Communications Basics
Lecture 3

More on Modulation

Orga

Textbook:

Rodger E. Ziemer & William H. Tranter, Principles of Communications **… READ!**

Our content comes from chps. 2-7 (according to edition 5)

Chp. 2 (Signals & Systems – separate course) … **polish your Fourier transforms**

Chp. 3 (Modulation + Demodulation)

Chp. 4 (Probability – separate course) … roll a couple of dice ☺ **… we'll bascially need Random variables, (multivariate) Gaussians, expected values, variances, covariances**

Chp. 5 (Random Processes & Noise)

Chp. 6 (Noise in Modulation Systems)

Chp. 7 (Binary Data Transmission)

Main platform: campusnet … course page !!!

Teaching in person … slides will be on campusnet … but make sure to take your own notes!

TA: Yasmine Ammouze … tutorials

Exam: Written, no cheat sheets (expected end of January), 2 hours, details as announced by the registrar (should show in campusnet)

Envelopes – Making Signals Analytic

``Ride the bike´´ :

Consider … make analytic, Hilbert transform, real envelope, …

1) $\mathrm{x}(t) = A\cos(t) + A\sin(t)$
2) $\mathrm{x}(t) = A\cos(99 \cdot t) + A\cos(101 \cdot t)$ … interpret the real envelope!

Reconsider our discussion on $\mathcal{H}^2\big(\mathrm{x}(t)\big)$ etc.
As mentioned before, we assumed that the signal $\mathrm{x}(t)$ has no DC-component.

Just on the side: What happens, if it does ?
Does ``$\mathcal{H}^4 = \text{Identity}$´´ still hold ?

Think: Besides $x_p(t)$, people also define another analytic signal related to a given signal $x(t)$:

$$x_n(t) = x(t) - j\hat{x}(t)$$

What is different, here? … What remains the same?

Envelopes – Making Signals Analytic

1) $\mathrm{x}(t) = A\cos(t) + A\sin(t)$ ➔ $\hat{x}(t) = A\sin(t) - A\cos(t)$

➔ $\mathrm{x_p(t)} = x(t) + j\hat{x}(t) = Ae^{jt} - jAe^{jt} = A(1-j)e^{jt}$ ➔ $|\mathrm{x_p(t)}| = |A|\sqrt{2}$

Or

$$|\mathrm{x_p(t)}| = |A|\sqrt{\left(\cos(t)+\sin(t)\right)^2 + \left(\sin(t)-\cos(t)\right)^2}$$

$$= \sqrt{\left(\cos(t)+\sin(t)\right)^2 + \left(\sin(t)-\cos(t)\right)^2} = |A|\sqrt{2}$$

2) $\mathrm{x}(t) = A\cos(99\,t) + A\cos(101\,t)$ … Interpret the real envelope!

$$\mathrm{x}(t) = 2A\cos(t)\cos(100\,t)$$

$$\Rightarrow \mathrm{x_p(t)} = 2A\cos(t)e^{j\,100\,t} \Rightarrow |\mathrm{x_p(t)}| = 2|A||\cos(t)|$$

$$\hat{x}(t) = ?$$

Envelopes – Making Signals Analytic

``Ride the bike´´ - chance to dig deaper (a long biking tour in a way):

$$\hat{x}(t) = \frac{1}{\pi t} * x(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(t'-t)}{t'} dt'$$

Usually, we would not use the time domain approach but go via the frequency domain. However, just for the fun of it, how can this integral be calculated … given the pole at zero?

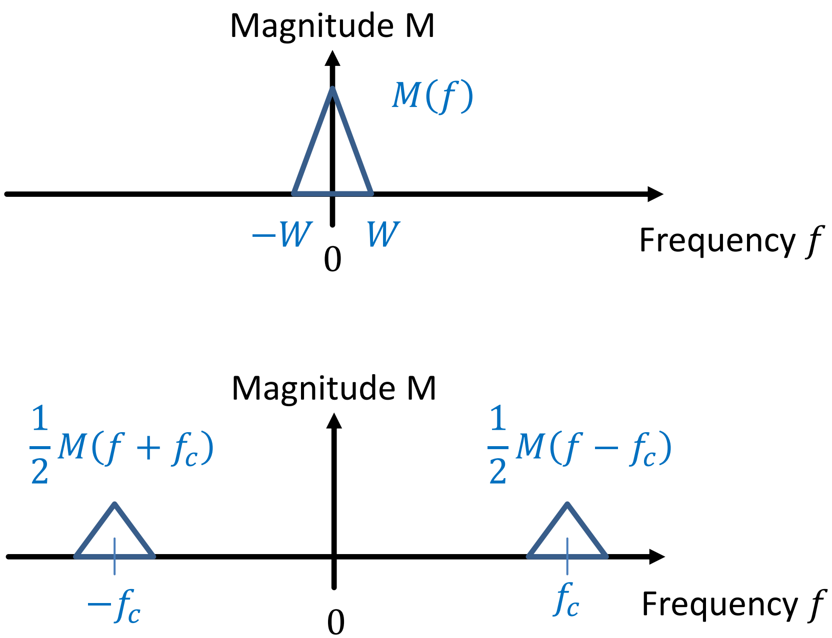
Such integrals pop up when you try to, systematically, find an adequate imaginary part for a given real function (defined along the real axis) … that is what we motivated, here.

A typical approach is via complex integrals … semi-cricle shaped integration paths … little circles around poles … residues … etc. This leads to the integral above in the sense of the so-called Cauchy principal value:

$$\lim_{\varepsilon \to 0} \left[\int_{-\infty}^{-\varepsilon} \frac{x(t'-t)}{t'} dt' + \int_{\varepsilon}^{\infty} \frac{x(t'-t)}{t'} dt' \right]$$

which, due to its symmetry, circumvents the trouble with the pole.

Linear Modulation



Symbolic spectra

Bandwidth used …

Reconsider DSB:

A message $m(t)$ with spectrum $M(f)$ and bandwidth W:

$$x_c(t) = A_c m(t)\cos(\omega_c t)$$

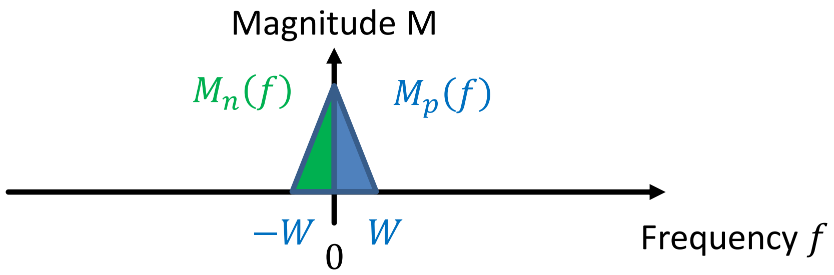
➔ Transmission covers a frequency interval (bandwidth) of size $2W$ … both for positive AND negative frequencies, of course (real signals)

Similar result for AM …

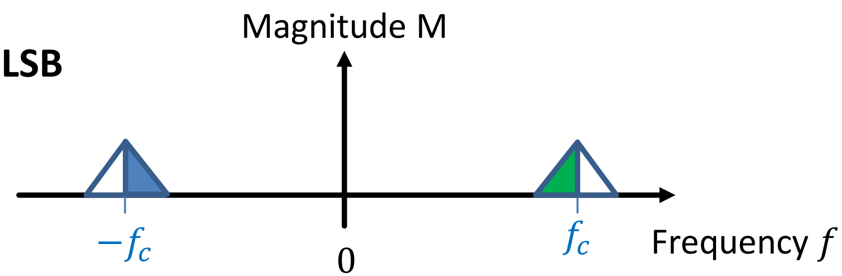
Can we do better? … Use less bandwidth, that is?

