Communications Basics
Fall term 2024
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Course CO-522-A
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Practice 1

The problems are based on Principles of Communications by Roger E. Ziemer and William H. Tranter, 5th ed., Wiley, 2002, problems 2.63-67. In the 6th ed., see problems 2.71-2.75.

Feel encouraged to discuss practice problems with your class mates. But it is your personal challenge to write up solutions by yourself – otherwise the learning effect will be close to zero - and do not forget to write appropriate explanations for your answers.

Task 1 (Hilbert Transforms, sec. 2.9, similar to problem 2.63)

Using appropriate Fourier transform techniques and pairs, express the spectrum Y(f) of

$$y(t) = x(t)\cos(\omega_0 t) + \hat{x}(t)\sin(\omega_0 t)$$

in terms of the spectrum X(f) of x(t), where X(f) is lowpass with bandwidth

$$B < f_0 = \frac{\omega_0}{2\pi}$$

Explain the result in general, and sketch Y(f) for a typical X(f).

Task 2 (Hilbert Transforms, sec. 2.9, similar to problem 2.64)

Show that $x(t)$ and $\hat{x}(t)$ are orthogonal for the following signals
a) $x(t) = \cos(\omega_0 t)$
b) $x(t) = 2\cos(\omega_0 t) + \sin(\omega_0 t)\cos^2(2\omega_0 t)$
Do NOT use the theorem on orthogonal Hilbert transforms, here. Find the transforms explicitly, and then check orthogonality directly.

Task 3 (Hilbert Transforms, sec. 2.9, similar to problem 2.65)

Assume that the Fourier transform of $x(t)$ is real and has a triangular shape: Maximum $X(f = 0) = A$, frequency range from –W to +W. Determine and plot the spectrum of each of the following signals:
a) $x_1(t) = \frac{3}{4}x(t) + \frac{1}{4}j\hat{x}(t)$
b) $x_2(t) = \left[\frac{3}{4}x(t) + \frac{1}{4}j\hat{x}(t)\right]e^{j2\pi f_0 t}$, where $f_0 \gg W$.

plot real and imaginary parts of the transforms in two separate figures. Also, mind: W is the signal bandwidth.

<u>Task 4</u> (Hilbert Transforms, sec. 2.9, similar to problem 2.66)

Consider the signal

$$x(t) = 2W\mathrm{sinc}(2Wt)\cos(2\pi f_0 t)$$

Assume $0 < W \ll f_0$

a) Obtain and sketch the spectrum of $x_p(t) = x(t) + j\hat{x}(t)$.

b) Obtain and sketch the spectrum of the complex envelope $\tilde{x}(t) = x_p(t)e^{-j2\pi f_0 t}$.

c) Find the complex envelope $\tilde{x}(t)$.

Hint: It is a common approach to solve such a problem approximately. Can you imagine what that means?