

BSMS VI<sup>th</sup> SEMESTER, END-TERM EXAMINATION-2019

NAME OF THE SUBJECT: OBJECT ORIENTED PROGRAMMING

CODE NO: DSMA36B19

Total Marks: 100

Time: 3 hours

Answer any ten (10) questions from the following:

[10 × 10 = 100 Marks]

State whether the following statements are true or false, justify:

[10 × 1]

- (i) Templates are declared inside classes or functions.
- (ii) Different types of parameter are required for constructors.
- (iii) Base class is known as the subclass and derived class as super-class.
- (iv) Static methods can be use non static members.
- (v) A member function can't be used in a derived class that override.
- (vi) In an array, the index value of the first element is void.
- (vii) The scope resolution operator can be overloaded.
- (viii) It is not possible to specify the accessibility modifier for methods inside the interface.
- (ix) Templates should be global and should not be local.
- (x) It is possible for a base to inherit the constructor of its abstract class.

2. Answer the following questions in one word.

[10 × 1]

- (i) How can you prevent your class to be inherited further?
- (ii) How many instances can be created for an abstract class?
- (iii) What is the index value of the first element in an array?
- (iv) Which keyword can be used for overloading?
- (v) Can you specify the accessibility modifier for methods inside the interface?
- (vi) What is the default access specifier in a class definition?
- (vii) Is it possible for a class to inherit the constructor of its base class?
- (viii) Which OOPS concept is used as reuse mechanism?
- (ix) Can you allow a class to be inherited, but prevent a method from being overridden in C#?
- (x) Which OOPS concept exposes only necessary information to the calling functions?

3. (a) Define Operator Overloading. What are the restrictions on Operator Overloading?

(b) What are the advantages &amp; disadvantages of structured analysis?

[(1 + 4) + 5]

4. (a) What are Constructors? What are different types of arguments present in Objective Oriented Programming?

(b) Explain the concept of destructor? What is the difference between procedural and object-oriented programming?

[(1 + 3) + (3 + 3)]

5. (a) Write an algorithm, to add two complex numbers using binary operator overloading?

(b) What is object-oriented programming (OOP)? Explain the basic features of OOPs.

[5 + 5]

6. (a) What are the advantages & disadvantages of object oriented analysis?  
(b) What do you mean by object oriented programming paradigm?  
[5 + 5]
7. (a) What are abstract classes? What are the distinct characteristics of an abstract class? Write down the difference between abstract Class and Interface?  
(b) What is access modifier? What are the various types of constructors in OOP? Explain  
[(2 + 2 + 3) + 3]
8. (a) What are the different phases in object-oriented software development?  
(b) Explain different types of inheritance and State the features of an interface in object oriented programming?  
[4 + 6]
9. (a) Define Virtual Function. Why are virtual functions needed?  
(b) What are similarities between a class and a structure?  
[(1 + 4) + 5]
10. (a) Is there anything that cannot be overloaded?  
(b) Define Stream. Write down some commonly used stream classes and explain them.  
[4 + (2 + 4)]
11. (a) What are the conditions required for Polymorphism.  
(b) What is a file? Write down the different types of parameters which are used in a file?  
[4 + (2 + 4)]
12. (a) Define a class and a structure. Write down the difference between a class and a structure.  
(b) What is a pointer? What are the uses of pointers?  
[(2 + 3) + (2 + 3)]
13. (a) Write down all the rules for overloading operator in OOP?  
(b) What are the models used in object-oriented analysis? Explain.  
[5 + 5]

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**Group-A**

[50 Marks]

Answer all the following questions

1. If  $R$  and  $S$  be relations on a set  $A$  represented by the matrices

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent (i)  $R \cup S$  (ii)  $R \cap S$  (iii)  $R \circ S$  (iv)  $S \circ R$  (v)  $R^{-1}$  (vi)  $S^{-1}$ .

[10]

2. Draw the digraph representing the partial ordering  $\{(a,b) | a \text{ divides } b\}$  on the set  $\{1, 2, 3, 4, 5, 6, 7, 8\}$ . Reduce it to the Hasse diagram representing the given partial ordering.