Single Sideband (SSB)

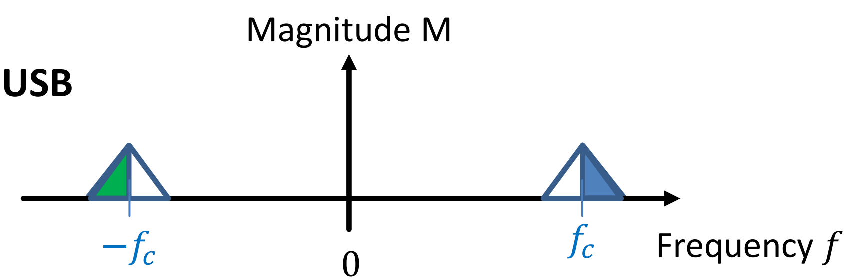
Consider SSB modulation in terms of pos. and neg. analytic parts of the message.



LSB



USB



Bandwidth used …

Consider SSB modulation:

Only a frequency interval of length W is required. ☺

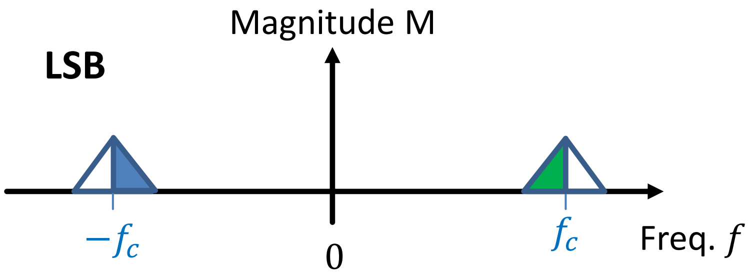
Sounds good! … What is the price?

Interpret Lower Sideband (LSB) modulation ($A_c = 1$):

$$x_c(t) = \frac{1}{2} m_p(t) e^{-j\omega_c t} + \frac{1}{2} m_n(t) e^{+j\omega_c t}$$

Can you see why?

Single Sideband (SSB)

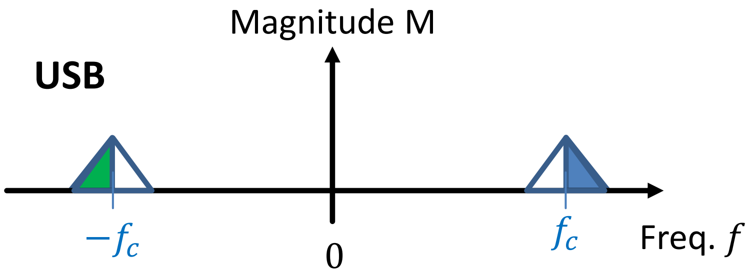


Lower Sideband (LSB) modulation $(A_c = 1)$:

$$x_c(t) = \frac{1}{2}m_p(t)e^{-j\omega_c t} + \frac{1}{2}m_n(t)e^{+j\omega_c t}$$

$$= \frac{1}{2}\big(m(t) + j\hat{m}(t)\big)(\cos(\omega_c t) - jsin(\omega_c t)) + \frac{1}{2}\big(m(t) - j\hat{m}(t)\big)(\cos(\omega_c t) + jsin(\omega_c t))$$

$$= m(t)\cos(\omega_c t) + \hat{m}(t)sin(\omega_c t)$$



Upper Sideband (USB) modulation $(A_c = 1)$:

$$x_c(t) = \frac{1}{2}m_p(t)e^{j\omega_c t} + \frac{1}{2}m_n(t)e^{-j\omega_c t}$$

$$= m(t)\cos(\omega_c t) - \hat{m}(t)sin(\omega_c t)$$

Single Sideband (SSB)

Bandwidth used …

Consider SSB modulation:
Only a frequency interval of length W is required.

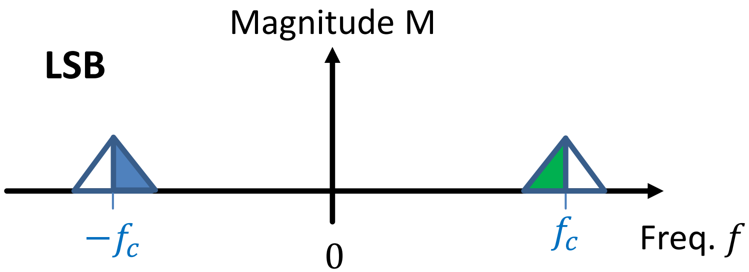
But we need both, cos + sin components:

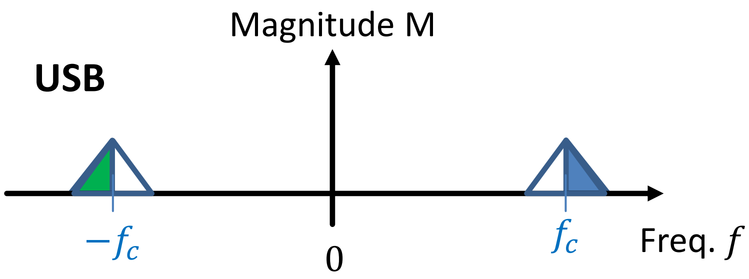
LSB

$$x_c(t) = m(t)\cos(\omega_c t) + \hat{m}(t) sin(\omega_c t)$$

USB

$$x_c(t) = m(t)\cos(\omega_c t) - \hat{m}(t) sin(\omega_c t)$$



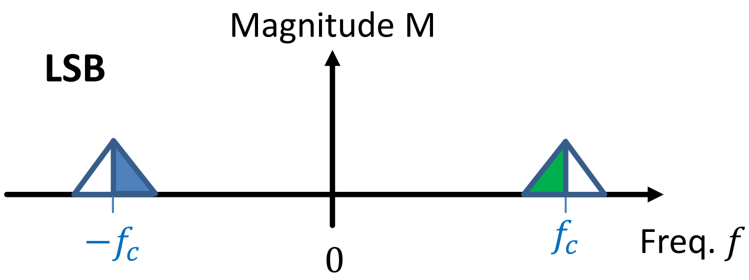


Compare: For DSB + AM, we need only one of those orthogonal components.
We might use the other for a different user.

