	6101
	Panel B: Equation (B.1)					
$\log(\text{AUM}_{it-1})$	**-3.33**	-0.02	**-1.55**	**0.78**	-0.02	**0.94**
	(0.139)	(0.111)	(0.158)	(0.054)	(0.065)	(0.057)
Cost_{it-1}	**-0.10**	-0.02	**0.05**	**0.02**	-0.01	**0.02**
	(0.018)	(0.008)	(0.009)	(0.004)	(0.006)	(0.005)
LiabRetired_{it}	**-7.05**	2.37	**3.67**	**1.00**	1.44	**1.60**
	(1.274)	(1.324)	(1.182)	(0.505)	(0.904)	(0.513)
Obs.	4764	4764	1859	2953	831	3751

Table A.7: **Average partial effects of size and cost on asset allocation**. This table shows average partial effect estimates from the fractional regression where share of asset A in the total AUM is modeled as $\omega_{iAt} = c_A + \lambda_{At} + \beta_{1,A}\log(\text{AUM}_{it-1}) + \beta_{2,A}\text{Cost}_{iAt-1} + \beta_{3,A}\text{Private}_i + \beta_{4,A}\text{nonUS}_i + \beta_{5,A}\text{LiabilityRetiree}_{it} + \epsilon_{iAt}$. ω_{iAt} indicates the plan i's share of total AUM allocated to asset class A at time t, $\log(\text{AUM}_{it-1})$ indicates the lagged log total AUM at time $t-1$. Cost_{iAt-1} is the lagged per-dollar cost for plan i in asset class A measured as a fraction of AUM and denoted in bps. LiabRetired_{is} is the fraction of a plan;s total liabilities owed to retirees. Private_i is a dummy that equals 1 if the plan is private, and Private_i is a dummy that equals 1 if the plan is located outside of the U.S. (NonUS_i). Boldface coefficients are statistically significant at the 5% level. **Panel A** of the table omits `LiabRetired` from the fractional regression and presents APE estimates of lag log size and cost. **Panel B** presents APE estimates of lag log size and cost where `LiabRetired` is included in the regression.

	Panel A: Cragg APE Estimates		
	Stocks	Fixed income	Hedge & multi ass.
$\log(\text{AUM}_{it-1})$	**9.85** (2.588)	**15.24** (2.701)	0.88 (0.835)
$\text{CostSpread}_{iAt-1}$	**29.55** (10.532)	**29.85** (9.185)	0.26 (0.64)
Private_i	-0.1 (4.297)	2.77 (6.74)	-0.48 (1.622)
NonUS_i	**12.06** (4.808)	**17.76** (5.249)	-1.95 (2.126)
LiabRetired_{it}	9.81 (13.935)	1.49 (21.661)	-4.47 (10.333)
Obs	6399	6413	1717
	Panel B: Cragg Selection Estimates		
	Stocks	Fixed income	Hedge & multi ass.
$\log(\text{AUM}_{it-1})$	**36.23** (4.431)	**25.4** (3.613)	**32.72** (9.131)
$\text{CostSpread}_{iAt-1}$	**94.62** (25.411)	23.28 (20.22)	**-13.99** (6.363)
LiabRetired_{it}	18.86 (31.665)	14.79 (27.608)	0.00 (0.00)
	$\Pr(\omega_{iAt}^{internal} > 0 \mid X = x)$		
Plan Size			
10th percentile	12.87	28.04	0.60
50th percentile	32.41	45.73	2.85
90th percentile	62.62	66.94	11.50

