

01/03

S<sub>6</sub>(DSMA36B16) MathematicsBSMS-6<sup>th</sup> SEMESTER, MID-TERM EXAMINATION-2019

NAME OF THE SUBJECT: Complex Analysis

SUBJECT CODE: DSMA36B16

Full Marks: 50

Time: 2 hours

BS-MATH  
(H.B.)

Answer all questions:

Group-A

Marks: 25

1. (a) If  $f(z)$  is analytic within and on a circle  $C$ , given by  $|z - a| = R$  and if  $|f(z)| \leq M$  for every  $z$  on  $C$ , then show that  $|f^n(a)| \leq \frac{M n!}{R^n}$
- (b) State and prove the Liouville's theorem.
- (c) Suppose a function  $f(z)$  is analytic in the closed ring bounded by two concentric circles  $C$  and  $C'$  of centre  $a$  and radii  $R$  and  $R'$ , ( $R' < R$ ). If  $z$  is any point of the annulus, then show that  $f(z) = \sum_{n=0}^{\infty} a_n (z - a)^n + \sum_{n=1}^{\infty} b_n (z - a)^{-n}$  where  $a_n = \frac{1}{2\pi i} \int_C \frac{f(t)}{(t-a)^{n+1}} dt$  and  $b_n = \frac{1}{2\pi i} \int_{C'} \frac{f(t)}{(t-a)^{-n+1}} dt$ .

[3+4+6=13]

2. (a) State and prove the Cauchy integral formula for higher order derivative.

(b) Calculate  $\int_C \frac{\sin z}{(z - \frac{\pi}{4})^3} dz$  where  $C$  is  $|z - \frac{\pi}{4}| = \frac{1}{2}$

- (c) Find the Laurent's series of  $\frac{1}{z^2 - 3z + 2}$  for  $1 < |z| < 2$ .

[6+3+3=12]

Group BAnswer all the following questions

Marks: 25

1. When a function  $f(z)$  of complex variable  $z$  is said to be uniformly continuous? A relation  $R$  on the set of complex numbers is defined by  $zRz' \Leftrightarrow \frac{z_1 - z_2}{z_1 + z_2}$  is real. Show that  $R$  is an equivalence relation.

2. State the necessary condition for a function to be analytic and prove it. [1+5]

3. Define Harmonic function. Prove that the function  $e^{-x}(x \cos y + y \sin y)$  is harmonic and find the harmonic conjugate. [5]

4. (a) State the principle of uniform convergence for a sequence of complex function. [6]

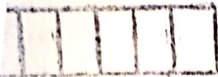
(b) If  $f(z) = u + iv$  is an analytic function of  $z = x + iy$  and  $u - v = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$ , find  $f(z)$  subject to the condition  $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$ .

[2+6]

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$$\frac{e^y - \cos x}{2i} + i \sin x$$

$$\frac{e^y + \cos x}{2} = \cosh y$$



S<sub>6</sub>(DSMA36B17) Mathematics

BS-MS 6<sup>th</sup> SEMESTER, MID-TERM EXAMINATION-2019

Subject Name: Discrete Mathematics

Subject Code: DSMA36B17

Marks: 50

Time: 2 hours

Symbols used here have their usual meanings

Group-A

1. The inverse of the composition of two functions is equal to the composition of the inverses of the functions in the reverse order. [5]
2. If  $f$  is a characteristic function of a set and  $A$  and  $B$  are any two subsets of  $U$ , then  $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_{A \cap B}(x)$ ,  $\forall x \in U$ . [5]
3. Define Hashing functions. Determine whether the functions  $f: \mathbb{Z} \rightarrow \mathbb{Z}$ , defined by  $f(x) = x^2 + 14x - 51$  is an injection and/or a surjection. [1+4=5]
4. Define characteristic function of a set, absolute value function and permutation function with an example. [2+1+2=5]
5. If  $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ , where  $\mathbb{Z}$  is the set of integers and  $f(x, y) = x * y = x + y - xy$ , show that the binary operation  $*$  is commutative and associative. Find the identity element and the inverse of each element. [2+3=5]

Group-B

Answer all the following questions

1. Define generating function. Using generating functions solve the recurrence relation  $a_n = 4a_{n-1} + 3$ , for  $n \geq 1$  with initial conditions and  $a_0 = 2$ . [1+4=5]
2. In how many ways we can distribute 7 objects into 3 distinct boxes in such a way that the first box contains 0, 1 or 3 balls, the second box contains 1, 2 or 3 balls and the third one contains 4 or 6 balls. [4]
3. Prove that a simple graph with  $n$  vertices and  $m$  components can have at most  $\frac{(n-m)(n-m+1)}{2}$  edges. [5]
4. Define cycles and circuit. Give one example of graph which is circuit but not a cycle. [1+1=2]
5. Solve the recurrence relation  $a_n = a_{n-1} + a_{n-2}$ ,  $n \geq 3$  with initial conditions  $a_1 = a_2 = 1$ . Find an explicit formula for  $\{a_n\}$ . [4]
6. Define Euler circuit with example. Prove that if a connected graph  $G$  is Eulerian, then every vertex of  $G$  has even degree.

$$\begin{aligned} -3) & \quad f(n) = 9 - 4 \cdot 2 - 51 \\ & \quad = 9 - 98 \\ & \quad = -89 \end{aligned}$$



S<sub>6</sub>(DSMA36B20): MA

BS-MS 6<sup>th</sup> SEMESTER, MID TERM EXAMINATION – 2019  
NAME OF THE SUBJECT: Number Theory and Cryptology  
CODE NO: DSMA36B20

Il Marks: 50

Time: 2 Hours

Symbols used here have their usual meanings

Group – A

Answer the following questions:

