Enrolment No. [| | | | | | |]

S$_4$(DSMA34B08): MA

BS-MS 4th SEMESTER, END TERM EXAMINATION – 2019
NAME OF THE SUBJECT: Linear Algebra
SUBJECT CODE: DSMA34B08

Full Marks: 100 Time: 3 Hours

Symbols used here have their usual meanings

Group - A

Marks: 50

Answer the following questions:

1. If $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be an orthogonal set of non-zero vectors in an inner product space V(K) and if β is any vector in V, then show that $\sum_{i=1}^{n} \left[\frac{|<\beta,\alpha_i>|^2}{\|\alpha_i\|^2}\right] \le \|\beta\|^2$

2. Define inner product space. If $\{u_1, u_2, \dots, u_n\}$ is any finite orthonormal set in an inner product space $V(K)$ and if u is any vector in V, then prove that $\sum_{i=1}^{n} |< u, u_i >|^2 \le \|u\|^2$.

$u = \sum_{i=1}^{n} a_i u_i$

$<u, u_i>$

Furthermore, equality holds iff u is in the subspace generated by $\{u_1, u_2, \dots, u_n\}$ or iff $\{u_1, u_2, \dots, u_n\}$ is a basis for V.

3. Find an orthonormal basis of the vector space V of all real polynomials of degree not greater than two, in which the inner product is defined as $[\varphi(x), \psi(x)] = \int_{-1}^{1} \varphi(x)\psi(x)dx$ where $\varphi(x), \psi(x) \in V$. $TT^* = T^*T$

4. Define self adjoint operator. Let S and T be linear operators on a finite dimensional inner product space $V(K)$ and $\alpha \in K$. Then show that (i) $(TS)^* = S^*T^*$, (ii) $(T^*)^* = T$

[6+(2+6)+6+(1+(2+2))=25]

5. Let $V(K)$ be a finite dimensional inner product space and f is a linear functional on V. Then prove that there exists a unique vector β in V such that $f(\alpha) =< \alpha, \beta > \forall\, \alpha \in$ V.

6. Every finite dimensional inner product space has an orthonormal basis. Gauss Schmidt

$|<u, v>| = \|u\|\|v\|$

7. Suppose P is the transition matrix from a basis $\{e_i\}$ to the basis $\{e_j'\}$ in a vector space V over a field K. Show that $P[v]_e' = [v]_e$, $v \in V$. Here also show that $[v]_e' = P^{-1}[v]_e$.

e_i'
A, P
v, $[v]_e'$
$[v_e]$
$P[u]_e'$

8. If the matrix of a linear map T on $V_3(C)$ w.r.t. the basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ is $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$. What is matrix of T w.r.t. basis $\{(1,1,-1),(-1,0,1),(1,2,1)\}$?

9. If W be a subspace of an inner product space V and $\{w_1, w_2, \dots, w_n\}$ is a basis for W. Then show that $w \in W$ if and only if $\langle w, w_i \rangle = 0$ for $i = 1,2,\dots,n$.

orthogonal

[6+6+6+4+3=25]

$\frac{6+2}{5} = \frac{8}{\frac{12}{5}}$

P.T.O

Group – B

Marks: 50

Answer the following questions:

1. a) Prove that every linearly independent sub-set of a finitely generated vector space V(F) is either a basis of V or can be extended to form a basis of V.
   b) Show that the row rank and the column rank of a matrix are equal. $R_i, S_k \quad R_1R_2 \cdots R_n \quad a_{ii} \cdots$
   c) If V(F) be a finite dimensional vector space then prove that any two bases of V have same number of elements. $n=m \quad S_1 \quad S_2 \quad S_3 \quad S_4$

   [5 + 4 + 4] = 13

2. a) State and prove **Replacement theorem** of basis.
   b) Find a basis containing the vectors **(1,1,0) , (1,1,1).** $\varphi(x,y) = ax+by \qquad \varphi(x,y) = cx+dy$
   c) If V has finite dimensions, then prove that the mapping $v \to \hat{v}$ is an isomorphism of V onto V^{**}.

