Fingerprint-MDS based Algorithm for Indoor Wireless Localization

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***Abstract*—Indoor wireless localization has emerged as a key wireless network technology and has been used for a variety of applications. In this paper, we examine the possibility to use one RF-based fingerprint system for indoor wireless localization, and show that there is room for improvement in its location sensing approach. We propose an indoor wireless localization solution by improving a fingerprinting localization algorithm with Multidimensional Scaling (MDS). In our approach, we configure RFID readers to receive signal strengths from both RFID tags and reference points, and use a fingerprinting localization algorithm for initial location estimation. We preprocess the received signal strength information to obtain the pairwise distances' estimation between the RFID tags and the reference points. Having estimated the pairwise squared distances, we apply MDS to reconstruct the RFID tags' distribution, and we subsequently use Procrustes analysis to refine the previously obtained fingerprinting location estimation. Simulation results show that our proposed localization algorithm improves the localization accuracy of the fingerprinting approach under different wireless network conditions.**

I. Introduction

Indoor wireless localization has emerged as a key wireless network technology, as recent advances in RF and MEMS technologies have made possible the use of large-scale wireless networks for numerous localization applications [1]. Recently, there has been a growing interest in fingerprinting indoor localization techniques, for the reason that it provides a low-cost and high-accuracy localization solution by utilizing in-building communications infrastructures. However, prevailing fingerprinting localization algorithms have an inherent problem that they assume the tracking object is located in a cell surrounded by its neighboring reference points, and hence they use a linear combination of the reference points' locations to determine the tracking object's location. Therefore, the estimated location of the tracking object's location always falls in the interior area of the reference points. This may result in significant localization error on condition that the tracking object does not fall in the interior area of the neighboring reference points.

The motivation of our work is to improve the accuracy of fingerprinting localization algorithm using Multidimensional Scaling (MDS). Based on the original fingerprinting location estimation, our proposed location sensing approach utilizes MDS and Procrustes analysis to improve the localization accuracy.

The remainder of the paper is organized as follows. Section II reviews some related work on indoor wireless localization algorithm. Section III presents the fingerprint-MDS based location sensing approach. Section IV discusses the performance of the proposed approach under different network settings. Finally, Section V concludes the paper with discussions on future research directions of this topic.

II. Related Work

A. Fingerprinting Localization System

Indoor localization systems that use fingerprinting technologies are introduced in [2], [3]. In a typical fingerprinting indoor location sensing system there are usually n RFID readers and m reference points as well as u RFID tags as tracking objects. The localization method is a range-based approach utilizing the received signal strength (RSS) information between the RFID readers and the reference points/RFID tags. It locates an unknown RFID tag using the coordinates of its neighboring reference points, determining the unknown RFID tag's coordinates $\mathbf{x}_0 = (x_0, y_0)$ as a linear combination of some of the reference points' coordinates $(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)$. A weighting factor is assigned to each of the reference points' coordinates. Intuitively, the nearer the RFID tag to a certain reference point, the larger the weighting factor is. It is not necessary to know the RFID readers' locations for localization purpose.

B. Multidimensional Scaling based Localization

MDS is a collection of statistical techniques which explores the similarities or dissimilarities in data. Recently there are related MDS-based works on range-based localization assuming the node-node pairwise distance is measurable, and on range-free localization only estimating node-node pairwise distance by connectivity/hop-count information. For certain range-based approaches, the objective function of MDS are replaced by the stress function using the least-square criterion for distance distortion measurement in [4]–[6]. In contrast, among the range-free approaches, a centralized MDS localization solution is presented in [7] for wireless sensor networks, using only the sensors' connectivity information. A decentralized version of [7] is presented in [8], and real-number hop count instead of integer hop count is exploited in [9] to achieve a better estimation of inter-node connectivity.

In our proposed algorithm, we represent Euclidean distances between the RFID tag and the reference points with the RSS-derived dissimilarity data. Hence we adopt classical MDS for location sensing in our wireless network.