Table A.8: **Asset allocation regression for internal vs. external management including** `LiabRetired`. **Panel A** of this table reports the average partial effects as shown in (6.5) of the regression (6.6): $\omega_{iAt}^{internal} = c_A + \lambda_{At} + \beta_{1,A}\log(\text{AUM})_{it-1} + \beta_{2,A}\text{CostSpread}_{iAt-1} + \beta_{3,A}\text{Private}_i + \beta_{4,A}\text{nonUS}_i + \beta_{5,A}\text{LiabRetired}_{it} + \epsilon_{iAt}$. $\omega_{iAt}^{internal}$ indicates the share of AUM that is internally managed by plan i in asset class A at time t ($\text{AUM}_{i,A,t}^{Internal}/(\text{AUM}_{i,A,t}^{Internal} + \text{AUM}_{i,A,t}^{External})$), λ_{At} is a time fixed effect, AUM_{it-1} denotes the lagged AUM of plan i, $\text{CostSpread}_{iAt-1}$ is plan i's cost spread between external and internal management in asset class A at time $t-1$, Private_i is a dummy equal to one if plan i is private, and nonUS_i is a dummy equal to one if the plan is domiciled outside the U.S., and LiabRetired_{it} is the percentage of liability due to retirees. In case a plan is fully internal (external) we impute the external (internal) cost as the median cost from plans that are similar in size, where size in a given year is either small (bottom 30th percentile), medium (between 30th and 70th percentile), or large (top 70th percentile) of total AUM. Robust standard errors are reported in parentheses and are clustered by plan. Boldface coefficients are statistically significant at the 5% level. The asset class "Private Debt" does not include time fixed effects due to the small sample size, and estimation for "Hedge Funds" start in 2000 due to lack observation prior to 2000. **Panel B** presents the results of the Cragg selection equation (6.2a). The bottom part of panel B shows the probability of allocating at least some portion of investments internally. We fix the cost spread at the mean cost spread across time and sponsors, and show the different probabilities based on size, using the 10th, 50th, and 90th percentile of AUM in 2019. All coefficients and standard errors are multiplied by 100.

	Stocks	Fixed income	Hedge & multi ass.	Private equity	Real assets
Aum allocation					
p-value: $\mu_1 = \mu_4$	0.000	0.000	0.000	0.000	0.000
p-value MR test	0.021	0.165	0.000	0.000	0.005
Relation	–	–	–	+	+

Table A.9: **Monotonicity test of asset allocation and size**. This table tests the monotonic relation between asset allocation and size for different asset classes. *p-value* $\mu_1 = \mu_4$ tests whether the mean on the first quartile portfolio (smallest plans) equals the mean on the fourth quartile portfolio (largest plans). *p-value MR test* denotes the p-value of the null hypothesis that $\min(\mu_i - \mu_{i-1}) \leq 0$ (positive relation) or $\min(\mu_{i-1} - \mu_i) \leq 0$ (negative relation). *Relation* signifies whether we test for a positive ("+") or negative ("–") monotonic relation. Portfolios are constructed as follows: we sort plans into quartiles based on size and use an equal weighted average of plans within a quartile and asset class. For a given year, we only include plans that also show up in the next's year database. At the end of the next year, the AUM allocation is calculated for each of the portfolios.

	External	Active	Private	nonUS	Obs	R^2
Stocks						
Canada	**0.13**	**0.17**	0.02		2615	0.25
	(0.014)	(0.016)	(0.011)			
EAFE	**0.29**	**0.41**	**0.05**	0.01	5769	0.24
	(0.028)	(0.015)	(0.014)	(0.017)		
U.S. Broad or All	**0.19**	**0.33**	**0.04**	0.01	5421	0.38
	(0.018)	(0.013)	(0.010)	(0.011)		
U.S. Large Cap	**0.13**	**0.31**	**0.04**	0.00	2509	0.39
	(0.026)	(0.017)	(0.014)	(0.014)		
U.S. Small Cap	**0.35**	**0.53**	**0.15**	-0.25	3288	0.01
	(0.065)	(0.043)	(0.064)	(0.414)		
Global	**0.29**	**0.41**	**0.05**	-0.01	2865	0.28
	(0.037)	(0.023)	(0.018)	(0.019)		
Emerging	**0.39**	**0.51**	**0.08**	**-0.05**	3778	0.28
	(0.040)	(0.024)	(0.021)	(0.022)		
ACWI x U.S.	**0.32**	**0.46**	**0.09**		1215	0.49
	(0.087)	(0.020)	(0.020)			
Fixed Income						
Canada	**0.10**	**0.10**	**0.02**		2326	0.40
	(0.009)	(0.010)	(0.008)			
Cash	**0.05**		-0.11	0.12	5374	0.00
	(0.008)		(0.175)	(0.206)		
U.S.	**0.15**	**0.13**	**0.05**	0.05	4407	0.10
	(0.011)	(0.014)	(0.010)	(0.029)		
Other	**0.41**	**0.22**	0.03	-0.05	1383	0.10
	(0.038)	(0.050)	(0.054)	(0.049)		
Long Bonds	**0.07**	**0.11**	0.01	**-0.02**	1651	0.36
	(0.012)	(0.009)	(0.010)	(0.009)		
Global	**0.25**	**0.23**	**0.06**	-0.03	1124	0.20
	(0.026)	(0.035)	(0.027)	(0.024)		
Inflation Indexed	**0.08**	**0.09**	0.02	-0.02	1870	0.21
	(0.010)	(0.010)	(0.011)	(0.011)		
High Yield	**0.27**	**0.24**	0.01	0.04	2006	0.04
	(0.055)	(0.070)	(0.023)	(0.027)		
Emerging	**0.45**	**0.38**	0.04	0.03	1301	0.15
	(0.060)	(0.056)	(0.040)	(0.047)		
Hedge & Multi-Asset						
Funded TAA	**0.60**		-0.03	0.14	1262	0.01
	(0.118)		(0.185)	(0.196)		
Hedge Funds			0.12	0.09	2630	0.00
			(0.083)	(0.083)		
Private Equity						
Diversified Private Equity	**5.28**		**-0.70**	0.52	4680	0.02
	(0.321)		(0.305)	(0.338)		
Other Private Equity	**3.33**		-0.90	0.35	1347	0.03
	(0.750)		(0.532)	(0.736)		
Real Assets						
Real Estate ex-REITs	**1.22**		-0.10	**-0.25**	6425	0.08
	(0.077)		(0.071)	(0.084)		
REITs	**0.40**	**0.31**	0.04	**-0.11**	1827	0.13
	(0.028)	(0.035)	(0.033)	(0.032)		
Infrastructure	**2.68**		-0.58	**-1.35**	1582	0.02
	(0.248)		(0.427)	(0.655)		

