university of groningen / faculty of economics and business / operations

Asset Management: Age-Based Maintenance

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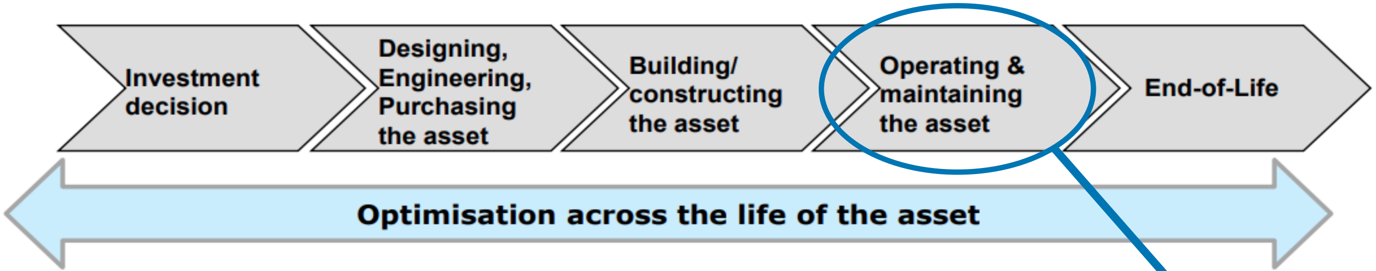
Introduction to Maintenance Policies

Introduction to Maintenance Policies



- Industrial system deteriorate with use and require maintenance throughout their life time.
- Maintenance restores systems to a state in which it can perform their intended function.
- **Maintenance optimization** is the scientific field that develops mathematic models that improve or optimize maintenance policies.

Why do we care about maintenance?

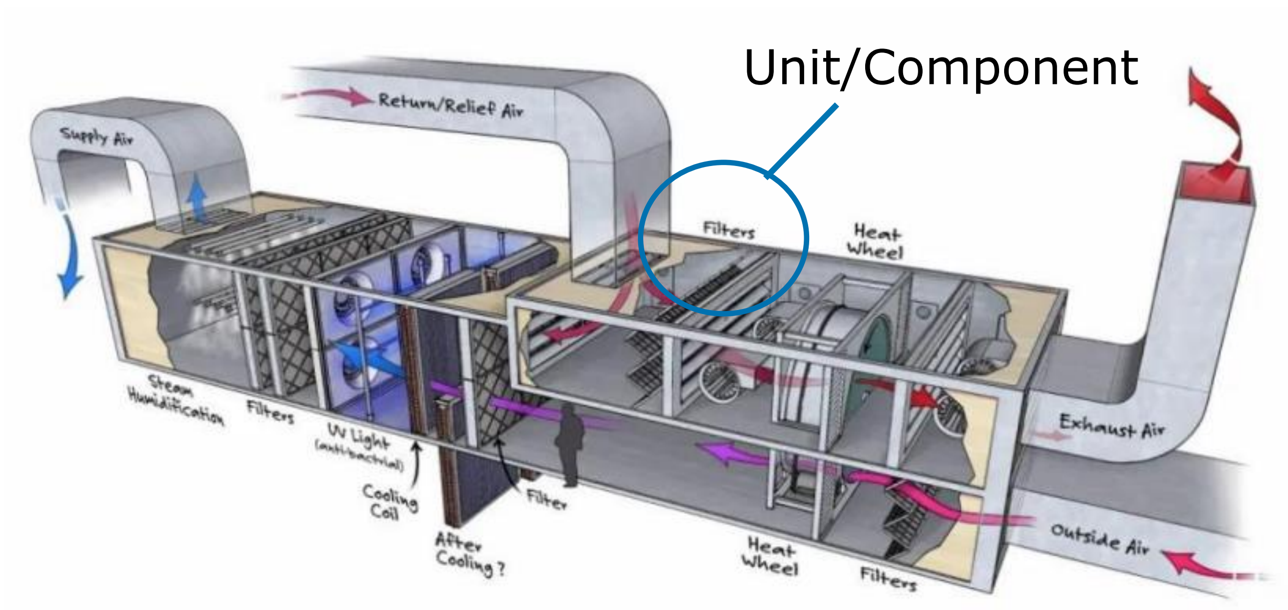
- Costs can be extremely large.
  - It is important to have an estimate before purchasing an asset.
  - Better maintenance strategies can result in large savings.
- Machine failures can result in lower utilization of your asset.
- Machine failures can be dangerous.

Example: Wind turbines at sea

About **25% of the total cost** of offshore wind is due to O&M [1]. Meanwhile, they are **available only 60-70% of the time** [2].

[1] Stehly, Beiter, Duffy. (2019) [2] Shafiee. (2015)

Systems vs. Units/Components



Unit/Component

System

Introduction to Maintenance Policies

Maintenance policies for multi-component systems

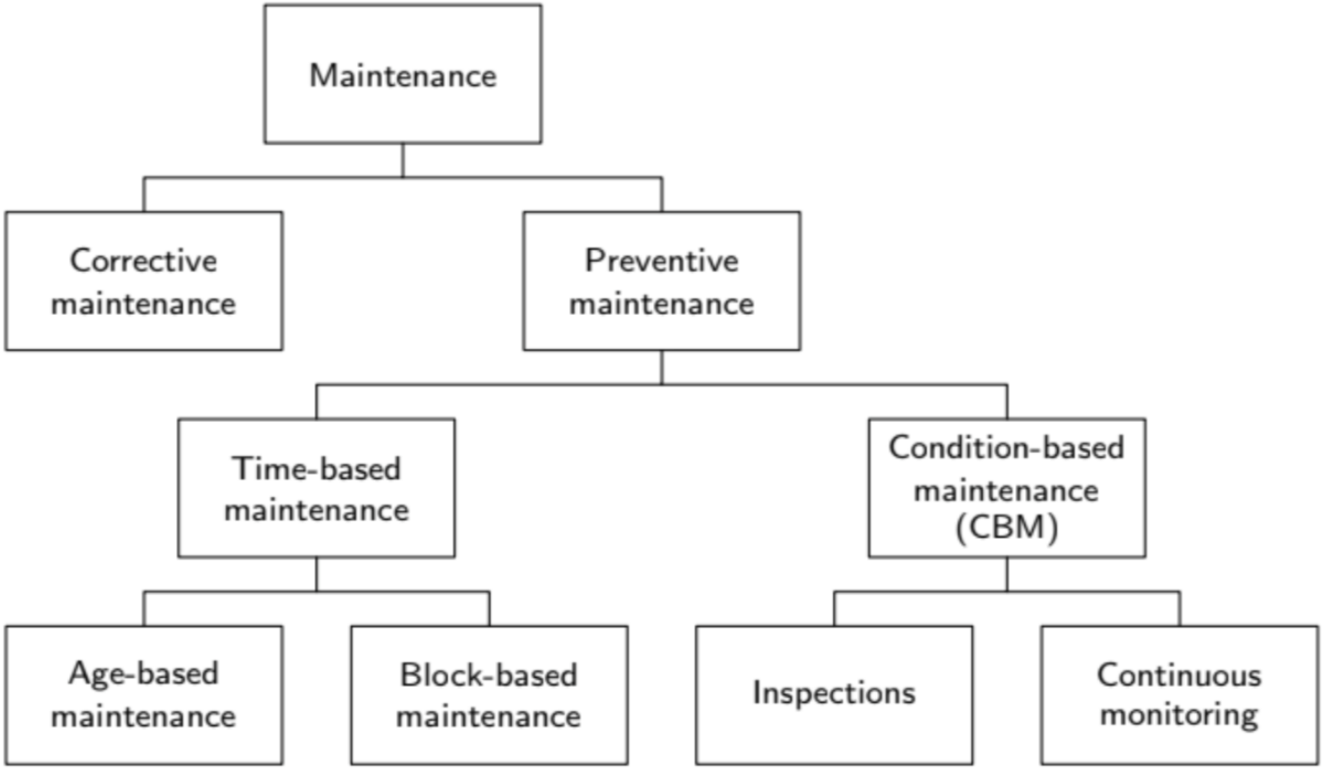
=/=

Maintenance policies for single-component system

Multi-component systems have a higher degree of complexity regarding component interdependencies.