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III. Proposed Fingerprint-MDS based Algorithm

In this section, we propose a novel algorithm which utilizes both fingerprinting localization methods and MDS. We set up a wireless network with a few RFID readers, and each reader's communication radius is sufficient to cover the whole area of the network. We first measure the RSS at several reference points offline and save them into computer's database. Next, we localize the RFID tag using both the RSS of the RFID tag itself and the RSS of reference points in the database. After using the fingerprinting localization algorithm for initial positioning estimation, we improve the fingerprinting localization estimation accuracy with MDS.

A. Fingerprinting Localization Algorithm

In our fingerprinting localization system, an RFID tag's signal strength vector is defined as $\vec{S} = (S_1, S_2, \ldots, S_n)$, where S_i denotes the signal strength perceived on the i^{th} reader, $i \in (1, n)$. Similarly, the signal strength vector at the j^{th} reference point is defined as $\vec{\theta}_j = (\theta_{j1}, \theta_{j2}, \ldots, \theta_{jn})$, where θ_{ji} denotes the signal strength the j^{th} reference point receives from the i^{th} reader [3].

Next, Euclidean distance in signal strength between an RFID tag and the reference point j is defined as

$$E_j = \sqrt{\sum_{i=1}^{n} (\theta_{ji} - S_i)^2}\ , \tag{1}$$

where E_j denotes the location relationship between the reference point j and the RFID tag. The nearer the reference point j to the RFID tag, the smaller E_j is.

In a 2D sensing network, the j^{th} reference point's coordinates is denoted as $\mathbf{x}_j = (x_j, y_j)$, $j \in (1, m)$. In signal strength space, we select m nearest neighboring reference points by choosing m smallest E_j. The unknown RFID tag's coordinates $\mathbf{x}_0 = (x_0, y_0)$ is estimated as

$$(\hat{x}_0, \hat{y}_0) = \sum_{j=1}^{m} w_j (x_j, y_j)\ ,$$

i.e.,

$$\hat{\mathbf{x}}_0 = \sum_{j=1}^{m} w_j \mathbf{x}_j\ , \tag{2}$$

where w_j is the weighting factor to the j^{th} neighboring reference point. In [3], the j^{th} neighboring reference point's weighting factor is empirically assigned as

$$w_j = \frac{1}{E_j^2} \Bigg/ \sum_{j=1}^{m} \frac{1}{E_j^2} \tag{3}$$

B. Classical Multidimensional Scaling

Assume there is a set of N measurements under consideration. Out of these N measurements, denote the pairwise distances between the i^{th} and the j^{th} measurement as

$$d_{ij} = d(\mathbf{x}_i, \mathbf{x}_j) = \|\mathbf{x}_i - \mathbf{x}_j\| = \sqrt{(\mathbf{x}_i - \mathbf{x}_j)^T (\mathbf{x}_i - \mathbf{x}_j)} \tag{4}$$

By writing the squared distances as $d_{ij}^2 = \mathbf{x}_i^T\mathbf{x}_i - 2\mathbf{x}_i^T\mathbf{x}_j + \mathbf{x}_j^T\mathbf{x}_j$, and defining $\psi = \left[\mathbf{x}_1^T\mathbf{x}_1, \ldots, \mathbf{x}_N^T\mathbf{x}_N\right]^T$, we can now write the squared distance matrix $D = \left\{d_{ij}^2\right\}_{i,j=1}^{N}$ as

$$D = \psi\mathbf{e}^T - 2\mathbf{X}^T\mathbf{X} + \mathbf{e}\psi^T, \tag{5}$$

where $\mathbf{e}$ is the N-dimensional vector of all ones, and $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N]$.

