Robotics

PS04 – Solutions

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Part 4: Actuators

Problem 1

Given a DC-motor with

- no-load speed N_0: $6{,}000\ rpm$
- stall torque T_s: $0.5\ gr \cdot \mathrm{cm}$

What is the maximum mechanical power of the motor (in Watt)?

Problem 1

maximum power

$$\boldsymbol{P_m^{max}} = T_{mp} \cdot \omega_{mp} = \frac{1}{2} \cdot T_{max} \cdot \frac{1}{2} \cdot \omega_{max} = \boldsymbol{\frac{1}{4} \cdot T_{max} \cdot \omega_{max}}$$

in Watt:

$$W = V \cdot A = \frac{kg \cdot m^2}{s^3} = \frac{N \cdot m}{s}$$

$$T \text{ in } Nm, \omega \text{ in } \frac{rad}{s} \text{ (note: } rad = \frac{1m}{1m}\text{)}$$

Problem 1

6,000 rpm
(rotations aka revolutions aka rounds per minute)

- $1\,rotation = 2\pi\,rad$
- $1\,rpm = 1/60 \cdot 2\pi \frac{rad}{\mathrm{sec}}$

$$\Rightarrow 6000rpm = 100 \cdot 2\pi \frac{rad}{sec} = 200\pi \frac{1}{s} = 200\pi Hz$$

Problem 1

torque „unit“: often as ***colloquial*** „weight“ times length

- proper physical weight is a force F_w derived from the mass
- F_w = mass · gravity value (g)

$$F_w = m \cdot g$$

with $g = 9.8 \frac{m}{s^2}$

i.e., $1N = 1kg \cdot 1 \frac{m}{s^2}$

Problem 1

i.e., „torque“: often as ***mass*** times length

- ensure SI units, i.e., convert to *kg*
- multiply mass (colloquially often called „weight“) with gravity value g to get a proper force (in N)

$$
\begin{aligned}
F_w &= 0.5gr \cdot 9.8\,{}^{m}/{}_{s^2} \\
&= 5 \cdot 10^{-1} \cdot 10^{-3} kg \cdot 9.8\,\frac{m}{s^2} \\
&= 5 \cdot 10^{-4} kg \cdot 9.8\,{}^{m}/{}_{s^2} \\
&= 4.9 \cdot 10^{-3} kg \cdot {}^{m}/{}_{s^2} \\
&= 4.9 \cdot 10^{-3} N
\end{aligned}
$$

Problem 1

torque: often as mass times ***length***

- ensure SI units, i.e., convert to *m*

$$T_{informal} = 0.5gr \cdot cm$$

$$\begin{aligned} T &= F_w \cdot l \\ &= 4.9 \cdot 10^{-3} N \cdot 1cm \\ &= 4.9 \cdot 10^{-3} N \cdot 10^{-2} m \\ &= 4.9 \cdot 10^{-5} Nm \end{aligned}$$

Problem 1

note: US/UK often even imperial units for “torque”

- 1 in·lb ≈ 0.113 Nm
- 1 ft·lb ≈ 1.356 Nm
- 1 in·oz ≈ 7.062 · 10^{-3} Nm

- 1 Nm ≈ 8.851 in·lb
- 1 Nm ≈ 0.737 ft·lb
- 1 Nm ≈ 141.59 in·oz

Problem 1

maximum power with proper SI-units

$$\boldsymbol{P^{max}} = \frac{1}{4} \cdot \omega^{max} \cdot T^{max} = \frac{1}{4} \cdot 200\pi \frac{1}{s} \cdot 4.9 \cdot 10^{-5} Nm$$

$$= \frac{1}{4} \cdot 2\pi \cdot 4.9 \cdot 10^{-3} \frac{Nm}{s} = 7.696902 \cdot 10^{-3} \frac{Nm}{s}$$

$$= \mathbf{7.696902 \cdot 10^{-3}}\ \boldsymbol{W}$$

Problem 2

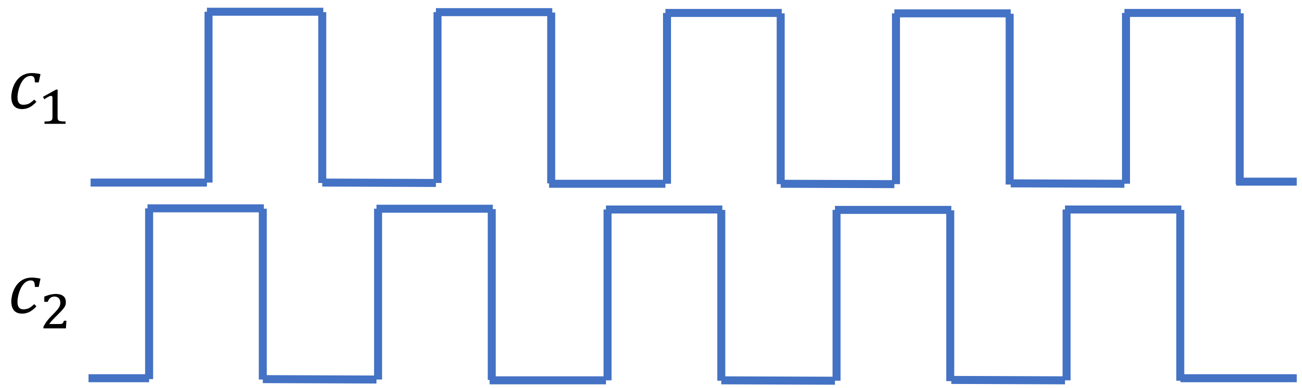
Given a rotational sensor in form of an incremental quadrature encoder with accordingly 2 channels c_1 and c_2, which each generate 1,024 pulses per revolution (PPR).

