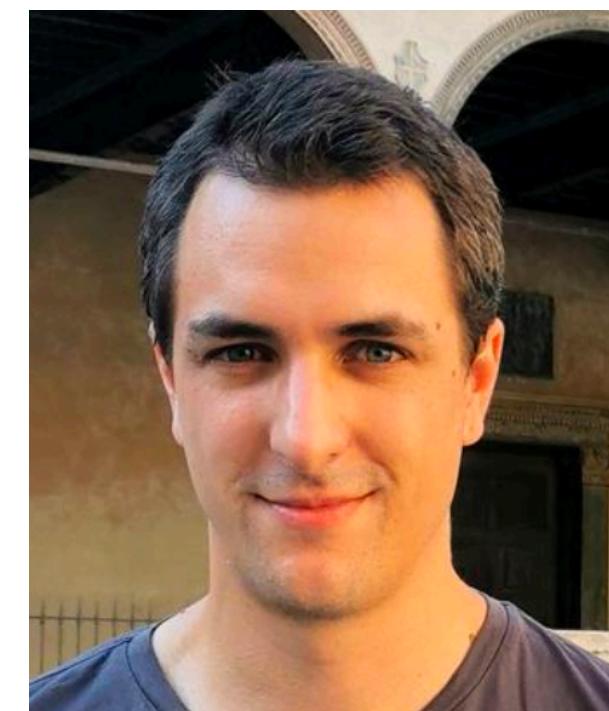


# Finding and Understanding Incompleteness Bugs in SMT Solvers

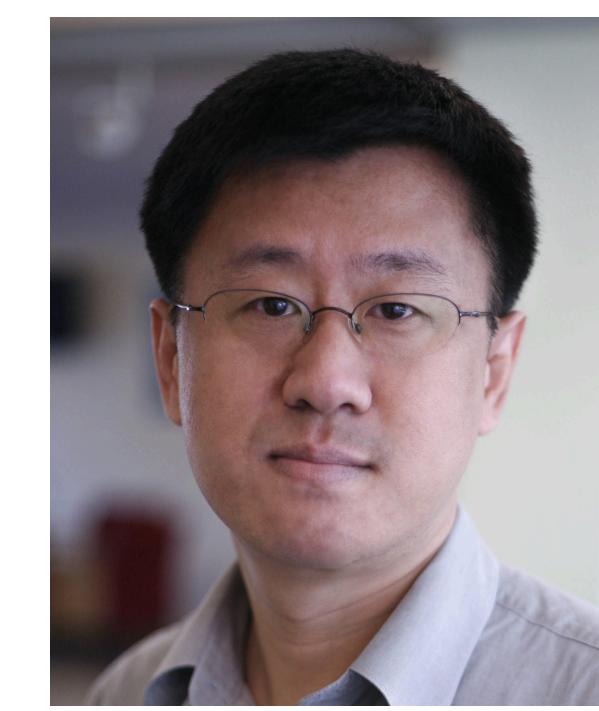
**Mauro Bringolf**  
ETH Zurich, Switzerland



**Dominik Winterer**  
ETH Zurich, Switzerland



**Zhendong Su**  
ETH Zurich, Switzerland



# SMT Problem

# SMT Problem

$$\varphi : x > 0 \wedge x < 0$$

# SMT Problem

$$\varphi : x > 0 \wedge x < 0$$

**UNSAT**

# SMT Problem

$$\varphi : x > 0 \wedge x < 1$$

# SMT Problem

$$\varphi : x > 0 \wedge x < 1$$

**SAT**

# SMT Problem

$$x = 0.5$$

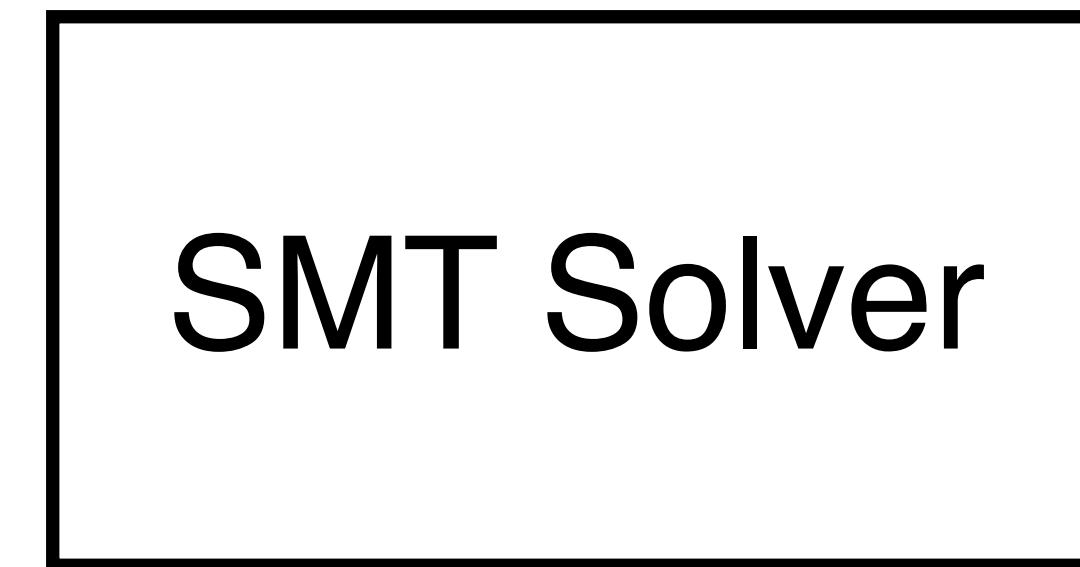
$$\varphi : x > 0 \wedge x < 1$$

**SAT**

# SMT Solver

SMT Solver

# SMT Solver

$$\varphi : x > 0 \wedge x < 1 \rightarrow$$


# SMT Solver

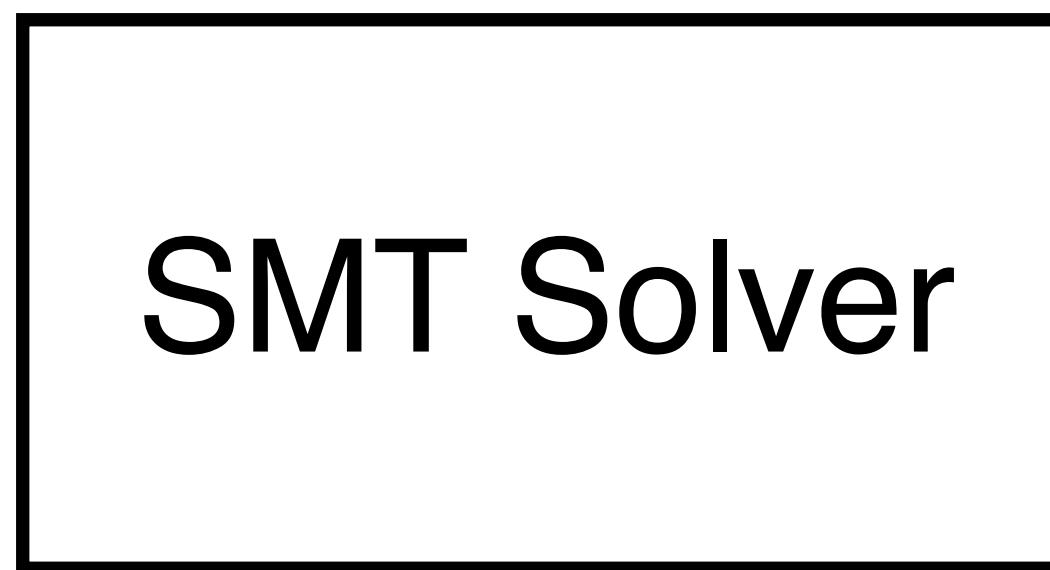
$$\varphi : x > 0 \wedge x < 1 \rightarrow \boxed{\text{SMT Solver}} \rightarrow \text{SAT}$$

# SMT Solver

$$\varphi : x > 0 \wedge x < 0 \rightarrow \boxed{\text{SMT Solver}} \rightarrow \text{UNSAT}$$