Marks: 25

1. What do you mean by the Principle of Mathematical Induction? Apply the same to show that  $\sum_{j=1}^{n-1} x^j = 1 + x + x^2 + \dots + x^{n-1} = \frac{x^n - 1}{x - 1}$ , for any real number  $x$  other than 1 and hence show that  $n < m^n$ , for any two positive integers with  $m > 1$ .

$$[1.5 + 3.5 + 2] = 7$$

2. State and prove the Basis Representation theorem. Moreover, if  $a_r k^r + a_{r-1} k^{r-1} + \dots + a_0$  is a representation of  $n$  to the base  $k$ , then show that  $0 < n \leq k^{r-1} - 1$ .

$$[1.5 + 4.5 + 3] = 9$$

3. Only state Euclid's Division lemma. Show that the smallest divisor (other than 1) of a composite number is a prime. Then establish the Fundamental Theorem of Arithmetic.

$$[1 + 2.5 + 5.5] = 9$$

Group - B

1. a) Let  $a$  and  $b$  be any integers at least one of them is non-zero. Then prove that there exist integers  $x$  and  $y$  such that  $\text{g.c.d.}(a, b) = ax + by$ .

b) Find the particular and general solutions to the equation  $21x + 14y = 35$ .

c) Using Extended Euclidean Algorithm find the multiplicative inverse of 23 in  $\mathbb{Z}_{100}$ .

d) Write a short note on Ciphers.

$$[3 + 3 + 3 + 3] = 12$$

2. a) Discuss different types of Cipher text only attacks.

b) Use additive cipher with key = 15 to decrypt the message "WTAAD".

c) Solve the following system of equations

$$3x + 2y \equiv 5 \pmod{7}$$

$$4x + 6y \equiv 4 \pmod{7}$$

d) Define Residue Matrix. Find the multiplicative inverse of the following residue matrix  $A$  over

$\mathbb{Z}_{10}$

$$A = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}.$$

$$[3 + 2 + 4 + (1 + 3)] = 13$$





S<sub>6</sub>(DSMA36B19) MA

**BSMS VI<sup>th</sup> SEMESTER, MID-TERM EXAMINATION-2019**  
**NAME OF THE SUBJECT: OBJECT ORIENTED PROGRAMMING**  
**CODE NO: DSMA36B19**

**Full Marks: 50**

**Time: 2 hours**

**Answer all questions from the following:**

**[10 × 5 = 50 Marks]**

State whether the following statements are true or false, justify:

- (i) Templates are declared inside classes or functions.
- (ii) Base class is known as the subclass and derived class as superclass.
- (iii) A member function can't be used in a derived class that override.
- (iv) The scope resolution operator can be overloaded.
- (v) Templates should be global and should not be local.

**[1 × 5]**

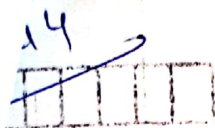
2. Answer the following questions in one word.

- (i) How can you prevent your class to be inherited further?
- (ii) What is the index value of the first element in an array?
- (iii) Can you specify the accessibility modifier for methods inside the interface?
- (iv) Is it possible for a class to inherit the constructor of its base class? ☒
- (v) Can you allow a class to be inherited, but prevent a method from being overridden in C#?

**[1 × 5]**

- 3. What is object-oriented programming (OOP)? Explain the basic features of OOPs.
- 4. What are the differences between a class and a structure?
- 5. What is a delegate? Write down all the features of an interface.
- 6. What are abstract classes? What are the distinct characteristics of an abstract class?
- 7. What are the differences between an abstract class and an interface.
- 8. What is access modifier? What are the various types of constructors in OOP? Explain.
- 9. Define Class. What is the relationship between a class and an object?
- 10. What are the different types of arguments? Explain different types of inheritance in OOP.

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BS/MS 6<sup>th</sup> SEMESTER, MID-TERM EXAMINATION-2019

SUBJECT NAME: STOCHASTIC PROCESSES

SUBJECT CODE NO: DSMA36B18

Marks: 50

Time: 2 Hours

Symbols used here have their usual meanings

## Group – A

er all of the following questions:

- (a) Define absorbing state with an example.  
(b) Describe Markov chain as a graph with an example. [2+3=5]
2. (a) Establish the relationship between probability distribution and transition probability.  
(b) Transition probability matrix of a Markov chain  $\{X_n, n = 1, 2, \dots\}$  having three states 1, 2 and 3 is given below:

$$\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

The initial distribution is  $\pi_0 = (0.7, 0.2, 0.1)$ .Find  $\Pr\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$ .

3. (a) Suppose that a coin with probability  $p$  for ahead is tossed infinite times. Let  $X_n$  be the outcome of the  $n$ th trial be  $k (= 0, 1, \dots, n)$  denotes that there is a run of  $k$  successes. Show that  $\{X_n: n \geq 0\}$  constitute a Markov Chain. Find out one step transition probabilities and hence form the transition matrix. [3+2=5]  
(b) Define class of state with example. [3+2=5]

4. (a) Define periodicity of a state.  
(b) State first entrance theorem. Verify first entrance theorem for the following transition matrix with state space  $S = \{1, 2, 3\}$  for the transition from the state 1 to 3 using 3 steps:

$$P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

5. Suppose that the probability of a dry day following a rainy day is  $1/3$  and probability of a rainy day following a dry day is  $1/2$ . Also probability of a dry day following a dry day is  $1/2$  and probability of a rainy day following a rainy day is  $2/3$ . Form two step transition matrix. Given that May 1 is dry day, find the probability that May 3 is a rainy day. Also find the probability that May 5 is a dry day under the given condition May 1 is a dry day. [1+(1+3)=5]

[5]