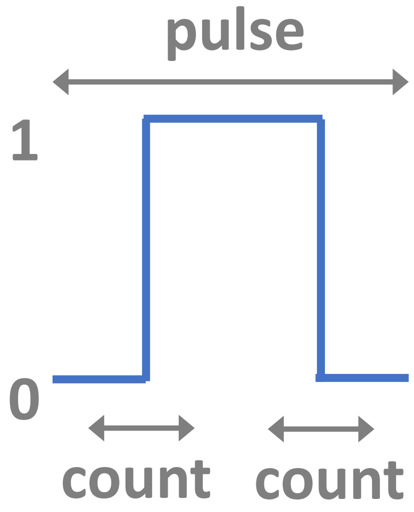
- What is the maximum resolution for angular velocity that we can get when using counts per revolution (CPR), i.e., when checking for the flanks of the square waves?
- What is the maximum resolution of measuring the sense of direction?

Problem 2

encoder with c_1 , c_2: 1,024 pulses per revolution (**PPR**)
max resolution for angular velocity using counts per revolution (**CPR**)

- **pulse**: signal is “high” (0->1->0)
- **count**: signal flanks, i.e., “transitions” (0->1 or 1->0), 2x #pulses





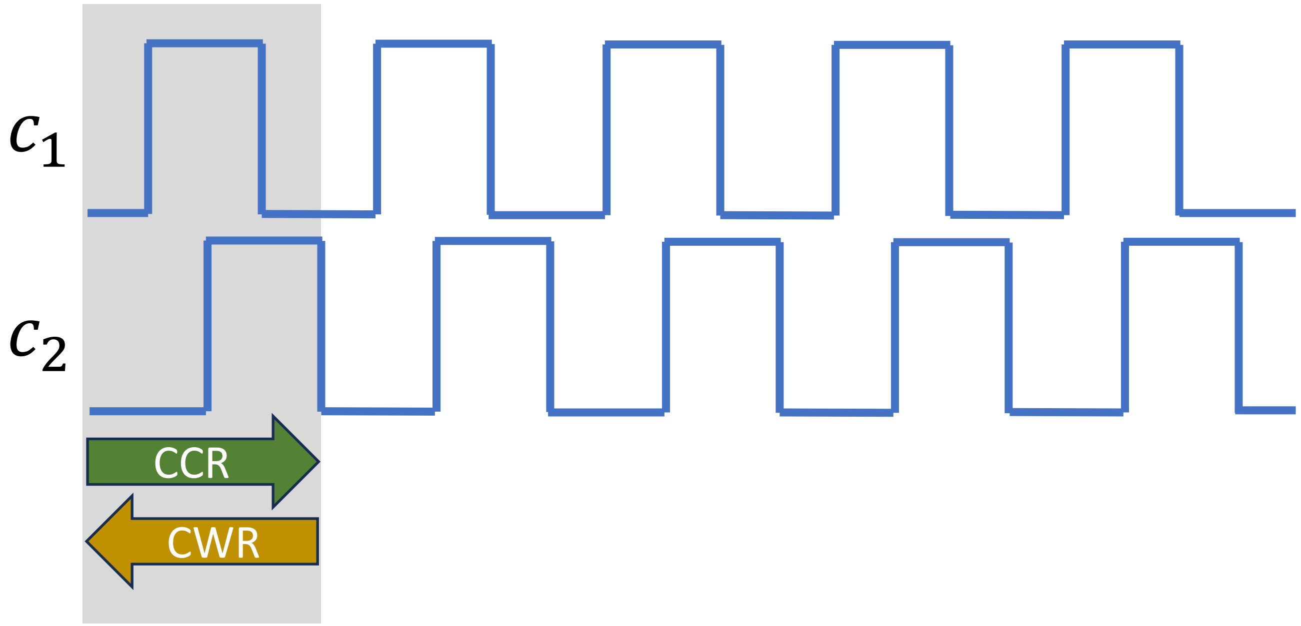
angular velocity measurement
2 channels with 1,024 PPR => **4096 CPR**