- Economic dependence
- Structural dependence
- Stochastic dependence
- Resource/logistical dependence

Maintenance strategies for single-component systems



Types of maintenance

<u>Corrective maintenance</u>

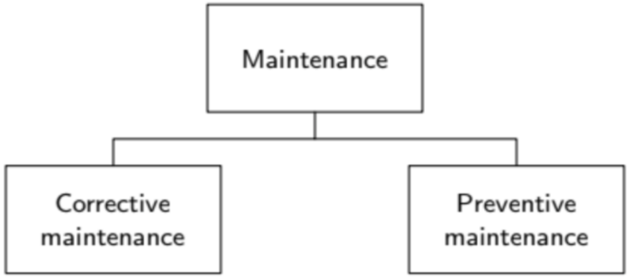
Also: reactive, failure based maintenance

Maintenance after breakdown/failure occurs.

Potential problems:

Safety issues, damage to secondary components, quality control, downtime, long repair times.

Typically more expensive than preventative maintenance.



Types of maintenance

<u>Preventative maintenance</u>

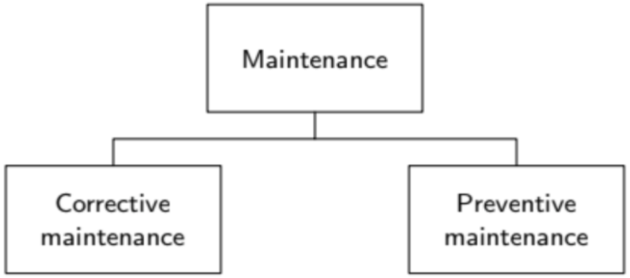
Maintenance *before* a breakdown/failure occurs.

Benefits over corrective maintenance:

Safer, cheaper, can be planned, repairs instead of replacements.

But, how do we plan this?

- Time-Based Maintenance
- Condition-Based Maintenance

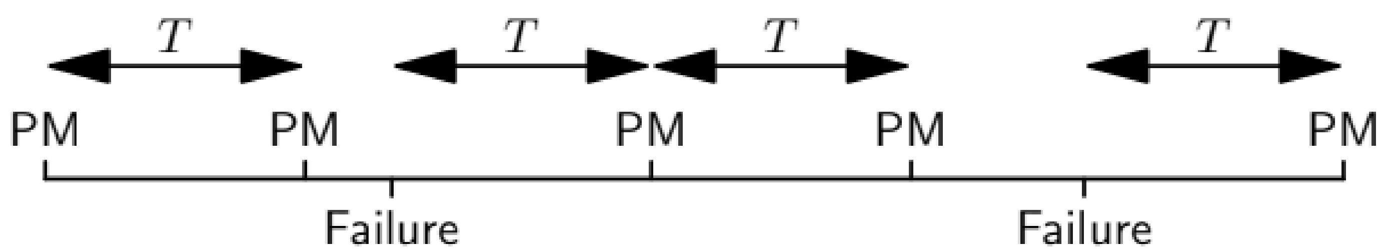


Time-based maintenance

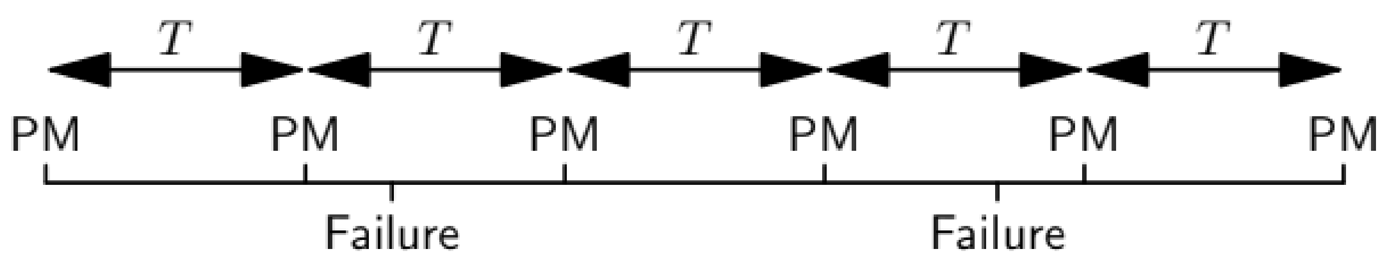
Plans preventive maintenance based on historical failure times

PM = preventative maintenance

- Age-based



- Block-based



Condition-based maintenance

Use available information to predict the condition of a machine, and maintain when we expect a failure.

› Sensor data
› Inspections

Pros: Potential cost savings due to just-in-time maintenance (no ‘unnecessary’ preventive maintenance)

Cons: Condition monitoring can be expensive, inaccurate or even impossible.

Organization of the remainder of the course

Course planning

- This week:
  - 2nd half of this lecture -> Age-based maintenance
  - Tutorial (Thursday 15:00-17:00) -> Dealing with optimization
  - Q&A session (Friday 11:00-15:00)
  - Closing session with EQUANS (Friday 15:00-17:00)
  - Drinks (Friday 17:00)

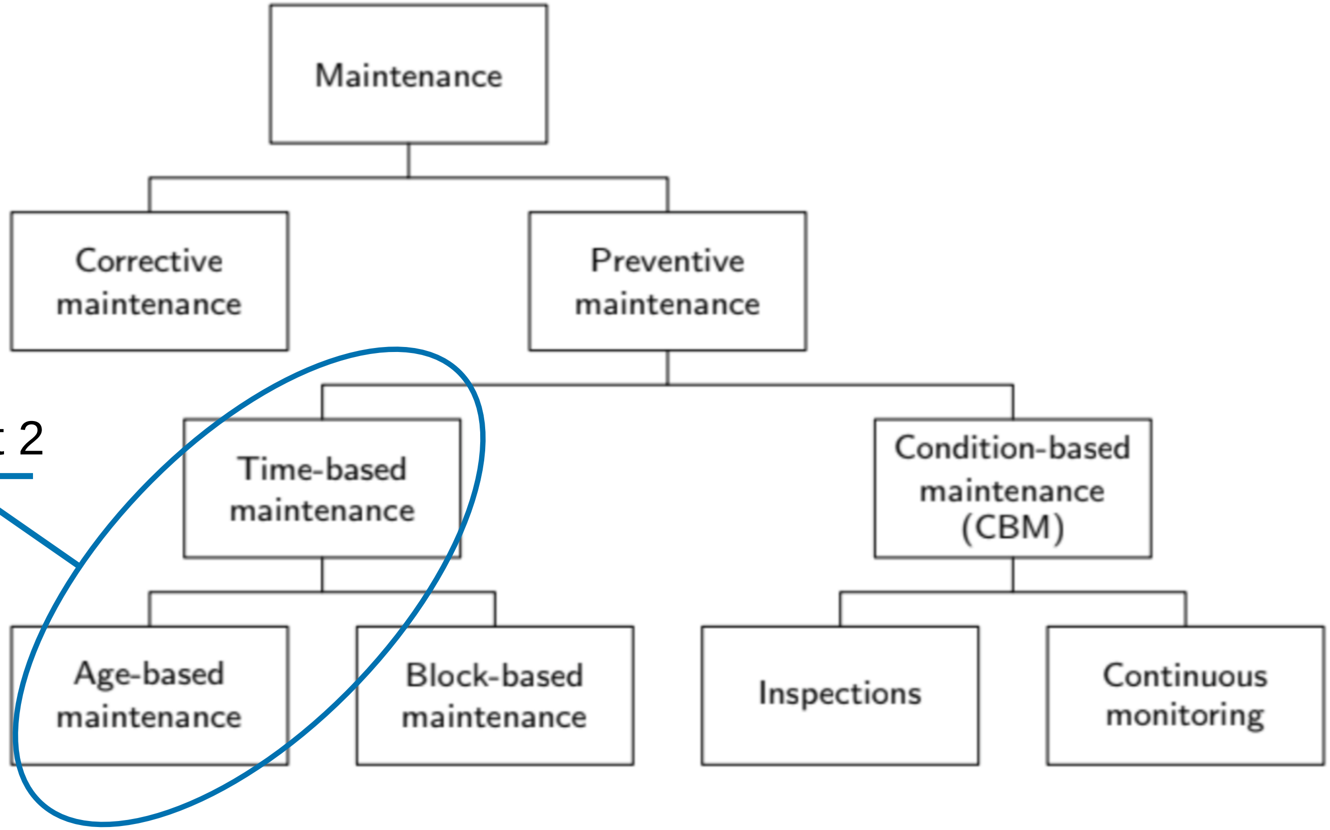
Course planning

- Next week:
  - 2nd lecture (Tuesday 17:00-19:00)-> Condition-based maintenance and lecture from EQUANS
  - Tutorial (Thursday 15:00-17:00) -> Simulation
  - Q&A session (Friday 11:00-17:00)
- Week 13:
  - Final Q&A session (Friday 11:00-17:00)
- Week 14:
  - Deadline assignment 2 (3rd of April, 17:00)

Assignment 2

Assignment 2

Focus of Assignment 2



Assignment 2

This assignment will test your data analytics skills. We want you to build a generic tool to create managerial insights, applicable to any dataset.