Table A.10: **Regression of cost on plan characteristics.** This table shows regression estimates of the model: $\text{Cost}_{iat} = c_a + \beta_{1,a}\text{External}_{iat} + \beta_{2,a}\text{Active}_{iat} + \beta_{3,a}\text{Private}_i + \beta_{4,a}\text{nonUS}_i + \varepsilon_{iat}$, where Cost_{iat} is the cost (in bps) of plan i in sub-asset class a at time t, External_{iat} (Active_{iat}) is a dummy equal to one if plan i manages sub-asset class a externally (actively) at time t, Private_i is a dummy equal to one if plan i is private, and nonUS_i is a dummy equal to one if the plan is domiciled outside the U.S. We only keep those sub-asset classes that have 1,000 observations or more. Robust standard errors are clustered by sponsor and reported in parentheses. Boldface coefficients are significant at the 5% level.

	$\log(\text{AUM}_{iats})$	Private_i	nonUS_i	Obs	R^2
EAFE (Stocks)					
IP	**0.74** (0.054)	0.33 (0.299)	0.51 (0.260)	956	0.69
EP	**0.76** (0.033)	0.05 (0.121)	-0.14 (0.148)	3999	0.67
IA	0.94 (0.062)	0.29 (0.330)	**0.48** (0.215)	1049	0.68
EA	**0.90** (0.010)	**0.06** (0.026)	**-0.08** (0.029)	10503	0.93
U.S. Broad/All (Stocks)					
IP	**0.77** (0.045)	0.26 (0.208)	**0.93** (0.191)	1780	0.74
EP	**0.75** (0.026)	0.07 (0.070)	**0.47** (0.082)	6899	0.69
IA	**0.87** (0.039)	**0.50** (0.183)	**0.63** (0.169)	2088	0.67
EA	**0.95** (0.015)	**0.13** (0.044)	-0.09 (0.054)	8172	0.88
Inflation Indexed (Fixed Income)					
IP	0.94 (0.079)	-0.44 (0.404)	**0.85** (0.334)	1072	0.72
EP	**0.78** (0.052)	0.19 (0.150)	**0.33** (0.146)	1384	0.66
IA	**0.76** (0.056)	-0.07 (0.234)	**0.72** (0.230)	954	0.67
EA	**0.88** (0.049)	0.01 (0.199)	0.05 (0.221)	1107	0.39
Diversified Private Equity					
IA	1.07 (0.038)	**0.54** (0.186)	**0.64** (0.216)	682	0.83
EA	**0.93** (0.008)	**-0.16** (0.029)	-0.04 (0.034)	5042	0.96
Other Private Equity					
IA	0.91 (0.063)	0.04 (0.260)	0.19 (0.319)	578	0.72
EA	**0.96** (0.019)	-0.10 (0.058)	**-0.14** (0.066)	1189	0.93
Mortgages (Private Debt)					
EA	1.01 (0.048)	0.20 (0.229)	-0.32 (0.235)	658	0.80
Real Estate ex-REITs (Real Assets)					
IA	1.04 (0.042)	-0.23 (0.189)	0.06 (0.133)	1945	0.74
EA	**0.95** (0.010)	**-0.10** (0.037)	**-0.20** (0.039)	7129	0.94
REITs (Real Assets)					
IA	0.97 (0.056)	0.40 (0.301)	0.45 (0.233)	606	0.78
EA	**0.88** (0.024)	0.01 (0.079)	**-0.32** (0.075)	1734	0.80