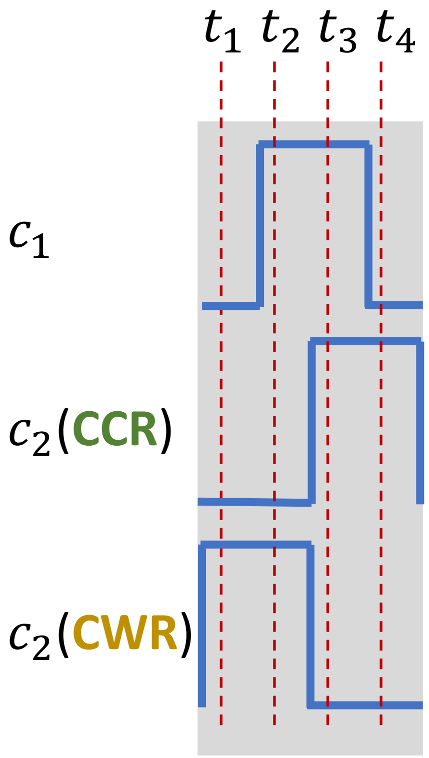
Problem 2

encoder with c_1 , c_2: 1,024 pulses per revolution (PPR)

maximum resolution of measuring the sense of direction

- need to look at **pairs of pulses**: (c_1, c_2)





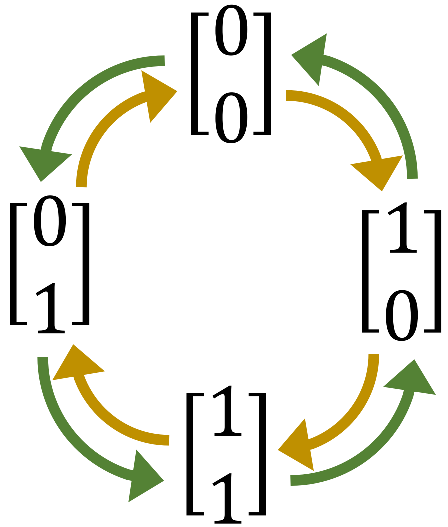
Problem 2

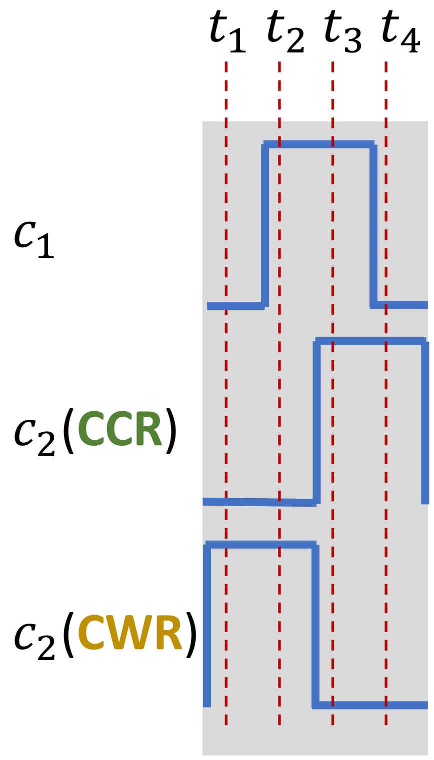
encoder with c_1 , c_2: 1,024 pulses per revolution (PPR)
maximum resolution of measuring the sense of direction

- need to look at **pairs of pulses**: $(\boldsymbol{c_1, c_2})$
- **every transition**, we get the sense of direction

=> sense of direction as often as counts:
4096 times per revolution

		$\boldsymbol{t_1}$	$\boldsymbol{t_2}$	$\boldsymbol{t_3}$	$\boldsymbol{t_4}$	$\boldsymbol{t_{5\sim1}}$
CCR	c1	0	1	1	0	0
	c2	0	0	1	1	0
CWR	c1	0	1	1	0	0
	c2	1	1	0	0	1





Problem 2

note:
the electronics that evaluates a quadrature *encoder* is usually denoted as a **quadrature decoder** (**QDEC)**

e.g., MC68332 micro-controller
with time processing unit (TPU)

- co-processor for TP
- e.g., PWM, QDEC
- 16 I/O-pins (8x QDEC)