Discuss: How realistic is that idea?

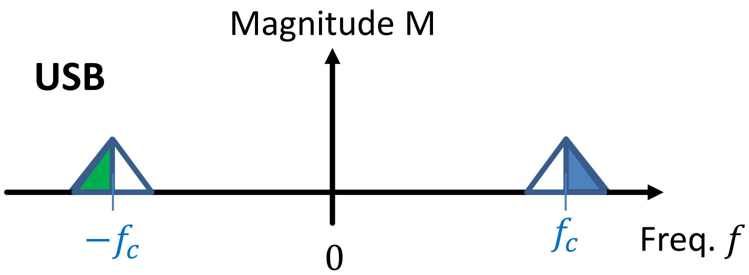
Single Sideband (SSB)

Demodulate SSB signals



LSB

$$x_c(t) = m(t)\cos(\omega_c t) + \hat{m}(t)sin(\omega_c t)$$



USB

$$x_c(t) = m(t)\cos(\omega_c t) - \hat{m}(t)sin(\omega_c t)$$

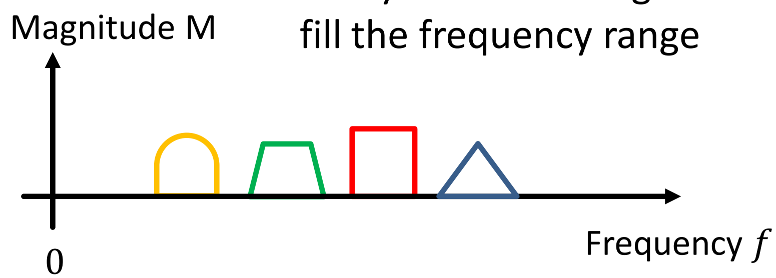
Realize: Coherent is possible.

Discuss:

What if we mess up with regard to the phase?

Frequency Translation & Mixing

Use a local oscillator to shift the carrier



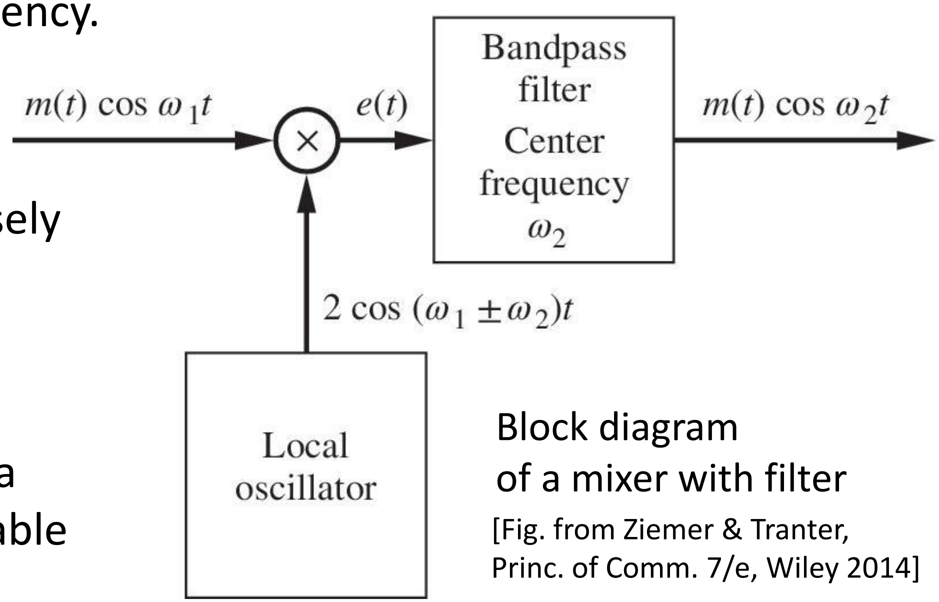
We want only one of them …
… how can we achieve that?
Tunable high performance filter?

Trigonometric identities make it relatively easy, via multiplication and filtering, to shift (bandlimited) signals along the frequency axis.

Notice: To achieve a required shift $\omega_1 \to \omega_2$, there are two options for the local oscillator frequency.

Mind:

In order to select only one out of many (closely packed) transmitted signals, it takes high-performance filtering.



Block diagram of a mixer with filter

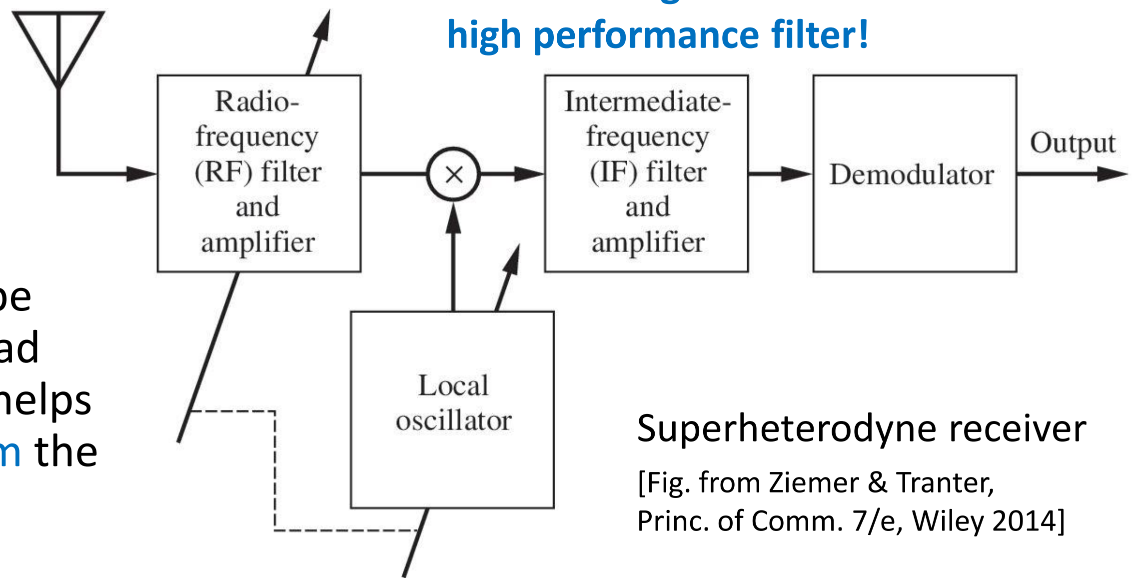
[Fig. from Ziemer & Tranter, Princ. of Comm. 7/e, Wiley 2014]

Frequency translation (based on a mixer) is a typical trick in order to avoid expensive tunable high-performance filters.

Frequency Translation & Mixing

Use a local oscillator to shift the carrier

Just a single fixed high performance filter!