[5]

3. State and prove well-ordering principle.

[5]

4. State generalization of the Pigeonhole principle. How many integers between 1 and 300 (both inclusive) are divisible by 5 but by neither 3 nor 7?

[2+8=10]

5. Define binary and  $n$ -ary operations. If  $*$  is a binary operation on a set  $S$  which is associative, the inverse of every invertible element  $a \in S$  is unique.

[2+5=7]

6. If  $A = \{x \in \mathbb{R} | x \neq \frac{1}{2}\}$  and  $f: A \rightarrow \mathbb{R}$  is defined by  $f(x) = \frac{4x}{2x-1}$ , (i) find the range ( $f$ ); (ii) show that  $f$  is invertible and (iii) find  $\text{dom}(f^{-1})$ ,  $\text{range}(f^{-1})$  and a formula for  $f^{-1}$ .

[3+3+3=9]

7. In how many ways can 2 letters be selected from the set  $\{a, b, c, d\}$  when repetition of the letters is allowed if the order does not matter?

[4]

**Group-B**

[50 Marks]

Answer all the following questions

1. (a) Define Boolean Algebra. In a Boolean Algebra  $B$ ,  $a, b, c \in B$  prove that  $a + a'b = a + b$ .

- (b) Define Conjunctive and Disjunctive normal form of a Boolean function with example.

- (c) In a Boolean Algebra  $B$ ,  $a, b, c \in B$  reduce the following Boolean function to its Conjunctive normal form  $abc + (a+b)(b+c)$ .

[(1+2)+3+4]

2. (a) Prove that if a connected graph  $G$  is Eulerian, then every vertex of  $G$  has even degree.

- (b) Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit.

$x$	$y$	$z$	$f(x, y, z)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

P.T.O.

$$\begin{aligned} & \frac{4}{3} \\ & 2\left(\frac{4}{3} - 2\right) \\ & = \frac{8}{3} - 4 \\ & = \frac{8}{3} - \frac{12}{3} \\ & = \frac{-4}{3} \\ & = -\frac{4}{3} \end{aligned}$$

$$10_1(20_2-1) = 1_2(24_3-1)$$

$$\frac{4a_1}{2a_1-1} = \frac{40_2}{20_2}$$

3. (a) Let G be a simple connected graph with 10 vertices and 9 edges. Does G contain a vertex of degree 1? Justify your answer.

(b) Find a minimal spanning tree for the following table of distances in kilometers between six villages

	A	B	C	D	E	F
A	—	6	8	9	5	4
B	6	—	3	6	7	5
C	8	3	—	9	10	2
D	9	6	9	—	12	8
E	5	7	10	12	—	7
F	4	5	(2)	8	7	—

20

4. (a) What is the chromatic number of  $K_{3,4}$ . (complete bipartite graph) 7

[4+1]

(b) Explain one application of graph coloring.

(c) Solve the difference equation for the given initial conditions

$$2a_n = 7a_{n-1} - 3a_{n-2}, n \geq 2, a_0 = 1, a_1 = 1.$$

[1+]

5. (a) Define spanning tree with suitable example.

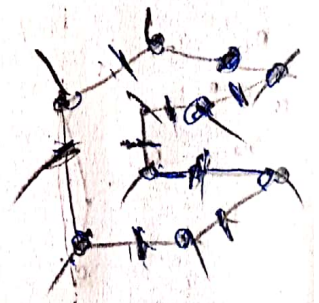
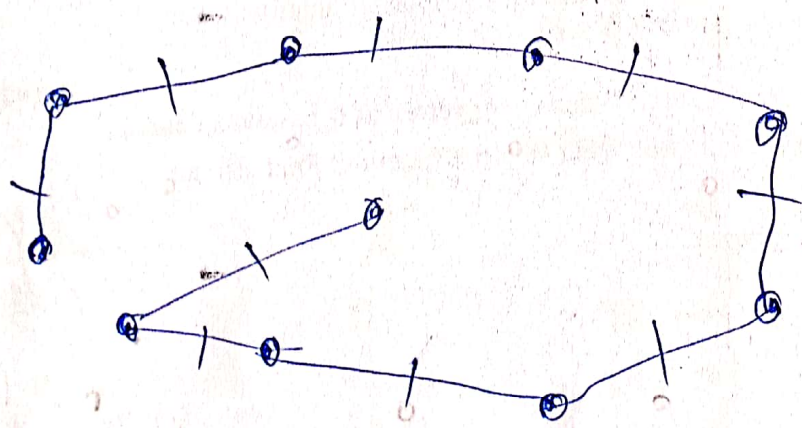
(b) Prove that every connected graph has at least one spanning tree.