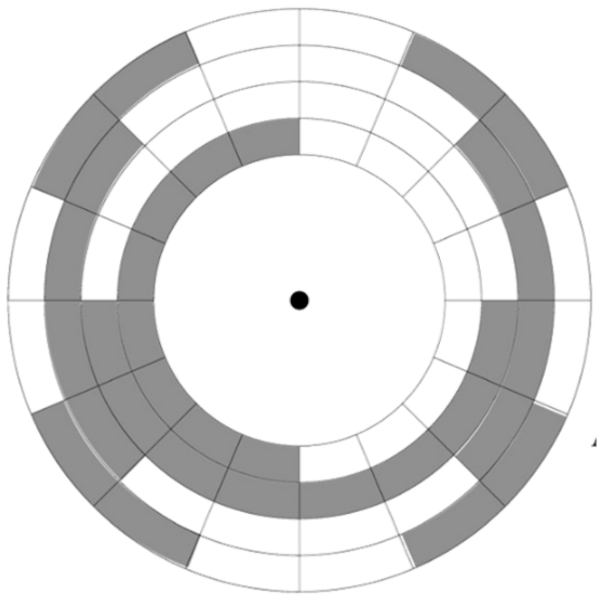
Problem 3

Given an absolute encoder with 16 channels $c_1, \ldots, c_{16}$, i.e., 16 bit encoding. What is the maximum angular resolution it can measure?

Suppose the encoder is once implemented with binary numbers and once with Gray code. What is the minimum and the maximum Hamming-distance between two sectors of the encoder in each case?

Problem 3

absolute encoder with b channels
- each sector represented by a unique bit-string s_i of length b
- hence, #sectors $n = 2^b$
- angular resolution $\Delta\theta = \frac{360^o}{n} = \frac{360^o}{2^b}$



e.g., 4 bit, 16 sectors

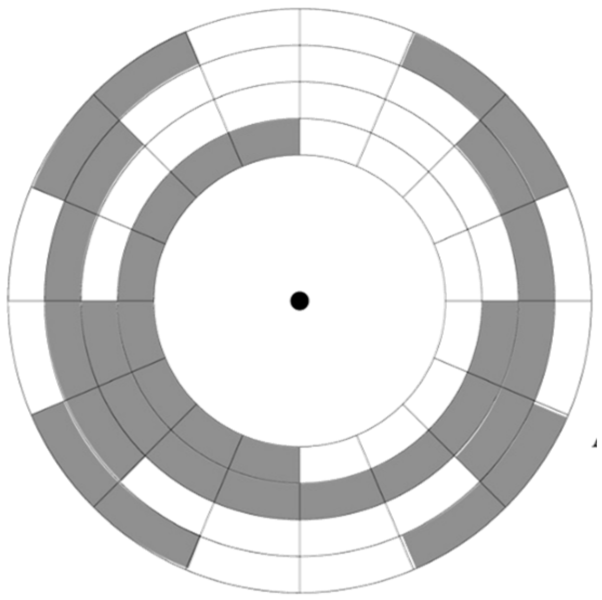
Problem 3

$b = 16$

- #sectors $n = 2^{16} = 65{,}536$
- angular resolution $\Delta\theta = \frac{360^o}{65{,}536}$

$$= 0.00549^o$$
$$= 0.329589844'$$
$$= 19.77539063''$$

$1^o = 60' = 3600''$
(arc minute & second)



e.g., 4 bit, 16 sectors

Problem 3

note: absolute encoder with b bit at a joint of an arm with length l

- 2^b sectors, i.e., $\Delta\theta = \frac{360^o}{2^b}$
- spatial resolution Δd at the end of arm $\Delta d = \sin(\Delta\theta) \cdot l$

example:

- $b = 16$
- 65K sectors, $\Delta\theta = 0.00549^o$
- length $l = 1.5m$

$\Delta d = 0.1438\,mm = 143.8\,\mu m$

Problem 3

given two strings $s^1 = (s_1^1, \dots, s_n^1)$ and $s^2 = (s_1^2, \dots, s_n^2)$
over alphabet A with length n

Hamming Distance $H(s^1, s^2) = \#i : s_i^1 \neq s_i^2$
(number of positions where the strings differ)

often, $A = \{0,1\}$, i.e., bit-strings

Problem 3

sequence of 2^n
$\boldsymbol{n}$-bit binary numbers s^i

$\min H(s^i, s^{i+1}) = 1$, e.g.,
- $s^1 = 0 \ldots 00 \ \ (0_{10})$
- $s^2 = 0 \ldots 01 \ \ (1_{10})$

$\max H(s^i, s^{i+1}) = n$, e.g.,
- $s^1 = 01 \ldots 11 \ \ (2^{n-1} - 1)$
- $s^2 = 10 \ldots 00 \ \ (2^{n-1})$

sequence of 2^n
$\boldsymbol{n}$-bit Gray code s^i

by definition
$\min H(s^i, s^{i+1}) = \max H(s^i, s^{i+1}) = 1$

note: a simple algorithm exists to recursively generate Gray code aka binary-reflected (Gray) code