Now define the inner product matrix $\mathbf{B} = \mathbf{X}\mathbf{X}^T$. $\mathbf{B}$ can be rewritten as

$$\mathbf{B} = -\frac{1}{2}\mathbf{J}D\mathbf{J}^T, \tag{6}$$

where $\mathbf{J} = \mathbf{I} - \mathbf{e}\mathbf{e}^T/N$ is the centering operator. Since $\mathbf{B}$ is symmetric and positive semidefinite, to recover $\mathbf{X}$ from $\mathbf{B}$, classical MDS performs singular value decomposition by extracting the eigenvalues and eigenvectors of $\mathbf{B}$ as follows.

$$\mathbf{B} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T \tag{7}$$

Retaining the first r of N eigenvectors, the estimated coordinate matrix $\mathbf{Y}$ is given by

$$\mathbf{Y} = \mathbf{V}_r\mathbf{\Lambda}_r^{1/2} \tag{8}$$

In conclusion, classical MDS reproduces the dissimilarities of $\mathbf{X}$ with $\mathbf{Y}$, in the sense that it minimizes the squared errors between d_{ij} of $\mathbf{Y}$ and d_{ij} of $\mathbf{X}$. Therefore, $\mathbf{Y}$ is the best lower-rank approximation of $\mathbf{X}$ [10].

C. Procrustes Analysis

As discussed in Section II-B, Classical MDS returns a configuration of points $\mathbf{Y}$. The interpoint distances of $\mathbf{Y}$ in this configuration reproduce those of the original data points $\mathbf{X}$. However, $\mathbf{Y}$ is not guaranteed to match $\mathbf{X}$ in the least-square sense. To solve this problem, we use Procrustes analysis [11] to determine a linear transformation of $\mathbf{Y}$ to best conform it to $\mathbf{X}$ under the criterion of least-squared-error.

Assume the point $\mathbf{y}_i$ in $\mathbf{Y}$ is linearly transformed to $\mathbf{z}_i = c\mathbf{T}\mathbf{y}_i + \mathbf{b}$, where the vector $\mathbf{b}$ is the translation component, the matrix $\mathbf{T}$ is the orthogonal rotation and reflection component, and the scalar c is the scale component. To best match the transformed matrix $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_N]$ to the original data points $\mathbf{X}$, we need to minimize the objective function

$$\Phi(c, \mathbf{T}, \mathbf{b}) = \sum_{i=1}^{N} (c\mathbf{T}\mathbf{y}_i + \mathbf{b} - \mathbf{x}_i)^T (c\mathbf{T}\mathbf{y}_i + \mathbf{b} - \mathbf{x}_i) \tag{9}$$

The solution to Procrustes analysis was described in detail in [11]. In our case, since $\mathbf{Z}$ is the least-square estimation of $\mathbf{X}$, we obtain the RFID tag's location estimation from the transformed matrix $\mathbf{Z}$.

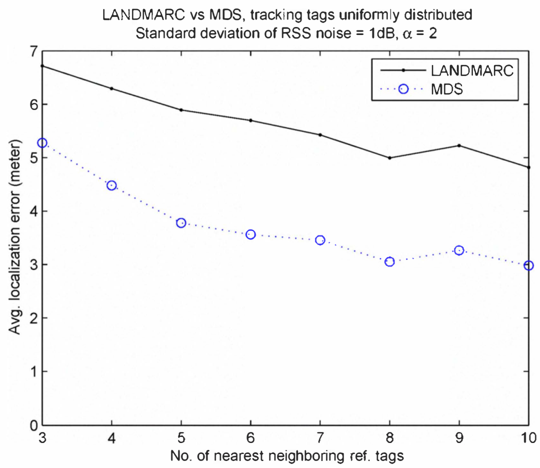

Fig. 1. Localization error distances with different numbers of reference tags in wireless network

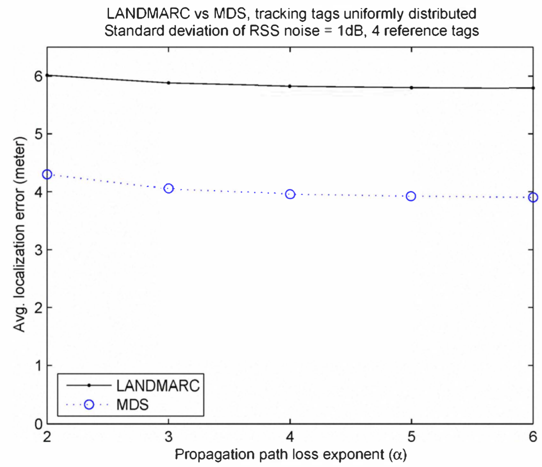

Fig. 2. Localization error distances with different propagation path loss exponents in wireless network

