Roll No. 1674744 01/03

S_6(DSMA36B16)Mathematics

BSMS-6th SEMESTER, MID-TERM EXAMINATION-2019

NAME OF THE SUBJECT: Complex Analysis

SUBJECT CODE: DSMA36B16

Full Marks: 50 Time: 2 hours

Bs.Math (HB)

Group-A

Answer all questions: Marks: 25

1. (a) If $f(z)$ is analytic within and on a circle C, given by $|z-a| = R$ and if $|f(z)| \leq M$ for every z on C, then show that $|f^n(a)| \leq \frac{M\,n!}{R^n}$

   (b) State and prove the Liouville's theorem.

   (c) Suppose a function $f(z)$ is analytic in the closed ring bounded by two concentric circles C and C' of centre a and radii R and R', $(R' < R)$. If z is any point of the annulus, then show that $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} b_n (z-a)^{-n}$ where $a_n = \frac{1}{2\pi i}\int_C \frac{f(t)}{(t-a)^{n+1}} dt$ and $b_n = \frac{1}{2\pi i}\int_{C'} \frac{f(t)}{(t-a)^{-n+1}} dt$.

[3+4+6=13]

2. (a) State and prove the Cauchy integral formula for higher order derivative.

   (b) Calculate $\int_C \frac{\sin z}{\left(z-\frac{\pi}{4}\right)^3} dz$ where C is $\left|z-\frac{\pi}{4}\right| = \frac{1}{2}$

   (c) Find the Laurent's series of $\frac{1}{z^2-3z+2}$ for $1< |z| < 2$.

[6+3+3=12]

Group B

Answer all the following questions

Marks: 25

1. When a function $f(z)$ of complex variable z is said to be uniformly continuous? A relation R on the set of complex numbers is defined by $zRz \Leftrightarrow \frac{z_1-z_2}{z_1+z_2}$ is real. Show that R is an equivalence relation.

[1+5]

2. State the necessary condition for a function to be analytic and prove it.

[5]

3. Define Harmonic function. Prove that the function $e^{-x}(x\cos y + y\sin y)$ is harmonic and find the harmonic conjugate.

[6]

4. (a) State the principle of uniform convergence for a sequence of complex function.

   (b) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$, find $f(z)$ subject to the condition $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$.

[2+6]

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$\frac{e^y - e^{-y}}{2i}$ = sinhy $\frac{e^y + e^{-y}}{2} = \cosh y$

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S6(DSMA36B17) Mathematics

BS-MS 6th SEMESTER, MID-TERM EXAMINATION-2019

Subject Name: Discrete Mathematics

Subject Code: DSMA36B17

larks: 50 — Time: 2 hours

Symbols used here have their usual meanings

Group-A

1. The inverse of the composition of two functions is equal to the composition of the inverses of the functions in the reverse order.

[5]

2. If f is a characteristic function of a set and A and B are any two subsets of U, then $f_{A\cup B}(x) = f_A(x) + f_B(x) - f_{A\cap B}(x)$, $\forall x \in U$.

[5]

3. Define Hashing functions. Determine whether the functions $f: Z \to Z$, defined by $f(x) = x^2 + 14x - 51$ is an injection and/or a surjection.

[1+4=5]

4. Define characteristic function of a set, absolute value function and permutation function with an example.

[2+1+2=5]

5. If $f: Z \times Z \to Z$, where Z is the set of integers and $f(x, y) = x * y = x + y - xy$, show that the binary operation * is commutative and associative. Find the identity element and the inverse of each element.

[2+3=5]

Group-B

Answer all the following questions

1. Define generating function. Using generating functions solve the recurrence relation $a_n = 4a_{n-1} + 3$, for $n \geq 1$ with initial conditions and $a_0 = 2$.

[1+4=5]

2. In how many ways we can distribute 7 objects into 3 distinct boxes in such a way that the first box contains 0, 1 or 3 balls, the second box contains 1, 2 or 3 balls and the third one contains 4 or 6 balls.

[4]

3. Prove that a simple graph with n vertices and m components can have atmost $\frac{(n-m)(n-m+1)}{2}$ edges.

[5]

4. Define cycles and circuit. Give one example of graph which is circuit but not a cycle.

[1+1=2]

5. Solve the recurrence relation $a_n = a_{n-1} + a_{n-2}, n \geq 3$ with initial conditions $a_1 = a_2 = 1$. Find an explicit formula for $\{a_n\}$.

[4]

6. Define Euler circuit with example. Prove that if a connected graph G is Eulerian, then every vertex of G has even degree.

-3, $f(x) = 9 - 42 - 51$
$= 9 - 93$
$= -84$

S_6(DSMA36B20): MA

BS-MS 6th SEMESTER, MID TERM EXAMINATION – 2019

NAME OF THE SUBJECT: Number Theory and Cryptology

CODE NO: DSMA36B20

ll Marks: 50 Time: 2 Hours

Symbols used here have their usual meanings

Group – A

Answer the following questions: **Marks: 25**

1. What do you mean by the Principle of Mathematical Induction? Apply the same to show that $\sum_{j=1}^{n-1} x^j = 1 + x + x^2 + \dots\dots\dots + x^{n-1} = \frac{x^n - 1}{x-1}$, for any real number x other than 1 and hence show that $n < m^n$, for any two positive integers with $m > 1$.