# SMT Solver

$$\varphi : x > 0 \wedge x < 0 \rightarrow$$



**→ UNKNOWN**

# SMT Solver

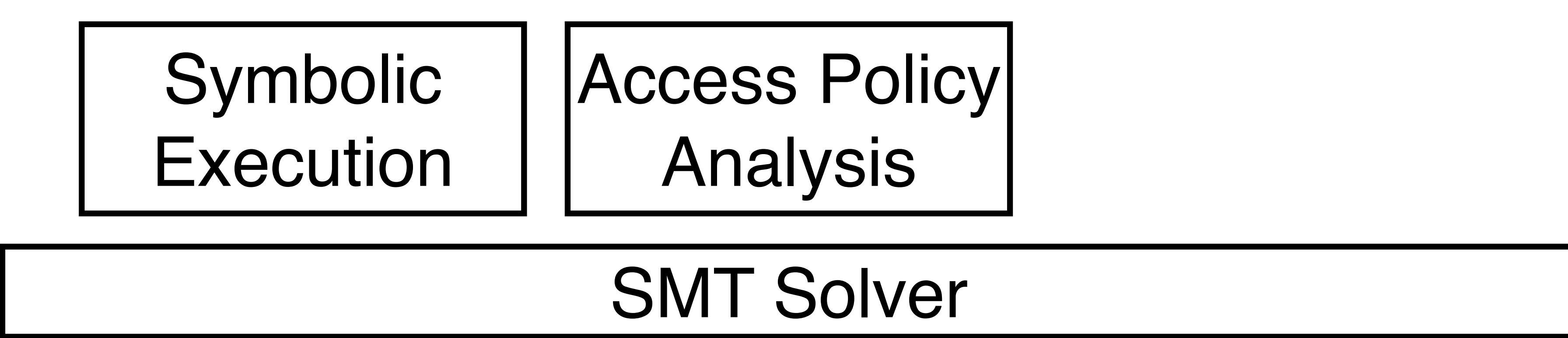
SMT Solver

# SMT Solver

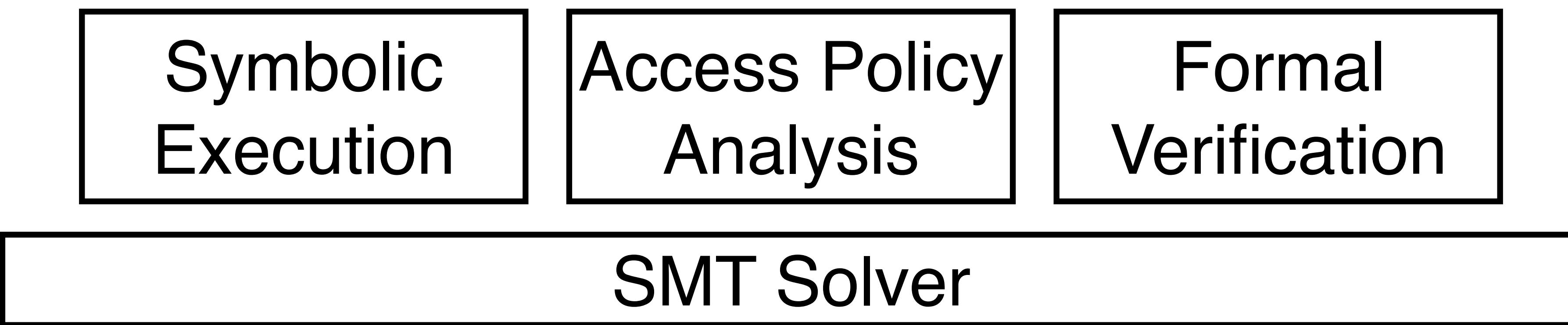
Symbolic  
Execution

SMT Solver

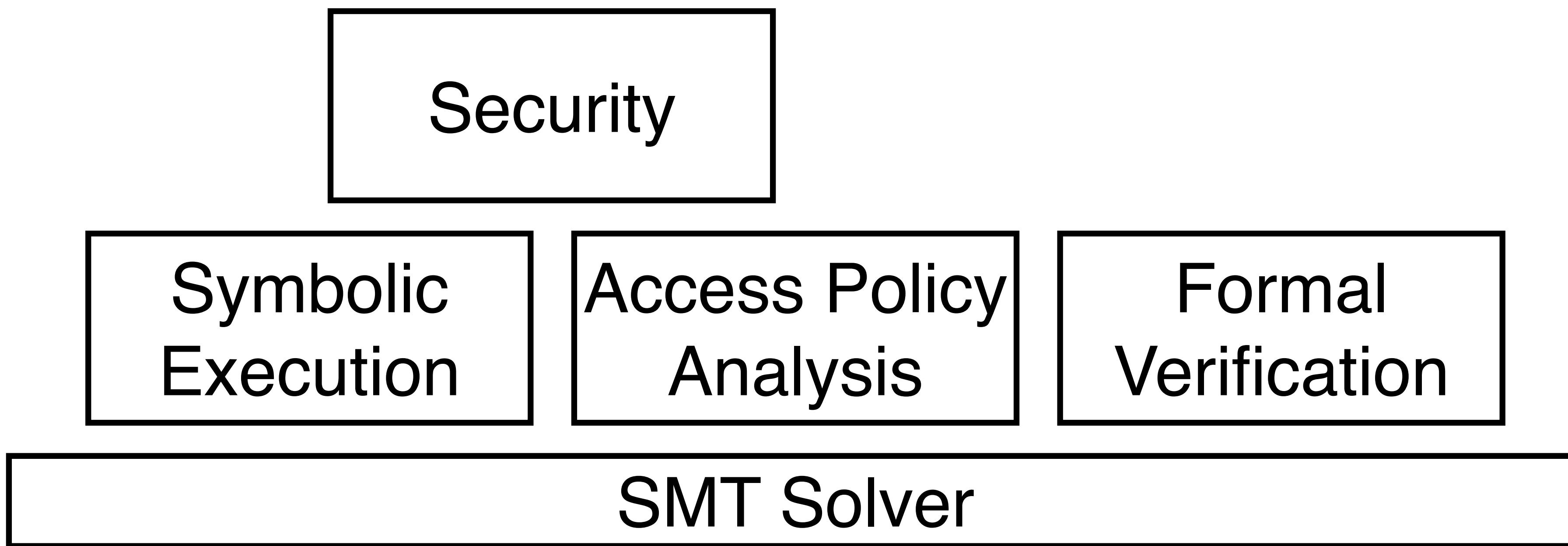
# SMT Solver



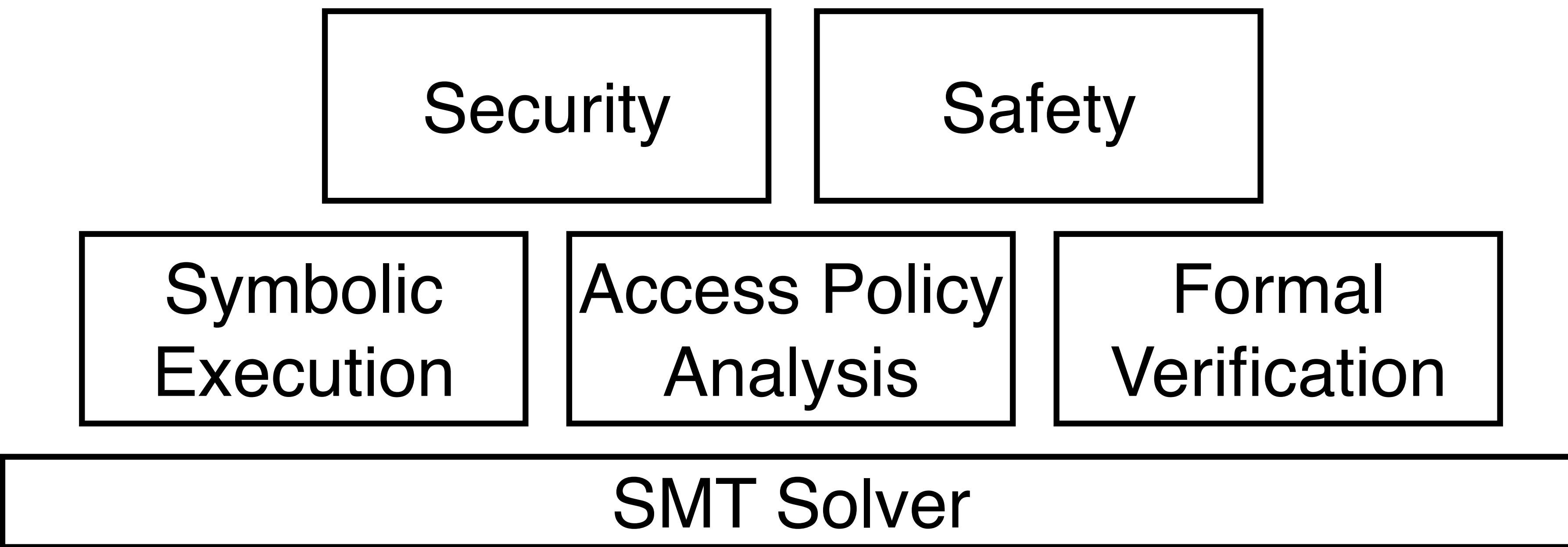
# SMT Solver



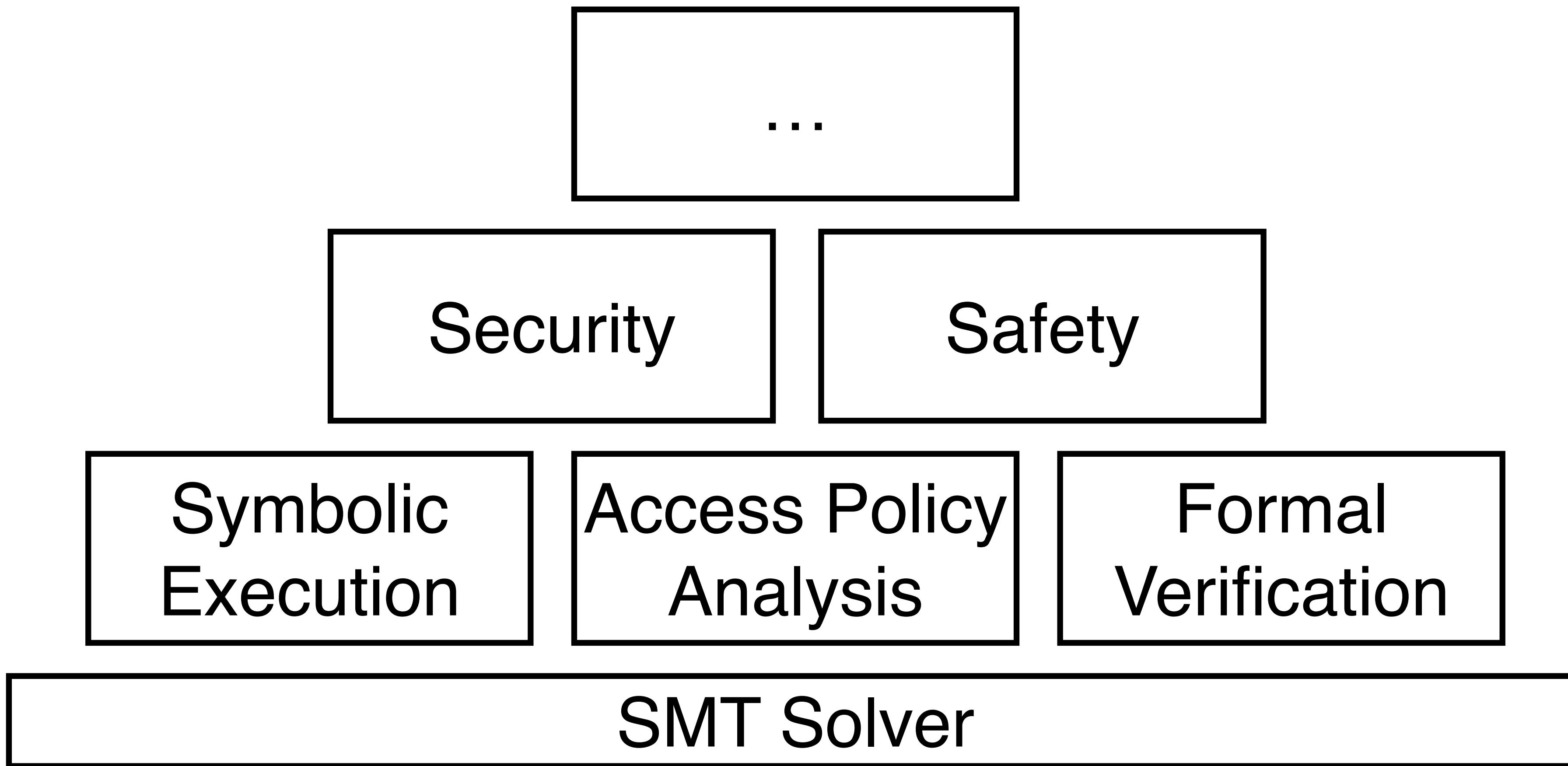
# SMT Solver



# SMT Solver



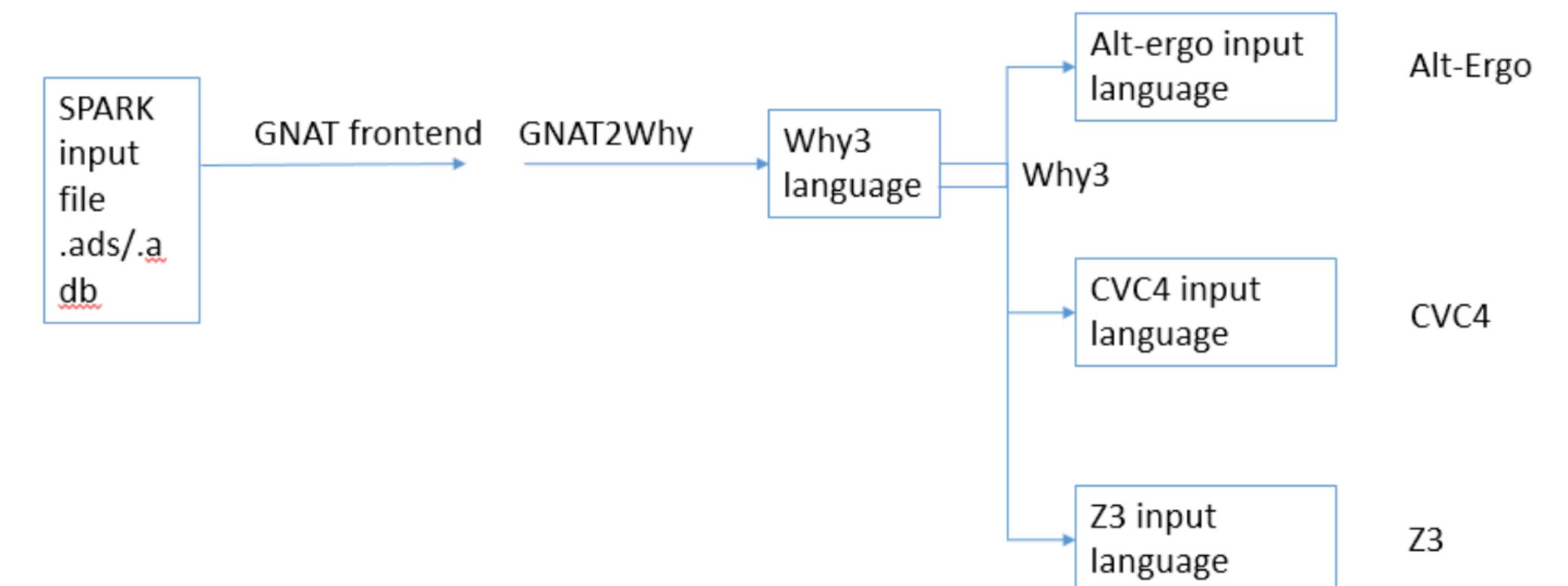
# SMT Solver



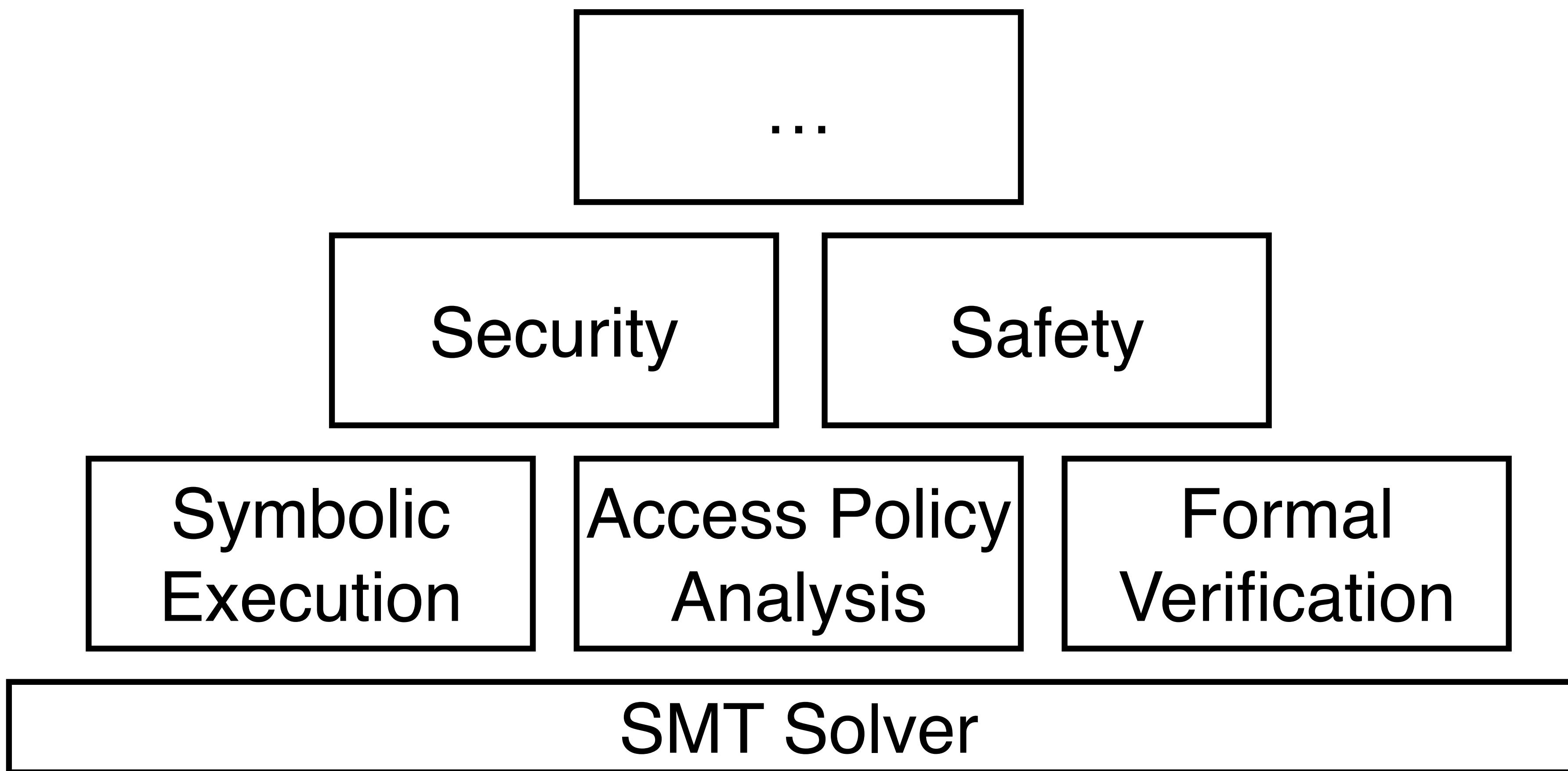
# Industrial Applications



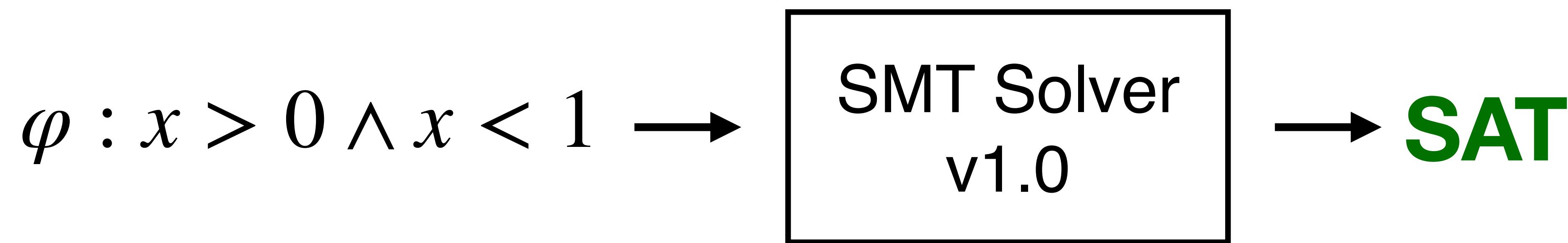
```
{  
  "Resource": "bucket1"  
  "Prohibited": { "AWS": "3333" },  
}  
  
(Resource = "bucket1")  $\wedge$  (Prohibited  $\neq$  3333)
```



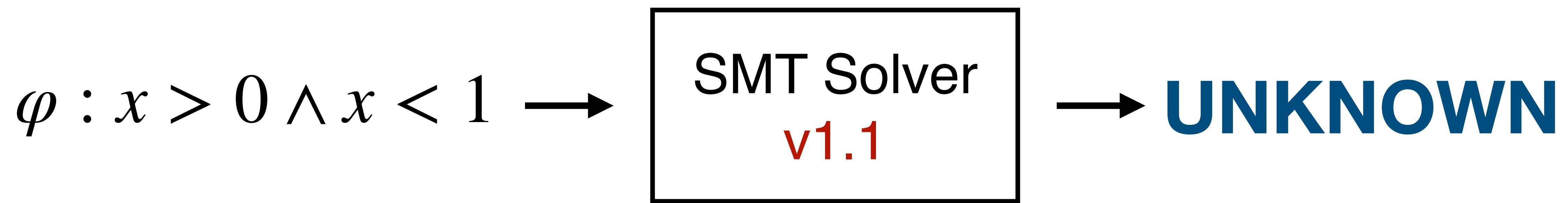
# SMT Solvers should be **reliable** and **fast!**



# Incompleteness Bug



# Incompleteness Bug



# Incompleteness Bug



# Incompleteness Bugs frustrate users!

"I'm seeing a regression [...], where a lot of simple formulas that used to be unsat now give unknown."

