Enrolment No. 17dsMA017

S5(DSMA33B03) MA

BS-MS 3rd SEMESTER, END-TERM EXAMINATION-2018

NAME OF THE SUBJECT: Ordinary Differential Equation

CODE NO: DSMA33B03

Full Marks: 100 **Time: 3 hours**

Symbols used here have their usual meanings

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Group A

Answer all the following questions

Marks: 50

1. (a) Prove that there exists two linearly independent solutions $y_1(x)$ and $y_2(x)$ of the equation

$$a_0(x)y''(x) + a_1(x)y'(x) + a_2(x)y = 0$$

such that its every solution $y(x)$ may be written as $y(x) = c_1y_1(x) + c_2y_2(x), x \in (a.b)$ where c_1 and c_2 are suitable chosen constants.

(b) Prove that Wronskian of the functions $e^{m_1x}, e^{m_2x}, e^{m_3x}$ is equal to $(m_1 - m_2)(m_2 - m_3)(m_3 - m_1)e^{(m_1+m_2+m_3)x}$. Are these functions linearly independent?

Contradiction

x_0 $u(x)$

$y_1(x) = 0$

$y_2(x) = 1$

$\neq 0$

[7+3]

y_1

y_2

2. (a) Prove that two solutions $y_1(x)$ and $y_2(x)$ of the equation

$$a_0(x)y''(x) + a_1(x)y'(x) + a_2(x)y = 0; a_0(x) \neq 0, x \in (a, b)$$

are linearly dependent if and only their Wronskian is identically zero.

y''_2

$x \in (a,b) \to a(x) \neq$

(b) Prove that $y_1(x) = e^{-x/2}\sin(\frac{x\sqrt{3}}{2})$ and $y_2(x) = e^{-x/2}\cos(\frac{x\sqrt{3}}{2})$ are linearly independent solutions of the equation $y'' + y' + y = 0$

[7+3]

3. (a) Define orthogonal and orthonormal set of functions. Given that $f_1(x) = a_0, f_2(x) = b_0 + b_1x, f_3(x) = c_0 + c_1x + c_2x^2$. Determine the constants $a_0, b_0, b_1, c_0, c_1, c_2$ such that the given functions form an orthonormal set on the interval $[-1, 1]$.

(b) Show that the set of functions $\{cosnx\}, n = 1, 2, 3, \ldots\ldots\ldots\ldots$ is orthogonal on the interval $-\pi \le x \le \pi$, and find the corresponding orthonormal set of functions.

[2+5+3]

4. (a) Define eigen values and eigen functions of Strum-Liouville problem.

(b) Find the eigen values and eigen functions of

$$X'' + \lambda X = 0, X(0) = 0 \text{ and } X'(L) = 0.$$

[2+8]

5. (a) Define ordinary point and singular point.

(b) Find the general power series solution near $x = 0$ of the Legendre's equation

$$(1 - x^2)y'' - 2xy' + p(p + 1)y = 0,$$

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Where p is an arbitrary constant.

[2+8]

Group B

Answer all the following questions

Marks: 50

1. Solve the following differential equation by the method of variation of parameter:

$$(D^3 - 6D^2 + 12D - 8)y = \frac{e^{2x}}{x}.$$

$a_1(x) = \int \frac{Q_2(x)}{W}$

$\int Q_1(x)$

[6]

2. Transform the differential equation $(cosx)y'' + (sinx)y' - 2ycos^3x = 2cos^5x$ into the one having z as independent variable, where $z = sinx$ and solve it.

[7]

3. The acceleration and velocity of a body falling in the air approximately satisfy the equation: $Acceleration = g - kv^2$, where v is the velocity of the body at time t and g, k are constants. Find the distance traversed as a function of the time, if the body falls from rest. Show that the value of v will never exceed $\sqrt{\frac{g}{k}}$.

$\frac{dv}{dt} = g - kv^2$ $\quad -k \log$

[8]

4. What do you mean by **Orthogonal Trajectories** of a family of curves? Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is a parameter.

$(x-c)^2 + (y-c)^2 = a^2$

[7]

5. Solve the equation: $x^2y'' - (x^2 + 2x)y' + (x+2)y = xe^x$.

[7]

6. Find the complete solution of $(D^2 + 5D + 6)y = e^{-2x}sec^2x(1 + 2tanx)$.

[7]

7. Define homogeneous and non-homogeneous differential equation with suitable example and then solve: $(x + y)dx + (3x + 3y - 4)dy = 0, y(1) = 0$.