That means your tool will handle data that you have not seen, and should rely on automation or user input.

Assignment 2

The scenario:

You work for an industrial company with many different machines. You are asked to evaluate the failure behaviour of three machines, based on historical data. Your manager would also like to have an improved maintenance strategy based on this data. You know that similar analyses will need to be done in the future, and therefore decide to automate the process.

Your goal:

Build a tool in Python that can evaluate any machine failure dataset that automatically produces the relevant managerial information regarding that machine's optimal maintenance policy.

Assignment 2

30% of grade

Deadline: **3rd of April, 17:00, 2023**

Scope limited to:

- Single-component systems
- Age-based and condition-based maintenance policies
- Decision optimization

In addition to the assignment, there is a manual which can help you build your tool. Please read this carefully even if you feel you can program without it, as it contains **mandatory** elements of your tool.

Age-based maintenance optimization

Analysis of historical failure data

Before we can make meaningful claims about age-based maintenance policies, we need to examine the failure behaviour of a system.

A good place to start, is the *Mean Time Between Failures* (MTBF)

Time	Duration
4.8	4.8
13.2	8.4
21.3	8.1
24.1	2.8
32.3	8.2
42.0	9.7

Failures

Time between subsequent failures

Analysis of historical failure data

In this case, calculating the MTBF is easy:

Time	Duration
4.8	4.8
13.2	8.4
21.3	8.1
24.1	2.8
32.3	8.2
42.0	9.7

MTBF

$$\frac{4.8 + 8.4 + 8.1 + 2.8 + 8.2 + 9.7}{6} = \frac{42}{6} = 7 \text{ hours.}$$

But, what if our historical data also contains preventive maintenance actions?

Analysis of historical failure data

What if our historical data also contains preventive maintenance actions?

After a preventive maintenance action, we "reset the clock" to the next failure. We correct the maintenance times:

Time	Duration	
4.8	4.8	Failure
10.7	**5.9**	Preventive maintenance
13.2	~~8.4~~ **2.5**	Failure
21.3	8.1	Failure
24.1	2.8	Failure
27.5	**3.4**	Preventive maintenance
32.3	~~8.2~~ **4.8**	Failure
42.0	9.7	Failure

Analysis of historical failure data

We can now calculate the MTBF by averaging the durations that ended in failure -> **easy**

But, preventive maintenance actions also give us information! If we did preventive maintenance after 5 weeks, then the system was functional for **at least** 5 weeks too.

Of course, we don’t know when the machine would have failed.

To improve our estimation of the MTBF, we use the **Kaplan-Meier Estimator**.

Kaplan-Meier Estimator

<u>Terminology:</u>

Failure -> Event duration

Preventive maintenance -> Censored duration

Sort your durations in ascending order:

Duration	Censored
4.8	No
5.9	Yes
2.5	No
8.1	No
2.8	No
3.4	Yes
4.8	No
9.7	No

→

Duration	Censored
2.5	No
2.8	No
3.4	Yes
4.8	No
4.8	No
5.9	Yes
8.1	No
9.7	No

Kaplan-Meier Estimator

If there are durations of equal length, event durations are listed **before** censored durations.

Duration	Censored
2.5	No
2.8	No
3.4	Yes
4.8	No
4.8	No
5.9	Yes
8.1	No
9.7	No

Equal length durations, but both are event durations; no change needed in this case!

Kaplan-Meier Estimator

Key concept to remember:

With corrective maintenance, every event duration carries the same weight in calculating MTBF [we calculate a simple average of all durations].

Now, we have more info about when machines do *not* break down. Therefore, we can appoint larger weights to the failures that occur *after* a preventative maintenance action.

Why? Because we know for sure the machine did not break down *before* that time; if the preventative maintenance didn't occur, it *would* have failed later at some point.

Kaplan-Meier Estimator

Start by giving all durations equal probability (equal weight).

Duration	Censored	Probability
2.5	No	0.125
2.8	No	0.125
3.4	Yes	0.125
4.8	No	0.125
4.8	No	0.125
5.9	Yes	0.125
8.1	No	0.125
9.7	No	0.125

Each probability equals 1/[number of durations].
Sum of all probabilities is 1.

Kaplan-Meier Estimator

Starting from the top, if you see a censored duration, spread its probability equally over all following durations.

Duration	Censored	Probability	
2.5	No	0.125	
2.8	No	0.125	
3.4	Yes	~~0.125~~	
4.8	No	0.125	+ 0.025
4.8	No	0.125	+ 0.025
5.9	Yes	0.125	+ 0.025
8.1	No	0.125	+ 0.025
9.7	No	0.125	+ 0.025

Duration	Censored	Probability	
2.5	No	0.125	
2.8	No	0.125	
3.4	Yes		
4.8	No	0.150	
4.8	No	0.150	
5.9	Yes	~~0.150~~	
8.1	No	0.150	+0.075
9.7	No	0.150	+0.075

Why? The unobserved failure is **equally likely** to be any of the event durations that is longer than the observed censored duration.

Kaplan-Meier Estimator

If there are equal event durations, the probabilities can be merged.

Duration	Censored	Probability
2.5	No	0.125
2.8	No	0.125
3.4	Yes	
4.8	No	0.150
4.8	No	0.150
5.9	Yes	
8.1	No	0.225
9.7	No	0.225

Duration	Censored	Probability
2.5	No	0.125
2.8	No	0.125
4.8	No	0.300
8.1	No	0.225
9.7	No	0.225

Kaplan-Meier Estimator

Now that we have the probabilities for each duration, we can calculate the MTBF again:

Duration	Censored	Probability
2.5	No	0.125
2.8	No	0.125
4.8	No	0.300
8.1	No	0.225
9.7	No	0.225

MTBF

$$0.125 \times 2.5 + 0.125 \times 2.8 + 0.300 \times 4.8 + 0.225 \times 8.1 + 0.225 \times 9.7 = 6.108 \text{ hours.}$$

Kaplan-Meier Estimator

We can use this data for more than just the MTBF.

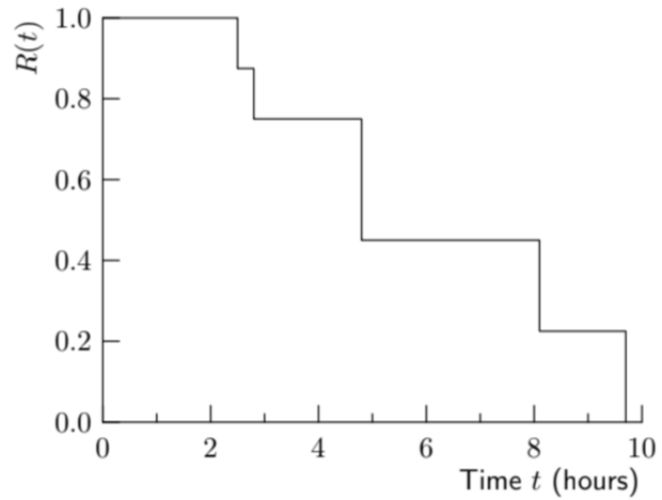
For example, we know that the probability that a machine fails at time 4.8 is exactly 30%.

But what is the probability the machine has (already) failed at time 4.8? Or vice versa: what is the chance the machine still works at time 4.8?

Duration	Censored	Probability
2.5	No	0.125
2.8	No	0.125
4.8	No	0.300
8.1	No	0.225
9.7	No	0.225

Kaplan-Meier Estimator

We can use the Kaplan-Meier Estimator data to determine the **cumulative** probability for **survival** of the machine. We call this the **reliability function: R(t).**



R(t) shows the probability (y-axis) of a machine working (*not* having failed!) at a certain time (x-axis)

Kaplan-Meier Estimator

Reliability, R(t): The probability of *survival* before time t.

Lifetime distribution function, F(t): The probability of a *failure* before time t. This is also the sum of the values in the “Probability”-column up to t.

