1706MA017

S4(DSMA34B08): MA

BS-MS 4th SEMESTER, MID TERM EXAMINATION – 2019
NAME OF THE SUBJECT: Linear Algebra
SUBJECT CODE: DSMA34B08

Full Marks: 50 Time: 2 Hours

Symbols used here have their usual meanings

Group - A

Marks: 25

Answer the following questions:

1. a) Define basis and dimension of a vector space. 2

b) State and prove the necessary and sufficient condition for a non-empty subset W of a vector space V over the field F to be a sub-space of V.

c) Determine k so that the vectors $(1,2,1), (k,1,1)$ and $(1,1,2)$ are linearly independent $(k \in R)$.

d) Let V be a vector space over the field F. Prove that the non-zero vectors $\alpha_1, \alpha_2, \ldots, \alpha_n \in V$ are linearly dependent if and only if one of them be a linear combination of the preceding vectors.

[(2+1) + (2+3) + 2 + 3] = 13

2. a) Define linear span of a vector space.

b) Prove that there exists a basis for each finite dimensional vector space.

c) Let $S = \{(x,y,z) \in R^3 : x + y + z = 0\}$. Show that the basis for this sub-space consists of the set of vectors {(1,0,-1), (0,1,-1)}. 4

d) Find the co-ordinate of the vector $\alpha = (-1,3,1)$ relative to the basis $B = \{(2,1,0), (2,1,1), (2,2,1)\}$. 2

[2+ 4 + 4+2] = 12

Group - B

Marks: 25

Answer the following questions:

1. (a) Define rank and nullity of a Linear Transformation. Let $F: R^4 \to R^3$ be the linear map defined by $F\langle x,y,z,t\rangle = \langle x - y + z + t, x + 2z - t, x + y + \;\; 3z - 3t\rangle$. Find a basis and the dimension of the image of F. 5

(b) Two finite dimensional vector spaces over the same field are isomorphic iff they have the same dimension, i.e., $U(K) \cong V(K) \Leftrightarrow \dim U = \dim V$.

(c) Let T be a linear map from a vector space $V(F)$ into a vector space $U(F)$. Then prove that range space $R(T)$ is a subspace of $U(F)$.

[(2+3) +4+4=13]

2. (a) Suppose $\{e_1, e_2, \ldots, e_n\}$ is a basis of a vector space V. Let F be a linear operator on V. Then show that $[F]_e[v]_e = [F(v)]_e, v \in V$.

(b)Let $T_a: R^3(R) \to R^3(R)$ be a map given by $T_a(x,y,z) = (x, ay, z), a \in R$ is fixed. Show that T_a is an isomorphism. What about T_0?

(c) Define Transition Matrix.

(d) Let V and U be the vector spaces over the field K. Let V be of finite dimension. Let $T: V \to U$ be a linear map. Then show that $dim\ V = dim\ R(T) + dim\ N(T)$.

[4+3+1+4=12]

Enrolment No. 1 7 d s m n 0 1 7.

S₄ (DSMA34B09) MA

BS-MS 4th Semester Mid-Term Examination, 2019
SUB: Partial Differential Equation
PAPER CODE: DSMA34B09

Full Marks: 50 **Time 2 Hours**

The figures in the margins indicates full marks in the questions
Candidate are required to give their answers in their own words as far as practicable

Part – A

Answer all the questions.

1. Define linear and quasi-linear partial differential equation with example. 2

2 ~~2.~~ Find the PDE of all planes which are at a constant distance 'a' from the origin. 4 $lx+my+nz=a$, $l^2+m^2+n^2=1$

5 ~~3.~~ Deduce Charpit's formula to obtain non-linear first order partial differential equation. 5

2 ~~4.~~ Find the PDE by eliminating the arbitrary function 'F' from the following equation:

$$F(xy+z^2, x+y+z)=0$$ 3

~~5.~~ Find the complete integral of the PDE $x^2p^2+y^2q^2-4=0$ Lagrange 4

6. Solve the following PDEs: 4 + 3

2 ~~(i)~~ $(x^2-y^2-z^2)p+2xyq=2zx$ Lag

~~(ii)~~ $(y+1)\frac{\partial z}{\partial x}+(x+1)\frac{\partial u}{\partial y}=z$ Lag

Part – B

Answer any five questions.