[8]

lment No. 17DSMN017

S3 (DSMA33B04): MA

BSMS 3rd SEMESTER, END-TERM EXAMINATION - 2018
NAME OF SUBJECT: ABSTRACT ALGEBRA-I
CODE NO: DSMA33B04

Full Marks: 100 **Time: 3 Hours**

Symbols used here have their usual meanings

Group A

Answer all the following questions: **Marks: 50**

1. a) Prove that a group G of prime order must be cyclic and every element of G other than identity can be taken as its generator.
   b) Show that in a cyclic group of order n, there exists $\varphi(m)$ elements of order m for every divisor m of n. Hence deduce that $n = \sum_{d|n} \varphi(d)$.
   c) Prove that a commutative ring R is an integral domain if and only if for all $a, b, c \in R$ $(a \neq 0)$ $ab = ac$ imply $b = c$.

[4 + 4+ 4] = 12

2. a) With an example prove that a division ring is not a field.
   b) Show that a Boolean ring is commutative.
   c) Define **unit** and **centre** of a ring.
   d) Prove that a non-zero idempotent cannot be nilpotent.
   e) If A and B are two ideals of a ring R then prove that $A + B$ is an ideal of R, containing both A and B.

[3 + 2+(2+2) + 2 + 4] = 15

3. a) If A and B are two ideals of a ring R then show that $A + B = \langle A \cup B \rangle$. A-B+B-A
   b) If $f: R \to R[x]$ such that $f(a) = a + 0.x + 0.x^2 + \cdots$, then show that R can be imbedded into R[x]. $f(a_1) = f(a_2)$ 1-1
   c) If R is an integral domain with unity, then prove that the units of R and R[x] are same.
   d) Prove that a ring R is commutative if and only if R[x] is commutative. $f(a)\cdot g(a)$

[4+ 3 + 3 + 3] = 13

4. a) Show that in a PID every non-zero prime ideal is maximal. $P \subseteq Q = q \subseteq R$
   b) Define **Unique Factorization Domain.** $p = qr$ $r = pt$
   c) Prove that in a UFD R, an element is a prime if and only if it is irreducible.

[4 + 2 + 4] = 10

Group B

Answer all the following questions: **Marks: 50**

1. a) If G and G_1 be two groups and $f : G \to G_1$ be a group homomorphism. Then show that $Img\, f$ is a subgroup of G_1 and $Ker\, f$ is a normal subgroup of G.

   Img f ~ kef

   b) State and prove **First Isomorphism theorem.**

   c) If G be a finite group, then show that $|G| = |Z(G)| + \sum_{a \notin Z(G)} [G : C(a)]$, where $Z(G)$ is the center of G and the summation runs over a complete set of distinct conjugacy class representations which don't belong to $Z(G)$.

[5+6+4=15]

P.T.O.

$\langle a \rangle = G$ $\quad f(a)=b$ $\quad G = \langle a \rangle$ $\quad G_1 = \langle b \rangle$ $\quad G_1 = f(G)$ $\quad u \in G_1$ $\quad y(u) = a$ $\quad y \in \langle a \rangle$ $\quad y = a^n$

2. a) State and prove Sylow's first theorem.

~~b)~~ If G and G_1 be two groups such that G_1 is a homomorphic image of G, then show that, (i) if G is a commutative group then G_1 is commutative, (ii) If G is a cyclic group, then so is G_1. $f(x)=a$ $\quad f(y)=b$ $\quad a=b$

~~c)~~ **State Second Isomorphism theorem.**

[7+6+2=15]

3. ~~a)~~ If G be a group and H be a subgroup of G and $S = \{aH \mid a \in G\}$ then show that there exist a homomorphism φ from G into $A(S)$ (the group of permutation of S) such that $Ker(\varphi) \subseteq H$. Extended cauchy

b) If G be a cyclic group of order n and H is a subgroup of G, then $o(H)$ divides $o(G)$. If m is a positive integer such that m divides n, then there exists a unique subgroup of G of order m.