Superheterodyne receiver

[Fig. from Ziemer & Tranter, Princ. of Comm. 7/e, Wiley 2014]

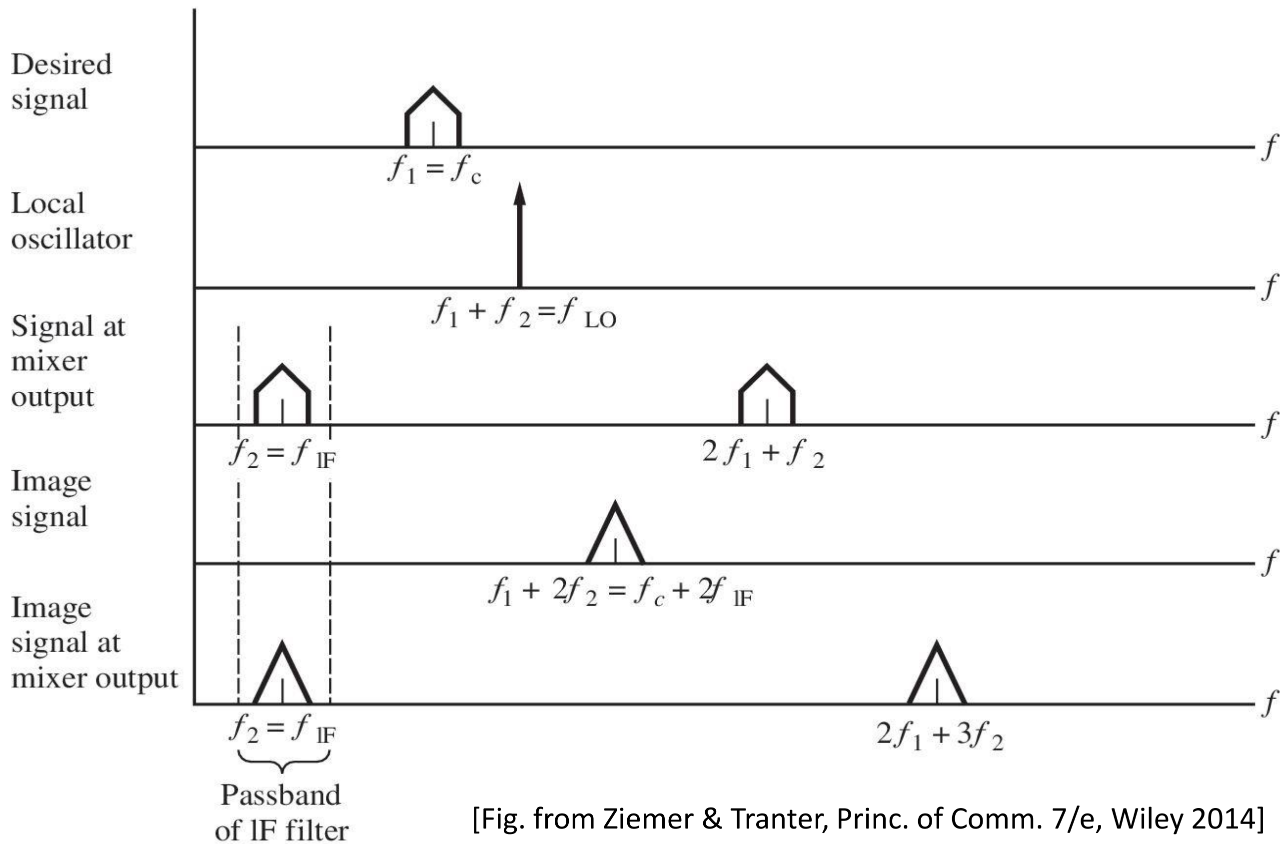
Only a single oscillator needs to be tuned … and usually another broad (low-performance) RF filter that helps to avoid interference coming from the image range … see below.

A properly tuned mixer shifts the desired incoming signal along the frequency axis to the postion of a (single) high performance intermediate frequency (IF) filter.

This IF filter is sharp and can stop neighboring signals.

Frequency Translation & Mixing

Use a local oscillator to shift the carrier



[Fig. from Ziemer & Tranter, Princ. of Comm. 7/e, Wiley 2014]

This is a high-side tuning example.

The mixer produces two copies … one that fits to the IF filter, and another relatively far away which would be stopped by the IF filter.

There is a second candidate frequency range from which signals would be mapped to the IF pass band. This looks a bit like a mirror image (the local oscillator frequeny being the mirror). Accordingly, the corresponding signal is called an image signal. The RF filter on the previous slide is supposed to stop such images even before they enter the process.

Lecture 3

Angle Modulation

Why is linear modulation called linear?

Example: DSB

$$m_1(t) \longmapsto A_c m_1(t)\cos(\omega_c t)$$

$$m_2(t) \longmapsto A_c m_2(t)\cos(\omega_c t)$$

$$\alpha_1 m_1(t) + \alpha_2 m_2(t) \longmapsto A_c\big(\alpha_1 m_1(t) + \alpha_2 m_2(t)\big)\cos(\omega_c t)$$

$$= \alpha_1 A_c m_1(t)\cos(\omega_c t) + \alpha_2 A_c m_2(t)\cos(\omega_c t)$$

This is what linearity means, mathematically.
Same thing for AM, LSB, USB.

Lecture 3

Angle Modulation

Angle modulation is a nonlinear technique

Rather than the amplitude, we vary the argument of the cos-term, now.

$$m(t) \longmapsto A_c \cos\big(\omega_c t + \phi(t)\big)$$

Instantaneous Phase

$$\theta_i(t) = \omega_c t + \phi(t)$$

$\phi(t)$: Phase deviation

Instantaneous Frequency

$$\omega_i(t) = \frac{d}{dt}\theta_i(t) = \dot{\theta}_i(t) = \omega_c + \frac{d}{dt}\phi(t) = \omega_c + \dot{\phi}(t)$$

