Robotics
Problem Sheet 2

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Notes

The homework serves as preparation for the exams. It is strongly recommended that you solve them before the given deadline - but you do not need to hand them in. Feel free to work on the problems as a group - this is even recommended.

1 Problem

Given the homogeneous matrix A with

$$A = \begin{pmatrix} 0.866 & -0.433 & -0.250 & 2 \\ 0 & -0.5 & 0.866 & -4 \\ -0.5 & -0.75 & -0.433 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

What is the rotation matrix part R_A of A? Is it a right- or a left-handed rotation?

What is the inverse A^{-1} of A (use an as simple as possible computation)?

2 Problem

Proof that when turning in circles you end up where you started. Or more concretely: given the motion `move(`α`, d)` (in 2D is sufficient) that turns with angle α and then makes a translation by a distance d, proof that the sequence of motions `move(90, d), move(90, d), move(90, d), move(90, d)` executed in pose p_{start} gets you into pose p_{end} with $p_{start} = p_{end}$.

3 Problem

Suppose an object, e.g., the earth, has the pose P_e and a 2nd object, e.g., the moon, with pose P_m is rotating around it with angle θ around the z-axis of P_e.

What is the new pose of P'_m for

$$\theta = 90^o, \quad p_e = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -4 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad p_m = \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & 7 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4 Problem

Given a world-frame F_w as identity matrix and an object with pose P_o with

$$p_o = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -4 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Suppose the object rotates by 90^o around the z-axis of F_w. What is the new pose P'_o of the object?
- Suppose world frame is an observer/sensor, who/which rotates by 90^o around its z-axis. What is the new pose P'_o of the object?

5 Problem

Given the quaternions $q_1 = (1, (2, 3, 4))$ and $q_2 = (0.4811480, (0.1984591, 0.7246066, 0.4517253))$.
Which of the two represents an orientation? And why?

6 Problem

Given point $p = (2, 3, 4)^T$. Use quaternions to rotate it

- by 30^o around the y-axis
- by 30^o around the axis $(1, -1, 3)^T$
- first by 30^o the y-axis, then by 90^o around the axis $(1, -1, 3)^T$

7 Problem

Use the Rodrigues formula to rotate $p = (2, 3, 4)^T$ by 30^o around the axis $(1, -1, 3)^T$.