Table A.11: **Economies of scale at the sub-asset class level**. This table shows estimates of the model: $\log(\text{Cost}^{\$}_{iats}) = c_{as} + \lambda_{ats} + \beta_{1,as}\log(\text{AUM}_{iats}) + \beta_{2,as}\text{Private}_i + \beta_{3,as}\text{nonUS}_i + \varepsilon_{iats}$, where $\text{Cost}^{\$}_{iats}$ is the (dollar) cost of plan i in sub-asset class a at time t for asset mandate s, c_{as} is a constant that varies with sub-asset class a and mandate s, λ_{ats} is the time fixed effect for sub-asset class a in investment management mandate s, $\log(\text{AUM}_{iats})$ is the log of total AUM of plan i in sub-asset class a at time t for mandate s, Private_i is a dummy equal to one if plan i is private and nonUS_i is a dummy equal to one if plan i is located outside the US. For stock and fixed income, we estimate the panel separately for the following styles s: Internal Passive (IP), Internal Active (IA), External Passive (EP) and External Active (EA). Robust standard errors are clustered by plan. The boldface coefficients on log(AUM) are significantly different from one at the 5% level and boldface coefficients on the other covariates are significantly different from zero. We only include sub-asset classes that have more than 400 observations.

Country	2009						2019					
	Stock	Fixed Income	Hedge & Multi Ass.	Private Equity	Private Credit	Real Asset	Stock	Fixed Income	Hedge & Multi Ass.	Private Equity	Private Credit	Real Asset
U.S.	0.79	0.48	5.36	13.47	2.40	3.28	0.67	0.44	4.29	9.34	4.85	3.68
Canada	0.73	0.28	5.69	14.29	0.62	2.38	0.76	0.31	3.84	9.26	3.15	3.12
Australia	0.67	0.21	6.47	12.12		2.68	0.25	0.22	2.34	7.22	1.96	1.54
China							0.34	0.30	3.19	7.65	4.73	1.97
Denmark	0.82	0.17	4.78	9.85		1.90						
Finland	0.52	0.16	6.86	11.78		2.37	0.64	0.47	6.15	9.48	1.96	2.36
France	0.38	0.24		31.61		0.24						
Netherlands	0.56	0.33	5.26	11.14	0.48	2.25	0.36	0.30	5.01	10.03	0.97	2.15
New Zealand	0.95	0.20	5.01	14.04		1.65	0.59	0.57	2.12	6.34		1.90
Norway	0.24	0.13					0.09	0.11				0.21
OtherUSD	1.14	1.16	8.36	13.99		1.94	1.04	0.77	4.16	9.13	7.27	3.13
Saudi Arabia							0.73	0.31	3.87	9.34	4.76	4.51
South Africa							0.72	0.16	5.38	4.71		1.35
South Korea	0.41	0.13	0.00	95.45		0.72	0.35	0.04	6.22	3.79		2.01
Sweden	0.29	0.13	3.76	12.32		0.92	0.37	0.09	3.31	10.10	2.94	1.27
UK	0.22	0.14	5.72	11.74		3.44	0.54	0.42	3.07	10.50	3.38	2.80

Table A.12: **Average scaled investment management costs by asset class and country in 2009 and 2019**. This table shows the average investment management costs measured relative to the grand average cost. The grand average cost ($\overline{\text{Cost}}$) is calculated as $\overline{\text{Cost}} = \frac{1}{N|A|(T-t+1)} \sum_{i=1}^{N} \sum_{A=1}^{|A|} \sum_{t=1991}^{T} \text{Cost}_{iAt}$, where i indicates plan sponsors, A indicates asset class, t indicates year. We calculate the scaled cost ($\overline{\text{Cost}}_{At}$) separately for each country, asset class, and time period as $\overline{\text{Cost}}_{At} = \frac{1}{N} \sum_{i=1}^{N} \text{Cost}_{iAt} / \overline{\text{Cost}}$.