$\frac{d}{dt}\phi(t)$: Frequency deviation

Angle Modulation

Phase Modulation

$$\phi(t) = k_p m(t)$$

k_p**:** Phase deviation constant

Frequency Modulation

$$\frac{d}{dt}\phi(t) = k_f m(t) = 2\pi f_d m(t)$$

d_f: Frequency deviation constant

k_f: Angular frequency deviation constant

Angle Modulation

Simple message signal

Original carrier

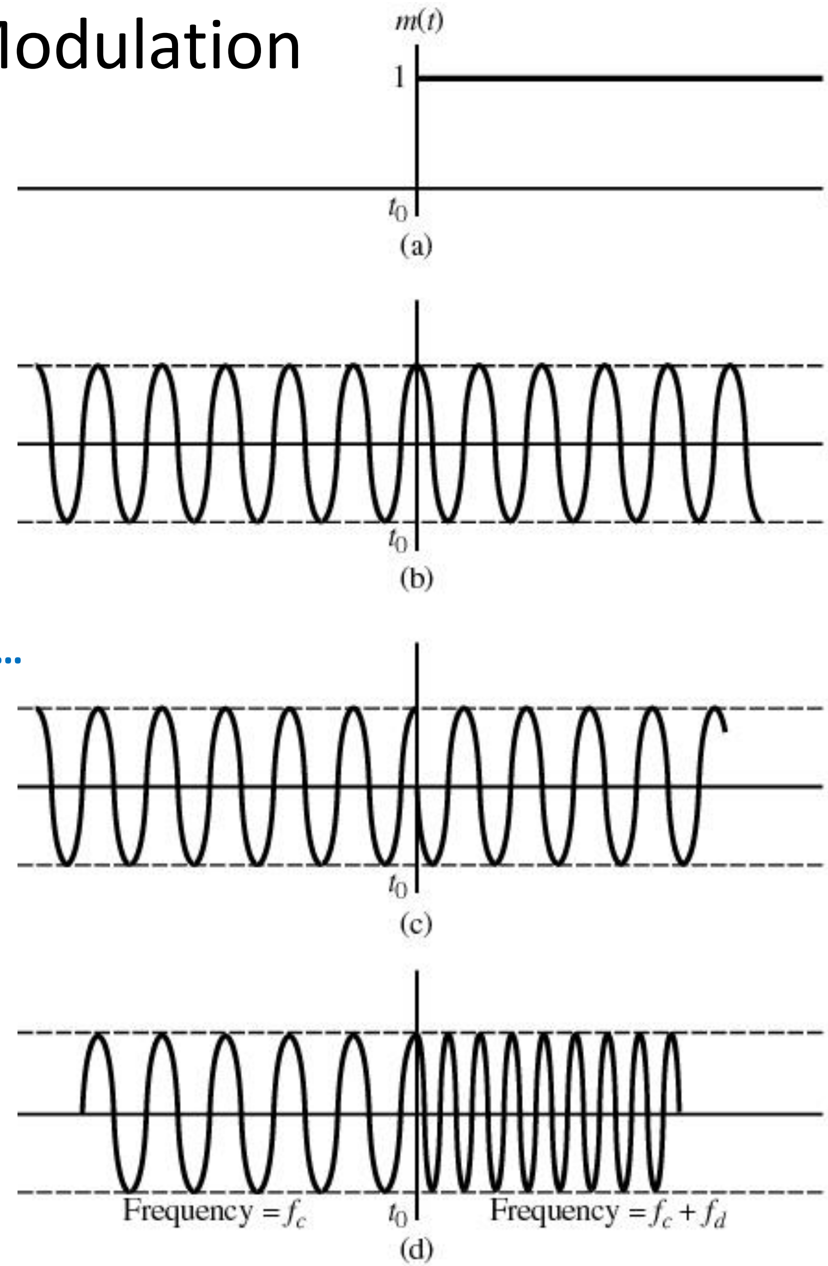
Understand:
Phase shift keying …

Phase Modulation

$$\phi(t) = k_p m(t)$$

Frequency Modulation

$$\frac{d}{dt}\phi(t) = k_f m(t) = 2\pi d_f m(t)$$



[Fig. from Ziemer & Tranter, Princ. of Comm 5/e., Wiley 2002]

Angle Modulation

How does angle modulation look like in frequency domain?

Calculating the Fourier transform of the modulated signal is diffcult, see the textbook for an example.

There is a very elegant way, though, to understand the basic situation.

Using a familiar approach, we re-write

Interpret as complex envelope

$$x_c(t) = A_c \cos\big(\omega_c t + \phi(t)\big) = \Re\{A_c \exp(j\omega_c t) \exp\big(j\phi(t)\big)\}$$

A power series expansion of the second exponential yields

$$x_c(t) = \Re\left\{A_c \exp(j\omega_c t)\left[1 + j\phi(t) - \frac{1}{2}\phi^2(t) - j\frac{1}{6}\phi^3(t) + \cdots\right]\right\}$$

Angle Modulation

Discuss: How does angle modulation look like in frequency domain?

$$x_c(t) = \Re\left\{A_c \exp(j\omega_c t)\left[1 + j\phi(t) - \frac{1}{2}\phi^2(t) - j\frac{1}{6}\phi^3(t) + \cdots\right]\right\}$$

For a message signal with frequency components $e^{\pm j\omega_1 t}$ and $e^{\pm j\omega_2 t}$, contributing to $\phi(t)$, this can be easily interpreted. Consider the terms in the square bracket:

0th order term = 1 ➔ just the carrier signal as in AM

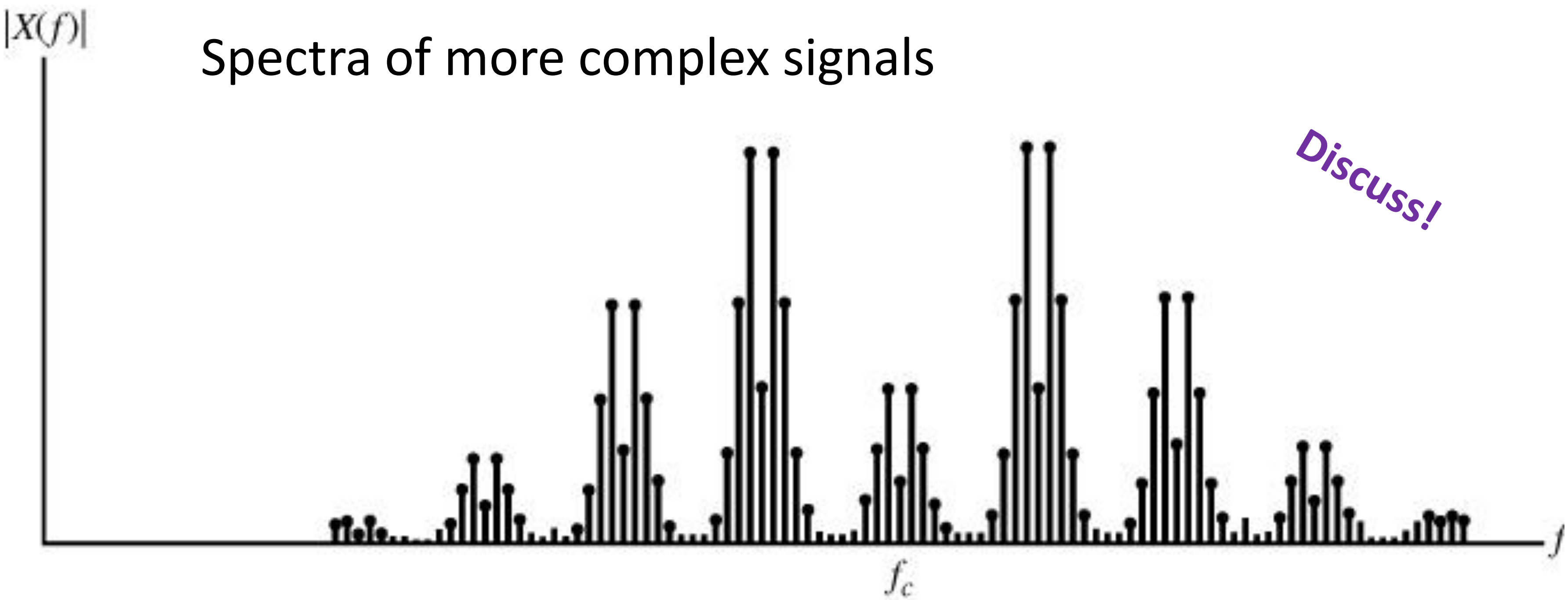
1st order term = $j\phi(t)$ ➔ single side band (left and right) similar to DSB but with extra 90°- phase shift