Problem 3

algorithm:
binary-reflected Gray code

$G_1 = (0,1)$

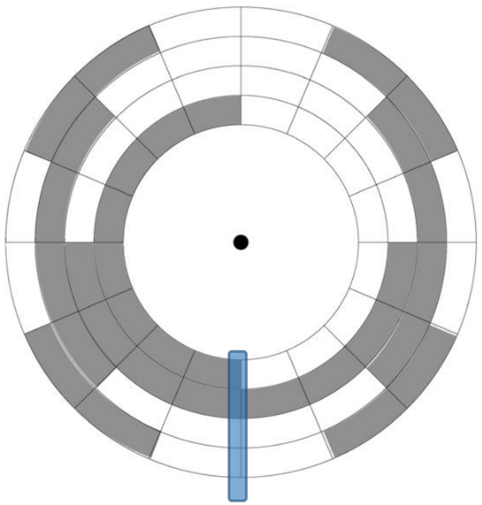
$G_n \to G_{n+1}$

- generate G'_n by reflecting G_n, i.e., list G_n in reverse order
- add prefix 0 to strings in G_n
- add prefix 1 to strings in G'_n
- concatenate G_n and G'_n

example: $G_1 \to G_2$

- $G_1 = (0,1)$
- $G'_1 = (1,0)$
- add prefix 0: $(00,01)$
- add prefix 1: $(11,10)$
- concatenate:
  $\mathrm{G}_2 = (00,01,11,10)$

Problem 3



sensors in between sectors:

possible reading of “left” or ”right” sector for every bit

degrees	Gray			
…	…			
157.5	0	1	0	0
180.0	1	1	0	0
…	…			

2 possible readings:

- 0100
- 1100

degrees	binary			
…	…			
157.5	0	1	1	1
180.0	1	0	0	0
…	…			

16 possible readings:

- 0000
- …
- 1111

degrees	binary				Gray			
0.0/360.0	0	0	0	0	0	0	0	0
22.5	0	0	0	1	0	0	0	1
45.0	0	0	1	0	0	0	1	1
67.5	0	0	1	1	0	0	1	0
90.0	0	1	0	0	0	1	1	0
112.5	0	1	0	1	0	1	1	1
135.0	0	1	1	0	0	1	0	1
157.5	0	1	1	1	0	1	0	0
180.0	1	0	0	0	1	1	0	0
202.5	1	0	0	1	1	1	0	1
225.0	1	0	1	0	1	1	1	1
247.5	1	0	1	1	1	1	1	0
270.0	1	1	0	0	1	0	1	0
292.5	1	1	0	1	1	0	1	1
315.0	1	1	1	0	1	0	0	1
337.5	1	1	1	1	1	0	0	0
0.0/360.0	0	0	0	0	0	0	0	0

Problem 3

note
- commercial absolute encoders internally operate with Gray code
- but **output as binary values** of the angle
- via standardized interfaces, e.g.,
  Synchronous Serial Interface (SSI)

Problem 4

Given a spur gear train with 3 axes a_1 to a_3. On each axis a_i is a gear or two gears $g_i.x$, $\mathrm{x} \in \{a, b\}$ with following numbers of teeth

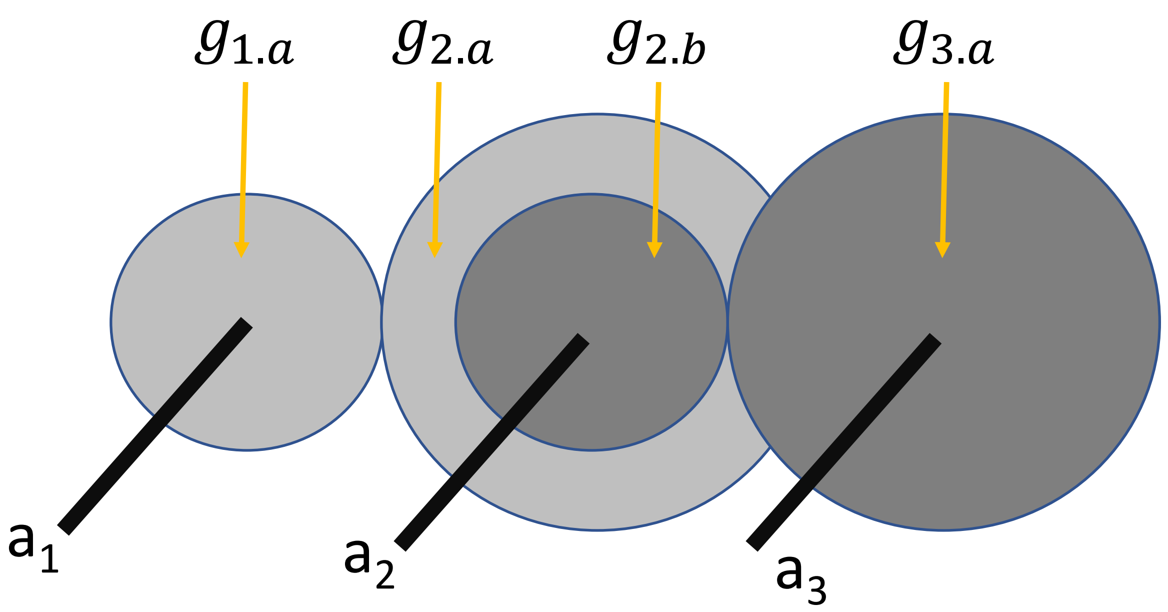
- $g_{1.a}$: 10
- $g_{2.a}$: 20
- $g_{2.b}$: 50
- $g_{3.a}$: 100

Gear $g_{1.a}$ drives $g_{2.a}$, $g_{2.b}$ drives $g_{3.a}$. Given input speed or torque on a_1 calculate the according output on a_3

- input torque a_1: 10 Nm; output torque a_3?
- input speed a_1: 100 rpm; output speed a_3?