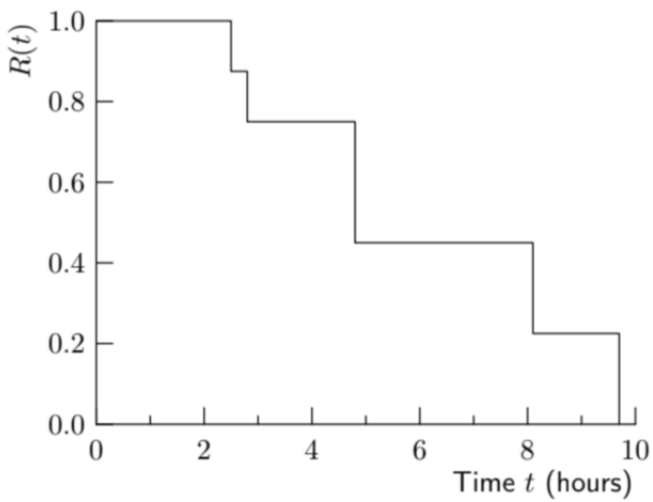
R(t) = 1-F(t)

In this case:

F(2.8) = 0.125 + 0.125 = 0.250

R(2.8) = 1- F(2.8) = 0.750

Duration	Censored	Probability	Reliability
2.5	No	0.125	0.875
2.8	No	0.125	0.750
4.8	No	0.300	0.450
8.1	No	0.225	0.225
9.7	No	0.225	0



Kaplan-Meier Estimator

What happens when the longest duration (last entry in table) is a censored duration?

- The probability of this censored event duration cannot be assigned to following event durations.
- This probability ‘disappears’ from the table and the R(t) step plot will never reach zero.
- MTBF can no longer be determined based on this dataset.

[sum of probability will no longer add up to 1!]

Kaplan-Meier Estimator

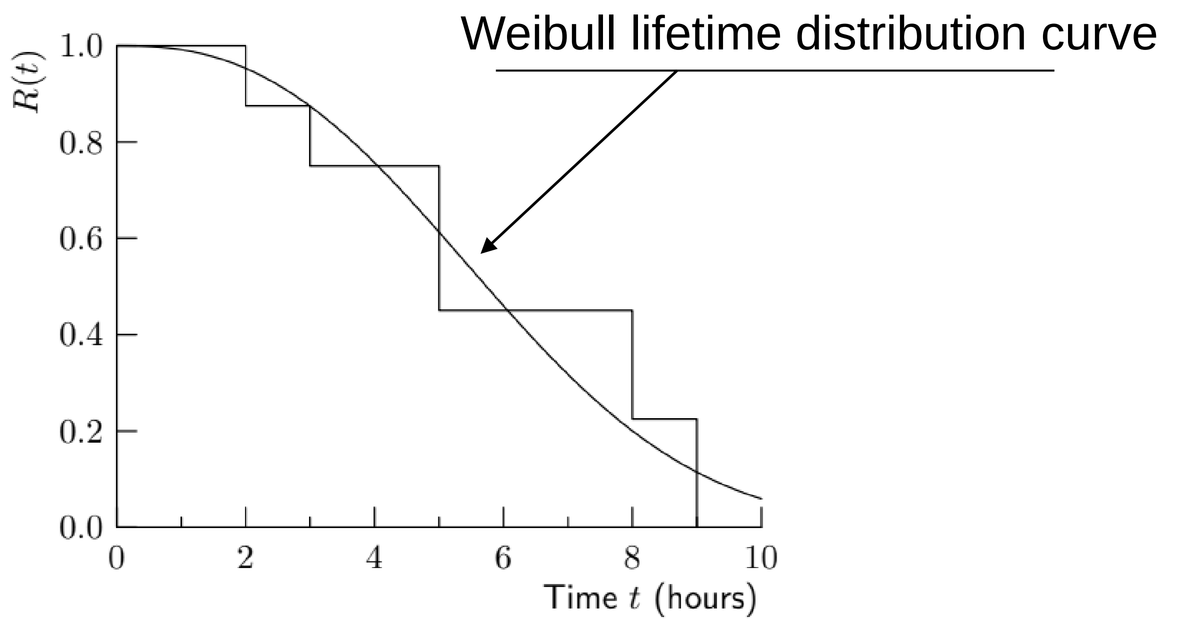
The Kaplan-Meier estimator has benefits, but is not without flaws.

- ✓ Easy to calculate; no complicated statistics.
- ✓ With enough data, the step plot shape gives a good impression of the machine's failure behaviour.

- × It is unrealistic to assign probabilities only to discrete points in time based on historical data. The probability that a failure happens after *exactly* the same time as earlier small.
- × It is difficult to develop (mathematical) optimization models to determine optimal maintenance policy based on step-wise R(t) data.

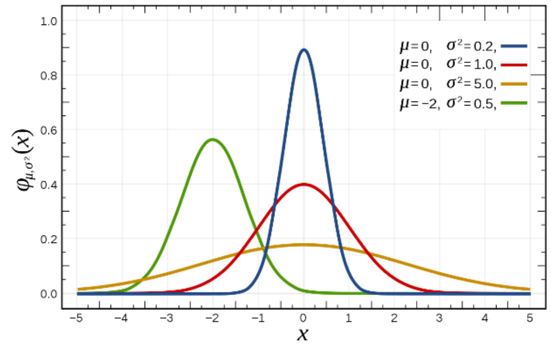
Weibull Distribution

To solve the problems on the previous slide, we can fit the Kaplan-Meier results to a continuous probability distribution.

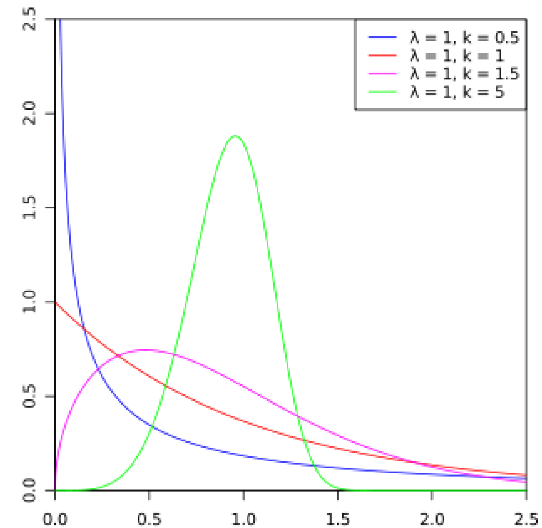


Weibull Distribution

Remember **normal** distributions? Normal distributions deal with probabilities and have particular qualities; symmetry around the mean, particular formulas to calculate properties, etc.



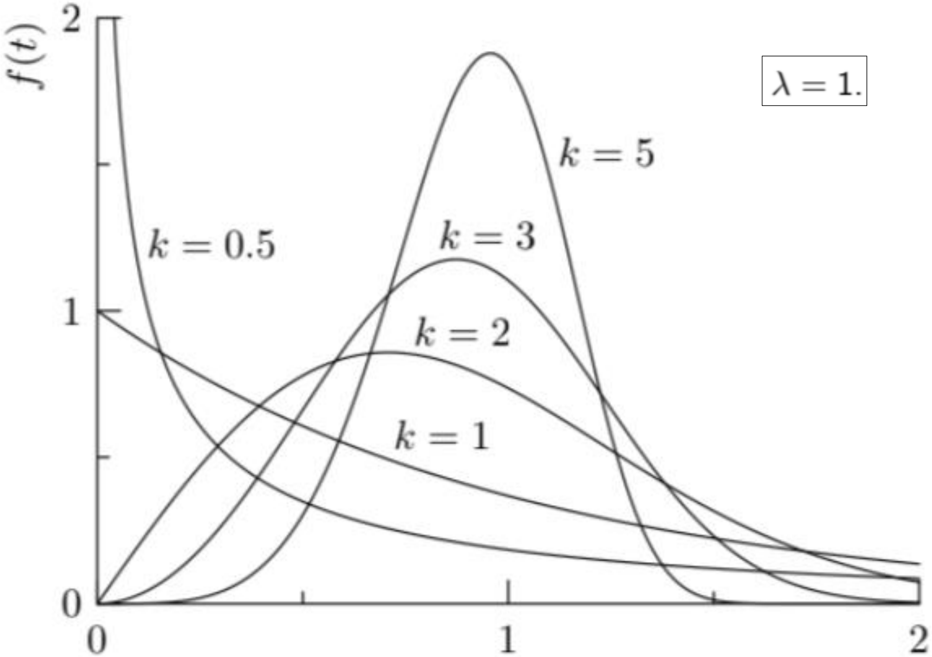
Just like the normal distribution, the **Weibull** distribution deals with probability and has a set of qualities that determine the shape and properties of the distribution.