(c) Find the number of solutions of the equation  $a + b + c = 17$ , where  $a, b, c$  are positive integers

$$2 \leq a \leq 5, 3 \leq b \leq 6, 4 \leq c \leq 7$$

$2-5n = A(1-3^n) + B(2^n)$   
 $A+B = A-12B = 2$   
 $-3A-B = -5$   
 $3A+B = 5$   
 $3A+6B = 6$   
 $-5B = -1$   
 $B = 1/5$   
 $A = 9/5$

[2+]





**BS-MS 6<sup>th</sup> SEMESTER, END TERM EXAMINATION – 2019**  
**NAME OF THE SUBJECT: Number Theory and Cryptology**  
**CODE NO: DSMA36B20**

S<sub>6</sub> (DSMA36B20): MA

Full Marks: 100

Time: 3 Hours

Symbols used here have their usual meanings

Group - A

Answer the following questions:

Marks: 50

1. (a) What do you mean by **Linear Diophantine equation**? Prove that for a linear Diophantine equation  $ax + by = c$ , for  $a, b, c$  being integers, has integer solution iff  $d|c$ , where  $d = g.c.d(a, b)$  and moreover if  $x = x_0, y = y_0$  is a particular solution, then any solution can be written as  $x = x_0 + \frac{b}{d}t$ ,  $y = y_0 - \frac{a}{d}t$ , where  $t$  is any integer.

- (b) Twenty three weary travellers entered the outskirts of a lush green and beautiful forest. They found 63 equal heaps of plantains (fruits) and 7 single fruits. They divided them equally. Find the number of fruits in each heap.

$[1 + 5 + 4] = 10$

2. (a) Define **Euler's phi function** and show that it is multiplicative.

- (b) Prove that  $\varphi(n) = \varphi(n+2)$  is satisfied by  $n = 2(2p-1)$ , whenever  $p$  and  $2p-1$  are both odd primes.

$[2 + 6 + 2] = 10$

3. (a) Prove that  $a \equiv b \pmod{m}$  iff  $a$  and  $b$  have the same remainders with respect to  $m$ .

- (b) Find the remainder when the sum

$S = 1! + 2! + 3! + \dots + 1000!$  is divided by 8.

- (c) State and prove the **Chinese Remainder Theorem**.

$[3 + 2 + (1 + 4)] = 10$

4. (a) Solve the system of linear congruence's

$$x \equiv 3 \pmod{11}$$

$$x \equiv 5 \pmod{19}$$

$$x \equiv 10 \pmod{29}$$

- (b) State and prove the **fundamental theorem of arithmetic** in its canonical form.

$[4 + 2 + 4] = 10$

P.T.O

5. (a) What is **Fermat's Little Theorem** and **Wilson's Theorem**?

(b) What do you mean by order of an integer? Find the order of 5 (mod 29).

(c) List the primitive roots of 10.

$$[3 + 2 + 2.5 + 2.5] = 1$$

### Group - B

Answer the following questions:

Marks: 50

1. a) Encrypt the message "Attack is today" by using Autokey Cipher where  $k_1 = 12$ .

b) Write a short note on **Playfair Cipher**.

c) Find  $(x^5 + x^2 + x) \otimes (x^7 + x^4 + x^3 + x^2 + x)$  in  $GF(2^8)$  with irreducible polynomial  $(x^8 + x^4 + x^3 + x + 1)$ .

d) Discuss **D-boxes** for modern block cipher.

$$[3 + 5 + 3 + 5] = 1$$

2. a) Prove that **mixer** in the **Feistel Cipher** is self-invertible.

b) Define **feedback shift register** and **Blowfish**.