D. Proposed Localization Algorithm

Assume that each time we need to localize one RFID tag in our wireless mesh network. Based on fingerprinting, classical MDS and Procrustes analysis, our localization algorithm is summarized in four steps as follows:

Step 1. Fingerprinting localization. Estimate the RFID tag's location using Equation 2 of the fingerprinting localization approach.

Step 2. Dissimilarities data preprocessing.

- Retrieve from **Step 1** the Euclidean distances in signal strength between the RFID tag and a reference point

$$\mathbf{E}_{trc} = \{E_{0i}\}_{i=1}^{m} \tag{10}$$

  Also obtain the Euclidean distances in signal strength between any 2 of the m reference tags,

$$\mathbf{E}_{ref} = \{E_{ij}\}_{i,j=1}^{m} \tag{11}$$

- Obtain the geographic distances between the i^{th} and the j^{th} reference points,

$$\mathbf{d}_{ref} = \{d_{ij}\}_{i,j=1}^{m} \tag{12}$$

- For determining the relationship between the geographic distances and the Euclidean distances in signal strength of the nearest m reference tags, calculate the scaling coefficient p and q by assuming

$$\mathbf{d}_{ref} = p\mathbf{E}_{ref} + q \tag{13}$$

- Use Equation 10 and 13 to estimate the distances between the RFID tag and the m reference tags.

$$\mathbf{d}_{trc} = \{d_{0i}\}_{i=1}^{m} = p\mathbf{E}_{trc} + q \tag{14}$$

Step 3. Classical MDS. Combine $\mathbf{d}_{trc}$ and $\mathbf{d}_{ref}$ to construct the pairwise squared distance matrix $D = \{d_{ij}^2\}_{i,j=0}^{m}$, and apply classical MDS to obtain $\mathbf{Y}$.

Step 4. Procrustes analysis. Given $\mathbf{Y}$ and $\mathbf{X} = [\hat{\mathbf{x}}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N]$, apply Procrustes analysis to obtain $\mathbf{Z} = [\mathbf{z}_0, \mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_N]$. The first column vector of $\mathbf{Z}$, i.e., $\mathbf{z}_0$, is the classical-MDS-refined estimation of the RFID tag's location.

IV. Experimental Evaluation

We evaluate the performance of our proposed localization algorithm using both simulation studies and real location sensing experiments in a 20×20 meter^2 wireless network in our lab. The coordinates of the four RFID readers are (0,0), (20,0), (20,20) and (0,20) respectively. We assume the reference points and RFID tags are uniformly distributed in the wireless network. All the reference points and RFID tags fall in the 20×20 meter^2 interior area of the four readers.

The performance metric of both simulation studies and real location sensing experiments is defined as the normalized error of the estimated location, i.e.,

$$\frac{1}{N}\sum_{i=1}^{N}\sqrt{(\mathbf{x}_0 - \hat{\mathbf{x}}_{i,0})^T(\mathbf{x}_0 - \hat{\mathbf{x}}_{i,0})} \tag{15}$$

where *N* is the number of the RFID tags, and $\hat{\mathbf{x}}_{i,0}$ represents the estimated value of the RFID tag's true position $\mathbf{x}_0$ at the i^{th} run.

A. Simulation Studies

In this section, we compare the performance of fingerprinting localization approach with that of our proposed algorithm. We use MATLAB simulation to discuss the effects of reference tag number, propagation path loss exponent α and Gaussian measurement noise on the two algorithms' localization accuracy. We also discuss the effect of reference tags' placement on localization accuracy, i.e., whether the reference tags are placed in such a way that the RFID tag falls in their interior area.