<https://github.com/Z3Prover/z3/issues/5516>

"The following code will produce unsat in z3 version 4.8.10.0 but is unknown in later versions."

<https://github.com/Z3Prover/z3/issues/5438>

"This is pretty unexpected since the query is small and does not contain features where we would expect to see performance regressions when updating releases."

<https://github.com/Z3Prover/z3/issues/5516>

# SMT-LIB Language

$$\varphi : x > 0 \wedge x < 1$$

# SMT-LIB Language

$$\varphi : x > 0 \wedge x < 1 \quad \rightarrow$$

```
(declare-fun x () Real)
(assert (and (> x 0)(< x 1)))
(check-sat)
```

# SMT-LIB Language

$$\varphi : x > 0 \wedge x < 1 \quad \rightarrow$$

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# SMT-LIB Language

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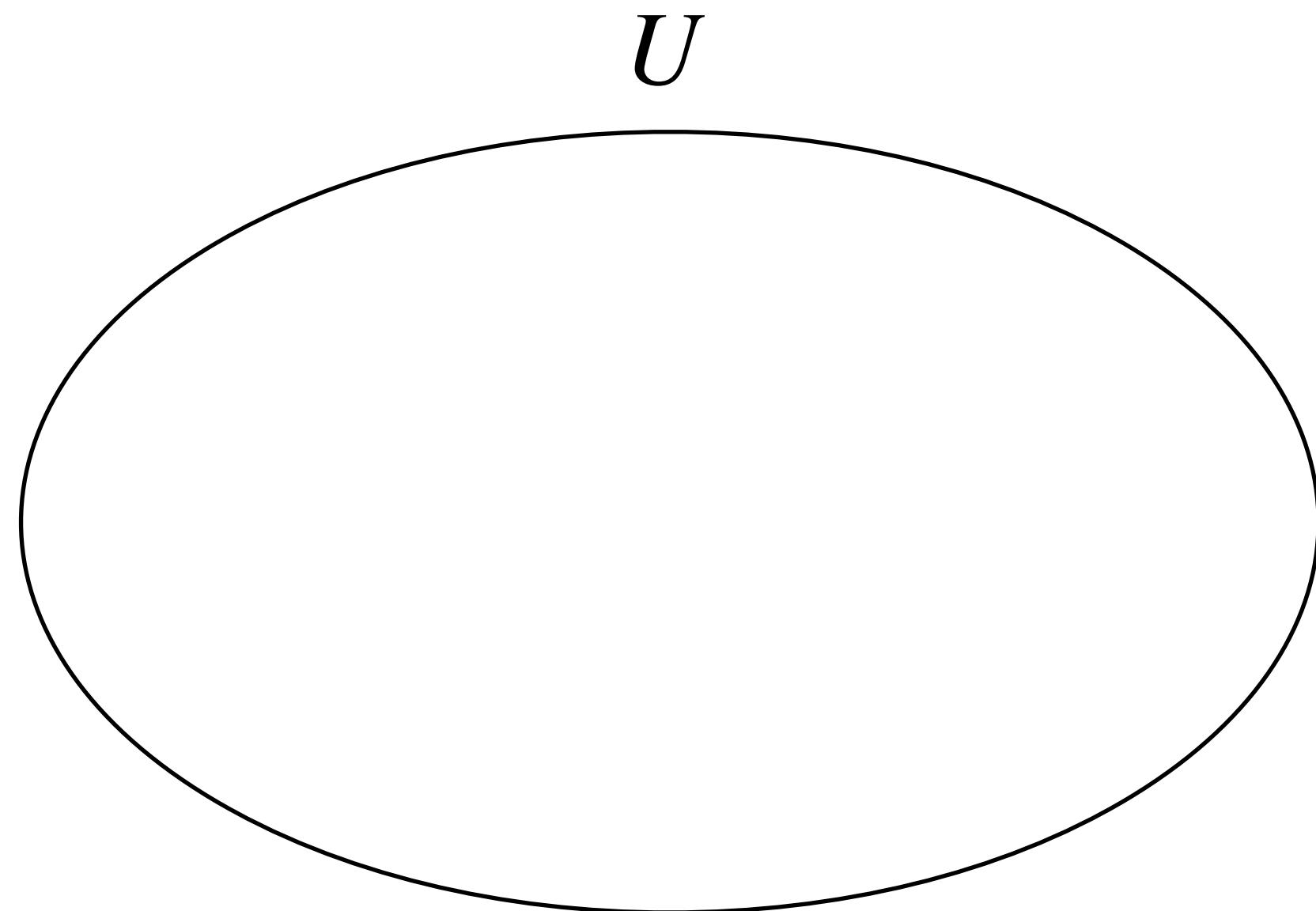
# SMT-LIB Language

$$\varphi : x > 0 \wedge x < 1 \quad \rightarrow$$

```
(declare-fun x () Real)
(assert (and (> x 0)(< x 1)))
(check-sat)
```