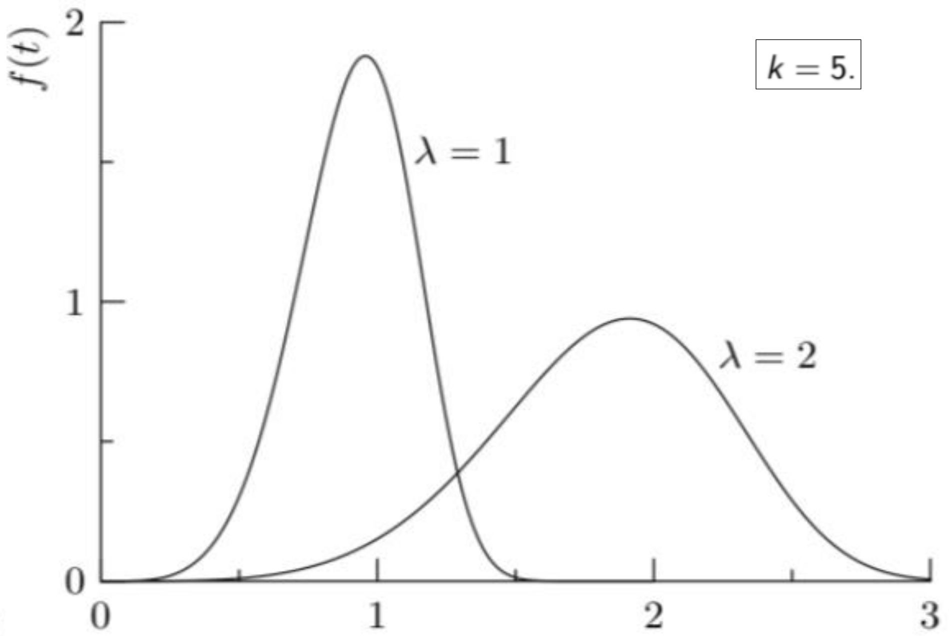
Weibull distributions can have a larger variety in shapes; making them very suitable to model lifetimes!

Weibull Distribution

Effect of κ (kappa) on f(t): [shape]



Effect of λ (lambda) on f(t): [scale]



Weibull Distribution

Just as before with the Kaplan-Meier Estimator, also for Weibull we have:

$$\boxed{R(t) = 1 - F(t)}$$

$R(t;\lambda,\kappa) \rightarrow$
Reliability function: probability of survival **before** time t

$F(t;\lambda,\kappa) \rightarrow$
Lifetime distribution function: probability of failure **before** time t

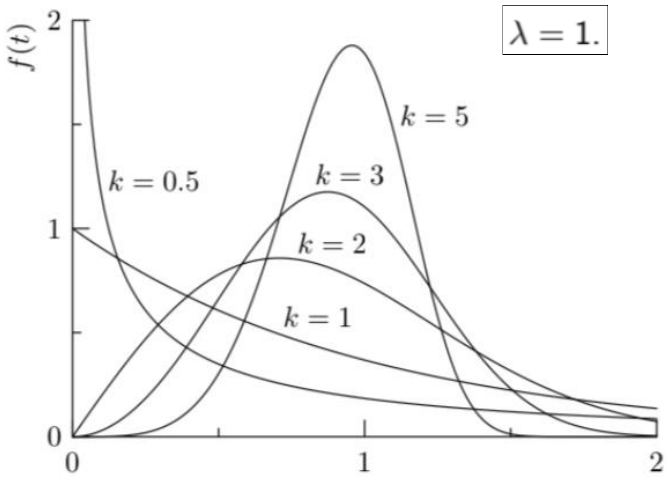
Also important:

$f(t;\lambda,\kappa) \rightarrow$ Density function: likelihood of failure **at** time t

$R(t;\lambda,\kappa)$ and $F(t;\lambda,\kappa)$ are cumulative probabilities, $f(t;\lambda,\kappa)$ is not.

Weibull Distribution

This is f(t):



The formula for f(t) is:

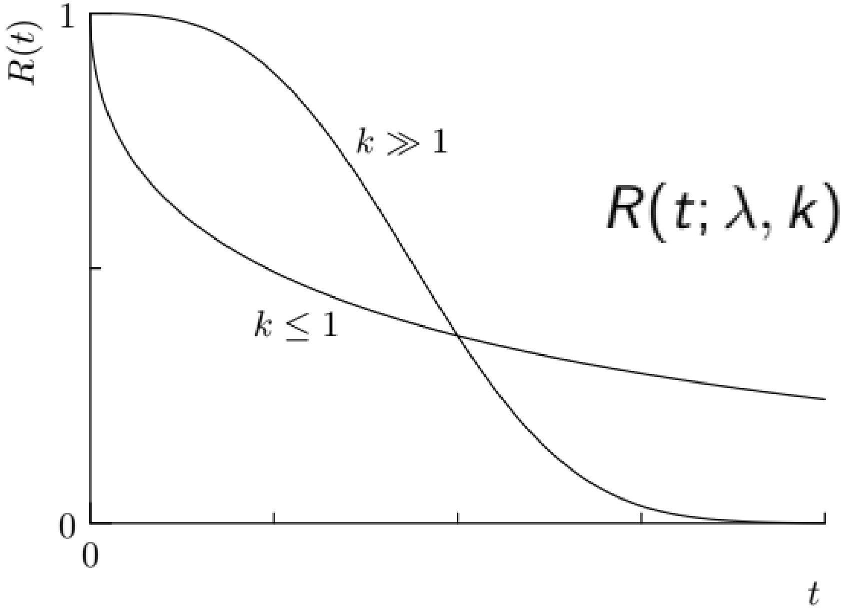
$$f(t;\lambda,k) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{-\left(\frac{t}{\lambda}\right)^k}, \quad t \geq 0.$$

This formula has two parameters:

- Scale parameter λ (lambda)
- Shape parameter κ (kappa)

Weibull Distribution

This is R(t):



The formula for F(t) and R(t) is:

$$F(t;\lambda,k) = 1 - e^{-\left(\frac{t}{\lambda}\right)^k}, \quad t \geq 0.$$

$$R(t;\lambda,k) = \underline{1 - F(t;\lambda,k)} = e^{-\left(\frac{t}{\lambda}\right)^k}, \quad t \geq 0.$$

R(t) = 1 – F(t)

Weibull Distribution

The MTBF of a Weibull distribution is directly linked to the parameters λ and κ.

MTBF of a Weibull distribution:

$$\lambda\Gamma\left(1+\frac{1}{k}\right)$$

Γ() is the gamma function. For this course, the mathematical details of the gamma function are not important.

The gamma function is built into Python.

Selecting the best Weibull Distribution

How to find the right Weibull distribution that fits your Kaplan-Meier Estimator data?

We want to know to **likelihood** of the fit between our Weibull distribution and the probabilities of the Kaplan-Meier Estimator.

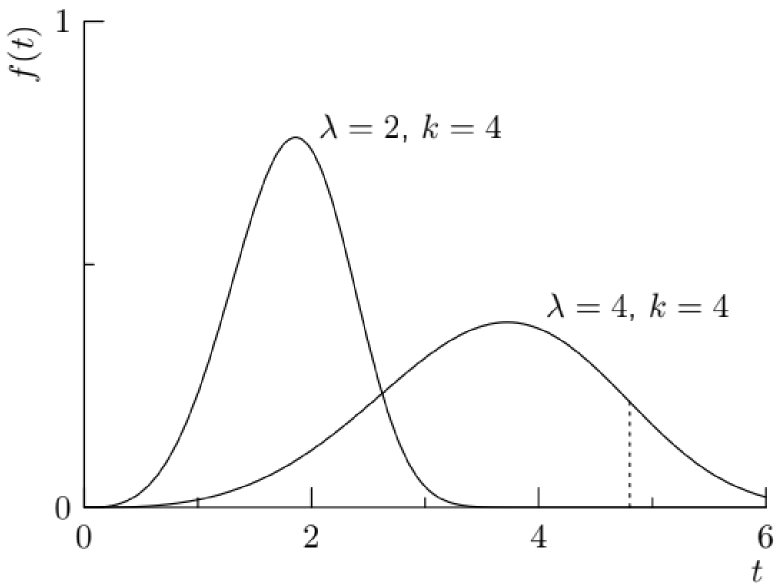
The likelihood of an event duration (failure) is modelled by $f(t,\lambda,\kappa)$.

The likelihood of a censored duration (PM) is modelled by $R(t,\lambda,\kappa)$.

Selecting the best Weibull Distribution

We evaluate 2 different Weibull distribution curves.
f(4.8, 2, 4) = 0 → Probability of zero; very unlikely!
f(4.8, 4, 4) = 0.21 → More likely than zero, but is it best fit?

