Enrolment No. 1674444

$\frac{2}{3} - \frac{2}{5} \times \frac{5}{3}$ $\frac{1}{3} - \frac{1}{5} \times \frac{5}{3}$

S₅(DSMA35B12): MA

BSMS-5th SEMESTER, END-TERM EXAMINATION-2018

Subject Name: Linear Programming and Game Theory

Subject Code: DSMA35B12

Full Marks: 100 **Time: 3 hours**

Symbols used here have their usual meanings

Group-A

Answer all the following questions **[50 Marks]**

1. (a) Examine whether the set $S = \{(x_1, x_2): 5x_1 + 3x_2 \geq 10, 2x_1 + 5x_2 \geq 10\}$ is convex or not.
(b) Prove that the optimum of objective function $z = f(x)$ of a LPP occurs at a vertex of the solution space S, provided S is bounded polyhedron. [5+5]

2. (a) If x be any feasible solution to the primal problem and v be any feasible solution to the dual problem, then prove that, $cx \leq b^*v$.
(b) Give the dual of the following LPP and hence solve it:

$$\text{Max } z = 3x_1 - 2x_2$$
$$\text{Subject to,} \quad x_1 \leq 4, \quad x_2 \leq 6, \quad x_1 + x_2 \leq 5, \quad -x_2 \leq -1, \quad x_1, x_2 \geq 0$$

$\frac{1}{3} + \frac{2}{5} \times \frac{1}{3} = \frac{1}{3}\left(\frac{2}{5} - 1\right)$

[5+5]

3. Find the optimal solution of the LPP:

$$\text{Max } z = 4x_1 + 3x_2$$
$$\text{Subject to,} \quad x_1 + x_2 \leq 5, \quad x_1 + 2x_2 \leq 10, \quad 3x_1 + x_2 \leq 7, \quad x_1, x_2 \geq 0$$

Show how to find the optimal solution of the problem, if
(i) The first component of the original requirement vector be increased by one unit and the third component be decreased by one unit
(ii) The second component of the original requirement vector is decreased by two units. [10]

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4. Use revised simplex method to solve the LPP:

$$\text{Max } z = 5x_1 + 3x_2$$
$$\text{Subject to,} \quad 5x_1 + 2x_2 \leq 10, \quad 3x_1 + 5x_2 \leq 15, \quad x_1, x_2 \geq 0$$

[10]

5. (a) When a solution is said to be degenerate solution?
(b) Define closed and open half spaces.
(c) Show that $x_1 = 5, x_2 = 0, x_3 = -1$ is a basic solution of the system of equations:

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

Find other basic solutions, if there be any. [2+2+6]

$\frac{8}{3} - \frac{2}{5} \times \frac{23}{3}$

$1+\frac{\cancel{5}}{19}\times\frac{-3}{\cancel{5}}$

$=1-3/19$

$-\frac{6}{5}+5$

$=\frac{-6+25}{5}$

Group-B

Answer all the following questions [50 Marks]

1. (a) Write down the algorithm for finding the total cost of a Transportation problem by VAM method. [5]

(b) Solve the following Transportation problem

	D_1	D_2	D_3	D_4	D_5	a_i
	3	4	6	8	8	20
	2	10	0	5	8	30
	7	11	20	40	3	15
	1	0	9	14	16	13
b_j	40	6	8	18	6	

$3-\frac{3}{5}\times 5$

$2+\frac{2}{5}$

[8]

(c) Find the minimum cost solution for the 5 x 5 assignment problems whose cost coefficients are as given in the adjacent table.

-2	-4	-8	-6	-1
0	-9	-5	-5	-4
-3	-8	-9	-2	-6
-4	-3	-1	0	-3
-9	-5	-8	-9	-5

$\frac{1}{5}+\frac{2}{19}\times\frac{3}{5}$

$\frac{1}{5}+\frac{2}{19}\frac{3}{5}$

[7]

2. (a) How to solve a unbalanced assignment problems having negative cost.
(b) Define loop in Transportation problem? How to remove degeneracy from a Transportation problem.
(c) Define Transportation and Assignment problems?