Panel A: Summary Statistics						
	All		Passive		Active	
	Mean	Sharpe	Mean	Sharpe	Mean	Sharpe
Stocks	0.108	0.525	0.100	0.459	0.108	0.500
Fixed Income	0.069	0.669	0.067	0.596	0.065	0.572
Hedge & multi ass.	0.071	0.546	0.129	1.263	0.068	0.450
Private Equity	0.159	0.631			0.156	0.644
Private debt	0.077	0.636			0.078	0.553
Real Asssets	0.084	0.537	0.054	0.220	0.086	0.531

Panel B: Correlation Matrix						
	Stocks	Fixed Income	Hedge & multi ass.	Private Equity	Private Debt	Real Assets
Stocks	0.169					
Fixed Income	0.277	0.074				
Hedge & multi ass.	**0.858**	**0.548**	0.124			
Private Equity	**0.413**	-0.050	**0.382**	0.216		
Private Debt	0.306	**0.598**	**0.528**	-0.006	0.115	
Real Assets	0.202	-0.162	0.124	**0.528**	0.204	0.127

Table A.13: **Summary statistics for asset class returns**. This table reports summary measures for returns on the six asset classes. Panel A presents summary statistics on the mean and Sharpe ratio. Mean returns are computed as the average return of the asset class across years and plan sponsors: $\bar{r}_A = \frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N} r_{iAt}$, where r_{iAt} is the gross return of plan i in asset class A at time t. Standard deviations of returns, reported on the diagonal in Panel B, are computed as follows: $\sqrt{(1/T)\sum_{t=1}^{T}(\bar{r}_{At} - \bar{r}_A)^2}$, where $\bar{r}_{At} = (1/N)\sum_{i=1}^{N} r_{iAt}$. The Sharpe Ratio is computed as the ratio of the mean excess return over the standard deviation of excess returns. In panel A, summary statistics are reported separately for all plans ("All"), and for passively managed assets ("Passive") and actively managed assets ("Active"). Because all Private Equity and Private Debt assets are actively managed, we do not provide any summary statistics for them in the "Passive" subheading. The asset class "Hedge & multi ass." includes hedge funds and multi-assets, hence also includes passively managed assets. The lower triangle of Panel B presents pairwise correlations between mean returns across aggregate asset classes. Boldface correlations are statistically significant at the 5% level.

Public asset classes

	Stocks			Fixed Income	
Factors	Portfolio	Pooled	Factors	Portfolio	Pooled
α	-0.69	**-0.01**	α	**0.65**	**0.01**
	(0.812)	(0.001)		(0.235)	(0.001)
Market	**0.95**	**0.95**	Bond Index	**0.78**	**0.79**
	(0.047)	(0.004)		(0.081)	(0.040)
SMB	**0.12**	**0.13**	Corp. Index	**0.44**	**0.42**
	(0.045)	(0.008)		(0.095)	(0.033)
HML	0.02	**0.02**	High Yield Index	0.04	**0.05**
	(0.048)	(0.006)		(0.044)	(0.015)
			Global Div. Index	0.01	**0.01**
				(0.011)	(0.005)
			Long Treasury	**0.24**	**0.23**
				(0.034)	(0.023)
R^2	0.95	0.92		0.97	0.58
Obs	29	4860		26	4617

Alternative asset classes

	Hedge & Multi Ass.		Private Equity		Private Credit		Real Assets	
Factors	Portfolio	Pooled	Portfolio	Pooled	Portfolio	Pooled	Portfolio	Pooled
α	**-9.01**	**-0.10**	2.60	0.02	5.46	**0.08**	2.44	**0.03**
	(2.894)	(0.012)	(3.217)	(0.010)	(3.624)	(0.010)	(3.502)	(0.006)
Market	**0.47**	**0.46**	**0.19**	**0.21**	**0.22**	**0.11**	0.15	**0.14**
	(0.035)	(0.017)	(0.073)	(0.021)	(0.072)	(0.027)	(0.083)	(0.014)
Market_{t-1}			**0.27**	**0.27**	0.09	0.02	**0.27**	**0.24**
			(0.048)	(0.017)	(0.124)	(0.043)	(0.130)	(0.019)
SizeSpread	**-0.17**	**-0.16**	-0.06	**-0.11**	-0.20	**0.15**	0.20	**0.14**
	(0.072)	(0.027)	(0.179)	(0.035)	(0.219)	(0.043)	(0.117)	(0.026)
SizeSpread_{t-1}			**-0.30**	**-0.18**	0.15	**0.21**	**0.26**	**0.24**
			(0.083)	(0.055)	(0.096)	(0.041)	(0.102)	(0.020)
BondMarket	**0.11**	**0.13**	**0.88**	**0.62**	-0.49	**-0.16**	0.01	0.06
	(0.030)	(0.012)	(0.273)	(0.067)	(0.297)	(0.068)	(0.268)	(0.037)
BondMarket_{t-1}			**-0.73**	**-0.46**	0.43	0.07	0.00	-0.06
			(0.270)	(0.057)	(0.303)	(0.060)	(0.251)	(0.035)
Bond trend	**-0.03**	**-0.03**						
	(0.008)	(0.003)						
Currency trend	0.02	**0.01**						
	(0.010)	(0.004)						
Commodity trend	0.00	0.00						
	(0.011)	(0.004)						
CreditSpread	0.11	**0.12**						
	(0.073)	(0.027)						
R^2	0.92	0.58	0.74	0.22	0.40	0.16	0.44	0.15
Obs	26	1801	25	2522	25	353	25	3105

