CONSTRUCTOR UNIVERSITY

Robotics

PS03 – Solutions

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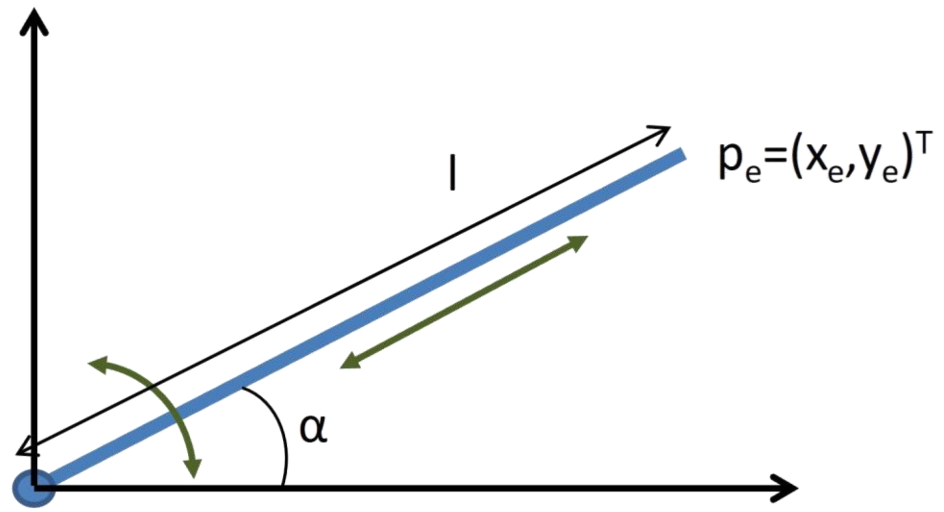
Part 3: Kinematics

Problem 1

Given the planar (2D) robot arm from the figure below with a rotational joint in the origin of the world frame and a prismatic joint linked to it with the respective DoF's α (rotation) and l (translation), with $l \in [500,1000]mm$.



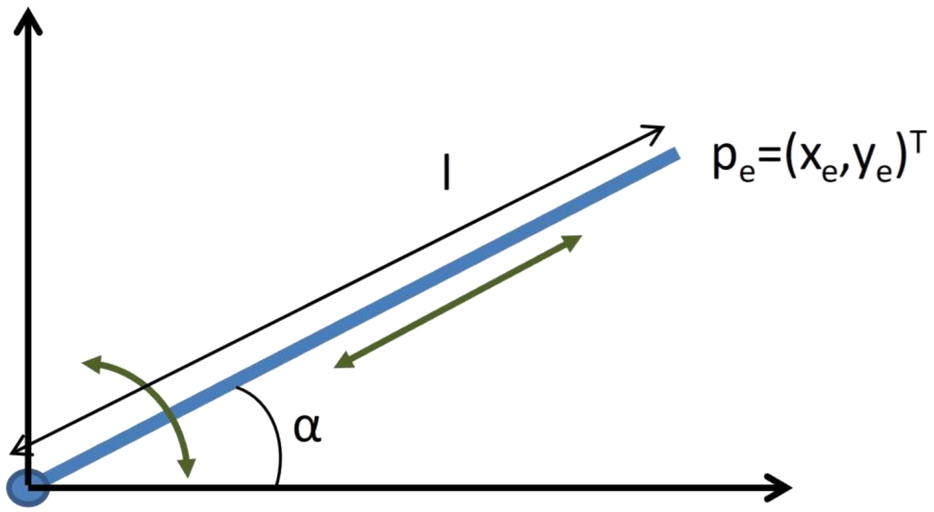
Provide the forward kinematics for the position $p_e = (x_e, y_e)$ of the end-effector of this robot.