[5+5=10]

4. ~~a)~~ Let G be a group and S be a G-set, then show that for all $a \in S$, $[G : G_a] = |[a]|$, where $G_a = \{g \in G : ga = a\}$ and $[a]$ is the orbit of a. $[a] = \{b \in R \mid a \sim b\}$ $\quad f : h \to [a]$ $\quad (gGa)$ $\quad = a$ $\quad b = ga$.

b) Show that $Z(G)$, the center of group G is a subgroup of G.

~~c)~~ If G be a non trivial group then show that G is a finite p-group if and only if $|G| = p^k$, for some positive integer k. $q \mid |G|$ $\quad$ P is only a prime divisor.

[4+3+3=10]

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$a, b \in G$

$T_g a = T_g b$ $\quad a^{-1}B$

$ga = gb$ $\quad a = b$

$a = b$ $\quad b = ga \; ga$

$g_1 g_2(a) = g_1(g_2 a)$ $\quad T_{g_1}(g$

$= T_{g_1}(T_{g_2} a)$

$A = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$

$A \cdot I = A \quad I \in R$

$\begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \in R.$

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Enrolment No. 17DSMA017.

S₃(DSMA33B06/DTMA33B06)

BSMS(PHYSICS,CHEMISTRY,MATH)/BTMT 3rd Semester End Term Examination, 2018

Name of Subject: Computer Programming

Paper code: DSMA33B06/DTMA33B06

Full Marks-100 **Time: 3:00 Hrs**

The figures in the margin indicate full marks for the questions

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A. Answer the following 10X2=20

a) What is function prototype? Give one example of function prototype
b) State four arithmetic operators and four logical operators. ++, --, *, % || &&
c) Give the meaning of declaration :int *ptr;
d) Declare one dimensional, 5 elements integer array 'List' and initialize all values
e) What is keyword? State two keywords of C.
f) Explain pointer variable.
g) What are formal parameters in functions?
h) Write the syntax of the if-else statement
i) State any four string handling functions.
j) What is the purpose of break statement?

B. Determine the value of each of the following logical expressions if a=0, b=10 and c=-1, 10

a) a<b && a<c || b>5 1
b) a<b && a>c || c<0 1
c) a==c || b>a 1
d) b>15 && c<0 0
e) (a==0.0 && b/2.0!=0.0)||c>0.0 1

C. 4X5=20

a) Write a program to take four integer values as input and print the average.
b) Explain the use of pointers in call by reference parameter passing for functions.
c) Explain the string handling function strcmp() in detail.
d) Write a program to swap (exchange) the values of two integer numbers.
e) Differentiate between while and do-while loop structures.
f) Write a program using array to read the ten integer numbers and find the counts for number of positive and negative numbers.

D. Write Answers 4X5=20

1. Explain the terms character set, tokens, constant and variables.
2. Find out errors in the following program component and justify the same:

```
float i;
int p = 0;
for (i=0; i = 10; i += 2)
{p = i * 2 ;
printf(" %d ", i, p) ;}
```

```
int i
float p=0
{
for(i=0; i=10; i+=2)
{ p=i*2;
printf("%d",&i,p);
}
```

3. Write a program to reverse the given integer number.(e.g. input : 2356, the reverse is 6532)
4. Explain with example a structure

5X6=30

E. Write codes

a) Write a program to find whether the given year is a leap or not.
b) Write a program to declare the structure 'point' with x, y coordinates as its members. Find the third member of structure 'quad' - the quadrant in which the point lies.
c) Explain the difference between structure and array with example.
d) Explain the meaning of following statements with reference to pointers
   int *ptr; int m = 8;*ptr = m ;ptr = &m ;
e) Write a program to calculate gross salary of an employee if basic pay=Rs 10000/- and DA=5% of Basic.

lment No. 17 PS MA 017

S3(DSMA33B05) MA

BS-MS 3rd SEMESTER, END-TERM EXAMINATION-2018
SUBJECT NAME: Probability & Statistics
SUBJECT CODE: DSMA33B05

Full Marks: 100 Time: 3 hours

Symbols used here have their usual meanings

Group-A Marks: 50

Answer all of the following questions:

1. A manager has two assistants and he bases his decision on information supplied independently by each one of them. The probability that he makes a mistake in his thinking is 0.005. The probability that an assistant gives wrong information is 0.3. Assuming that the mistakes made by the manager are independent of the information given by the assistants, find the probability that he reaches a wrong decision.