Table A.14: **Regression of net returns on risk factors**. This table shows estimates of alphas and betas (factor loadings) from regressions of U.S. plans' annual average net returns on risk factors for different asset classes (see Equation (D.2) in the main text). In the column "Portfolio", the returns are constructed as an equally weighted average over the individual plans' net returns in a specific year and asset class. In the column "pooled", returns are pooled across all U.S. plans. Cluster robust standard errors are reported in parentheses for pooled regression and robust standard errors are reported in parentheses for the portfolio regressions. Boldface coefficients are significant at the 5% level.

	Stocks		Fixed Income	
	α	β	α	β
Min.	-0.0367	-0.3373	-0.0214	-1.3565
1st Qu.	-0.0010	-0.0234	-0.0013	-0.0929
Median	0.0034	0.0031	0.0020	0.0015
Mean	0.0038	-0.0003	0.0029	-0.0042
3rd Qu.	0.0082	0.0252	0.0064	0.1038
Max.	0.0319	0.2257	0.0248	1.4038
# of Plans	199		203	

Table A.15: **Regressions of policy-adjusted gross returns on a single risk factor.** This table shows summary statistics of plan-level policy-adjusted gross returns regressed on a single factor (see (D.1)), where α denotes a plan's "alpha" and β denotes the factor loading. For stocks we use the excess market return factor, and for fixed income we use the U.S. aggregate bond index factor. We require plans to have at least 10 years of observations to be included in the regression and only consider U.S. plans.

	Gross	**Net**						
Stocks	log(AUM)	log(AUM)	External	Active	Private	nonUS	Obs	R^2
Canada	-0.07	-0.04	-0.02	**1.59**	0.20		2568	0.35
	(0.079)	(0.078)	(0.336)	(0.294)	(0.174)			
EAFE	0.08	**0.12**	-0.12	**0.46**	0.00	-0.31	5571	0.20
	(0.053)	(0.052)	(0.587)	(0.220)	(0.162)	(0.186)		
U.S. Broad or All	0.12	0.13	0.24	-0.34	**0.28**	-0.11	5217	0.11
	(0.074)	(0.074)	(0.350)	(0.188)	(0.137)	(0.217)		
U.S. Large Cap	0.11	**0.13**	0.29	-0.18	0.22	**0.45**	2439	0.09
	(0.060)	(0.060)	(0.200)	(0.178)	(0.123)	(0.220)		
U.S. Small Cap	**0.28**	0.18	-0.64	0.52	0.26	0.86	3142	0.10
	(0.113)	(0.180)	(0.772)	(0.311)	(0.316)	(0.481)		
Global	**0.11**	**0.16**	-0.04	**0.61**	0.13	-0.29	2714	0.07
	(0.055)	(0.055)	(0.315)	(0.259)	(0.228)	(0.256)		
Emerging	0.10	**0.15**	0.13	**0.86**	-0.21	-0.25	3568	0.08
	(0.056)	(0.056)	(0.378)	(0.234)	(0.213)	(0.197)		
ACWI x U.S.	0.14	0.20	0.25	**0.94**	0.12		1173	0.20
	(0.122)	(0.123)	(0.609)	(0.324)	(0.335)			
Fixed Income								
Canada	0.03	**0.05**	0.02	0.10	-0.05		2270	0.12
	(0.026)	(0.025)	(0.089)	(0.164)	(0.078)			
Cash	-0.02	0.01	-0.17		0.07	**-0.40**	5979	0.01
	(0.043)	(0.045)	(0.188)		(0.114)	(0.180)		
U.S.	-0.03	-0.01	0.06	**0.46**	-0.04	0.46	4278	0.30
	(0.052)	(0.051)	(0.184)	(0.188)	(0.107)	(0.404)		
Other	-0.40	-0.37	-3.18	0.41	-1.14	-2.19	1199	0.03
	(0.448)	(0.448)	(2.305)	(0.910)	(1.261)	(1.634)		
Long Bonds	0.06	0.08	0.16	**0.42**	-0.09	**-0.38**	1594	0.05
	(0.056)	(0.056)	(0.234)	(0.182)	(0.201)	(0.156)		
Global	**0.27**	**0.30**	0.58	0.37	0.51	-0.52	1036	0.15
	(0.097)	(0.098)	(0.462)	(0.653)	(0.430)	(0.421)		
Inflation Indexed	**0.13**	**0.14**	0.20	0.22	-0.11	0.16	1754	0.03
	(0.062)	(0.062)	(0.184)	(0.209)	(0.227)	(0.185)		
High Yield	0.15	0.17	-0.24	-0.21	-0.05	0.43	1897	0.22
	(0.096)	(0.095)	(0.522)	(0.501)	(0.242)	(0.307)		
Emerging	-0.13	-0.05	**-0.79**	0.07	0.30	**-0.54**	1226	0.26
	(0.105)	(0.109)	(0.335)	(0.360)	(0.244)	(0.237)		
Hedge & multi ass.								
Funded TAA	-0.10	0.08	-1.97		-0.45	0.07	1123	0.14
	(0.415)	(0.288)	(1.126)		(0.711)	(0.709)		
Hedge Funds	**0.37**	**0.46**			0.48	-0.40	2406	0.17
	(0.101)	(0.099)			(0.402)	(0.395)		
Private Equity								
Diversified	0.31	**0.79**	2.08		**1.22**	**2.57**	4212	0.22
	(0.215)	(0.190)	(1.614)		(0.620)	(0.609)		
Other	**1.22**	**1.29**	0.21		4.21	3.45	1176	0.07
	(0.583)	(0.579)	(3.661)		(3.067)	(2.078)		
Real Assets								
Real Estate ex-REITs	0.19	0.29	**-1.16**		0.49	**0.72**	6076	0.07
	(0.157)	(0.159)	(0.420)		(0.356)	(0.292)		
REITs	0.22	**0.27**	**1.52**	-0.12	0.50	0.27	1688	0.06
	(0.130)	(0.131)	(0.465)	(0.531)	(0.389)	(0.350)		
Infrastructure	0.05	0.56	1.84		0.92	0.79	1443	0.11
	(0.383)	(0.378)	(1.279)		(0.816)	(0.813)		