Problem 1

$$^{F_2}p_e = {}^{F_1}_{F_2}T(l)\,{}^{F_0}_{F_1}R(\alpha)\,{}^{F_0}o$$

$$\Rightarrow$$

$$^{F_0}p_e = {}^{F_0}R(\alpha)\,{}^{F_0}T(l)\,{}^{F_0}o$$



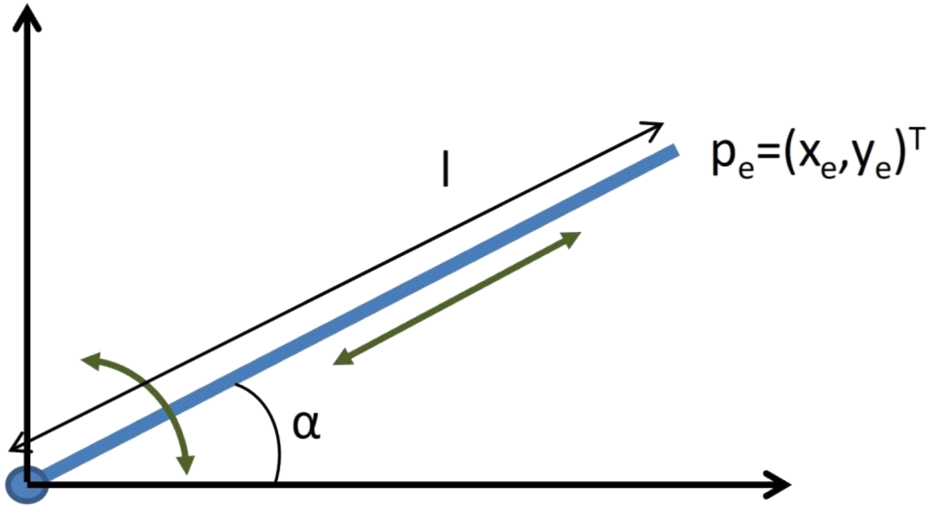
$$p_e = \begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = {}^{F_0}R(\alpha)\,{}^{F_0}T(l)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & l \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Problem 1

$$\begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & l \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha & -s\alpha & c\alpha \cdot l \\ s\alpha & c\alpha & s\alpha \cdot l \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha \cdot l \\ s\alpha \cdot l \\ 1 \end{pmatrix}$$



note: which is also in-line with solving it with high-school geometry ☺

Problem 1

$$\begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & l \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha & -s\alpha & c\alpha \cdot l \\ s\alpha & c\alpha & s\alpha \cdot l \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha \cdot l \\ s\alpha \cdot l \\ 1 \end{pmatrix}$$

note: the order in which
the matrices are multiplied out
is up to your personal preferences

$R(\alpha) \cdot T(l)$ always
leads to “just l rotated”

$A \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ always leads to
“just select last column in A”

Problem 1

$$\begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & l \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} l \\ 0 \\ 1 \end{pmatrix}$$

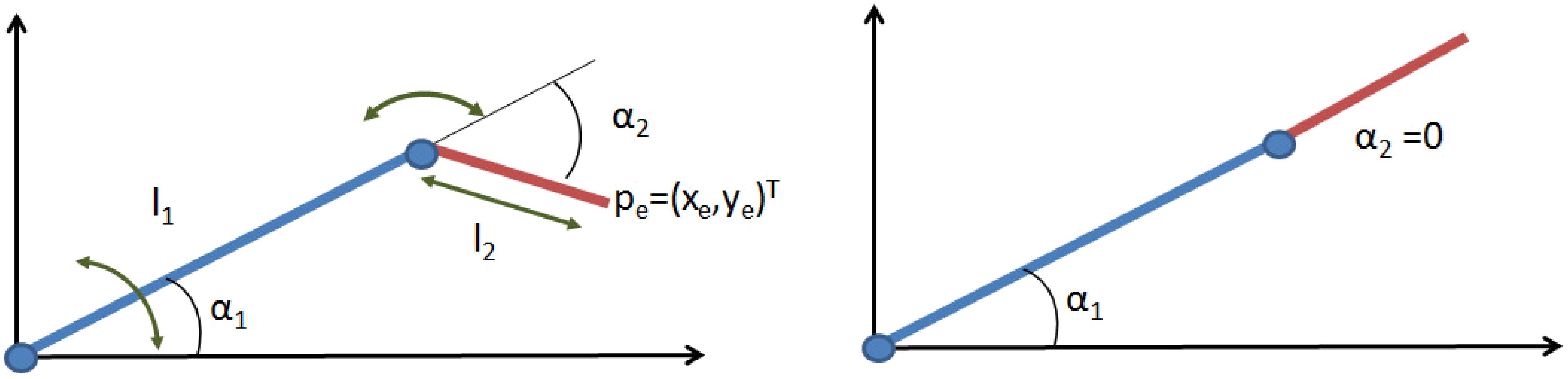
$$= \begin{pmatrix} c\alpha \cdot l \\ s\alpha \cdot l \\ 1 \end{pmatrix}$$