c) Write a short note on **Round** of a product cipher.

d) Briefly discuss data units of **AES**.

$$[3 + [2+2] + 5 + 5] = 17$$

3. a) Prove that in **RSA cryptosystem**, encryption and decryption algorithms are inverses of each other.

b) Discuss encryption and decryption algorithms of **Rabin cryptosystem**.

c) Do a comparison between **symmetric** and **asymmetric key cryptography**.

d) Prove that the remainder of an integer when divided by 8 is the same as the remainder of division of the rightmost three digits by 8.

$$[3 + (3+4) + 4 + 3] = 17$$

2, 3

12

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a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
s	t	u	v	w	x	y	z										
18	19	20	21	22	23	24	25										

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**BS/MS 6<sup>th</sup> SEMESTER, END-TERM EXAMINATION-2019**  
**SUBJECT NAME: STOCHASTIC PROCESSES**  
**SUBJECT CODE NO: DSMA36B18**

S<sub>1</sub> (DSMA36B18): MA

Full Marks: 100

Time: 3 Hours

Symbols used here have their usual meanings

**Group – A**

**[Marks: 50]**

Answer any five (05) from the following questions:

1. A) Prove that in an irreducible chain, all the states are of the same type and also they have the same period.

B) If state  $j$  is persistent, then show that for every state  $k$  that can be reached from state  $j$ ,  $F_{kj} = 1$ .

[6+4=10]

2. A) Let  $\{X_n: n \geq 0\}$  be a Markov chain having state space  $S = \{1, 2, 3, 4\}$  and transition matrix

$$P = \begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$$

Classify the nature of each state. Prove that state 1 and 2 are ergodic.

B) Define martingale.

[8+2=10]

3. A) State and prove Ergodic theorem.

B) Show that Gambler's ruin problem is a martingale.

[7+3=10]

4. A) Define intree of a point  $j$ . Find the limiting distribution  $\{v_k\}$  for the following transition matrix

$$P = \begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

B) State and prove Chapman-kolmogorov equation for higher transition probabilities.

[(1+4)+5=10]

Describe briefly the Wiener process for continuous state space.

[10]

Consider the two state Markov chain with transition matrix  $P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$ ,  $0 < a, b < 1$ ,

$a+b < 1$ . Find  $P^n$  and hence find  $\lim_{n \rightarrow \infty} P^n$ .

[10]

P.T.O.

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Group - B

Marks:50

Answer all the following questions:

1. Define birth and death processes. Derive steady state solutions of birth and death processes. [3+7]
2. Describe application of birth and death processes in M/M/s queue. [7]
3. Describe Erlang's loss model. [7]
4. Describe Renewal process in discrete time. Derive relation between  $F(s)$  and  $P(s)$ . [6+4]
5. Show that if  $N(t)$  is a Poisson process and  $s < t$ , then  $\Pr\{N(s) = k | N(t) = n\} = \binom{n}{k} \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}$  [4]
6. Show that under the postulates of a Poisson process, a stochastic process  $N(t)$  follows Poisson distribution with mean  $\lambda t$ , i.e.,  $p_n(t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$ ,  $n = 0, 1, 2, \dots$  [7]
7. Show that random selection from a Poisson process yields a Poisson Process. [5]

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BSMS-6<sup>th</sup> SEMESTER, END-TERM EXAMINATION-2019S<sub>6</sub>(DSMA36B16): MA

NAME OF THE SUBJECT: Complex Analysis

SUBJECT CODE: DSMA36B16

Full Marks: 100

Time: 3 hours

Group-A

Marks: 50

Answer all questions:

1. (a) If  $f(z)$  is a continuous function in a domain  $D$  and if for every closed contour  $C$  in the domain  $D$ , then prove that  $\int_C f(z) dz = 0$ .

(b) Obtain Laurent's series which represents the function  $\frac{z^2-1}{(z+2)(z+3)}$  in the regions

(i)  $2 < |z| < 3$ , (ii)  $|z| > 3$

(c) If  $f(z)$  is analytic within and on a simple closed contour  $C$  and  $f(z)$  is not constant. Then prove that  $|f(z)|$  reaches its maximum value on  $C$ .