Table A.16: **Regression of sub-asset class returns on plan characteristics.** This table shows estimates of the model: $\widetilde{r}_{iat} = \lambda_{at} + \beta_{1,a}\log(\text{AUM}_{iat-1}) + \beta_{2,a}\text{Active}_{iat} + \beta_{3,a}\text{External}_{iat} + \beta_{4,a}\text{Private}_i + \beta_{5,a}\text{nonUS}_i + \varepsilon_{iat}$, where $\widetilde{r}_{iat}$ denotes the policy-adjusted **net** return, λ_{at} is a time fixed effect, AUM_{iat-1} is plan i's total AUM allocated to sub-asset class a at time $t-1$, External_{iat} and Active_{iat} are dummies equal to one if the plan manages a external or active, Private_i is a dummy equal to one if plan i is private, and nonUS_i is a dummy equal to one if plan i is domiciled outside the U.S. For comparison, the first column reports results when running the same regression using **gross** returns. We keep only those sub-asset classes that have 1,000 observations or more. Robust standard errors are clustered by sponsor and reported in parentheses. Boldface coefficients are significant at the 5% level.

Figures

Figure A.1: **Asset allocation over time**: This figure shows the share of total AUM allocated to each of the six asset classes within a year. The shares are reported separately for U.S. plans (top panel) and non-U.S. plans (bottom panel).

Figure A.2: **Sub-asset class allocation over time for U.S. plans.** This figure shows the share of total AUM allocated to each sub-asset class for a given year and asset class for U.S. plans only.

(a) U.S. Plans

(b) Non-U.S. Plans

Figure A.3: **Total AUM by asset class and year for U.S. and non-U.S. plans**. This figure presents total AUM (in billion dollars) allocated to stocks, fixed income, hedge fund and multi assets, private equity, private debt, and real assets for U.S. and non-U.S. plans. Total AUM is defined as $\text{AUM}_{At} = \sum_i \text{AUM}_{iAt}$, where AUM_{iAt} indicates the AUM of plan i in asset class A at time t.

(a) Stocks

(b) Fixed Income

(c) Hedge & Multi-Asset

(d) Private Equity

(e) Private Debt

(f) Real Assets

Figure A.4: **Sub-asset class allocation over time for non-U.S. plans.** This figure shows the share of total AUM allocated to each sub-asset class for a given year and asset class for plans domiciled outside the U.S.