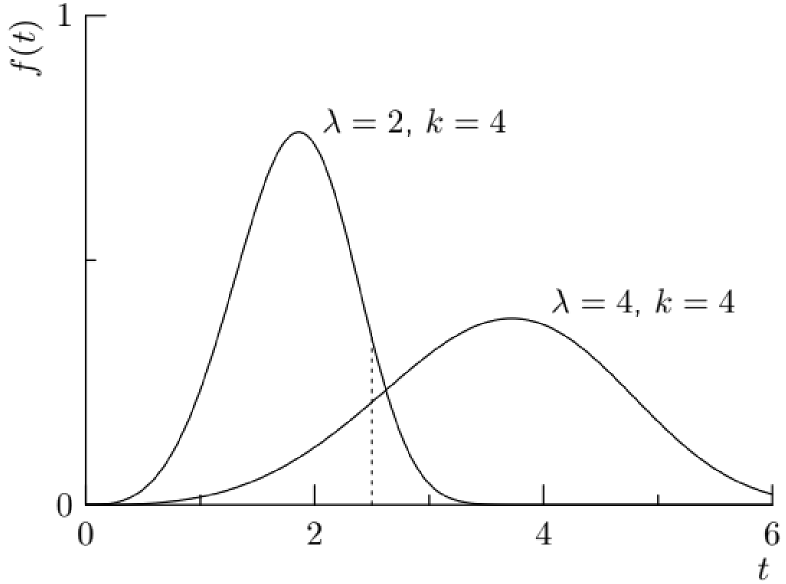
Duration	Censored
4.8	No
5.9	Yes
2.5	No
8.1	No
2.8	No
3.4	Yes
4.8	No
9.7	No



Selecting the best Weibull Distribution

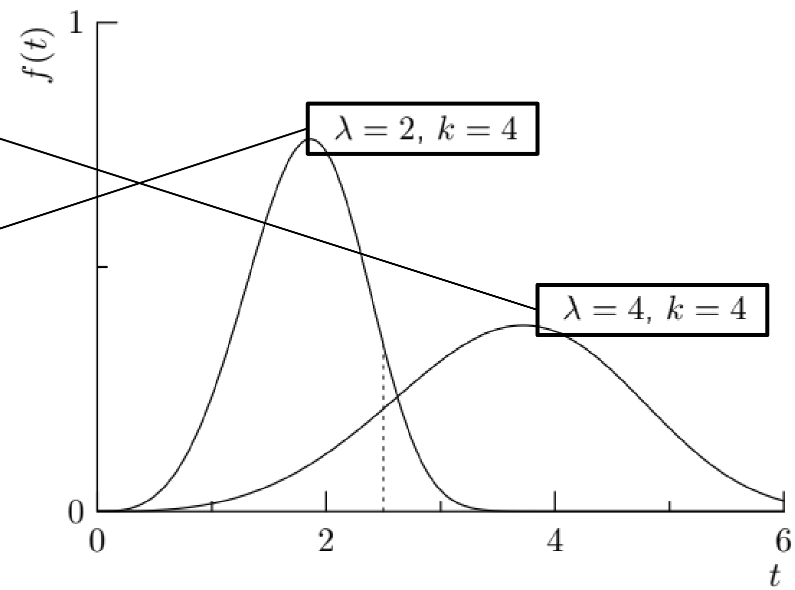
We evaluate 2 different Weibull distribution curves.
f(2.5, 2, 4) = 0.34 → Best fit out of 2
f(2.5, 4, 4) = 0.21

Duration	Censored
4.8	No
5.9	Yes
$\boxed{2.5}$	$\boxed{\text{No}}$
8.1	No
2.8	No
3.4	Yes
4.8	No
9.7	No



Selecting the best Weibull Distribution

Duration	Censored
4.8	No
5.9	Yes
2.5	No
8.1	No
2.8	No
3.4	Yes
4.8	No
9.7	No



'Best' fit of each individual duration is different!
How to determine the best λ and κ for the **whole** dataset?

Selecting the best Weibull Distribution

Duration	Censored	Likelihood	
4.8	No	$f(4.8; \lambda, k)$	
5.9	Yes	$R(5.9; \lambda, k)$	$= 1 - F(5.9; \lambda, k)$
2.5	No	$f(2.5; \lambda, k)$	
8.1	No	$f(8.1; \lambda, k)$	
2.8	No	$f(2.8; \lambda, k)$	
3.4	Yes	$R(3.4; \lambda, k)$	$= 1 - F(3.4; \lambda, k)$
4.8	No	$f(4.8; \lambda, k)$	
9.7	No	$f(9.7; \lambda, k)$	

For each duration, we can calculate the likelihood at that particular value for t.

We can also calculate the likelihood for the whole dataset, called L(λ,κ). We want L(λ,κ) to lead to an overall maximum match for each individual duration. L(λ,κ)

Selecting the best Weibull Distribution

Duration	Censored	Likelihood	
4.8	No	$f(4.8; \lambda, k)$	
5.9	Yes	$R(5.9; \lambda, k)$	$= 1 - F(5.9; \lambda, k)$
2.5	No	$f(2.5; \lambda, k)$	
8.1	No	$f(8.1; \lambda, k)$	
2.8	No	$f(2.8; \lambda, k)$	
3.4	Yes	$R(3.4; \lambda, k)$	$= 1 - F(3.4; \lambda, k)$
4.8	No	$f(4.8; \lambda, k)$	
9.7	No	$f(9.7; \lambda, k)$	

$$\begin{aligned} L(\lambda, k) &= f(4.8; \lambda, k) \times R(5.9; \lambda, k) \times f(2.5; \lambda, k) \times f(8.1; \lambda, k) \\ &\quad \times f(2.8; \lambda, k) \times R(3.4; \lambda, k) \times f(4.8; \lambda, k) \times f(9.7; \lambda, k). \end{aligned}$$

Problem: f(t,λ,κ) and R(t,λ,κ) are small numbers. Multiplication of small numbers leads to *very* small numbers. This leads to computational problems.

Selecting the best Weibull Distribution

Solution:

Mathematical trick to change multiplication to summation.

$$\begin{aligned} L(\lambda, k) &= f(4.8;\lambda,k) \times R(5.9;\lambda,k) \times f(2.5;\lambda,k) \times f(8.1;\lambda,k) \\ &\quad \times f(2.8;\lambda,k) \times R(3.4;\lambda,k) \times f(4.8;\lambda,k) \times f(9.7;\lambda,k). \end{aligned}$$

$$\begin{aligned} \log L(\lambda, k) &= \log(f(4.8;\lambda,k) \times R(5.9;\lambda,k) \times f(2.5;\lambda,k) \\ &\quad \times f(8.1;\lambda,k) \times f(2.8;\lambda,k) \times R(3.4;\lambda,k) \\ &\quad \times f(4.8;\lambda,k) \times f(9.7;\lambda,k)). \end{aligned}$$

$$\begin{aligned} \log L(\lambda, k) &= \log f(4.8;\lambda,k) + \log R(5.9;\lambda,k) + \log f(2.5;\lambda,k) \\ &\quad + \log f(8.1;\lambda,k) + \log f(2.8;\lambda,k) + \log R(3.4;\lambda,k) \\ &\quad + \log f(4.8;\lambda,k) + \log f(9.7;\lambda,k). \end{aligned}$$

Selecting the best Weibull Distribution

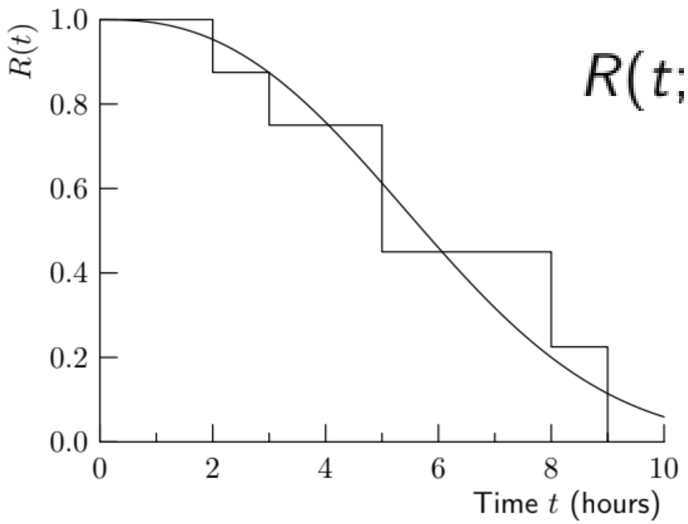
$$
\begin{aligned}
\log L(\lambda, k) \quad = \quad & \log f(4.8; \lambda, k) + \log R(5.9; \lambda, k) + \log f(2.5; \lambda, k) \\
& + \log f(8.1; \lambda, k) + \log f(2.8; \lambda, k) + \log R(3.4; \lambda, k) \\
& + \log f(4.8; \lambda, k) + \log f(9.7; \lambda, k).
\end{aligned}
$$

This can (quite easily) be numerically maximalised.