Problem 4

- $z_{1.a} = 10$
- $z_{2.a} = 20$
- $z_{2.b} = 50$
- $z_{3.a} = 100$



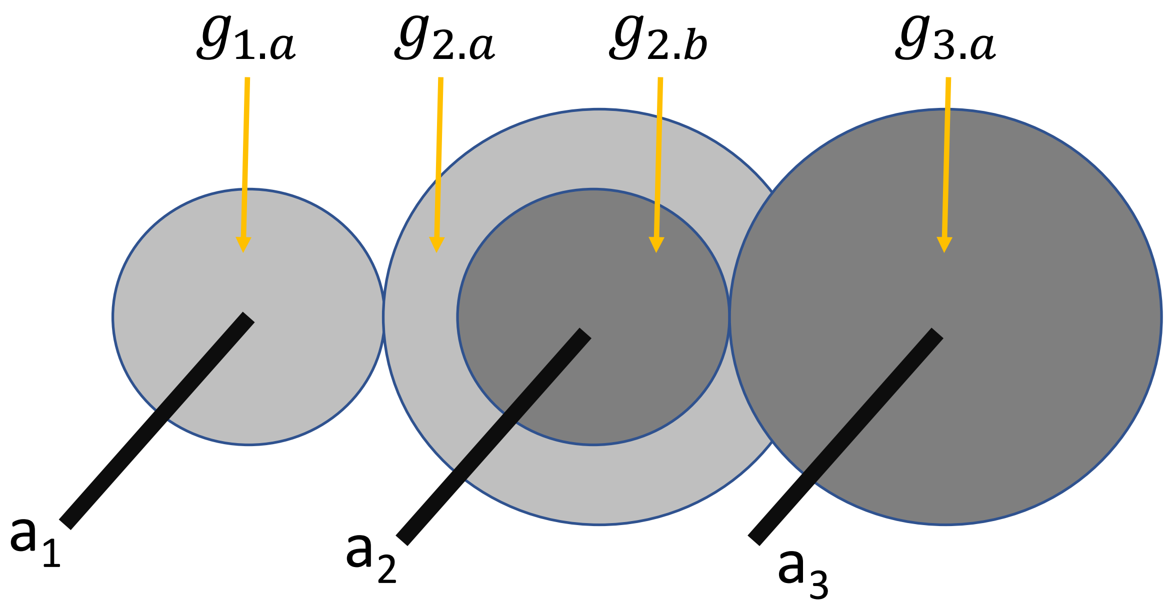
$$-\frac{z_{2.a}}{z_{1.a}} = -\frac{20}{10} = -2 \;\; (GR_1 = 2:1)$$

$$-\frac{z_{3.a}}{z_{2.b}} = -\frac{100}{50} = -2 \;\; (GR_2 = 2:1)$$

Problem 4

$$N_3 = (-2)^{-1} \cdot N_2 = (-2 \cdot -2)^{-1} \cdot N_1 = \frac{1}{4} \cdot N_1$$