2nd order term = $-\frac{1}{2}\phi^2(t)$ ➔ a) second side band (left and right) *other* than in all linear techniques PLUS b) contribution at the center frequency and contributions at mixed message frequencies …

$$\exp(j(\pm\omega_1 \pm \omega_2)t)$$

Angle Modulation

Spectra of more complex signals



Discuss!

Example with two frequencies

$$\phi(t) = \sin\omega_1 t + \sin\omega_2 t \text{, with } \omega_2 = 12\omega_1$$

[Fig. from Ziemer & Tranter, p. 139
Princ. of Comm. 5/e, Wiley 2002]

Angle Modulation

Bandwidth for transmission (first idea …)

$$x_c(t) = A_c \cos\big(\omega_c t + \phi(t)\big)$$

Instantaneous Frequency

$$\frac{d}{dt}\theta_i(t) = \dot{\theta}_i(t) = \omega_c + \frac{d}{dt}\phi(t)$$

Phase Modulation

$$\phi(t) = k_p m(t) \Rightarrow \frac{d}{dt}\theta_i(t) = \omega_c + k_p \frac{d}{dt} m(t)$$

Bandwidth (PM) $\approx k_p[max\,\dot{\boldsymbol{m}}(t) - min\,\dot{\boldsymbol{m}}(t)]$ **typically** $= \mathbf{2}k_p\, max\,\dot{\boldsymbol{m}}(t)$

$= \mathbf{2} \cdot \boldsymbol{peak\ frequency\ deviation}$ (scales with k_p)

Angle Modulation

Bandwidth for transmission (first idea …)

$$x_c(t) = A_c \cos\big(\omega_c t + \phi(t)\big)$$

Instantaneous Frequency

$$\frac{d}{dt}\theta_i(t) = \dot{\theta}_i(t) = \omega_c + \frac{d}{dt}\phi(t)$$

Frequency Modulation

$$\frac{d}{dt}\phi(t) = k_f m(t) = 2\pi f_d m(t) \Rightarrow \frac{d}{dt}\theta_i(t) = \omega_c + k_f m(t)$$

Bandwidth (FM) $\approx k_f[max\,\boldsymbol{m}(t) - min\,\boldsymbol{m}(t)]$ **typically** $= \boldsymbol{2}k_f\,max\,\boldsymbol{m}(t)$

$= \boldsymbol{2} \cdot \boldsymbol{peak\ frequency\ deviation}$ (scales with k_f)

Angle Modulation

Bandwidth – Carson‘s Rule

$$x_c(t) = A_c \cos\big(\omega_c t + \phi(t)\big)$$

Think & Discuss:

Can we be super efficient, and transmit with bandwidth → 0 by using very small deviation constants? ??

$$\boldsymbol{2 \cdot peak\ frequency\ deviation = 2 \cdot \Delta f}$$

works for **large** frequency deviations

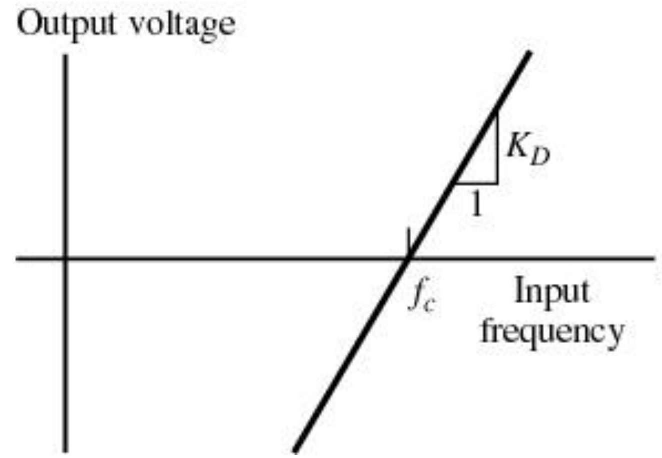
For **small** frequency deviations, make sure to keep at least the first sideband.

→ **Carson‘s Rule:**

In order to transmit a message with bandwidth W, use a transmission bandwidth $\mathbf{B = 2(\Delta f + W)}$

Discuss

Angle Modulation



Demodulation-1

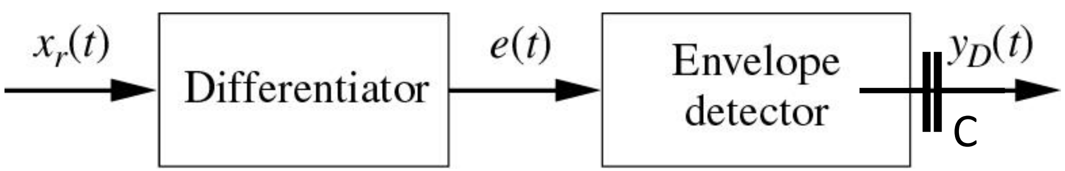
Received signal: $x_r(t) = A_c \cos\big(\omega_c t + \phi(t)\big)$

Assume FM … we need: $y_D(t) \propto \frac{d}{dt}\phi(t) \propto m(t)$

Use a differentiator to obtain:

$$e(t) = -A_c\left(\omega_c + \frac{d}{dt}\phi(t)\right)\sin\big(\omega_c t + \phi(t)\big)$$

FM discriminator:



Discuss DC-part

Don't forget to remove the dc component!