Python (but also Excel etc.) can find the λ and κ combination that leads to the highest log L(λ,κ) value. This λ and κ combination represents the Weibull distribution curve that best fits your Kaplan-Meier Estimator data.

Selecting the best Weibull Distribution

Once the best fitting λ and κ are know, you can easily make the corresponding R(t,λ,κ) graph to fit your Kaplan-Meier Estimator data. Also, λ and κ can be used to calculate MTBF (and compare it to the Kaplan-Meier MTBF!)



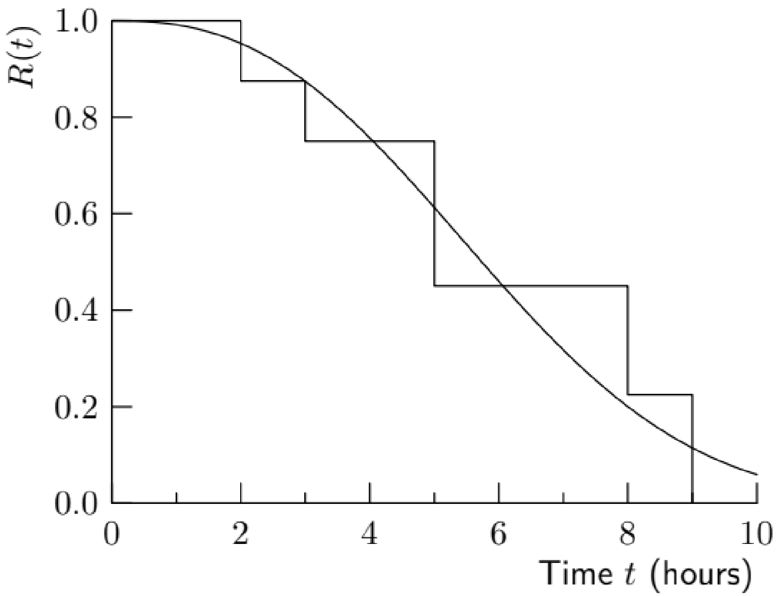
$$R(t;\lambda,k) = 1 - F(t;\lambda,k) = e^{-\left(\frac{t}{\lambda}\right)^k}, \quad t \geq 0.$$

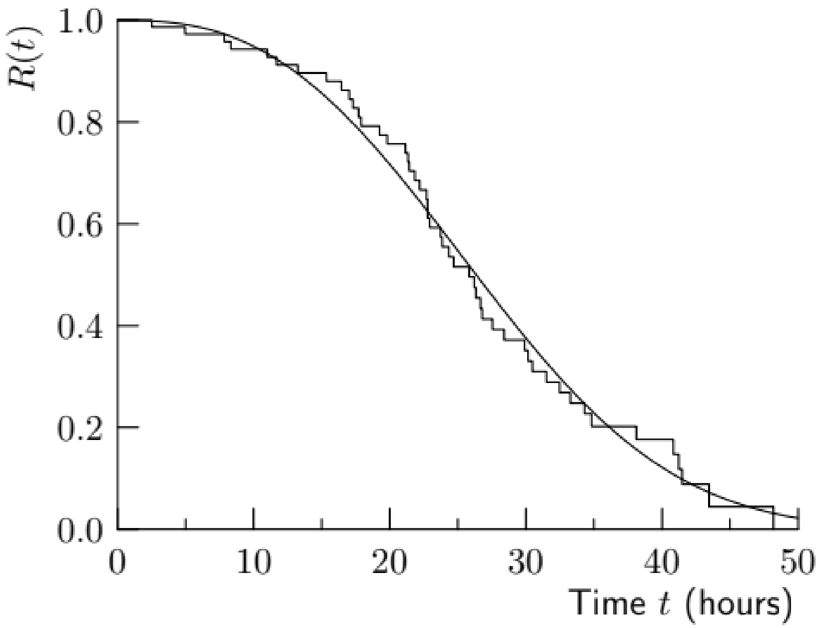
MTBF (Weibull):

$$MTBF = \lambda \Gamma\left(1 + \frac{1}{k}\right)$$

Selecting the best Weibull Distribution

Note that a good fit requires sufficient data points!





Interpreting the Weibull Distribution

Now that we have a well-fitting Weibull distribution;

What can we do with it?

The distribution tells us the failure rate behaviour of the machine, based on which we can decide a preventative maintenance policy.

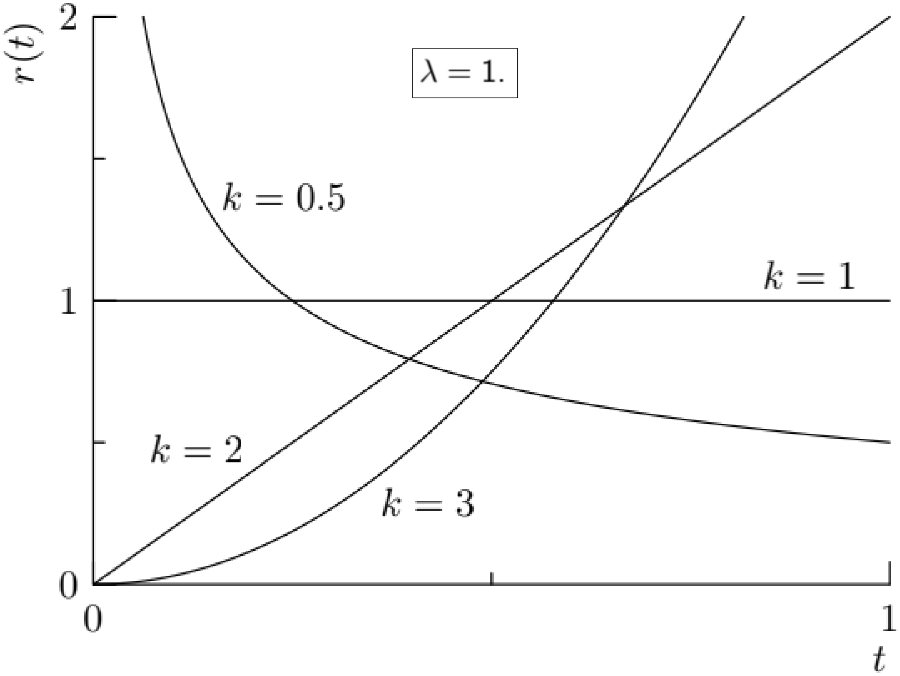
- Increasing Failure Rate. Probability of failure <u>increases</u> over time. Policy: prevent failure by implementing preventative maintenance
- Constant Failure Rate. Probability of failure stays <u>constant</u> over time. Policy: preventative maintenance has no effect.
- Decreasing Failure Rate. Probability of failure <u>decreases</u> over time. Policy: No preventative maintenance!

Interpreting the Weibull Distribution

Failure rate: r(t)

$$r(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)}.$$

$$r(t) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1}$$



Increasing if κ > 1
Constant if κ = 1
Decreasing if κ < 1

Interpreting the Weibull Distribution

Now that we have a well-fitting Weibull distribution;
What can we do with it?

We can use Weibull to calculate <u>costs</u> associated with maintenance actions and determine:

› *When* to perform preventative maintenance to minimize costs
› What the cost savings are of preventative maintenance vs. corrective maintenance.

Age-based maintenance

Cost of corrective maintenance: c_{cm}
Cost of preventative maintenance: c_{pm}

We do preventative maintenance, because it is cheaper: $c_{pm} < c_{cm}$

There is a trade-off:
› Maintain earlier -> We will maintain more often
› Maintain later -> Failures are more likely to happen

Age-based maintenance

We determine a maintenance age T, which is the time at which we do maintenance (if no failure happened beforehand).

<u>Goal of optimization:</u>

Find the maintenance age T that minimizes the **mean** cost per unit time η(T).

$$\eta(T) = \frac{\text{Mean cost per cycle}}{\text{Mean cycle length}}$$

Cycle; the **duration** from good-as-new until either preventative maintenance or failure (whichever occurs first).

Age-based maintenance

Calculating the mean cost per cycle is straightforward:

R(T) →
Reliability function [probability of survival **before** time T]
➔Preventative maintenance

F(T) →
Lifetime distribution function [probability of failure **before** time T]
➔Corrective Maintenance

Mean cost per cycle: $c_{cm}F(T) + c_{pm}R(T) = c_{cm}F(T) + c_{pm}(1 - F(T))$

Age-based maintenance

Calculating the mean cycle length is more complex:

Mean cycle length is **not** the same as T! -> It is possible we do preventive maintenance at time T, but the machine can also fail before this time.