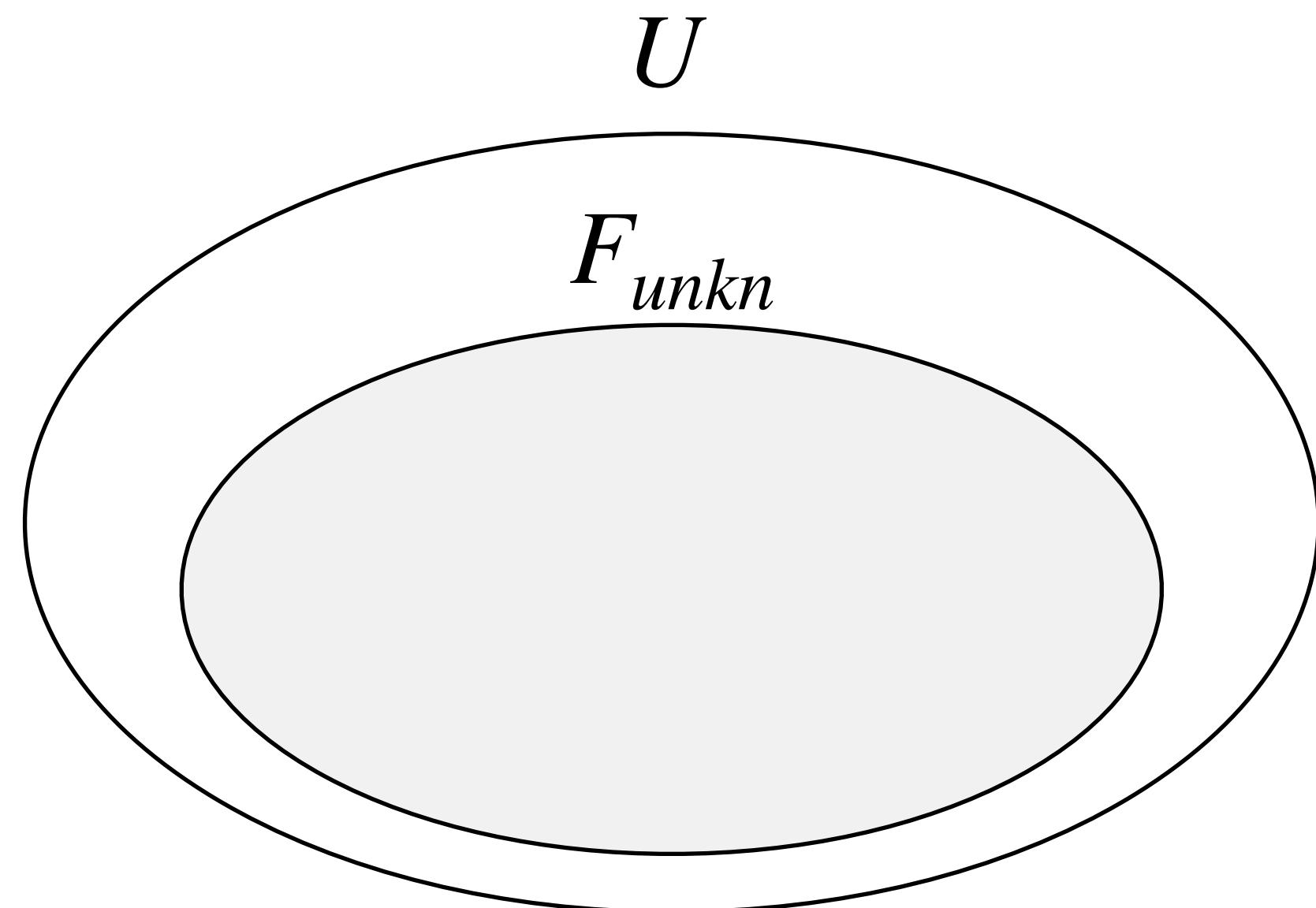
# Problem Statement

# Problem Statement



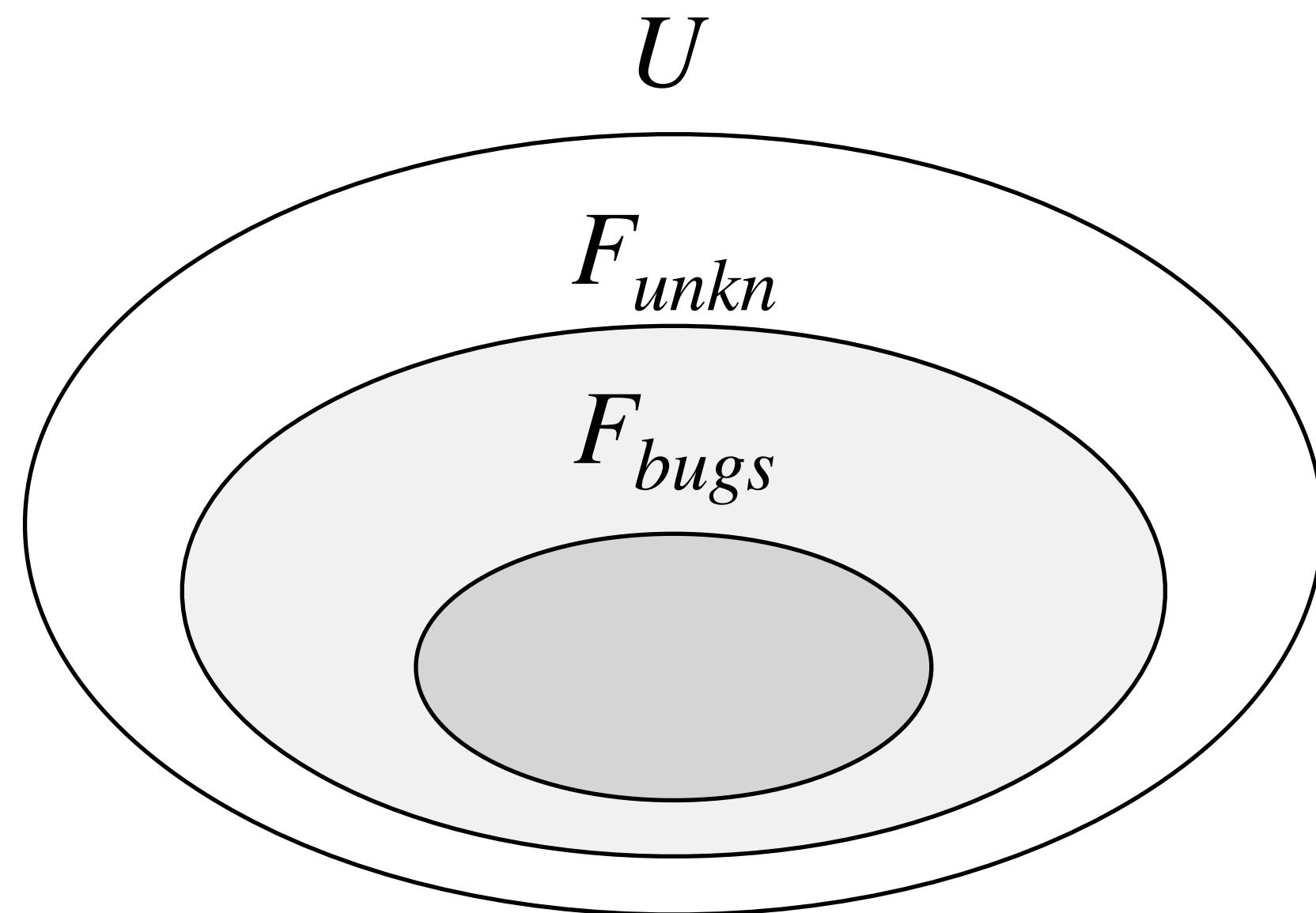
$U$ : Universe of SMT formulas

# Problem Statement



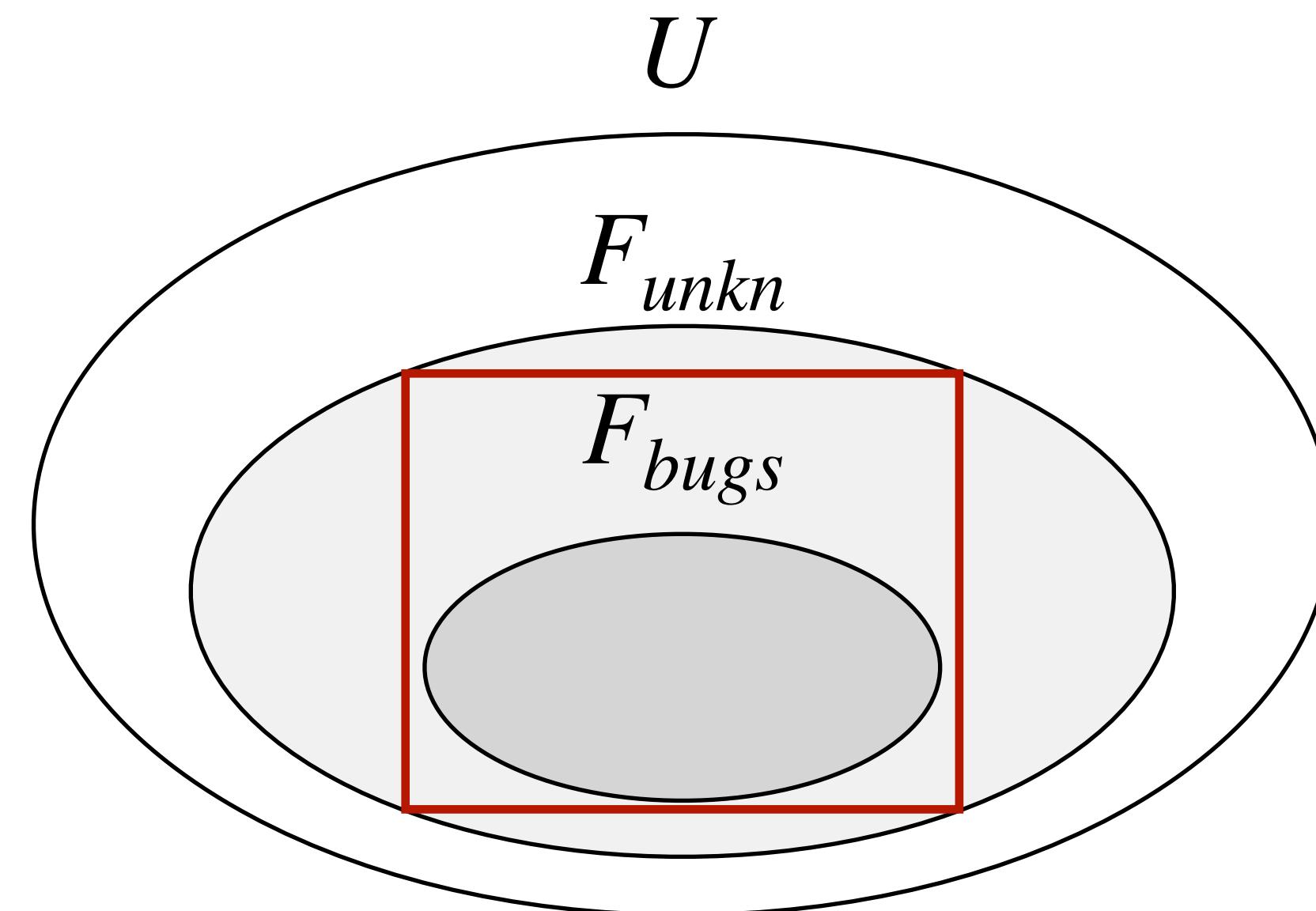
$F_{unkn}$ : All unknown formulas

# Problem Statement



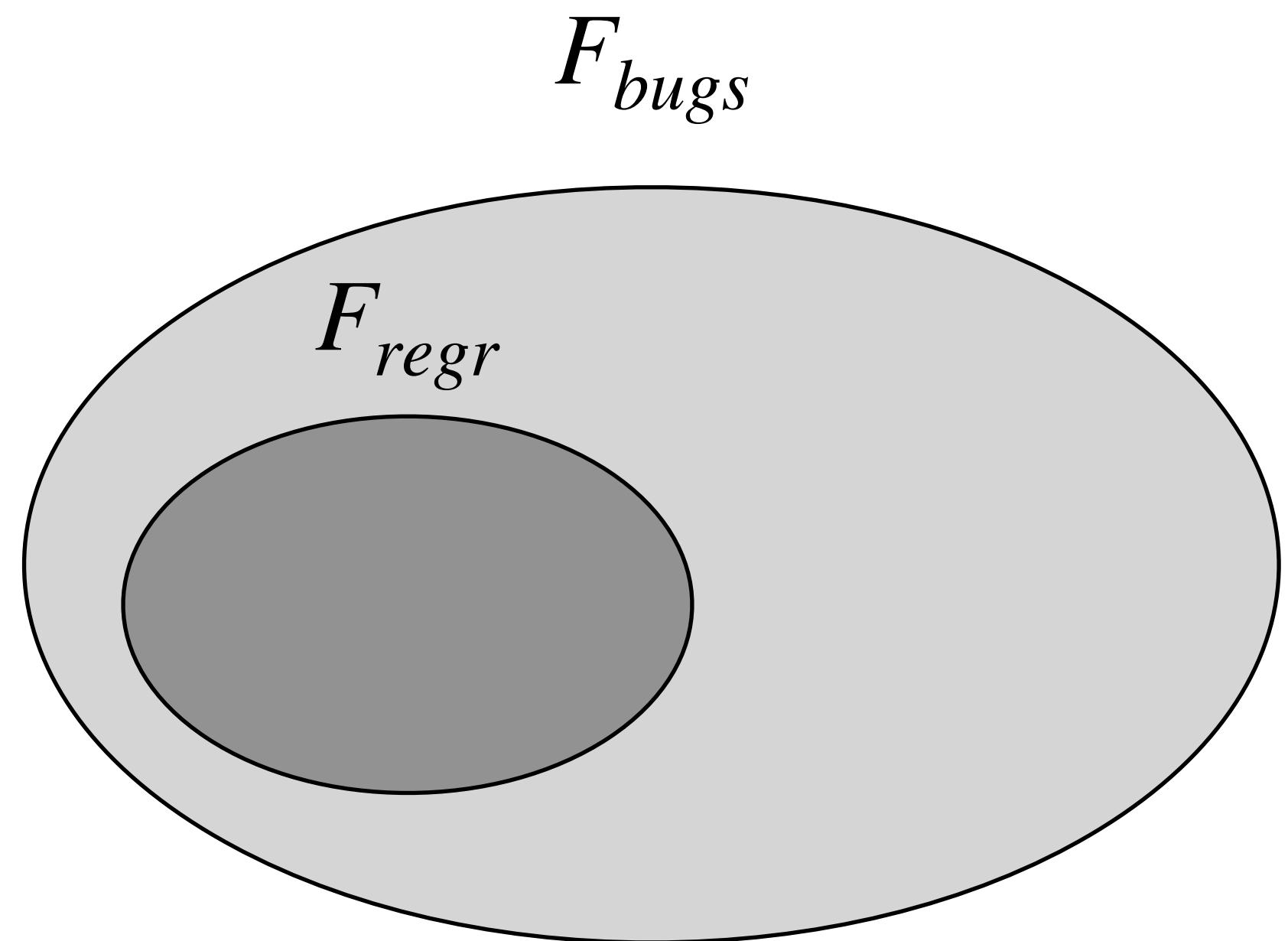
$F_{bugs}$ : Incompleteness bugs

# Problem Statement



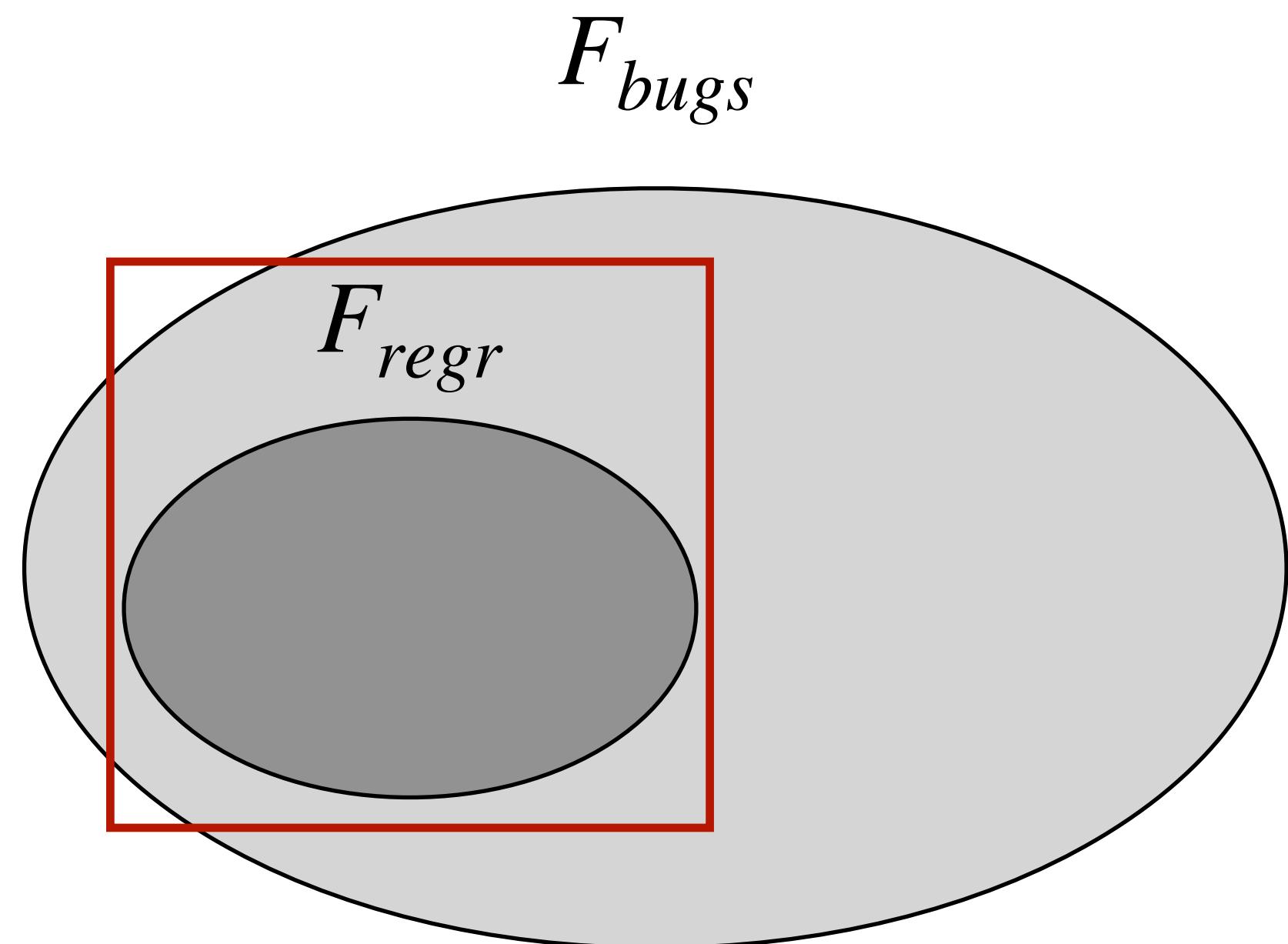
$F_{bugs}$ : Incompleteness bugs

# Problem Statement



$F_{regr}$ : Regression incompleteness bugs

# Problem Statement



$F_{regr}$ : Regression incompleteness bugs

# Regression Incompleteness

```
$ cat formula.smt2
(assert (forall ((v Int)) (= 0 v)))
(assert (= 0 (mod 0 0)))
(check-sat)
```

# Regression Incompleteness

```
$ cat formula.smt2
```

```
(assert (forall ((v Int)) (= 0 v)))
(assert (= 0 (mod 0 0)))
(check-sat)
```

*“All integers are equal to 0”*

# Regression Incompleteness

```
$ cat formula.smt2
```

```
(assert (forall ((v Int)) (= 0 v)))
(assert (= 0 (mod 0 0)))
(check-sat)
```

*“All integers are equal to 0”*

```
$ z3-4.8.10 formula.smt2
```

```
unsat
```

# Regression Incompleteness

```
$ cat formula.smt2
```

```
(assert (forall ((v Int)) (= 0 v)))
(assert (= 0 (mod 0 0)))
(check-sat)
```

*“All integers are equal to 0”*

```
$ z3-4.8.10 formula.smt2
```

```
unsat
```

```
$ z3-trunk formula.smt2
```

```
unknown
```

# Regression Incompleteness

```
$ cat formula.smt2
```

```
(assert (forall ((v Int)) (= 0 v)))  
(assert (= 0 (mod 0 0)))  
(check-sat)
```

*“All integers are equal to 0”*

```
$ z3-4.8.10 formula.smt2
```

```
unsat
```

```
$ z3-trunk formula.smt2  
unknown
```



# Regression Incompleteness

```
$ cat formula.smt2
```

```
(assert (forall ((v Int)) (= 0 v)))  
(assert (= 0 (mod 0 0)))  
(check-sat)
```

*“All integers are equal to 0”*