(a) (b)

Figure A.5: **Frequency of internal and external active management in 2019**. This figure shows a histogram of the number of sub-asset class observations by plan for internal active (IA) and external active (EA) management in 2019 for stocks and fixed income.

Figure A.6: **Distribution of allocation to asset classes**. The figure shows the histogram total portfolio shares allocated to different asset classes.

Figure A.7: **Fractional regression estimates of the relation between plan size and AUM allocation**. This figure shows the mean of AUM allocation (ω_{iAt}) conditional on the observed range of $\log(\text{AUM}_{iAt-1})$ for different asset classes, over the sample period 1991–2019. The vertical blue bars indicate the 95 percent confidence interval based on the fractional regression.

Figure A.8: **Median cost by asset management style**. The figure shows a time series plot of the (scaled) median cost across plans by asset management style for the public asset classes, and the pooled private asset classes ("Alternative"). The four asset management styles considered are: Internal Passive, External Passive, Internal Active, and External Active management. We only include years that have enough plan observations for each asset class and style. Median cost are scaled by the average cost across years and plans.

(a) (b)

Figure A.9: **Investment management costs by mandate for stocks and fixed income holdings.** The figure shows boxplots of scaled cost by management mandate for public asset classes in 2019. The different type of management styles include: Internal Passive (IP), External Passive (EP), Internal Active (IA) and External Active (EA). Cost are scaled by the average cost across plans, years, and asset classes.

Figure A.10: **Evolution in investment management costs by mandate.** The figure shows box plots of total (scaled) cost for internal, external, passive, and active management across plans over the sample period 1991–2019. Costs are averaged over the asset classes (by AUM) to get a plan level measure. Finally, we divide the cost by the average cost computed across plans, years, and asset classes.

Figure A.11: **Evolution of stock investment management costs by mandate.** The figure shows box plots of scaled cost in stock investments for the mandates: Internal Passive, External Passive, Internal Active and External Active. Cost are defined as the weighted average (by AUM) of all costs attributed to a particular investment style for a specific plan/year. The cost are scaled by the average cost across years, asset classes, and plans.

Figure A.12: **Evolution of fixed income investment management costs by mandate.** The figure shows boxplots of scaled cost in fixed income investments for the mandates: Internal Passive, External Passive, Internal Active and External Active. Cost are defined as the weighted average (by AUM) of all costs attributed to a particular investment style for a specific plan/year. The cost are scaled by the average cost across years, asset classes, and plans.

Figure A.13: **Median management costs by mandate in public sub-asset classes.** This figure shows median (scaled) investment management costs at the sub-asset class level for four different management mandates: Internal Passive, External Passive, Internal Active, External Active. Median costs represent the median of average cost across plans for a given year. We only include sub-asset classes that have enough time series observations for all management mandates. Finally, we scale the costs by the average cost across years, asset classes, and plans.

Figure A.14: **External active cost in sub-asset classes for small and large plans.** This figure shows the (scaled) median cost of externally actively managed assets in 2019. Small and large plans are defined as the bottom 30% and top 70% percentiles of total AUM within each asset class. Only sub-asset classes with more than five invested plans are shown. Costs are scaled by the average cost across years, asset classes, and plans.

Figure A.15: **Relation between log cost and log AUM within the same product market.** This figure shows scatter plots and fitted regression lines of $\log(\text{AUM}_{iats})$ versus $\log(\text{Cost}^{\$}_{iats})$ within matched fund/product markets. Here, AUM_{iats} and $\text{Cost}^{\$}_{iats}$ denote the dollar AUM holdings and dollar cost, respectively, of plan i in sub-asset class a at time t and management style s. Product markets are identified using the matching procedure described in Section 5.3. We display the 11 most common matched funds/products across all years. Panel headers indicate the year, sub-asset class, mandate type (EP/EA/IP/IA), and asset category, where S denotes stocks, F denotes fixed income, and real estate falls under the real asset category.

(a) Stocks

(b) Fixed Income

(c) Hedge Fund

(d) Private Equity

(e) Private Debt

(f) Real Assets

Figure A.16: **Policy-adjusted gross returns**. This figure shows box plots of gross policy-adjusted returns pooled across plans in a given year for different asset classes.

(a) Stocks

(b) Fixed Income

(c) Hedge Fund

(d) Private Equity

(e) Private Debt

(f) Real Assets

Figure A.17: **Policy return box Plots**. This figure presents the time series box plots for policy gross returns across plans and asset classes.