[Figs. from Ziemer & Tranter, Princ. of Comm. 5/e, Wiley 2002]

Lecture 3

Angle Modulation

Demodulation-2 (Phase locked loop - PLL)

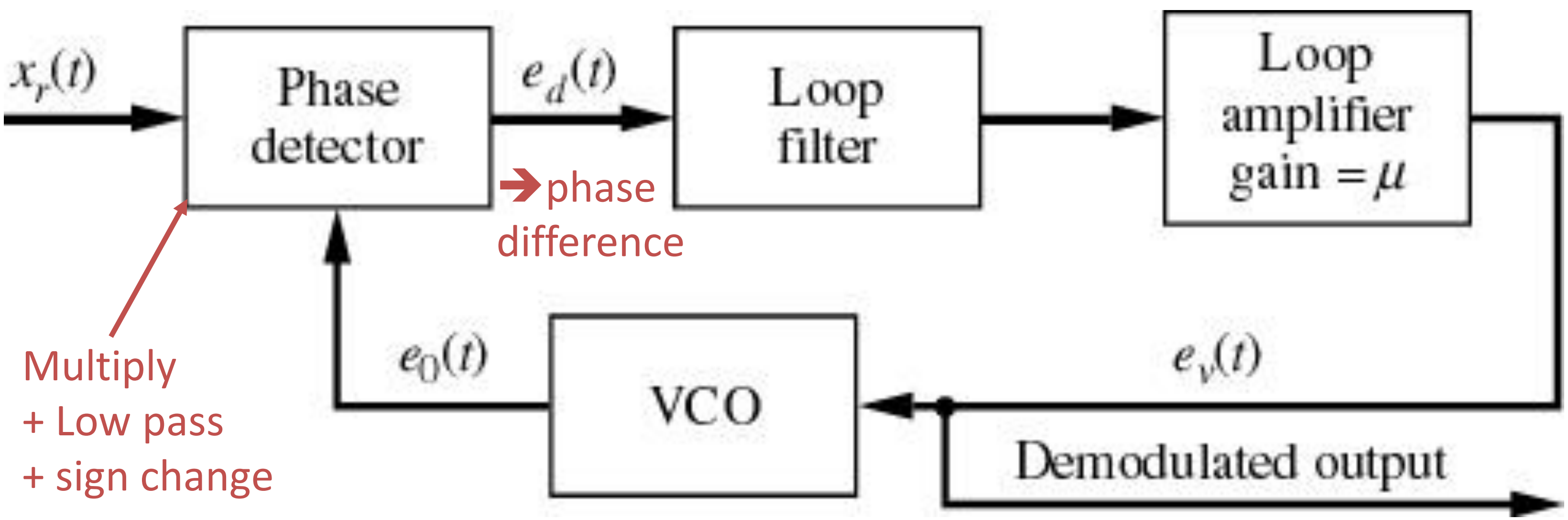
This is a ``feedback copying machine´´!

Received signal: $x_r(t) = A_c \cos\big(\omega_c t + \phi(t)\big)$

Use a feedback-loop with a
voltage controlled oscillator (VCO):

$$e_0(t) = A_v \sin\big(\omega_c t + \theta(t)\big)$$

Find the voltage that makes the VCO copy the input



➔ phase difference

Multiply
+ Low pass
+ sign change

[Fig. from Ziemer & Tranter, Princ. of Comm. 5/e, Wiley 2002]

Angle Modulation

Verify:

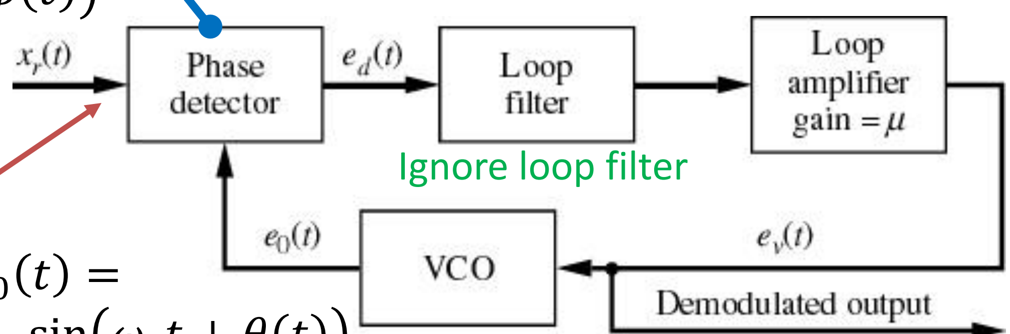
Low pass

$$\sin(\phi - \theta) - \sin(2\omega_c t + \phi + \theta)$$
$$= \sin(\omega_c t + \phi - \omega_c t - \theta) - \sin(\omega_c t + \phi + \omega_c t + \theta)$$
$$= - 2\cos(\omega_c t + \phi)\sin(\omega_c t + \theta)$$

Inverter

$x_r(t) =$
$A_c \cos\big(\omega_c t + \phi(t)\big)$

$$e_d(t) = \frac{1}{2} A_c A_v \sin\big(\phi(t) - \theta(t)\big)$$



Ignore loop filter

Idea:
If input is ahead, speed up the VCO

$e_0(t) =$
$A_v \sin\big(\omega_c t + \theta(t)\big)$

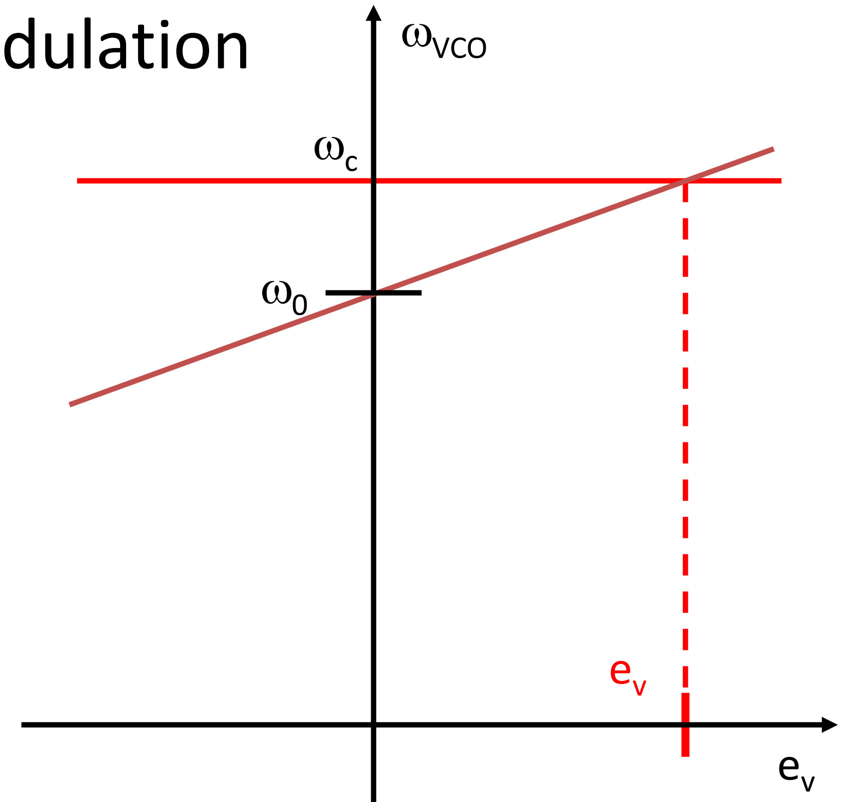
Multiply
\+ Low pass
\+ Inverter (sign change)

FM-case: use $e_v(t)$ as output

[Fig. from Ziemer & Tranter,
Princ. of Comm. 5/e, Wiley 2002]