   [[2 + 5] + 4 + 3 = 14]

3. a) Define **Annihilator** and **Dual Space** of a vector space.
   b) Suppose $\{v_1, v_2, \dots, v_n\}$ be a basis of V over K. Let $\varphi_1, \varphi_2, \dots, \varphi_n \in V^*$ be the linear functional as defined by $\varphi_i(v_j) = \delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$. Then prove that $\{\varphi_1, \varphi_2, \dots, \varphi_n\}$ is a basis of V^*.
   c) Find the dual basis of **{(1,-2,3), (1,-1,1) , (2,-4,7)}.**

   [[2+2] + 4 + 3] = 11

4. a) Find the minimal polynomial of A = $\begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$. $|A - \lambda I|$
   b) Let $\{v_1, v_2, \dots, v_n\}$ be a basis of V and let $\{\varphi_1, \varphi_2, \dots, \varphi_n\}$ be the dual basis of V^*. Then prove that for any linear functional $\sigma \in V^*, \sigma = \sum_{i=1}^{n} \sigma(v_i)\varphi_i$.
   c) Suppose V has finite dimension and W is a subspace of V. Then prove that

   $$\dim W + \dim W^0 = \dim V$$

   d) Let W be the subspace of R^4 spanned by {(1,2,-3,4), (1,3,-2,6), (1,4,-1,8)}. Find a basis of the annihilator of W.

   [3 + 3 + 3 + 3] = 12

5
4
4
3

12
18
$\boxed{20}$

Enrolment No. [ ][ ][ ][ ][ ][ ][ ][ ][ ]

S$_4$(DSMA34B09): BS-MS

BSMS 4th Semester, End-Term Examination-2019

Name of Subject: **Partial Differential Equation**

Code No: **DSMA34B09**

Full Marks: 100 **Times: 3 Hours**

Symbols used here have their usual meanings

Group-A

Answer all questions:

1. (a) Find the complete integral, singular integral and general integral of the following PDE:

~~(i)~~ $x^2p^2 + y^2q^2 = z^2$. $f(z, a, q)$ $p = \frac{\partial z}{\partial u}$ $q = a\frac{\partial z}{\partial u}$

(ii) $p^2 + q^2 = (x^2 + y^2)z$

(b) Show that the PDE $z = px + qy$ *and* $2xy\,(p^2 + q^2) = z(yp + xq)$ are compatible and find their solution.

(4+4) + 7 = 15

2. (a) Solve (by Charpit's Method): $xp + 3yq = 2\,(z - x^2q^2)$

(b) Obtain the boundary value problem: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, if $u(x, 0) = 6e^{-3x}$ $u(0,t) = u(l,t) = 0.$

(c) Using the method of separation of variables, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$,where $u(0, t) = 0$, $u(4, t) = 0$ and $u(x, 0) = 6\sin(\pi x/2) + 3\sin(\pi x)$. TX $X''T$

4+5+6 =15

3. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in the equilibrium position. It is set vibrating by giving to each of its point a velocity of $v_0 \sin^3\frac{\pi x}{l}$. Find the displacement $y\,(x, t)$.

(b) A bar AB of 10 cm length has its ends A and B kept at 30°C and 100°C respectively, until steady-state condition is reached. Then the temperature at A is lowered to 20°C and that at B to 40°C and these temperatures are maintained. Find the subsequent temperature distribution in the bar.

10+10 = 20

Group-B

Answer any five questions: **10*5=50**

4. (a) Describe the method of reducing Euler-Cauchy type equation to a linear partial differential equation with constant coefficients. 5

(b) Solve: $ys + p = \cos(x + y) - y\sin(x + y)$ 5

5. (a) Solve: $xr + ys + p = 10x^3 y$. 5

(b) Solve the following PDE, $yt + 2q = (9y+6)e^{2x+3y}$ 5

6. (a) Find the general solution of the PDE

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} + nz = n(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}) + x^2 + y^2 + x^3 .$$ 5

(b) Solve $(D^2 - D'^2 3D + 3D')z = xy$. 5

7. (a) Solve $D(D+D'-1)(D+3D'-2)z = x^2 - 4xy + 2y^2$. 5

(b) Define reducible and irreducible linear partial differential equations with constant coefficients. Write down the working rule for finding complementary function of reducible non-homogeneous linear partial differential equation with constant coefficients. 5

8. (a) Solve $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y} + xy + \sin(2x+y)$ 5