[6]

2. Suppose that there is a chance for a newly construction building to collapse, whether the design is faulty or not. The chance that the design is faulty is 10%. The chance that building collapses is 95% if the design is faulty and otherwise it is 45%. It is seen that the building collapsed. What is the probability that it is due to faulty design? (Using Baye's theorem)

[5]

3. Show that for the symmetric distribution $f(x) = \frac{2a}{\pi}\left(\frac{1}{a^2+x^2}\right), -a \le x \le a;$ $\mu_2 = \frac{a^2(4-\pi)}{\pi}$ and $\mu_4 = a^4\left(1 - \frac{8}{3\pi}\right)$.

[5]

4. If t is any positive real number, show that the function defined by $p(x) = e^{-t}(1-e^{-t})^{x-1}$ can represent a probability function of a random variable X assuming the values 1, 2, 3,... Find $E(X)$ and $Var(X)$ of the distribution.

[5]

5. Under which conditions the Poisson distribution is a limiting case of Binomial distribution. State and prove Renovsky's formula for Binomial distribution.

[2+5=7]

6. Prove that the sum of two independent Poisson variates is a Poisson variate. Is the result true for the differences also? Give reasons.

[6]

7. The marks obtained by a number of students for the certain subject are assumed to the approximately normally distributed with mean value 65 and with a standard deviation of 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70? σ

[5]

8. Define correlation and scatter diagram with example. Show that correlation is independent of change of origin and scale.

[3+4=7]

9. Define line of regression of Y on X. Show that if one of the regression coefficients is greater than unity, then the other must be less than unity.

[1+3=4]

Group-B

Marks: 50

Answer question no. 1 and any three from the rest:

1. a. Define **unbiased estimator** of a parameter $\boldsymbol{\theta}$ in random sampling.

b. If $\{x_1, x_2, \dots, x_n\}$ is a random sample from an infinite population having mean μ and variance σ^2. If $\bar{x}$ is the sample mean then show that $E(\bar{x}) = \mu$ and $V(\bar{x}) = \frac{\sigma^2}{n}$.

c. In a large city A, 20% of a random sample of 900 school children had defective eye-sight. In other large city B, 15% of random sample of 1,600 children had the same defect. Is this difference between the two proportions significant? Obtain 95% confidence limits for the differences in the population proportions.

d. It is believed that the precision measure by the variance of an instrument is no more than 0.16. Write down the null and alternative hypothesis for testing this mean. Carry out the test at 1% level. Given that measurements of the same subject on the instruments as: 2.5, 2.3, 2.4, 2.3, 2.5, 2.7, 2.5, 2.6, 2.6, 2.7, 2.5.

$x \quad (x-\bar{x})^2$

[1 + 5+ 6+8= 20]

2. a. A simple random sample (x_1, x_2, x_3, x_4) of size 4 drawn from a infinite population with mean μ, variance σ^2. Define the two estimators of μ as follows:

$T_1 = \frac{1}{3}(x_1 + x_2) + \frac{1}{6}(x_3 + x_4)$, $T_2 = \frac{1}{10}(x_1 + 2x_2 + 3x_3 + 4x_4)$, which one is better and why? Which one is best among T_1, T_2, $\bar{x}$(the sample mean), as an estimator of μ?

b. Obtain a Maximum likelihood estimator of the mean of a Poisson distribution.

[5+5=10]

3. a. A die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the die cannot be regarded as an unbiased one. Find the limits between which probability of throws 3 or 4 lies.

b. Describe the process of goodness of fit test using Chi-square distribution.

$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$ $\quad n = 9000$ $\quad x = 3240$ $\quad p = \frac{1}{2}$ $\quad p = \frac{x}{n}$

[6+4=10]

4. Define consistency with an example. Suppose that $\{x_1, x_2, \dots, x_n\}$ be a simple random sample of size n drawn from a normal population having parameters μ and σ^2. Obtain moment estimator of μ and σ^2, and verify if they are unbiased. If not, suggest an unbiased estimator for the parameter.

[2+8=10]

5. A survey of 800 families with 4 children each reveal the following distribution:

No. of boys:	0	1	2	3	4
No. of girls:	4	3	2	1	0
No. of families:	32	178	290	236	64

Is this result consistent with the hypothesis that male and female births are equally probable?

[10]

^nC_r

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$^nC_r \, p^r q^{r-1}$

$= 50$

$c(0) =$ $\quad = 200$

$= 500$