$\frac{9\times 5}{19}$

[5+5+5]

3. (a) In a 2 X 2 game if the largest and second largest elements lie along a diagonal, then show that the game has no saddle point. [5]

(b)For what value of 'a', the game with the following payoff matrix is strictly determinable?

a	5	2
-1	a	-8
-2	3	a

$\frac{5}{a}\times 10-\frac{2}{19}\times 15$ $\quad -6+15 \quad -\frac{3}{\cancel{5}}\times\frac{\cancel{5}}{19}$

[5]

(c) Write down the algorithm for solving a game problem by dominance method. [5]

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$-\frac{6}{5}+5$

$\frac{1}{5}+\frac{2}{19}\frac{3}{5}=\frac{1}{5}+\frac{6}{\quad}$ $\quad \frac{2}{5}-\frac{19}{5}\times\frac{2}{19}$

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BSMS 5th Semester, End–Term Examination–2018

Name of Subject: **Integral Transforms & Applications**

Code No: **DSMA35B14**

Full Marks: 100 Times: 3 Hours

Symbols used here have their usual meanings

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Group-A

Answer all the following questions

1. Find the potential $V(r,z)$ of a field due to a flat circular disc with centre at origin and radius 1. The axis of the disc is along z-axis and is satisfies the partial differential equation

$$\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{\partial^2 V}{\partial z^2}=0, 0\le r<\infty, z\geq 0$$

And boundary conditions are $V=V_0$, when $z=0, 0\le r<1$, $\frac{\partial V}{\partial z}=0$, when $z=0, r>1$. [10]

2. State first and shifting theorem for inverse Laplace transform. Solve $ty''-2y'+ty=0, y(0)=0, y'(0)=6$. (using Laplace transform) [3+7=10]

3. Define Hankel transform. State and prove inversion formula for Hankel transform. [12]

4. Prove that $H_n\left\{\frac{d^2f}{dx^2}+\frac{1}{x}\frac{df}{dx}-\frac{n^2}{x^2}f\right\}=-s^2F_n(s)$. [10]

5. Show that $\int_0^\infty e^{-t}\,\frac{\sin^2 t}{t}dt=\frac{1}{4}\log 5$. [4]

6. Find the inverse Laplace transform of the function $F(s)=\frac{s}{s^4+s^2+1}$ using partial fraction expansion. [4]

Group-B

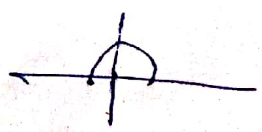
1. a. If $f(x)=\{\pi-|x|\}^2$ on $[-\pi,\pi]$, prove that the Fourier series of f is given by $\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{cosnx}{n^2}$. Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ and $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$.

b. Expand the following function in cosine series in the interval $\left(\frac{3}{2}, 3\right)$:

$\{\pi - |x|\}^2$

$\{\pi - |-x|\}^2 = \{\pi - |x|\}^2$

$f(x) = f(-x)$



$$f(x) = \begin{cases} 1, & \frac{3}{2} < x \le 2 \\ 3 - x, & 2 < x < 3 \end{cases}$$

Hence show that

$$\frac{\pi^2}{9} = \sum_{n=1}^{\infty} \frac{1}{n^2} [Cos \frac{n\pi}{3} - Cos\, n\pi]$$

c. Define Fourier integral, Fourier Sine and Cosine integral. Express the function

$$f(x) = \begin{cases} Sin\, x, & if\ 0 \le x \le \pi \\ 0, & if\ x > \pi \end{cases}$$

as a Fourier Sine integral and show that

$$\int_0^{\infty} \frac{Sin\, wx\ Sin\, \pi w}{1 - w^2} dw = \frac{\pi}{2} Sin\, x, \quad 0 \le x \le \pi$$

8+5+7=20

7. a. State and prove Modulation Theorem for Fourier Transform.
b. Find the Fourier Transform of

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

and hence evaluate

(i) $\int_{-\infty}^{\infty} \frac{Sin\, sa\ Cos\, sx}{s} ds$

(ii) $\int_0^{\infty} \frac{Sin\, s}{s} ds$

c. Find the Fourier Transform of $e^{-|x|}$.

d. Solve the integral equation,

$$\int_0^{\infty} F(x) Cos\, \lambda x\, dx = \begin{cases} 1 - \lambda, & for\ 0 \le \lambda \le 1 \\ 0, & for\ \lambda > 1 \end{cases}$$

e. Find *f(x)* if its Fourier Sine transform is $\frac{s}{1+s^2}$

3+5+3+4+5=20

8. a. Find the Finite Fourier Cosine transform of $\left(1 - \frac{x}{\pi}\right)^2$
b. State and prove Parseval's identity for Fourier Transform.