$$T_3 = -2 \cdot T_2 = -2 \cdot -2 \cdot T_1 = 4 \cdot T_1$$

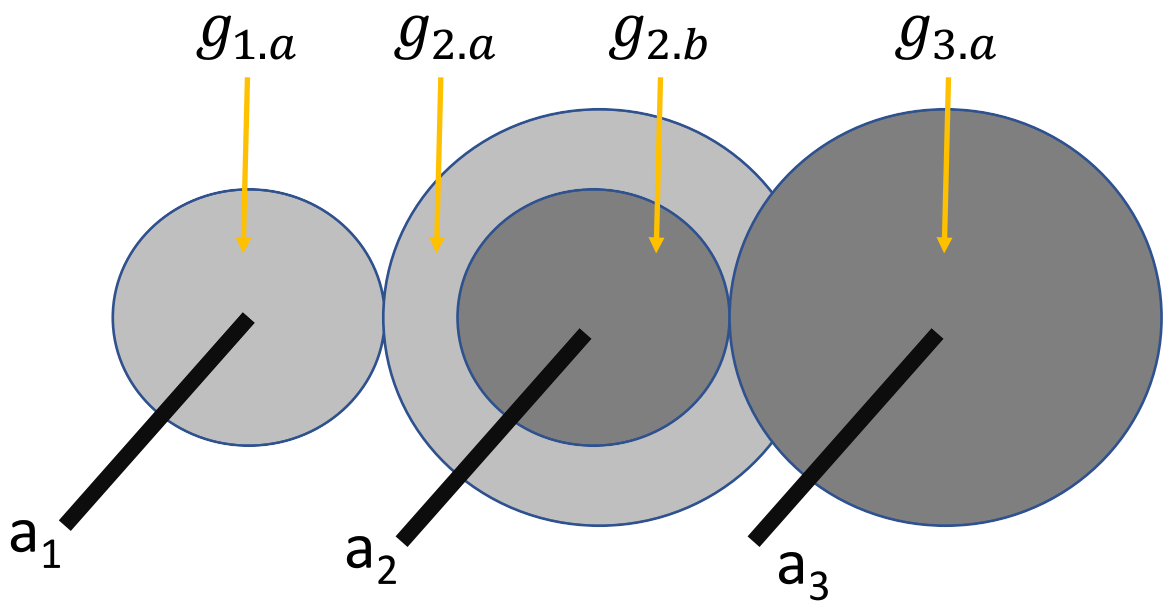


Problem 4

note: if you are not explicitly asked to provide the solution in SI units, you do not have to do it

$$N_3 = \frac{1}{4} \cdot 100\, rpm = \mathbf{25\, rpm}$$

$$T_3 = 4 \cdot 10\, Nm = \mathbf{40\, Nm}$$



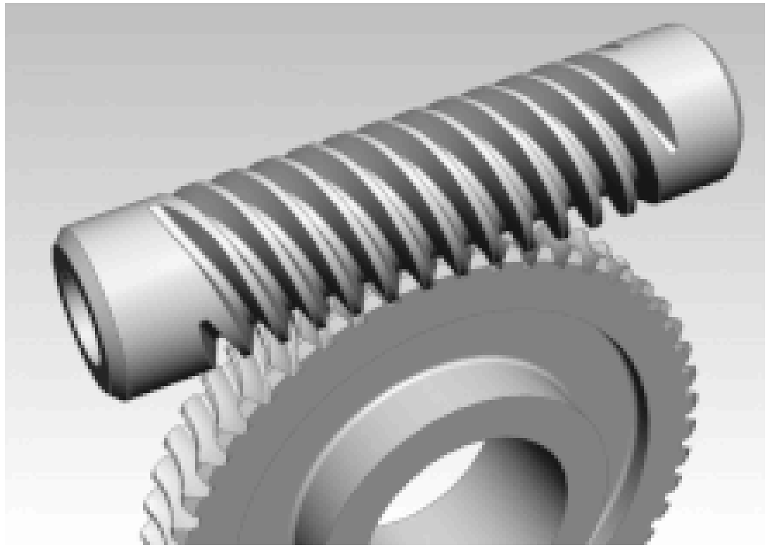
Problem 5

Given a worm gear G_w with $z_w = 100$ teeth on the worm wheel. What is its gear ratio?

Problem 5

- nominal #teeth of a worm is 1
- z =#teeth of the spur gear, respectively worm wheel

=> gear ratio $gr_w = \frac{z}{1} = z$



here: $z = 100$, i.e.,

$$\boldsymbol{gr_w = \frac{100}{1} = 100}$$

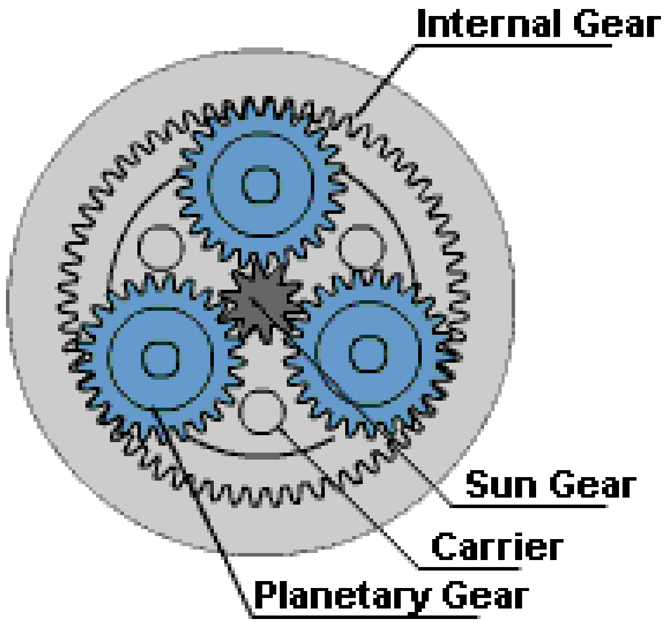
Problem 6

Given a planetary gear G_p with

- 4 planet gears
- $z_p = 50$ teeth on each planet
- $z_i = 200$ teeth on the internal gear
- $z_s = 10$ teeth on the sun

What is its gear ratio?