$[1.5 + 3.5 + 2]$=7

2. State and prove the Basis Representation theorem. Moreover, if $a_r k^r + a_{r-1} k^{r-1} + \dots\dots\dots + a_0$ is a representation of n to the base k, then show that $0 < n \le k^{r-1} - 1$.

$[1.5 + 4.5 + 3] = 9$

3. Only state Euclid's Division lemma. Show that the smallest divisor (other than 1) of a composite number is a prime. Then establish the Fundamental Theorem of Arithmetic.

$[1 + 2.5 + 5.5] = 9$

Group - B

1. a) Let a and b be any integers at least one of them is non-zero. Then prove that there exist integers x and y such that $g.c.d\,(a, b) = ax + by$.

   b) Find the particular and general solutions to the equation $21x + 14y = 35$.

   c) Using Extended Euclidean Algorithm find the multiplicative inverse of 23 in Z_{100}.

   d) Write a short note on **Ciphers**.

[3 + 3 + 3 + 3] = 12

2. a) Discuss different types of **Cipher text only attacks.**

   b) Use additive cipher with **key** = **15** to decrypt the message "**WTAAD**".

   c) Solve the following system of equations

$$3x + 2y \equiv 5(mod\ 7)$$
$$4x + 6y \equiv 4(mod\ 7)$$

   d) Define **Residue Matrix.** Find the multiplicative inverse of the following residue matrix A over Z_{10}

$$A = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}.$$

[3 + 2 + 4 + (1 + 3)] = 13

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S6(DSMA36B19) MA

BSMS VIth SEMESTER, MID-TERM EXAMINATION-2019

NAME OF THE SUBJECT: OBJECT ORIENTED PROGRAMMING

CODE NO: DSMA36B19

ull Marks: 50 Time: 2 hours

wer all questions from the following: [10 × 5 = 50 Marks]

State whether the following statements are true or false, justify:

(i) Templates are declared inside classes or functions.

(ii) Base class is known as the subclass and derived class as superclass.

(iii) A member function can't be used in a derived class that override.

(iv) The scope resolution operator can be overloaded.

(v) Templates should be global and should not be local.

[1 × 5]

2. Answer the following questions in one word.

(i) How can you prevent your class to be inherited further?

(ii) What is the index value of the first element in an array?

(iii) Can you specify the accessibility modifier for methods inside the interface?

(iv) Is it possible for a class to inherit the constructor of its base class? no

(v) Can you allow a class to be inherited, but prevent a method from being overridden in C#?

[1 × 5]

3. What is object-oriented programming (OOP)? Explain the basic features of OOPs.

4. What down the differences between a class and a structure?

5. What is a delegate? Write down all the features of an interface.

6. What are abstract classes? What are the distinct characteristics of an abstract class?

7. What are the differences an abstract class and an interface.

8. What is access modifier? What are the various types of constructors in OOP? Explain.

9. Define Class. What is the relationship between a class and an object?

10. What are the different types of arguments? Explain different types of inheritance in OOP.

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BS/MS 6th SEMESTER, MID-TERM EXAMINATION-2019

SUBJECT NAME: STOCHASTIC PROCESSES

SUBJECT CODE NO: DSMA36B18

Marks: 50 Time: 2 Hours

Symbols used here have their usual meanings

Group – A

er all of the following questions:

1. (a) Define absorbing state with an example.
(b) Describe Markov chain as a graph with an example.

[2+3=5]

2. (a) Establish the relationship between probability distribution and transition probability.
(b) Transition probability matrix of a Markov chain $\{X_n, n = 1, 2, ..\}$ having three states 1, 2 and 3 is given below:

$$\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

The initial distribution is $\pi_0 = (0.7, 0.2, 0.1)$.
Find $\Pr\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

[3+2=5]

3. (a) Suppose that a coin with probability p for ahead is tossed infinite times. Let X_n be the outcome of the nth trial be $k(= 0,1,..,n)$ denotes that there is a run of k successes. Show that $\{X_n: n > 0\}$ constitute a Markov Chain. Find out one step transition probabilities and hence form the transition matrix.
(b) Define class of state with example.

[3+2=5]

4. (a) Define periodicity of a state.
(b) State first entrance theorem. Verify first entrance theorem for the following transition matrix with state space $S = \{1, 2, 3\}$ for the transition from the state 1 to 3 using 3 steps:

$$P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

[1+(1+3)=5]

5. Suppose that the probability of a dry day following a rainy day is 1/3 and probability of a rainy day following a dry day is 1/2. Also probability of a dry day following a dry day is 1/2 and probability of a rainy day following a rainy day is 2/3. Form two step transition matrix. Given that May1 is dry day, find the probability that May 3 is a rainy day. Also find the probability that May 5 is a dry day under the given condition May 1 is a dry day.

[5]