5 ~~1.~~ Describe the method of finding complementary function (C.F) of the linear homogeneous partial differential equation with constant coefficients of order 'n', namely $F(D,D')z=f(x,y)$. (5)

~~2.~~ Prove that: $\frac{1}{(bD-aD')^n}\phi(ax+by)=\frac{x^n}{b^n \angle n}\phi(ax+by)$ (5)

2 ~~3.~~ Solve: $(D^3-7DD'^2-6D'^3)z=x^2+xy^2+y^3+\cos(x-y)$ 1, 2, −3 (5)

4. Solve: $r-t=6=\tan^3 x\tan y-\tan x\tan^3 y.$ (5)

~~5.~~ Solve: $(D^2-2DD'-15D'^2)z=12xy$ (5)

$-5DD'+3$

5 ~~6.~~ Solve, $(D^3-4D^2D'+5DD'^2-2D'^3)z=e^{y+2x}+(y+x)^{\frac{1}{2}}$ (5)

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12
11

Enrolment No. 1 7 9 0 1 9

S4(DSMA34B11) MA

BSMS 4th SEMESTER, MID-TERM EXAMINATION-2019

NAME OF THE SUBJECT: DATA STRUCTURE AND ALGORITHMS

CODE NO: DSMA34B11

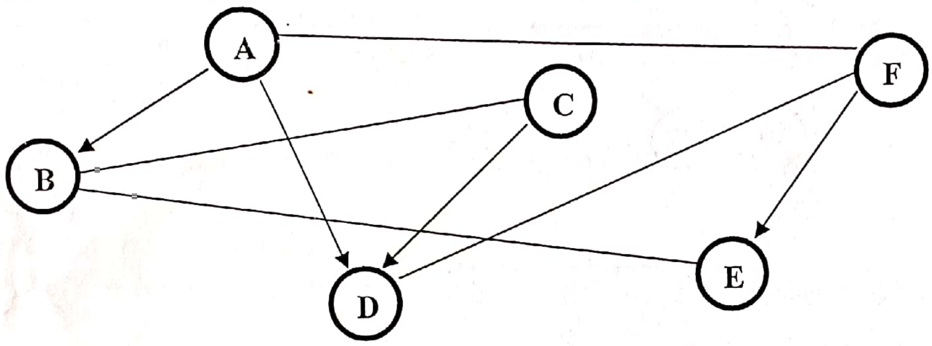
Full Marks: 50 **Time: 2 hours**

Answer all the questions from the following: **[10 × 5 = 50 Marks]**

1. (a) Write three the difference between B+ tree and B tree.

(b) Find the values of $\lfloor \sqrt[3]{50} \rfloor$ and $\lceil e^e \rceil$?

5 ~~(c)~~ Draw the adjacency matrix for the following graph.



[3 + 2 + 5 = 10]

2. ~~(a)~~ Write an algorithm to delete an element from any position in a single linked list?

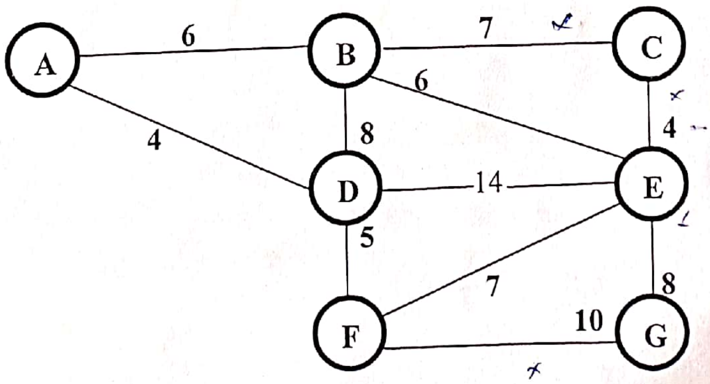
5 ~~(b)~~ Construct an AVL tree by inserting the following elements in the given order.