(d) Evaluate:  $\int_{|z|=4} \frac{(3z^3+2)}{(z-1)(z^2+9)} dz$

[5 + (2.5 + 2.5) + 6 + 4 = 20]

2. (a) By the method of contour integration prove that  $\int_0^{2\pi} e^{-\cos\theta} \cos(n\theta + \sin\theta) d\theta = \frac{2\pi(-1)^n}{n!}$ ,

where  $n$  is a positive integer.

(b) State and prove Cauchy's integral formula.

(c) Evaluate:  $\int_C \frac{\tan(\frac{z}{2})}{(z-x_0)^2} dz$ , where  $C$  is the square whose sides lie along the lines  $x = \pm 2, y = \pm 2$  and it is described in positive sense, where  $|x_0| < 2$ .

[5 + 5 + 5 = 15]

3. (a) Let  $f(z)$  be analytic on  $D: |z - z_0| < R$  and  $|f(z)| \leq |f(z_0)|$ , for all  $z \in D$ , then prove that  $f(z)$  is a constant function on  $D$  having the constant value  $f(z_0)$ .

(b) Evaluate the residues of  $\frac{z^3}{(z-1)(z-2)(z-3)}$  at  $z = 1, 2, 3$  and infinity.

(c) By the method of contour integration prove that  $\int_0^\infty \frac{\cos mx}{x^2+a^2} dx = \frac{\pi e^{-ma}}{2a}$ , where  $m > 0, a > 0$ .

[6 + 4 + 5 = 15]

P.T.O

Group B

Answer all the following questions

Marks: 50

1. (a) Which of the function is discontinuous at  $z = 0$ ?

- (i)  $\sin z$ , (ii)  $\cos z$ , (iii)  $\tan z$ , (iv)  $\frac{1}{z}$

(b) A single valued function  $f(z)$  defined in a domain  $D$  is said to be analytic at a point  $z_0$  of  $D$  if it is differentiable:

- (i) at  $z_0$ , (ii) at the origin, (iii) at some neighbourhood of  $z_0$ , (iv) at some deleted neighbourhood of  $z_0$ .

(c) A harmonic conjugate of  $u(x, y) = e^x \sin y$  is:

- (i)  $e^y \cos x$ , (ii)  $e^x \cos y$ , (iii)  $-e^x \cos y + 1$ , (iv)  $\frac{e^y}{\sin x}$

(d) Which of the following is not correct for analytic functions  $f(z)$  and  $g(z)$  in a region  $R$ ?

- (i)  $f(z) + g(z)$  is analytic in  $R$ , (ii)  $f(z) - g(z)$  is analytic in  $R$ , (iii)  $f(z) \cdot g(z)$  is analytic in  $R$ , (iv)  $f(z)/g(z)$  is analytic in  $R$ .

(e) An analytic function with constant modulus is:

- (i) Variable, (ii) may be variable or constant, (iii) constant, (iv) none of these.

[10]

2. What do you mean by critical points? If the mapping  $w = f(z)$  is conformal, then show that  $f(z)$  is an analytic function of  $z$ .

[2+6]

3. Establish the relation  $w = \frac{iz+2}{4z+i}$  transforms the real axis in  $z$ -plane to a circle in the  $w$ -plane. Find the centre and the radius of the circle and the point in the  $z$ -plane which is mapped on the centre of the circle.

[6]

4. State Cauchy-Riemann equation in polar form. Find the orthogonal trajectory of the family of curves  $x^2 - y^2 + x = c$ .

[2+5]

5. Find the analytic function whose imaginary part is  $\frac{x-y}{x^2+y^2}$  and find the real part also.

[6]

6. Define fixed points. Find the fixed points and the normal form of the bilinear transformation  $w = \frac{3iz+1}{z+i}$ . Discuss the nature of this transformation.

[2+5]

7. Find the transformation which maps outside  $|z| = 1$ , on the half plane  $R(w) \geq 0$ , so that the points  $z = 1, -i, -1$  corresponds to  $w = i, 0, -i$  respectively.

[6]

do: dz \*\*\*\*\*