```
$ z3-4.8.10 formula.smt2
```

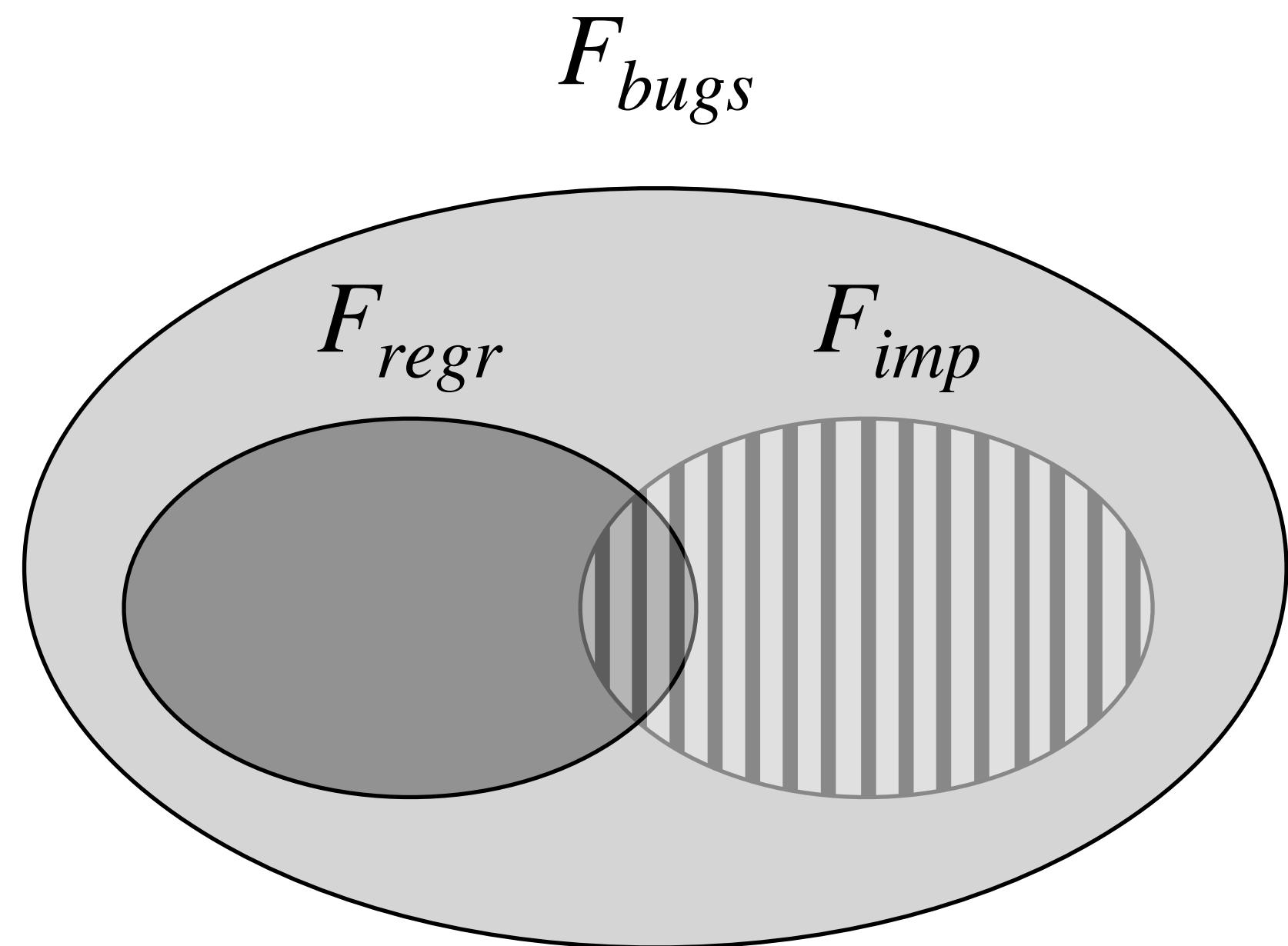
```
unsat
```

```
$ z3-trunk formula.smt2  
unknown
```



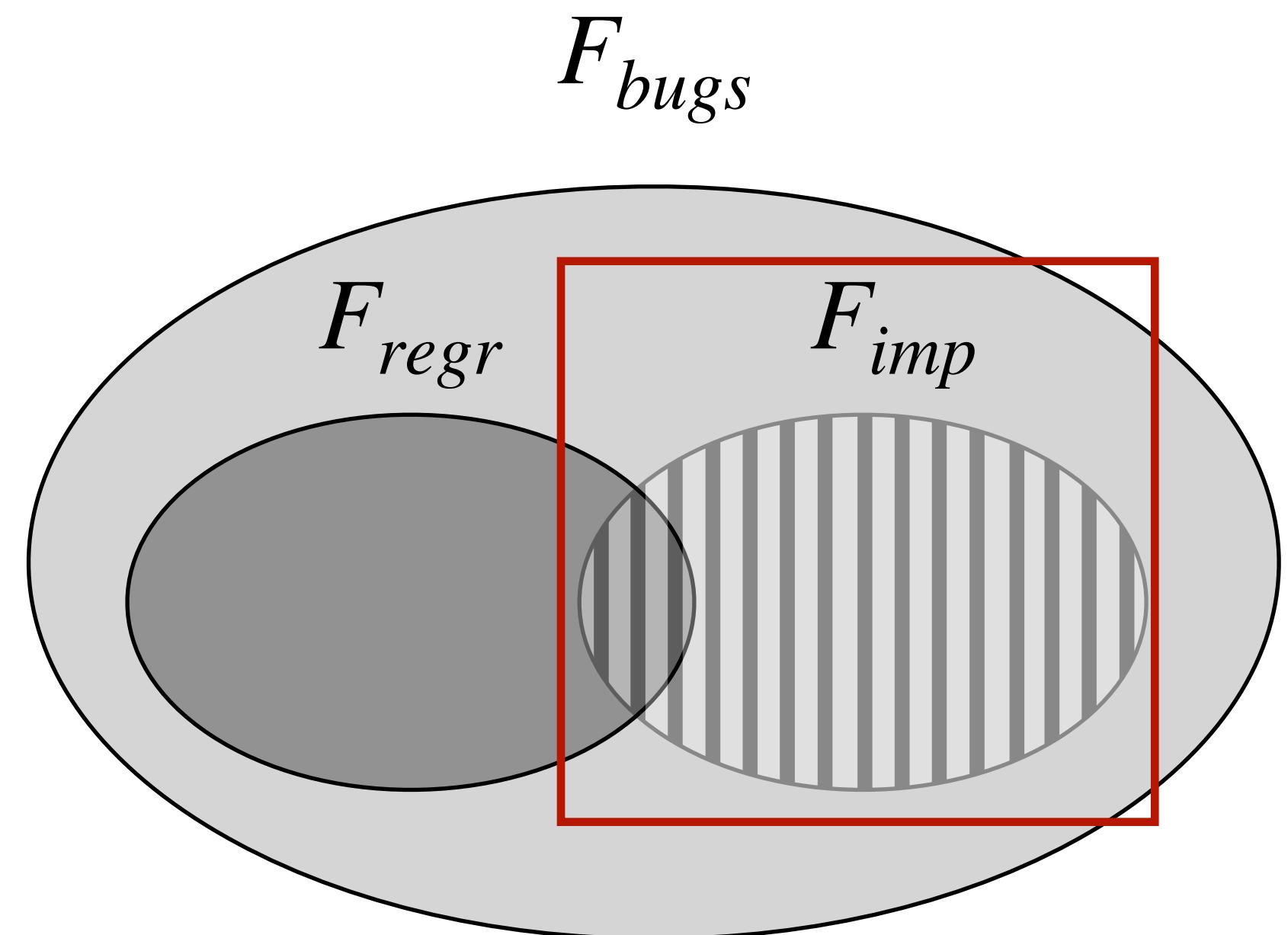
<https://github.com/Z3Prover/z3/issues/5338>

# Problem Statement



$F_{imp}$ : Implication incompleteness bugs

# Problem Statement



$F_{imp}$ : Implication incompleteness bugs

# Implication Incompleteness

```
$ cat formula.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (= (* s k) 1))
(check-sat)
```

# Implication Incompleteness

```
$ cat formula.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (= (* s k) 1))
(check-sat)
```

*“Are there two reals multiplying to 1”*

# Implication Incompleteness

```
$ cat formula.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (= (* s k) 1))
(check-sat)
```

```
$ cvc5 -q formula.smt2
sat
```

# Implication Incompleteness

```
$ cat formula.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (= (* s k) 1))
(check-sat)
```

```
$ cvc5 -q formula.smt2
sat
```

```
$ cat formula-new.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (>= (* s k) 1))
(check-sat)
```

# Implication Incompleteness

*“Are there two reals multiplying to greater/eq. 1”*

```
$ cat formula.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (= (* s k) 1))
(check-sat)
```

```
$ cvc5 -q formula.smt2
sat
```

```
$ cat formula-new.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (>= (* s k) 1))
(check-sat)
```

# Implication Incompleteness

*“Are there two reals multiplying to greater/eq. 1”*

```
$ cat formula.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (= (* s k) 1))
(check-sat)
```

```
$ cvc5 -q formula.smt2
sat
```

```
$ cat formula-new.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (>= (* s k) 1))
(check-sat)
```

```
$ cvc5 -q formula-new.smt2
unknown
```