5+5=10

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Enrolment No. 167444

S5(DSMA35B15):MA

BSMS 5th SEMESTER END TERMEXAMINATION – 2018

SUBJECT NAME: Fuzzy Mathematics, Rough Sets and Applications

SUBJECT CODE :DSMA35B15

Full Marks: 100 Time: 3 Hours

Symbols used here have their usual meanings

Group-A

Answer all the following questions: 5×10=50

1. a) Define Fuzzy relation with example.
   b) Define union and intersection of two fuzzy relation with example.
   c) Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ be the set of earthquake regions and the following table represents the ratio of damages in different regions.

Regions	x_1	x_2	x_3	x_4	x_5
x_{i1} (no damages)	0.3	0.2	0.1	0.7	0.4
x_{i2} (medium damages)	0.6	0.4	0.6	0.2	0.6
x_{i3} (severe damages)	0.1	0.4	0.3	0.1	0

Compute r_{12} using cosine amplitude similarity method. [2+5+3]

2. Discuss different types of fuzzy functions with example. [10]

3. a) Define reflexive and symmetric fuzzy relation with example.
   b) Let $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ are two reflexive fuzzy relations. Is $\boldsymbol{R_1 o R_2}$ reflexive? Justify your answer with a suitable example.
   c) Let $\check{f} = \{(f_1, 0.4), (f_2 0.7), (f_3, 0.4\}$ and $f_1 = x, f_2 = x^2, f_3 = x + 1$.
   Compute $\int_1^2 \check{f}\, dx$. [2+5+3]

4. a) Consider a bus trip from Agartala to NITA campus subject to the following:
   - the distance between the two places is nearly 20 km
   - the speed cannot exceed 50km/h
   - the traffic is usually intense and the speed decreases at the toll booths
   - the bus usually leaves late but not more than 30 minutes

   Compute the total time spent on a trip from Agartala to NITA.

b) Define upper approximation, lower approximation and boundary region using rough membership function.

[7+3]

5. a) Define intuitionistic fuzzy number.

b) Derive the membership and non membership function of addition of two intuitionistic fuzzy numbers.

c) Define interval valued fuzzy set and L-fuzzy set with interpretation.

[1+7+2]

Group-B

Answer all the following questions: 5×10=50

1. **a.** Define **i) Cartesian Product of fuzzy sets**

ii) Fuzzy Point

iii) s-norm

b. Let $\tilde{A}(x) = \{(3,.5),(5,1),(7,.6)\}$ and $\tilde{B}(x) = \{(3,1),(5,.6)\}$. Find the followings:

i) $\tilde{A} \times \tilde{B}$ **and ii)** $\tilde{A} \oplus \tilde{B}$.

c. Define **L-R representation** of a fuzzy number.

d. Let $A = \begin{bmatrix} 0.7 & 0.5 \\ 0.4 & 1 \\ 0 & 0.8 \end{bmatrix}$ be a fuzzy matrix. Find $A \leftarrow A'$.

e. If μ_1, μ_2 and μ_3 are fuzzy subsets of the set X then prove that $(\mu_1 + \mu_2) + \mu_3 \equiv \mu_1 + (\mu_2 + \mu_3)$.

[(2+2+2) + (2+2) + 2 + 3 + 4] = 19

2. a) Define **Fuzzy Decision** and **Efficient Solution.**

b) Solve symmetric fuzzy linear programming problem using **Zimmermann's approach.**

c) Discuss Verdegay's approach to solve fuzzy LPP.

[(2+2) + 7 + 5] = 16

3. a) Briefly discuss **Multi-Objective Optimization Problem.**

b) Discuss a solution procedure to solve **Multi-Objective Linear Programming Problem** where the objectives are fuzzy.

c) Define **ideal point** and **ideal solution** of a Multi-Objective Optimization Problem.