just always going
matrix · *vector*
is a good option, too

Problem 2

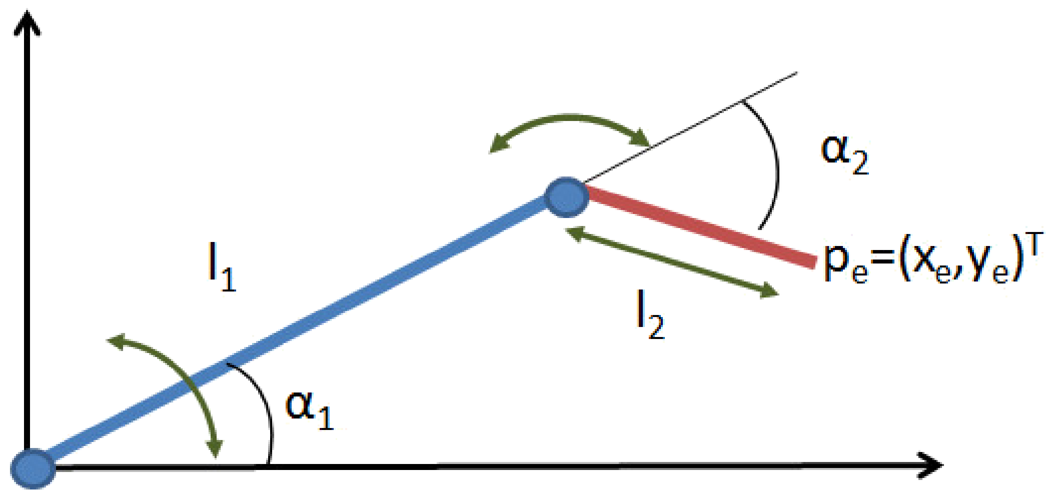
Given a planar (2D) robot arm with 3 DoF:

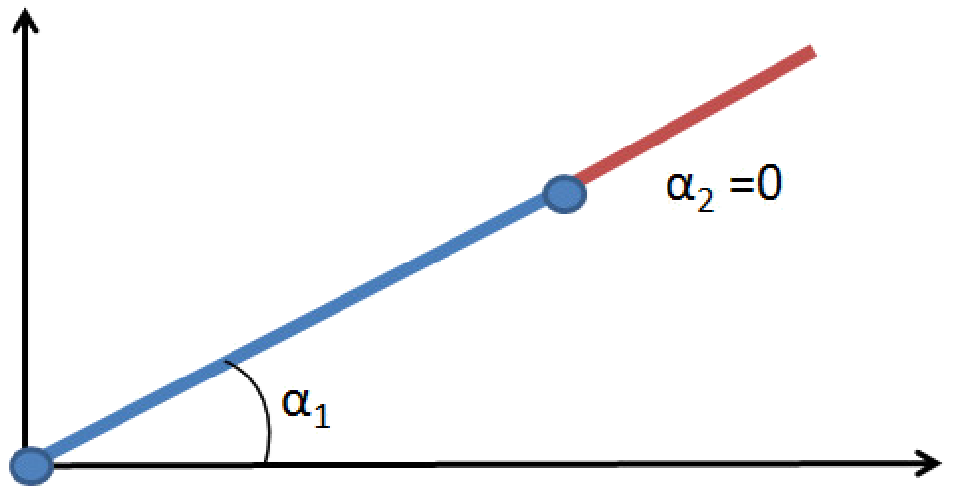
- a rotational joint in the origin of the world frame with DoF α_1,
- followed by a fixed link of length $l_1 = 10$ with rotational joint at its end with DoF α_2,
- and a prismatic joint linked to it with the DoF l_2 with $l_2 \in [5,10]$, which is co-aligned with l_1 for $\alpha_2 = 0^o$



Provide the FK for the position $p_e = (x_e, y_e)$ of the end-effector of this robot.

Problem 2





$$p_e = \begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = {}^{F_0}R(\alpha_1)\,{}^{F_0}T(l_1)\,{}^{F_0}R(\alpha_2)\,{}^{F_0}T(l_2)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Problem 2

$$\begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = {}^{F_0}R(\alpha_1)\,{}^{F_0}T(l_1)\,{}^{F_0}R(\alpha_2)\,{}^{F_0}T(l_2)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha_1 & -s\alpha_1 & 0 \\ s\alpha_1 & c\alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c\alpha_2 & -s\alpha_2 & 0 \\ s\alpha_2 & c\alpha_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & l_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha_1 & -s\alpha_1 & c\alpha_1 \cdot 10 \\ s\alpha_1 & c\alpha_1 & s\alpha_1 \cdot 10 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c\alpha_2 & -s\alpha_2 & c\alpha_2 \cdot l_2 \\ s\alpha_2 & c\alpha_2 & s\alpha_2 \cdot l_2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha_1 & -s\alpha_1 & c\alpha_1 \cdot 10 \\ s\alpha_1 & c\alpha_1 & s\alpha_1 \cdot 10 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c\alpha_2 \cdot l_2 \\ s\alpha_2 \cdot l_2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha_1 c\alpha_2 \cdot l_2 - s\alpha_1 s\alpha_2 \cdot l_2 + c\alpha_1 \cdot 10 \\ s\alpha_1 c\alpha_2 \cdot l_2 + c\alpha_1 s\alpha_2 \cdot l_2 + s\alpha_1 \cdot 10 \\ 1 \end{pmatrix}$$