(b) Solve $: r - t = tan^3 x \tan y - tanx \tan^3 y$ 5

$f(x, c-mx)$

9. (a) Describe the method of finding the complementary function of the linear homogeneous partial differential equation with constant coefficient, namely $F(D,D')z = f(x,y)$ 6

(b) Solve $(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x} + (y+x)^{1/2}$ $(D-2D')(D-D')$ 4

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$r = \frac{\partial^2 z}{\partial x^2}$

$p = \partial$ $s = \frac{\partial^2 z}{\partial x \partial y}$ $\frac{\partial z}{\partial q}$ $\frac{1}{2}\tan x \tan y.$

$t = \frac{\partial^2 z}{\partial y^2}$

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

17DSMA017

Enrolment No.

$S_{4(DSMA34B10)}$ MA

BSMS 4th SEMESTER END TERM EXAMINATION 2019

Subject Name: Numerical Analysis

Subject Code: DSMA34B10

Full Marks: 100 Time: 3 hours

Symbols used here have their usual meanings

Group A

Answer all the following questions 5× 10

1. a) Find the root of the equation $x \ln x = 1$, by Regula Falsi method correct to 5 significant figures.

   b) State and prove condition for convergence of iteration method.

   [6+4]

2. a) Explain Power method.

   b) Find all eigen values and eigenvectors of the matrix using Jacobi method (**perform 2 iteration**):

$$\begin{pmatrix} 1 & 1 & 0.5 \\ 1 & 1 & 0.25 \\ 0.5 & 0.25 & 2 \end{pmatrix}$$

   [4 + 7]

3. a) Show that Newton-Raphson method converges with constant error $\frac{1}{2}\left|\frac{f''(\alpha)}{f'(\alpha)}\right|$, where α is a root of $f(x) = 0$.

   b) If $f(x)$ is a quadratic polynomial, show that $\int_1^3 f(x)dx = \frac{1}{12}[f(0) + 22f(2) + f(4)]$.

   x_1 x_2

   [5+5]

4. a) Derive Gauss 2-point formula and hence evaluate $\int_0^2 \frac{dx}{1+x^2}$.

   $t_0=0$ $(b-a)^2$

   b) Write down the expression for Newton-Cote's coefficient $\left(H_i^{(n)}\right)$ and hence show that $\sum_{i=0}^{n} H_i^{(n)} = 1$.

   [7+3]

5. a) Evaluate $\int_0^{0.6} e^x dx$ correct to 5 decimal places by Simpson's 1/3 formula with 7 ordinate. Also compute absolute error and relative error. Give the geometrical interpretation of trapezoidal formula.

   b) Derive truncation error in Trapezoidal formula.

   [7+3]

Group B

Answer all the following questions

Marks: 50

1. (a) Solve the system of equation by Gauss-Seidel iteration method,

$$3x_1 + 9x_2 - 2x_3 = 11$$
$$4x_1 + 2x_2 + 13x_3 = 24$$
$$4x_1 - 2x_2 + x_3 = -8$$

Correct upto four significant figures.

(b) Find by *Newton's divided difference formula*, the interpolating polynomial from the following table:

x	0	1	2	5
$f(x)$	2	3	12	147

$y(x_0) + (x - x_0)[x_0, x_1]$

Hence find $f(4)$.

[6+6]

2. (a) Find the value of $y(1.2)$, using Improved Euler's method with $h = 0.2$, given that $\frac{dy}{dx} = \frac{2y}{x} + x^3$, $y(1) = 0.5$. Compare the values with the exact solution.

(b) Find the inverse of the matrix, by Gauss Jordan elimination method:

$$A = \begin{bmatrix} 3 & -1 & 10 & 2 \\ 5 & 1 & 20 & 3 \\ 9 & 7 & 39 & 4 \\ 1 & -2 & 2 & 1 \end{bmatrix}$$

[6+6]

3. Derive the relation between differential operator (D) and the shift operator (E). Solve the equation $\frac{d^2y}{dx^2} = xy^2, y(0) = 1, y'(0) = 0$ for $y(0.2)$ **and** $y(0.4)$ by Runge-Kutta method of the fourth order.

[3+6]

4. State Lagrange's inverse interpolation formula. Find the value of $y(0.3)$, using Adam-Bash forth's predictor corrector method, given that $\frac{dy}{dx} = (x + y)e^{-x}$, $y(-0.1) = 0.9053$, $y(0) = 1$, $y(0.1) = 1.1046$, $y(0.2) = 1.2173$.