[4 + 7+ (2+2)] = 15

$(a_1, b_1, c_1; a_1', b_1', c_1')$

16DSMA0029

Enrolment No. [| | | | | | |]

S5(DSMA35B13) MA

BS-MS 5th SEMESTER, END-TERM EXAMINATION-2018

NAME OF THE SUBJECT: Statics and Dynamics

CODE NO: DSMA35B13

Full Marks: 100

Time: 3 hours

Symbols used here have their usual meanings

Group-A

Answer all the following questions [50 Marks]

1. What do you mean by **Catenary of uniform strength**? If the densities at any point of a cord vary as the radius of curvature of the curve in which it hangs, show that this is the catenary of uniform strength. [2+5]

2. ABC is a triangle. The forces P, Q, R act along OA, OB, OC and be in equilibrium. Prove that $\frac{P}{BC} = \frac{Q}{CA} = \frac{R}{AB}$, where O is the orthocenter of the triangle ABC. [6]

3. Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 3, 2x + y = 1,\ x - y + 1 = 0$. Find the equation of the line of action of the resultant. [6]

4. When a body is said to be in **Stable equilibrium**? A string of length a form the shorter diagonal of a rhombus formed of four uniform rods; each of length and weight W, which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension of the string is $\frac{2W(2b^2-a^2)}{b\sqrt{(4b^2-a^2)}}$. [2+6]

5. Define **Poinsot's central axis.** Equal forces act along two perpendicular diagonals of opposite faces of a cube of side a. Show that they are equivalent to a single force R acting along a line through the centre of the cube and a couple $\frac{1}{2}aR$ with the same line for axis. [2+6]

6. A system of coplanar forces acts on a rigid body and the moment of the system about three non-collinear points in the plane of the forces are α, β, γ. Prove that
(i) if α, β, γ are not equal and not zero, the system is equivalent to a single force.
(ii) if $\alpha = \beta = \gamma = k, (k \neq 0)$, the system is equivalent to a couple,
(iii) if $\alpha = \beta = \gamma = 0$, the system is in equilibrium. [8]

$G^0 = G - E_s Y + \eta X$

7. State and prove the **Energy test of stability**. [7]

Group-B

[50 Marks]

Answer all from the following questions:

1. Consider the motion of a particle under a force which is directed towards a fixed point and varies inversely as the square of the distance from that point. Prove that the equation of the elliptical orbit is given by $v^2 = \mu(\frac{2}{r} - \frac{1}{a})$. [7]

2. Explain stability of orbits for the motion of a particle moving under a central force. [7]

3. (a) Write Kepler's laws of planetary motion.

(b)If the velocity of a body in an elliptic orbit, major axis **2a**, is the same at a certain point **P**, whether the orbit being described in a periodic time T about one focus S or in periodic time T′about the other focus S′, prove that $SP = \frac{2aT'}{T+T'}$ and $S'P = \frac{2aT}{T+T'}$

[3+7=10]

4. A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to infinity. Show that the time it takes in reaching a height is $\frac{1}{3}\sqrt{\frac{2a}{g}}\left\{(1+\frac{h}{a})^{\frac{3}{2}}-1\right\}$, where a is the radius of the earth.

[7]

5. What is Simple Harmonic Motion? A particle of mass m is executing a S.H.M in a straight line under a force equal to $mn^2 \times (distance)$ towards a fixed point in the line in a medium which offers a resistance $mn \times (velocity)$ interrupted by a periodic force $mg\cos pt$. If the period of free vibration be equal to one half of that of the forced vibration, then show that $3n^2 = 16p^2$.

[2+5=7]

6. A particle is projected with a velocity u at an angle α to the horizon in a medium whose resistance is $mk \times (velocity)$; obtain the equation of motion of the particle. Show also that the direction of its velocity will make an angle $\frac{\alpha}{2}$above the horizon after a time $\frac{1}{k}\log\left(1+\frac{kV}{g}\ tan\frac{\alpha}{2}\right)$.

[7+5=12]

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