Lecture 3

Angle Modulation



<u>VCO Model:</u>

$$\omega_{VCO}(t) = \omega_0 + Ke_v(t)$$

<u>Idea:</u> This is a feedback copying machine! ☺
Make the VCO copy the instantaneous frequency of the input $x_r(t)$:

$$\omega_c + \frac{d}{dt}\theta(t) = \omega_{VCO}(t) = \omega_0 + Ke_v(t)$$

$$\Rightarrow \frac{d}{dt}\theta(t) = \omega_0 - \omega_c + Ke_v(t) = \omega_0 - \omega_c + \frac{1}{2}K\mu A_c A_v \sin\big(\phi(t) - \theta(t)\big)$$

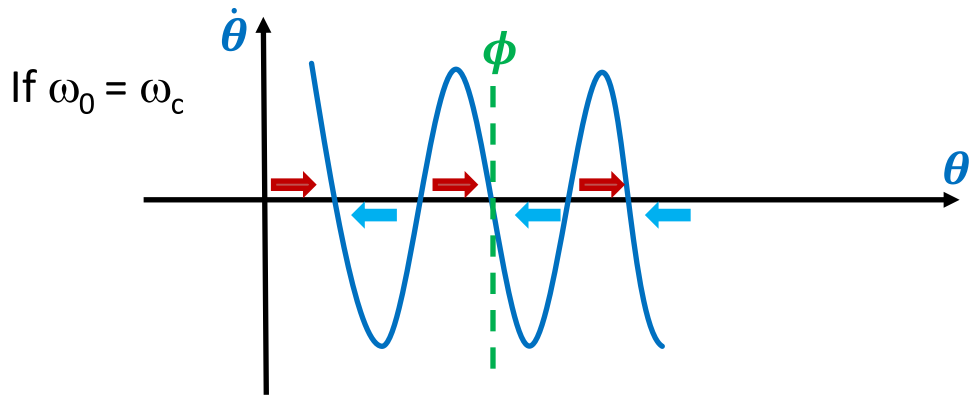
Discuss
1) DC-part
2) PM

$e_v(t)$ **varies the VCO frequency …**
FM-case: use $e_v(t)$ as demodulated signal
PM-case: we need the phase … so integrate $e_v(t)$ in order to get the demodulated signal.

Angle Modulation

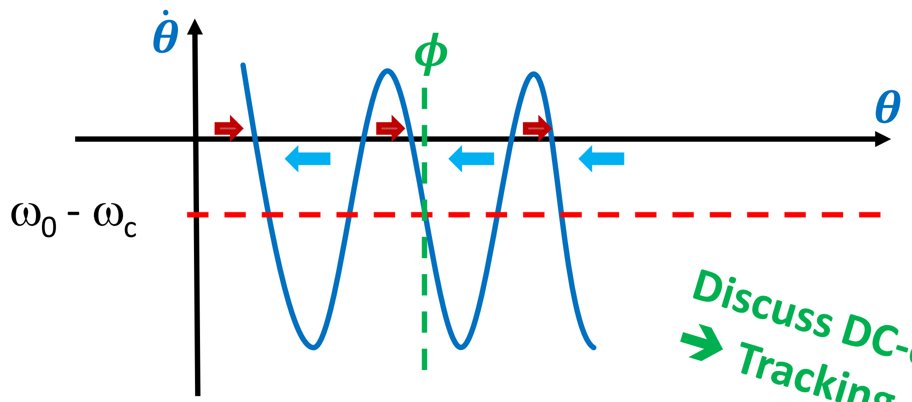
Use the result:

$$\frac{d}{dt}\theta(t) = \omega_0 - \omega_c + \frac{1}{2}K\mu A_c A_v \sin\big(\phi(t) - \theta(t)\big)$$



Consider
a fixed ϕ … or
a slowly changing one

What does θ do?



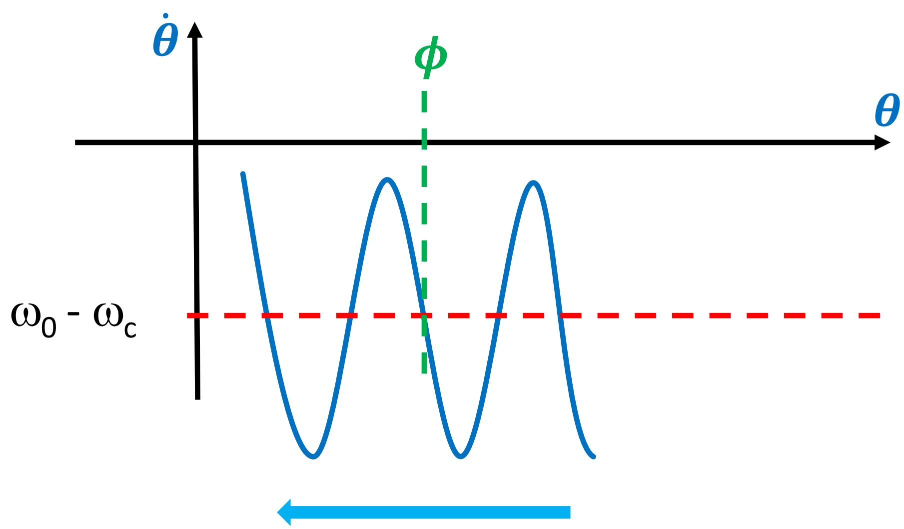
Discuss DC-offset of $\theta(t)$
➔ Tracking error.

Angle Modulation

Use the result:

$$\frac{d}{dt}\theta(t) = \omega_0 - \omega_c + \frac{1}{2}K\mu A_c A_v \sin\left(\phi(t) - \theta(t)\right)$$

Does this always work?



Finite locking range in terms of ω_c for this type of a PLL.
There are many others …

Lecture 3

Envelopes – Making Signals Analytic

``Ride the bike´´ :

Practice …

A little more on Hilbert, making signals analytic, real envelope

1) $\mathrm{x}(t) = \cos(t) + 0.1\sin(t)$
2) $\mathrm{x}(t) = A\cos\big(100 \cdot t + \beta \cdot sin(t)\big)$ … find an approximate solution

Mixing … **understand via Euler!** … Notice that $j = e^{j\frac{\pi}{2}}$

1) $\mathrm{x}(t) = A\cos(t) + A\sin(t)$ *… rewrite as … $A\cos(t+\varphi)$*
2) $\mathrm{x}(t) = A\cos(100 \cdot t)\cos(3 \cdot t)$ *… rewrite as a sum of sinusoids*

Carson … $\mathrm{m}(t) = a\cos(1000Hz \cdot 2\pi t)$, $f_c = 1MHz$ … find the used bandwidth

1) **FM**: $k_f = 3rad/s$
2) **PM**: $k_p = 7rad$

Thank you for your attention!

See you soon …