[3+7]

5. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, at $x = 3$ and 6 for the function $y = f(x)$ given in the table:

x	1	2	3	4	5	6
$f(x)$	2.7183	3.3210	4.0552	4.9530	6.0496	7.3891

[7]

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BSMS 4th SEMESTER, END-TERM EXAMINATION-2019

NAME OF THE SUBJECT: DATA STRUCTURE AND ALGORITHMS

CODE NO: DSMA34B11

Full Marks: 100 Time: 3 hours

Answer any ten (10) questions from the following: [10 × 10 = 100 Marks]

1. **Fill in the blanks:** [10 × 1]
   a) The ASCII code for A–Z varies from ______.
   b) ______ is the address of the first element in the array.
   c) Memory is allocated for a structure when ______ is done.
   d) Inserting a node at the end of the circular linked list needs to modify ______ pointers.
   e) ______ returns the position in the string where the string pattern first occurs.
   f) A function is said to be ______ recursive if it explicitly calls itself.
   g) Each array element is accessed using a ______.
   h) Inserting a node at the beginning of the circular doubly linked list needs to modify ______ pointers.
   i) A structure is declared using the keyword struct followed by a ______.
   j) toupper() is used to ______.

2. **Tick the correct option for the following questions:** [10 × 1]

(a) If an array is declared as arr[] = {1,3,5,7,9}; then what is the value of arr[3]?
(i) 1 (ii) 7 (iii) 9 (iv) 5

(b) Which function adds a string to the end of another string?
(i) stradd() (ii) strcat() (iii) strtok() (iv) strcpy()

(c) The index of U in Oxford University Press is?
(i) 5 (ii) 6 (iii) 7 (iv) 8

(d) A string can be read using which function(s)?
(i) gets() (ii) scanf() (iii) getchar() (iv) all of these

(e) A structure member variable is generally accessed using
(i) Address operator (ii) Dot operator (iii) Comma operator (iv) Ternary operator

(f) typedef can be used with which of these data types?
(i) struct (ii) union (iii) enum (iv) all of these

(g) Linked list is used to implement data structures like
(i) Stacks (ii) Queues (iii) Trees (iv) All of these

(h) Typical time requirement for operations on queues is
(i) O(1) (ii) O(n) (iii) O(log n) (iv) O(n^2)

(i) When a node N is accessed it is splayed to make it the
(i) Root node (ii) Parent node (iii) Child node (iv) Sibling node

(j) Total number of nodes at the nth level of a binary tree can be given as
(i) 2^n (ii) 2^n-1 (iii) 2^{n+1} (iv) 2^{n-1}

3. **Which of the following statement is true or false, justify your results?** **[10 × 1]**

(a) C permits copying of one structure variable to another.

(b) A linked list can grow and shrink during run time.

(c) A loop is used to access all the elements of an array.

(d) Assignment operator can be used to copy the contents of one string into another.

(e) If the thread appears in the right field, then it will point to the in-order successor of the node.

(f) When we insert a new node in a binary search tree, it will be added as an internal node

(g) The dereference operator is used to select a particular member of the structure.

(h) Every node in a linked list contains an integer part and a pointer.

(i) A node that has no successors is called the root node.

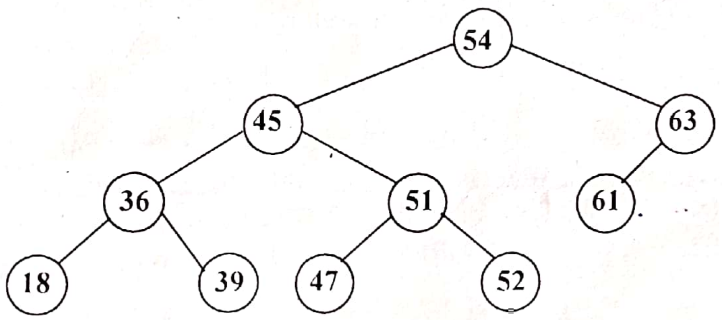
(j) It is possible to pass an entire array as a function argument.

4. (a) Write an algorithm to delete an element from any position in a single linked list?

(b) Write down the overview of Data Structure and explain the terms related to non-linear data structures with examples.