Problem 6



planetary gear

- number of planets does not matter
- number of planet teeth does not matter (idler)

$$gr = \frac{N_S}{N_C} = \frac{z_I + z_S}{z_S} = \frac{210}{10} = 21$$

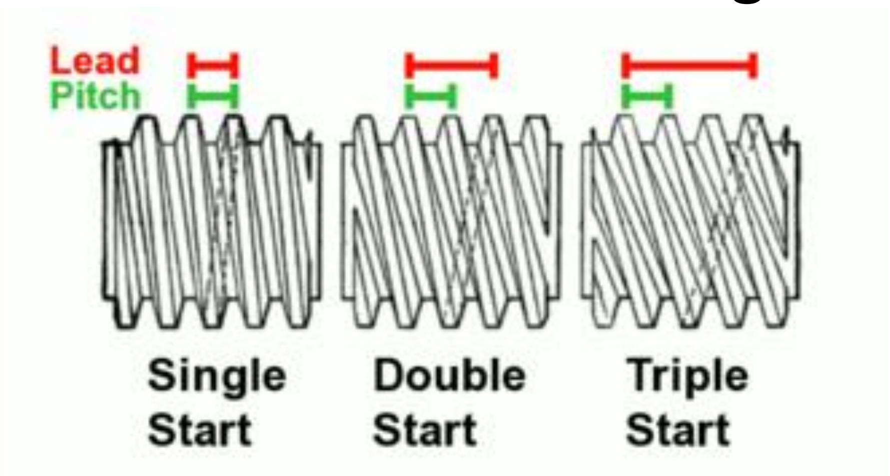
(sun: input, carrier: output)

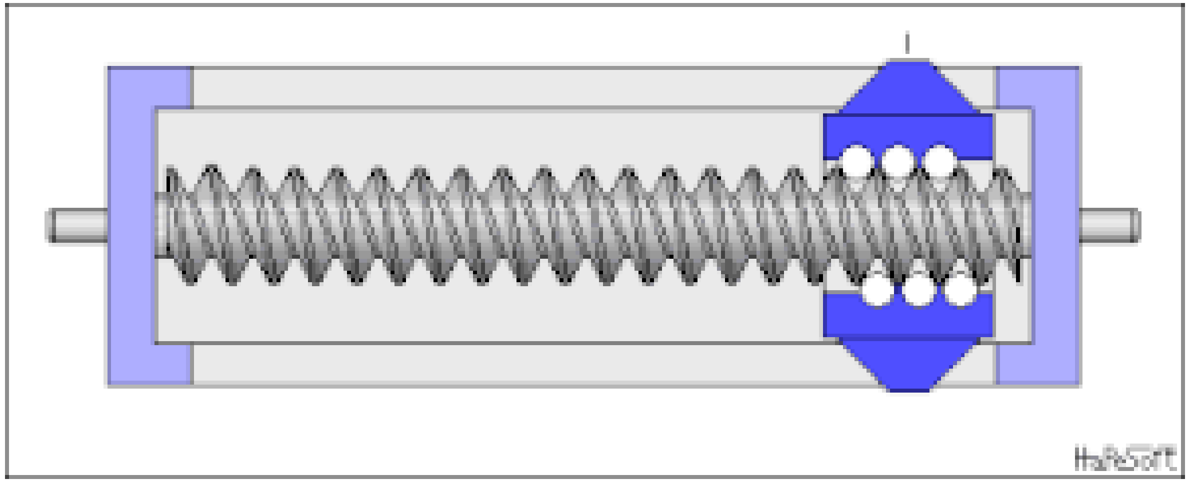
Problem 7

A double-start lead-screw with a diameter of $d = 15\ mm$ and pitch $p = 2\ mm$ is driven by a geared DC-motor with angular velocity $\omega = 10\frac{rad}{s}$ and torque $T = 0.15\ Nm$. The efficiency η of the lead-srew is 87%. What is the velocity v of the nut, respectively its load f?

Problem 7

#starts = #helix windings





#starts s, lead l, pitch p: $l = s \cdot p$

here: $l = 2 \cdot 2\, mm = 0.004\, m$

note: often incorrect use of term pitch as synonym for lead

Leadscrew

velocity v of the nut:

$$v = \omega \cdot l = 10 \frac{rad}{s} \cdot 0.004\, m = 0.04 \frac{m}{s}$$

load, i.e., force f produced by the nut:

$$f = \eta \frac{T}{l} = 0.87 \cdot \frac{0.15\, Nm}{0.004\, m} = 0.87 \cdot 37.5\, N = 32.625\, N$$

(as $P_{out} = \eta P_{in} \Leftrightarrow vf = \eta \cdot \omega T = \eta \cdot \frac{v}{l} T$)