# Implication Incompleteness

*“Are there two reals multiplying to greater/eq. 1”*

```
$ cat formula.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (= (* s k) 1))
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(declare-fun k () Real)
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```

```
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unknown
```



# Implication Incompleteness

*“Are there two reals multiplying to greater/eq. 1”*

```
$ cat formula.smt2
(declare-fun s () Real)
(declare-fun k () Real)
(assert (= (* s k) 1))
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sat
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```
$ cat formula-new.smt2
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(assert (>= (* s k) 1))
(check-sat)
```

```
$ cvc5 -q formula-new.smt2
unknown
```



<https://github.com/cvc5/cvc5/issues/7009>

# Approach

# Janus

- Tool: Janus



# Janus

- Tool: Janus
- Can both bug types: regression and implication



# Janus

- Tool: Janus
- Can both bug types: regression and implication
- Technique: Weakening and strengthening



# Janus

- Tool: Janus
- Can both bug types: regression and implication
- Technique: Weakening and strengthening
  - Idea:  $\varphi_1 \implies \varphi_2 \dots \implies \varphi_{n-1} \implies \varphi_n$



# Janus

- Tool: Janus
- Can both bug types: regression and implication
- Technique: Weakening and strengthening
  - Idea:  $\varphi_1 \implies \varphi_2 \dots \implies \varphi_{n-1} \implies \varphi_n$
  - Label AST with polarities +,-



# Janus

- Tool: Janus
- Can both bug types: regression and implication
- Technique: Weakening and strengthening
  - Idea:  $\varphi_1 \implies \varphi_2 \dots \implies \varphi_{n-1} \implies \varphi_n$
  - Label AST with polarities +,-
  - Randomly choose an expression



# Janus

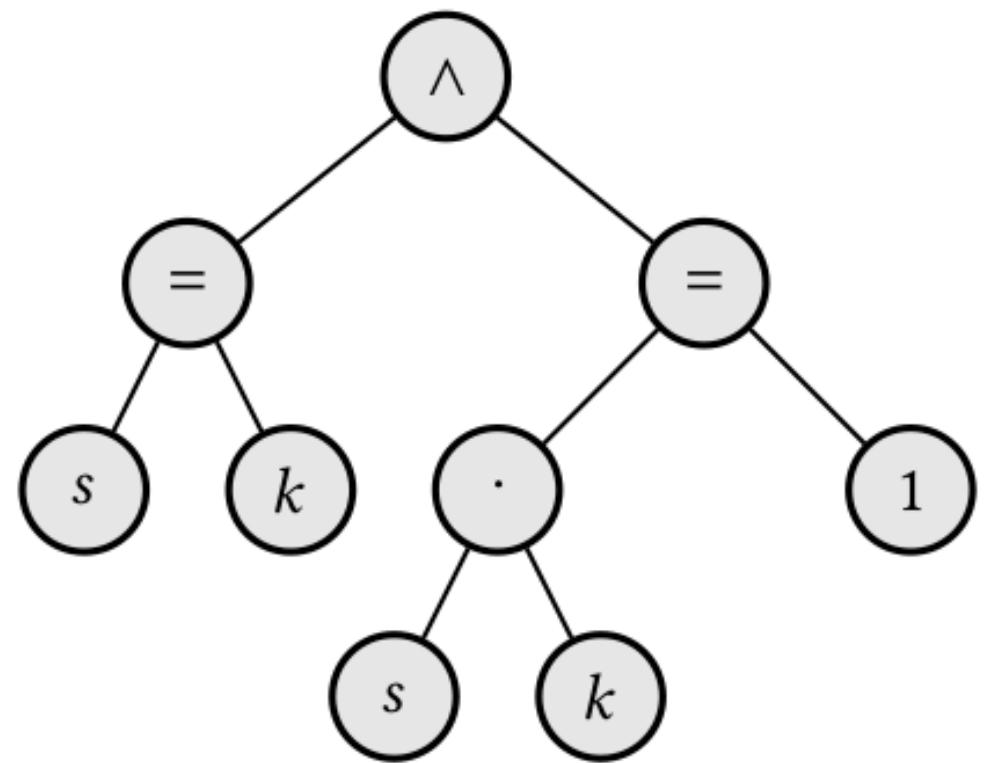
- Tool: **Janus**
- Can both bug types: **regression** and **implication**
- Technique: **Weakening and strengthening**
  - Idea:  $\varphi_1 \implies \varphi_2 \dots \implies \varphi_{n-1} \implies \varphi_n$
  - Label AST with polarities +,-
  - Randomly choose an expression
  - Apply weakening/strengthening rule to transform expression



# Weakening & Strengthening

1

```
(declare-const s Real)
(declare-const k Real)
(assert (and (= s k)
             (= (* s k) 1)))
(check-sat)
```



CVC5

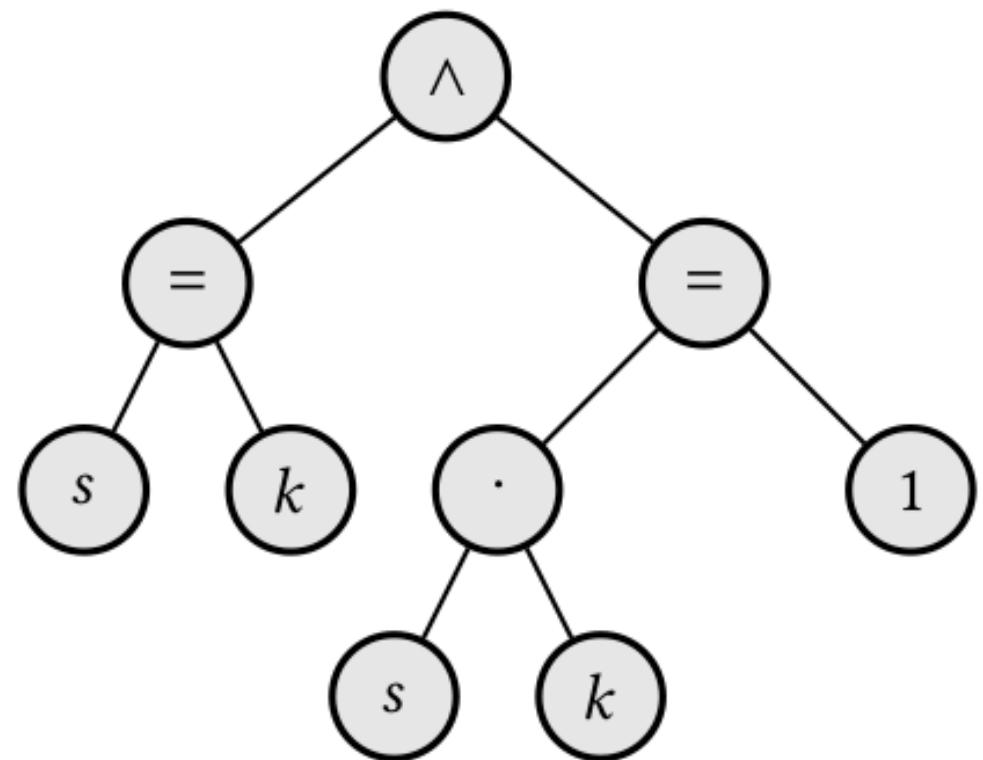
sat

✓

# Weakening & Strengthening

1

```
(declare-const s Real)
(declare-const k Real)
(assert (and (= s k)
             (= (* s k) 1)))
(check-sat)
```



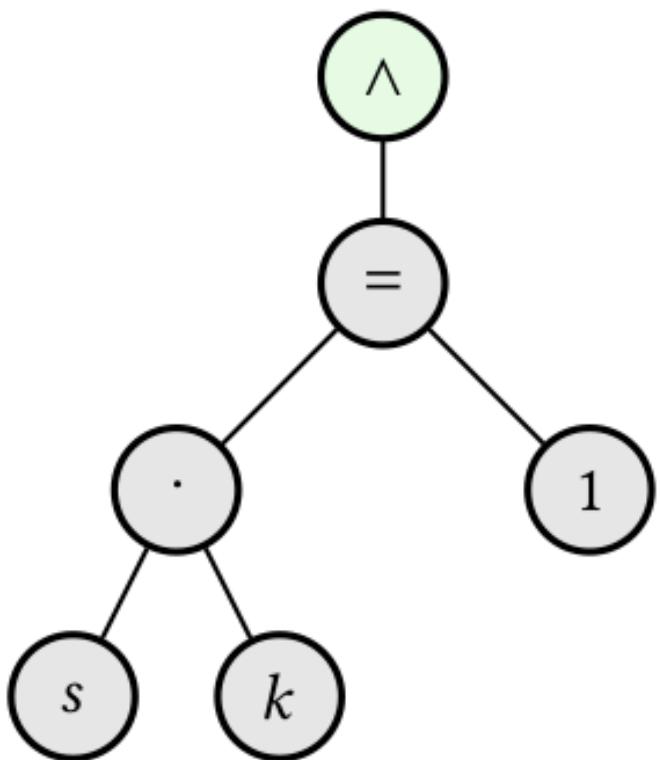
CVC5

sat

✓

2