The mean cycle length takes both options into account.

Mean cycle length: $\int_0^T R(t)\,dt.$

Mean cycle length

$$\int_0^T R(t)\,\mathrm{d}t.$$

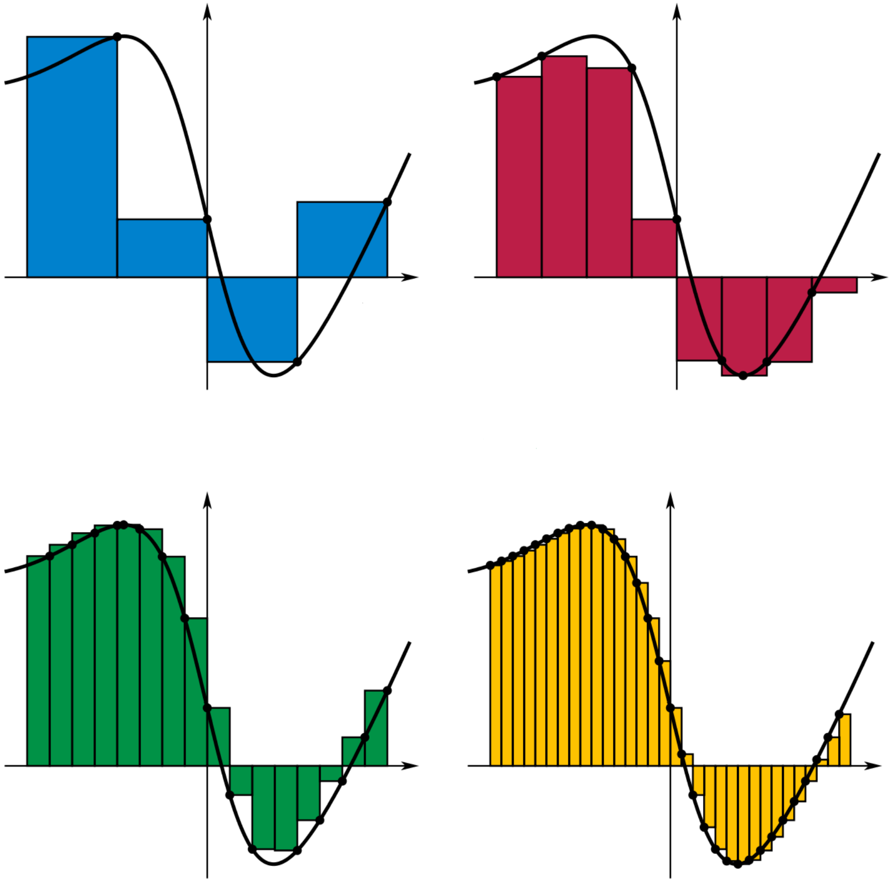
Problem: this integral does not have an <u>explicit</u> solution.
Solution: we need to find an <u>approximate</u> solution.

Riemann sum

Integral →
Area <u>underneath</u> a curve.

If we can't find the **exact** area underneath the curve, we can **approximate** it by using rectangles. Smaller widths = better approximation

The **sum** of all rectangles is an approximation of the area. This is called 'Riemann sum'.



Riemann sum

The width of our rectangles is determined by the x-axis: time.

Let's take a very small width and call this width Δ.

We want to know the (approximate) area underneath the curve between 0 and a certain time T on the x-axis. We do this by making rectangles that are Δ time long.

$$\int_0^T R(t)\,\mathrm{d}t \approx \Delta R(\Delta) + \Delta R(2\Delta) + \Delta R(3\Delta) + \ldots + \Delta R(T-\Delta)) + \Delta R(T).$$

This we *can* use!

Age-based maintenance

Now, we can calculate the cost rate for any T!

$$\eta(T) = \frac{\text{Mean cost per cycle}}{\text{Mean cycle length}} = \frac{c_{cm}F(T) + c_{pm}R(T)}{\int_0^T R(t)\,\mathrm{d}t}.$$

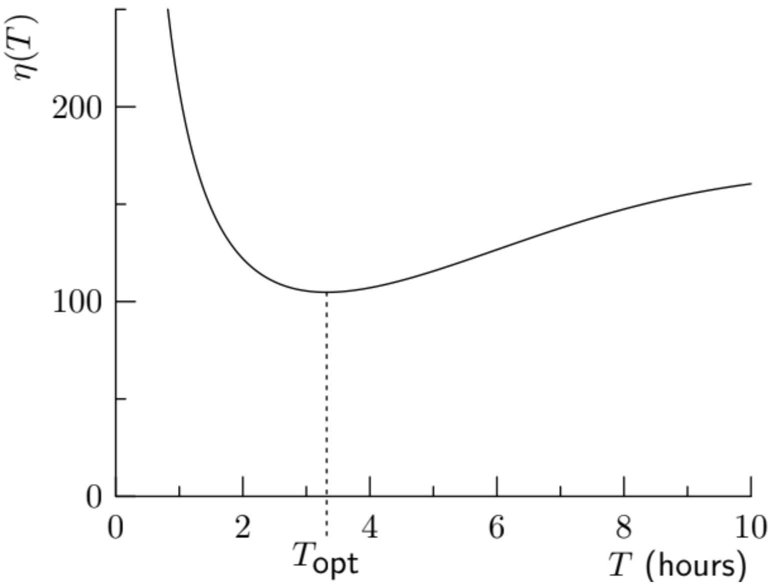
For which:

$$\int_0^T R(t)\,\mathrm{d}t \approx \Delta R(\Delta) + \Delta R(2\Delta) + \Delta R(3\Delta) + \ldots + \Delta R(T-\Delta)) + \Delta R(T).$$

Age-based maintenance

We can solve η(T) for the T that minimizes the cost.

We can *also* solve it for all possible maintenance times T to create a continuous curve that we can plot.



$c_{pm} = 200.$

$c_{cm} = 1000.$

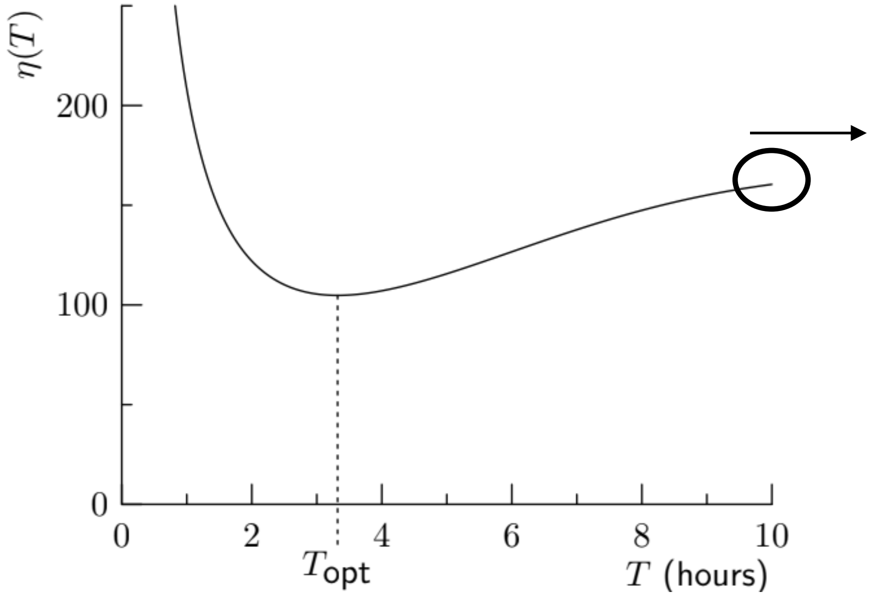
Found optimum:

T_{opt} = 3.32 hours

η(3.32) = 104.78 cost/hour

Age-based maintenance

If we take an extremely large T, failures will almost certainly happen before this time. So, as T increases, the cost rate will increase to a stable rate, which is equal to a policy where we only do corrective maintenance.



$$\frac{c_{cm}}{MTBF} = \frac{c_{cm}}{\lambda\Gamma\left(1+\frac{1}{k}\right)}$$

$$\frac{1000}{5.97} = 167.50.$$

MTBF-Weibull for this dataset

That’s all for age-based maintenance

You will be applying the math you have seen today in your assignment to do your own analysis of failure behaviour and create management insight into maintenance policy.

During the tutorial this Thursday I will go into more detail on selecting the optimal Weibull distribution parameters in Python.