63, 9, 19, 27, 18, 108, 99, 81 .

[5 + 5 = 10]

3. (a) What is a minimal spanning tree?

~~(b)~~ Construct a minimal spanning tree for the following graph. (Taking D as the initial vertex)



[5 + 5 = 10]

4. (a) Define Stacks with examples.

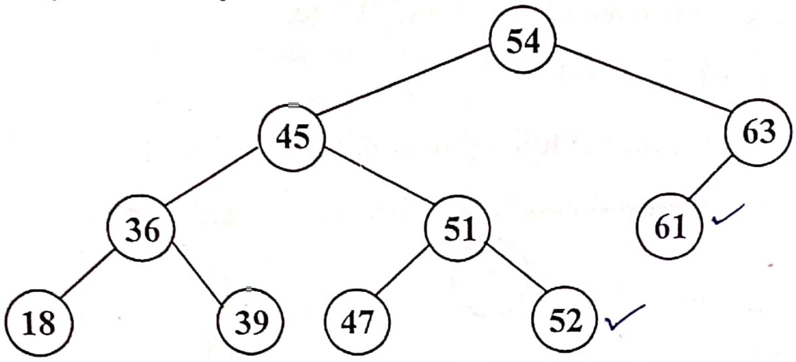
3 ~~(b)~~ Define Ω, Θ and O with examples.

(c) Write an algorithm to search an element in single linked list?

[2 + 3 + 5 = 10]

3 5. ~~(a)~~ Write down the overview of Data Structure and explain the terms related to non-linear data structures with examples.

5 ~~(b)~~ What do you mean by AVL tree? Delete 52 and 61 from the following tree.



[5 + 5 = 10]

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Enrolment No. 179010 S4(DSMA34B07): MA

BS/MS. 4th SEMESTER, MID-TERM EXAMINATION-2019

SUBJECT NAME: REAL ANALYSIS

SUBJECT CODE NO: DSMA34B07

Full Marks: 50 Time: 2 Hours

Symbols used here have their usual meanings

Group – A [Marks: 25]

Answer the following questions:

1. Prove that every bounded infinite subset of R has at least one limit point (in R). Weier

[5]

2. (a) Define limit point of a set and derived set.

   (b) Let $S = \{\frac{1}{m} + \frac{1}{n} : m, n \in N\}$. Show that 0 is a limit point of S. If $k \in N$, show that $\frac{1}{k}$ is a limit point of S.

[2+3=5]

3. Let A and B be subsets of R. Then prove that $(A \cup B)' = A' \cup B'$.

[5]

4. (a) Define neighbourhood of a point.

   (b) Let $c \in R$. Prove that the union of two neighbourhoods of c is a neighbourhood of c. $F_1 \cup F_2 \cdots F_n$

[1+4=5]

5. (a) Using one example show that intersection of infinite number of neighbourhoods of a point may not be a neighbourhood of that point.

   (b) Let G be an open set in R. The show that complement of G (in R) is a closed set in R.

[2+3=5]

Group – B [Marks: 25]

Answer the following questions:

1. (a) Define convergence of a sequence.

   (b) Prove that a sequence cannot converge to more than one limit.

   (c) If $\{a_n\}$ be a sequence such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = l$, where $|l| < 1$ then prove that $\lim_{n\to\infty} a_n = 0$.

   Hence evaluate the value of $\lim_{n\to\infty} \frac{x^n}{n!}$.

[1+4+5+2]

2. (a) Prove that the set of rational number is not order complete.

   (b) Let S be a non-empty bounded subset of R with $\sup S = M$ and $\inf S = m$. Prove that the set $T = \{|x - y| : x \in S, y \in S\}$ is bounded above and $\sup T = M - m$.

   (c) Find the glb and lub of $\{1 + (-1)^n \frac{1}{n}\}$.

[6+5+2]

5