[5 + 5]

5. (a) What do you mean by AVL tree? Delete 52 and 61 from the following tree.



(b) Construct an AVL tree by inserting the following elements in the given order.

45, 39, 56, 12, 34, 78, 32, 10, 89, 54, 67, 61

[5 + 5]

6. (a) Write three the difference between B+ tree and B tree.

(b) Find the values of $\lfloor \sqrt[3]{50} \rfloor$ and $\lceil e^e \rceil$?

(c) Write an algorithm to search an element in single linked list?

[3 + 2 + 5]

7. (a) Write three differences between **worst-fit allocation** and **next-fit allocation?**

(b) What do you mean by the term **internal fragmentation**?

(c) Find the number of elements in the **αβ - band** matrix.

[3 + 4 + 3]

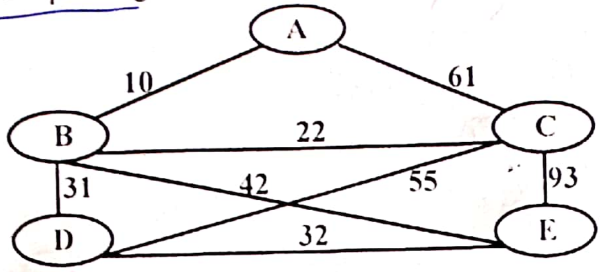
8. (a) Write two advantages and disadvantages of sequential representation of a binary tree respectively?

(b) How many types of rotations are there? Explain with diagram.

[5 + 5]

9. (a) Is a flowchart is a graphical representation of a program? Give reason.

(b) Construct a minimal spanning tree for the following graph. (Taking A as the initial vertex)



[5 + 5]

10. (a) Write the algorithm to insert the first node of a single linked list.

(b) Draw all the possible non-similar binary trees having five nodes.

(c) Show that,

(i) $4n^2 = o(n^3)$.

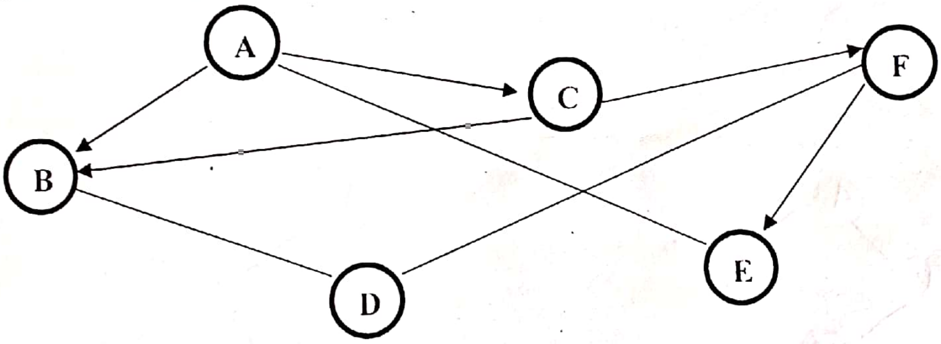
(ii) $400n^3 + 20n^2 = o(n^3)$.

[4 + 2 + 4]

11. (a) Write an algorithm for de-queue in a circular queue.

(b) What do you mean by **divide and conquer** rule? Explain with example.

(c) Draw the adjacency matrix for the following graph.



[5 + 3 + 2]

12. (a) When a Graph is said to be complete? Draw a complete Graph with five vertices.

(b) Write the difference between **ENQEUE** & **DEQEUE**.

(c) Write an algorithm to traverse an element in an array?

[4 + 2 + 4]

13. (a) Why **quick sort algorithm** is used in data structure and algorithm?

(b) What is an **asymptotic notation**?

(c) Define **average queuing time, average queue length** and **total service time.**

(d) Write an algorithm for tower of Hanoi problem?

[2 + 1 + 3 + 4]

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Enrolment No. [| | | | | | | |]

$S_{4(DSMA34B07)}$: MA

BS/MS. 4th SEMESTER, END-TERM EXAMINATION-2019

SUBJECT NAME: REAL ANALYSIS

SUBJECT CODE NO: DSMA34B07

Full Marks: 100 **Time: 3 Hours**

Symbols used here have their usual meanings

Group – A **[Marks: 50]**

Answer all of the following questions:

1. A) Show that any finite set has no limit points.
   B) Show that union of infinite number of closed set in R may not be a closed set.
   C) A function f is defined on $[0, 1]$ by $f(x) = \begin{cases} 1, \text{if } x \text{ is rational} \\ 0, \text{if } x \text{ is irrational} \end{cases}$. Find $\overline{\int_0^1} f\,dx$ and $\underline{\int_0^1} f\,dx$ and hence show that f is not integrable on $[0, 1]$.