```
(declare-const s Real)
(declare-const k Real)
(assert (and (= (* s k) 1)))
(check-sat)
```



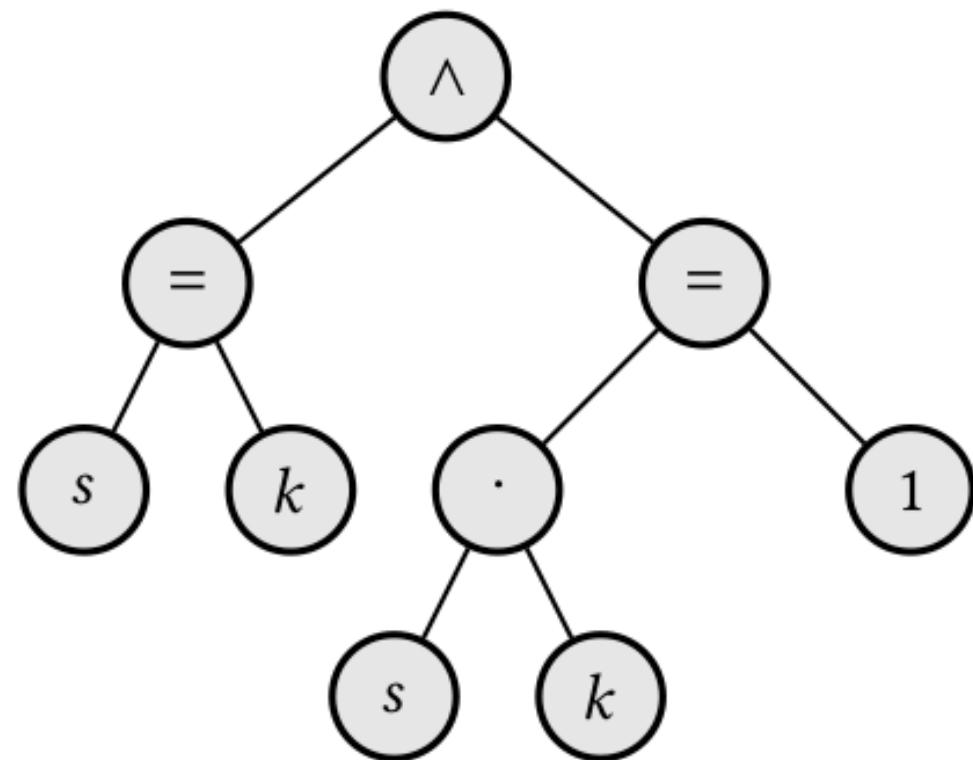
sat

✓

Weakening rule: Drop Conjunct

# Weakening & Strengthening

1  
(**declare-const** s Real)  
(**declare-const** k Real)  
(**assert** (and (= s k)  
          (= (\* s k) 1)))  
(**check-sat**)

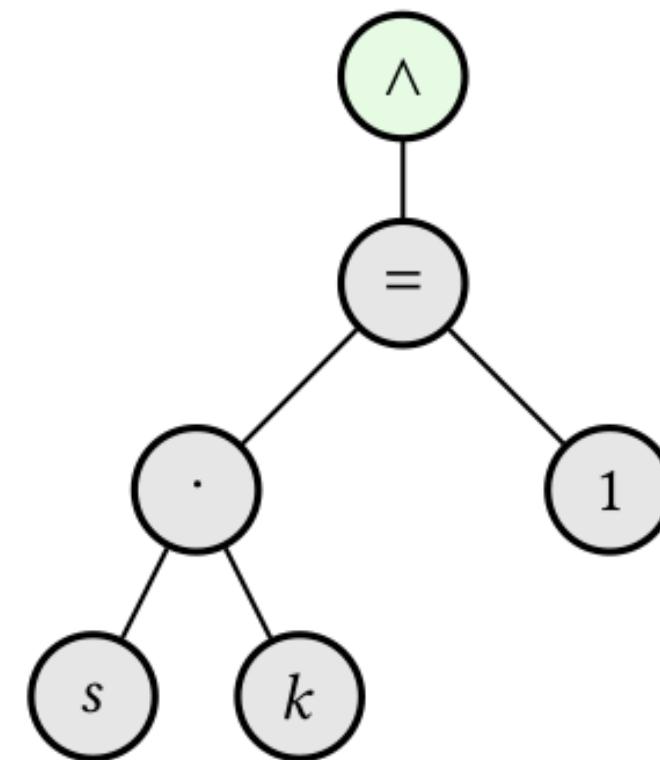


CVC5

sat



2  
(**declare-const** s Real)  
(**declare-const** k Real)  
(**assert** (and (= (\* s k) 1)))  
(**check-sat**)

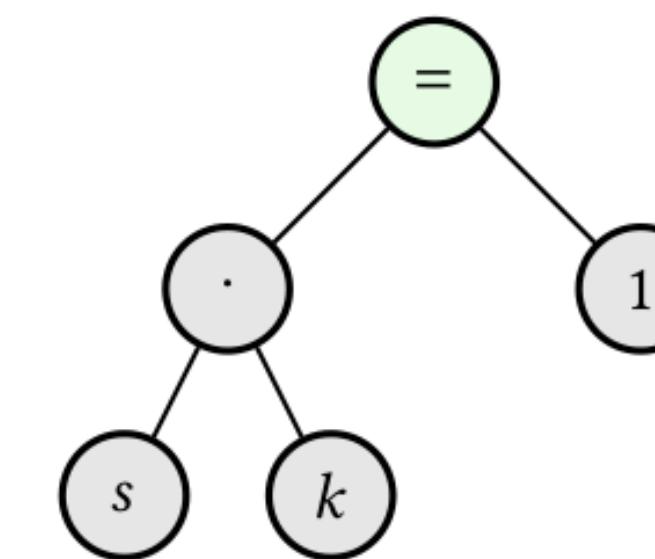


sat



...

**n - 1**  
(**declare-const** k Real)  
(**declare-const** s Real)  
...  
(**assert** (= (\* s k) 1))  
(**check-sat**)



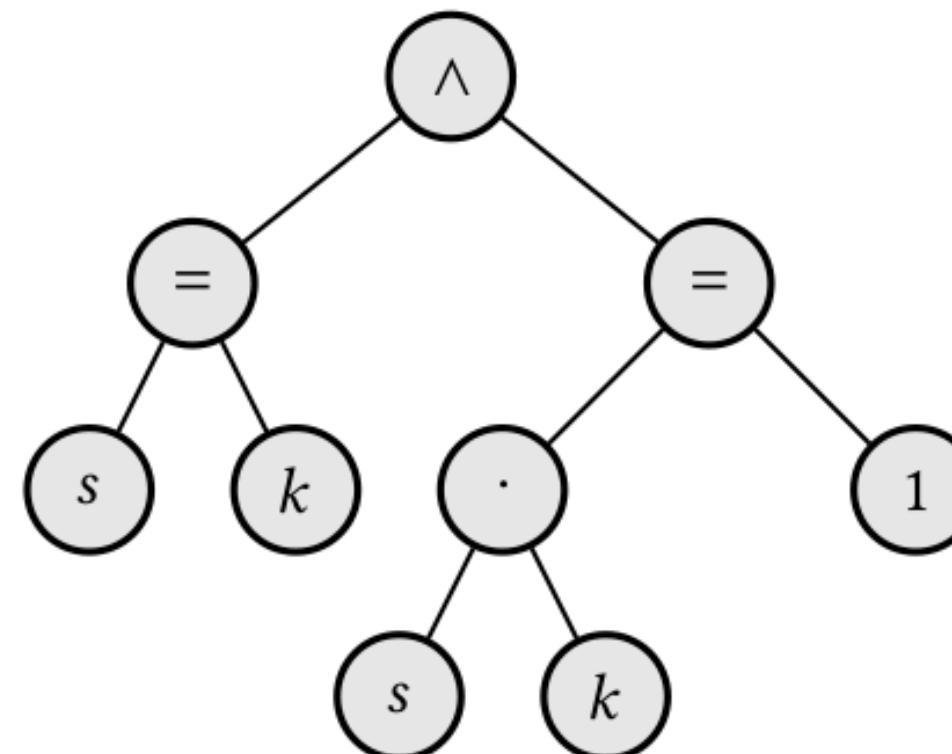
...

sat



# Weakening & Strengthening

1  
`(declare-const s Real)  
 (declare-const k Real)  
 (assert (and (= s k)  
 (= (* s k) 1)))  
 (check-sat)`

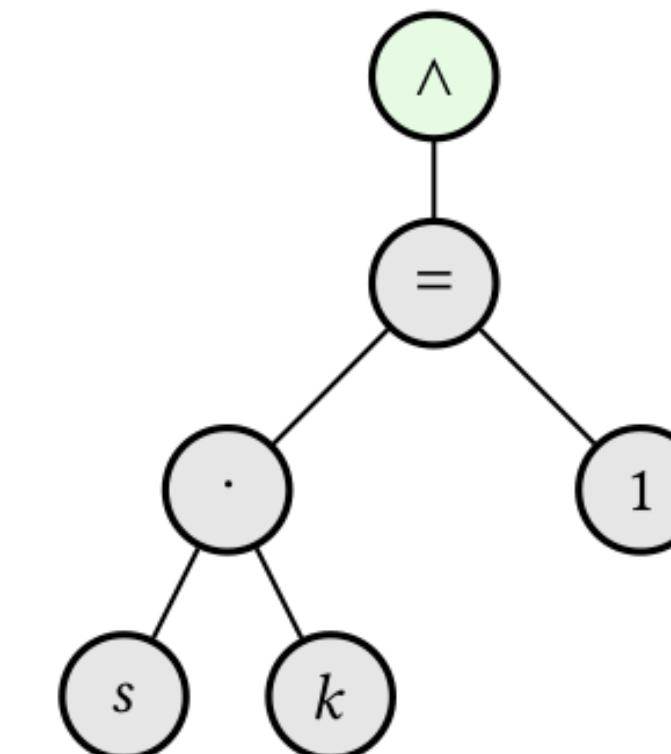


CVC5

sat



2  
`(declare-const s Real)  
 (declare-const k Real)  
 (assert (and (= (* s k) 1)))  
 (check-sat)`

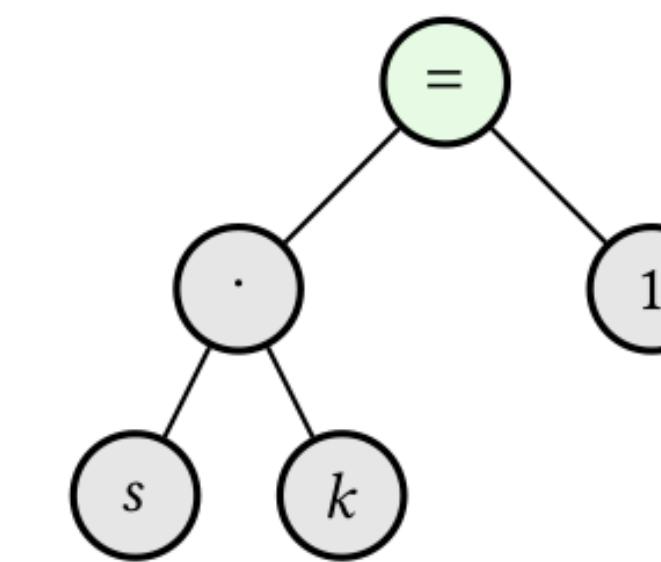


sat



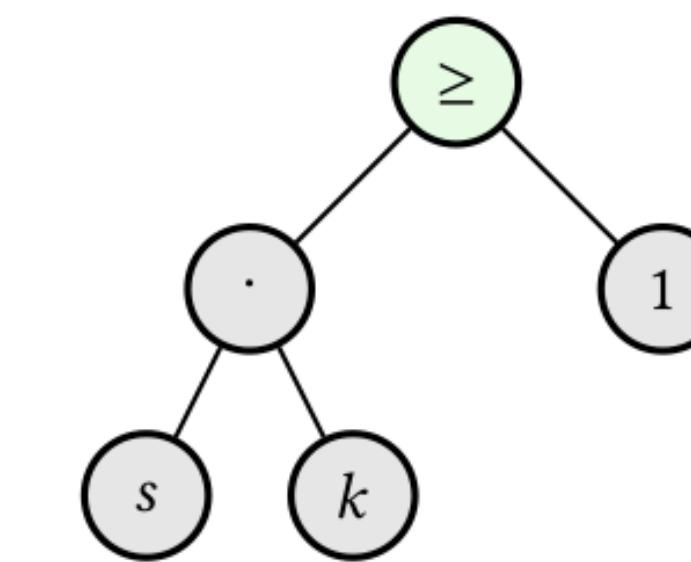
...

$n - 1$   
`(declare-const k Real)  
 (declare-const s Real)  
 (assert (= (* s k) 1))  
 (check-sat)`



...

$n$   
`(declare-const k Real)  
 (declare-const s Real)  
 (assert (>= (* s k) 1))  
 (check-sat)`



sat



unknown



Weakening rule: = to  $\geq$

# Rules and Proofs

**Finding and Understanding Incompleteness Bugs in SMT Solvers**

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**ABSTRACT**

We propose *Janus*, an approach for finding incompleteness bugs in SMT solvers. The key insight is to mutate SMT formulas with local weakening and strengthening rules that preserve the satisfiability of the seed formula. The generated mutants are used to test SMT solvers for incompleteness bugs, i.e., inputs on which SMT solvers unexpectedly return unknown. We realized *Janus* on top of the SMT solver fuzzing framework YinYang. From June to August 2021, we stress-tested the two state-of-the-art SMT solvers Z3 and CVC5 with *Janus* and totally reported 31 incompleteness bugs. Out of these, 26 have been confirmed as unique bugs and 19 are already fixed by the developers. Our diverse bug findings uncovered functional, regression, and performance bugs—several triggered discussions among the developers sharing their in-depth analysis.

**ACM Reference Format:**  
Mauro Bringolf, Dominik Winterer, and Zhendong Su. 2022. Finding and Understanding Incompleteness Bugs in SMT Solvers. In *37th IEEE/ACM International Conference on Automated Software Engineering (ASE '22), October 10–14, 2022, Rochester, MI, USA*. ACM, New York, NY, USA, 10 pages. <https://doi.org/10.1145/3551349.3560435>

**1 INTRODUCTION**

Satisfiability Modulo Theories (SMT) solvers are fundamental tools for software engineering and programming language advances e.g., symbolic execution [7, 12], program synthesis [23], solver-aided programming [24], and program verification [9, 10]. An SMT solver returns sat on an input formula  $\varphi$  if there is an assignment to  $\varphi$ 's variables that evaluates the formula to true, unsat if there is no such assignment and unknown if the SMT solver cannot decide the formula. Incompleteness bugs, i.e. unexpected unknown-results, impact the performance of SMT solvers' client applications frustrating their developers—especially since SMT solvers are usually at the very core of their client software solving NP-hard problems. Formula  $\varphi$  may realize a path constraint in a symbolic execution engine (e.g. KLEE [7], Microsoft's SAGE [13]), an access policy of a web service (e.g. AWS's Zelkova [2]), or a model of a safety-critical system (e.g. AdaCore's Spark [1]). Potential consequences of incompleteness bugs include missed bugs in the software under test, slow (or even non-terminating) verification of safety-critical or security-critical properties and other undesirable effects. Fig. 1 shows an incompleteness bug in Z3 [8] which returns unknown on a simple SMT formula. The first statement of the script declares an integer variable, the second specifies a constraint, and the third queries the SMT solver. Since we cannot choose an admissible value for  $x$  to satisfy the constraint (as there is no square number equal to all integers), the formula is unsatisfiable, and Z3 should return unsat. Why does Z3 fail at solving such a simple formula? As it turns out, it is caused by a bug in the implementation of model-based quantifier instantiation (MBQI). MBQI guesses values for the universal quantifiers ( $v$  in our case) to check whether the formula is unsatisfiable. On inspecting the issue deeper, we noticed that even if we run MBQI a million iterations, Z3 could not decide the formula.<sup>1</sup> However, fixing a random integer  $v$  which is not a square number would have been enough to determine unsatisfiability. We reported this bug to the issue tracker of Z3. Z3's main developer promptly fixed it.

**Incompleteness in SMT solvers.** Not every unknown-result indicates a bug. As SMT solvers support undecidable logics, they are necessarily incomplete. An SMT solver returns unknown on a formula if it has no decision procedure to solve the formula or to avoid a timeout. In practice, SMT solver can solve most problem instances from undecidable logics relevant to users. Similar to decidable logics, SMT solver developers enhance their solvers by rewriter rules, pre-processors etc. However, distinguishing expected from unexpected incompletenesses is difficult and confuses users:

*"I'm seeing a regression [...], where a lot of simple formulas that used to be unsat now give unknown."*  
<https://github.com/Z3Prover/z3/issues/5516>

*"The following code will produce unsat in z3 version 4.8.10.0 but is unknown in later versions."*  
<https://github.com/Z3Prover/z3/issues/5438>

<sup>1</sup>z3 smt.mbqi.max\_iterations=1000000 bug.smt2

ASE '22, October 10–14, 2022, Rochester, MI, USA  
Mauro Bringolf, Dominik Winterer, and Zhendong Su

**Table 1: Weakening and strengthening rules for core logic, reals and integers, strings, and regexes. Symmetric cases are omitted for brevity. The legend column describes newly introduced symbols per group.**

Type	Strong	Weak	Legend
<i>Real/Int</i>			
	$n_1 = n_2$	$n_1 \geq n_2 \mid n_1 \leq n_2$	$n_1, n_2 \in \mathbb{R}$ or $n_1, n_2 \in \mathbb{N}$
	$n_1 > n_2$	$n_1 \geq n_2 \mid n_1 \neq n_2$	
	$n_1 < n_2$	$n_1 \leq n_2 \mid n_1 \neq n_2$	
	$n_1 \odot n_2$	$(n_1 + c) \odot (n_2 + c) \mid n_1 \odot (n_2 + c) \mid n_1 \odot (n_2 + c)$	$c \in \mathbb{N}$ or $c \in \mathbb{R}$ $\odot \in \{=, >, <, \leq, \geq\}$

Type	Strong	Weak	Legend
<i>Bool</i>			
	$\varphi_1 \wedge \varphi_2$	$\varphi_1 \vee \varphi_2 \mid \varphi_1$	$\varphi, \varphi_1, \varphi_2$ are boolean formulas
	$\varphi_1 \oplus \varphi_2$	$\varphi_1 \vee \varphi_2$	$\oplus$ is the logical xor
	$\forall x: \varphi \mid \varphi[x \mapsto B]$	$\exists x: \varphi$	$B$ is an expression of same type as $x$
	$x_1 = \dots = x_n$	$f(x_1) = \dots = f(x_n)$	$x_1, \dots, x_n$ are terms of arbitrary type $f$ : SMT-LIB built-in function
	$\varphi_1 \vee \varphi_2$	$\exists b: \text{ite}(b, \varphi_1, \varphi_2)$	$ite$ is the if-then-else operator $b$ is a boolean variable
	$\text{ite}(B, \varphi_1, \varphi_2)$	$B \rightarrow \varphi_1 \mid \neg B \rightarrow \varphi_2$	
	$\varphi_1 \rightarrow \varphi_2$	$\text{ite}(\varphi_1, \varphi_2, \top) \mid \text{ite}(\neg\varphi_1, \top, \varphi_2) \mid \forall b: (\varphi_1 \wedge b \rightarrow \varphi_2 \wedge b)$	
	$\varphi_1 \vee \varphi_2$	$\neg\varphi_1 \rightarrow \varphi_2$	
	$\forall x. \varphi$	$\varphi[x \mapsto B]$	
	$\varphi_1$	$\varphi_1 \vee \varphi_2$	

Type	Strong	Weak	Legend
<i>String</i>			
	$s_1 = s_2$	$s_1 \neq (s_1 ++ s_3) \mid s_1 \leq_s s_2 \mid \text{prefixof}(s_1, s_2) \wedge \text{suffixof}(s_1, s_2) \mid \text{prefixof}(s_1, s_2) \wedge \text{prefixof}(s_2, s_1) \mid \text{contains}(s_1, s_2) \mid \text{suffixof}(s_1, s_2) \wedge \text{suffixof}(s_2, s_1) \mid \text{prefixof}(s_1, s_2) \mid \text{suffixof}(s_1, s_2)$	$s_1, s_2, s_3$ are strings $++$ : string concatenation
	$s_1 < s_2$	$s_1 \neq s_2 \mid s_1 \leq_s s_2$	$\leq_s$ : lexicographical ordering
	$s_1 \leq_s s_2$	$\text{substr}(s_1, 0, \text{ite}(0 \leq i \leq \text{len}(s_1) - 1, i, \text{len}(s_1))) \leq_s s_2 \mid s_1 \leq_s (s_2 ++ s_3)$	
	$\text{contains}(s_1, s_2)$	$\text{len}(s_1) \geq \text{len}(s_2)$	

Type	Strong	Weak	Legend
<i>Regex</i>			
	$r$	$r^+$	$r, r_1, \dots, r_n$ are regexes, $n \in \mathbb{N}$
	$r$	$\text{loop}(1, n, r)$	$+$ : Kleene plus $\text{loop}(i, n, r) = L(r)^i \cup \dots \cup L(r)^n$
	$r$	$\text{opt}(r)$	$\text{opt}(r) := \text{union}(r, (\text{str.to\_re } ""))$
	$r_1 + r_2 \dots + r_n$	$\text{union}(r_1, \dots, r_n)^n$	$++_r$ : regex concat
	$r$	$\forall x: \text{union}(r, s)$	for an arbitrary string $s$
	$r^*$	$r^*$	$*$ : Kleene star
	$\text{range}(s_1, s_2)$	$\text{range}(s_3, s_4)$	for fixed strings $s_1, s_2$ choose strings $s_3, s_4$ s.t. $\text{range}(s_1, s_2) \text{range}(s_3, s_4)$

# Evaluation

# Empirical Evaluation

- Tool: **Janus**



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- Bug hunting: Jun 2021 - Aug 2021



# Empirical Evaluation

- Tool: **Janus**
- Bug hunting: Jun 2021 - Aug 2021
- Testing targets: **Z3** **CVC5**



# Empirical Evaluation

- Tool: **Janus**
- Bug hunting: Jun 2021 - Aug 2021
- Testing targets: **Z3** **CVC5**
- Seeds: preprocessed SMT-LIB files (sat/unsat)



# Bug Findings

Status	Z3	CVC5	Total
Reported	13	18	31
Confirmed	8	18	26
Fixed	8	11	19
Rejected	5	0	5

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# Bug Samples

# Z3 #5376