[2+3+5=10]

2. A) Define norm of a partition. Let a function $f: [a, b] \to R$ be bounded on $[a, b]$. If $\{P_n\}$ be a sequence of partitions of $[a, b]$ such that sequence $\{\|P_n\|\}$ converges to 0, then show that $\lim_{n\to\infty} U(P_n, f) = \overline{\int_a^b} f$.
   B) A function f is defined on $[a, b]$ by $f(x) = e^x$. Find $\overline{\int_a^b} f\,dx$ and $\underline{\int_a^b} f\,dx$. Deduce that f is integrable on $[a, b]$.

[(1+4)+5=10]

3. A) Let a function $f: [a, b] \to R$ be bounded on $[a, b]$ and let f be continuous on $[a, b]$ except for a finite number of points in $[a, b]$. Then show that f is integrable on $[a, b]$.
   B) Let $f(x) = sgn\, x, x \in [-2, 2]$. Then show that f is integrable on $[-2, 2]$.

[8+2=10]

4. A) If $f(x) = \begin{cases} \frac{1}{n}, & \frac{1}{n+1} < x \le \frac{1}{n}\ (n = 1, 2, 3, \ldots) \\ 0, & x = 0 \end{cases}$, then prove that f is integrable on $[0, 1]$ and hence evaluate $\int_0^1 f$.
   B) Let f be defined on [-2, 2] by $f(x) = \begin{cases} 3x^2 cos\frac{\pi}{x^2} + 2\pi sin\frac{\pi}{x^2}, & x \ne 0 \\ 0, & x = 0 \end{cases}$. Show that f is integrable on $[-2, 2]$. Evaluate $\int_{-2}^{2} f$ by using its primitive.

[6+4=10]

5. A) Define primitive of a function with example.
   B) State first mean value theorem of integral calculus. Use first mean value theorem to prove that $\frac{\pi}{6} \le \int_0^{1/2} \frac{1}{\sqrt{(1-x^2)(1-k^2x^2)}} dx \le \frac{\pi}{6} \frac{1}{\sqrt{1-k^2/4}}, k^2 < 1$.

[2+2+6=10]

P.T.O

Group – B

[Marks: 50]

Answer all of the following questions:

1. (a) Define monotonic sequence.

(b) Show that the function f defined by

$$f(x) = \begin{cases} [x+1]sin\frac{1}{x} & ,x \in (-1,0) \cup (0,1) \\ 0 & , \quad otherwise \end{cases}$$

has discontinuity of the second kind at $x = 0$ and discontinuity of the first kind at $x = 1$.

(c) Prove that a function f is defined on an interval I is continuous at a point $c \in I$ iff for every sequence $\{c_n\}$ in I converging to c.

[1+4+5=10]

2. (a) Define continuity and uniform continuity of a function.

(b) Prove that every uniform continuous function is continuous function but the converse may not be true in general.

[2+(3+5)=10]

3. (a) Prove that every bounded sequence has a limit point.

(b) If $f(x)$ be any polynomial then prove that between any pair of roots $f(x) = 0$ lies a root of $f'(x) = 0$.

[5+5=10]

4. (a) Show that $2x < log\,\frac{1+x}{1-x} < 2x\{1 + \frac{x^2}{3(1-x^2)}\}, 0 < x < 1$.

(b) If $f(x), \phi(x)$ and $\psi(x)$ are continuous on $[a, b]$ and derivable on (a, b) then prove that there exist a value $c \in (a, b)$ such that

$$\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f'(c) & \phi'(c) & \psi'(c) \end{vmatrix} = 0$$

(c) Write down the geometric interpretation of Lagrange's Mean Value theorem.

[5+3+2=10]

5. (a) State and prove Cauchy's first theorem on limits.

(b) Expand $\sqrt{1+x+x^2}$ in powers of $(x-1)$.

[5+5=10]

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