```
$ cat formula.smt2
(declare-const x Int)
(assert (forall ((v Int)) (= v (* x x))))
(check-sat)
```

# Z3 #5376

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(declare-const x Int)
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```

*“Every integer is a square number”*

# Z3 #5376

```
$ cat formula.smt2
(declare-const x Int)
(assert (forall ((v Int)) (= v (* x x))))
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```
(check-sat)
```

```
$ z3-4.8.10 formula.smt2
```

```
unsat
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*“Every integer is a square number”*

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```

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```

```
unsat
```

```
$z3-trunk formula.smt2
unknown
```



*“Every integer is a square number”*

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$ cat formula.smt2
(declare-const x Int)
(assert (forall ((v Int)) (= v (* x x))))
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```

```
unsat
```

```
$ z3-trunk formula.smt2
unknown
```



*“Every integer is a square number”*

Root Cause: **Functional bug in MBQI hidden by “unknown”**

<https://github.com/Z3Prover/z3/issues/5376>

# CVC5 #6717

```
$ cat formula.smt2
(declare-const v String)
(assert true)
(assert (or T (and (str.prefixof v ")
(exists ((x Int)) (= "t"
(str.substr v 0 x))))))
(check-sat)
```

# CVC5 #6717

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$ cat formula.smt2
(declare-const v String)
(assert true)
(assert (or T (and (str.prefixof v ")
(exists ((x Int)) (= "t"
(str.substr v 0 x))))))
(check-sat)
```

```
$ cvc4-1.8 formula.smt2
sat
```

# CVC5 #6717

```
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(check-sat)
```

```
$ cvc5 formula.smt2
unknown
```



# CVC5 #6717

*"Commit 11c1fba added new rewrites for ITE. Due to the new rewrites taking precedence over existing rewrites, it could happen that some of the previous rewrites did not apply anymore even though they would have further simplified the ITE"*

CVC5 developer

# Janus 0.1.0 Release



[github.com/testsmt/janus](https://github.com/testsmt/janus)



# Key Takeaways

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# Key Takeaways

- Incompleteness bugs frustrate tool developers & users
- Janus: pioneering approach to detect them
- SMT solvers
  - Can solve **very hard problems**
  - But: can (sometimes) **not solve** simple classroom problems



# SMT Solver

$\varphi : x > 0 \wedge x < 0 \rightarrow$  SMT Solver → UNKNOWN

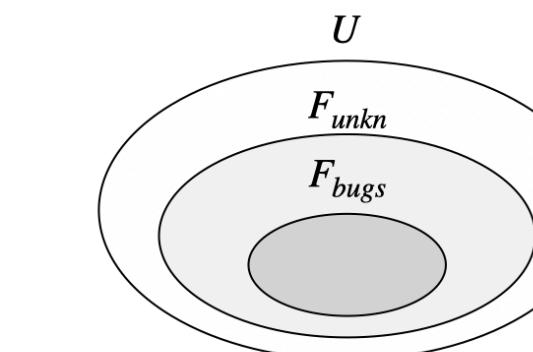
# Incompleteness Bug

$$\varphi : x > 0 \wedge x < 1 \rightarrow$$

SMT Solver  
v1.1

$$\rightarrow \text{UNKNOWN}$$

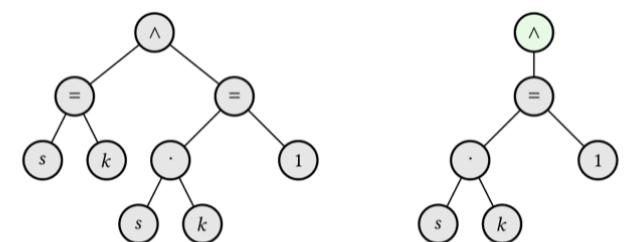

## Problem Statement



## $F_{bugs}$ : Incompleteness bugs

# Weakening & Strengthening

1	2
<pre>(declare-const s Real) (declare-const k Real) (assert (and (= s k)             (= (* s k) 1))) (check-sat)</pre>	<pre>(declare-const s Real) (declare-const k Real) (assert (and (= (* s k) 1))) (check-sat)</pre>

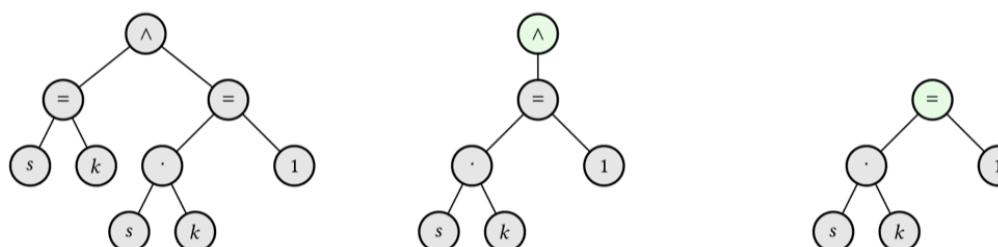


CVC5 sat sat

## Weakening rule: Drop Conjunct

# Weakening & Strengthening

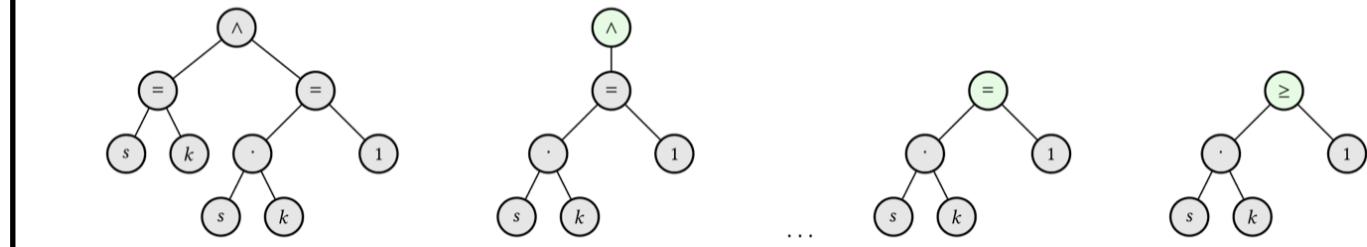
1	2	...
$  \begin{array}{l}  (\text{declare-const } s \text{ Real}) \\  (\text{declare-const } k \text{ Real}) \\  (\text{assert } (\text{and } (= s k) \\  \quad (= (* s k) 1))) \\  (\text{check-sat})  \end{array}  $	$  \begin{array}{l}  (\text{declare-const } s \text{ Real}) \\  (\text{declare-const } k \text{ Real}) \\  (\text{assert } (\text{and } (= (* s k) 1))) \\  (\text{check-sat})  \end{array}  $	$  \begin{array}{l}  (\text{declare-const } k \text{ Re} \\  (\text{declare-const } s \text{ Re} \\  \dots \\  (\text{assert } (= (* s k) \\  (\text{check-sat})  \end{array}  $



sat                    sat                    ...                    sat

## Weakening & Strengthening

1	2	...	$n - 1$	n
$\begin{array}{l} (\text{declare-const } s \text{ Real}) \\ (\text{declare-const } k \text{ Real}) \\ (\text{assert } (\text{and } (= s k) \\ \quad (= (* s k) 1))) \\ (\text{check-sat}) \end{array}$	$\begin{array}{l} (\text{declare-const } s \text{ Real}) \\ (\text{declare-const } k \text{ Real}) \\ (\text{assert } (\text{and } (= (* s k) 1))) \\ (\text{check-sat}) \end{array}$	$\dots$	$\begin{array}{l} (\text{declare-const } k \text{ Real}) \\ (\text{declare-const } s \text{ Real}) \\ (\text{assert } (= (* s k) 1)) \\ (\text{check-sat}) \end{array}$	$\begin{array}{l} (\text{declare-const } k \text{ Real}) \\ (\text{declare-const } s \text{ Real}) \\ (\text{assert } (>= (* s k) 1)) \\ (\text{check-sat}) \end{array}$



Weakening rule:  $\vdash$  to  $\supseteq$

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# Backup Slides

# Bug Findings