1) Reference Tag Number: In order to evaluate the performance of fingerprinting localization approach and the proposed MDS approach, we randomly distribute $m = 3, 4, ..., 9, 10$ reference tags and 1 RFID tag in the network.

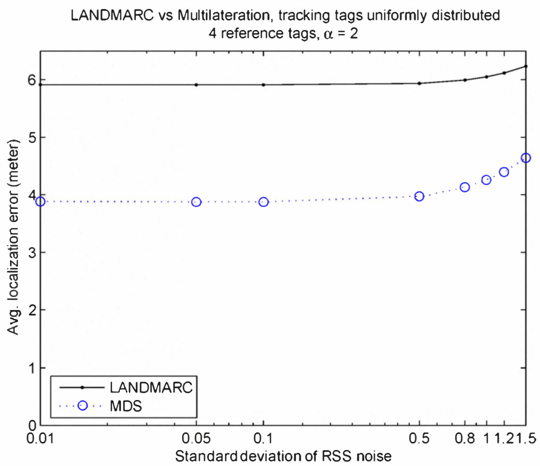

Fig. 3. Localization error distances with different Gaussian measurement noise levels in wireless network

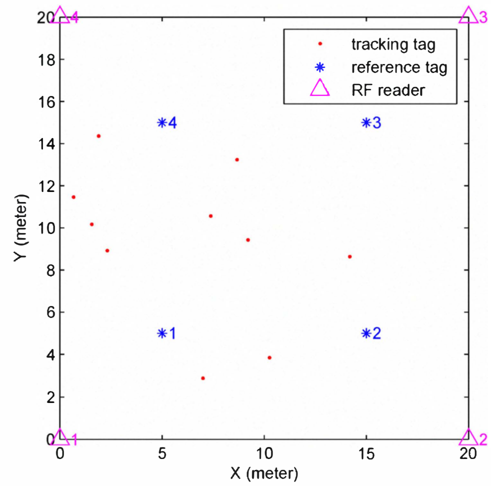

Fig. 4. Placement of 10 uniformly distributed RFID tags in the interior area of the four RFID readers

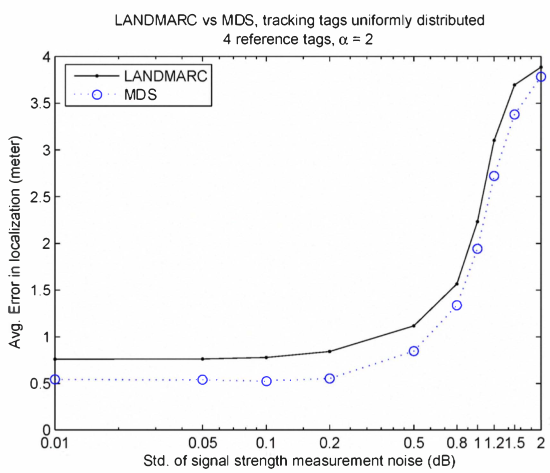

Fig. 5. Localization error distances with different Gaussian measurement noise levels, with all RFID tags in the reference points' interior area

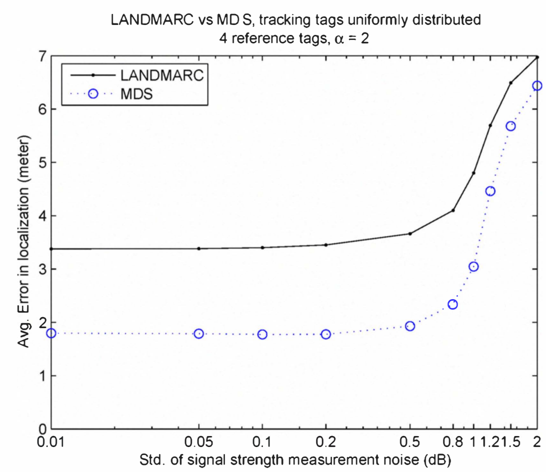

Fig. 6. Localization error distances with different Gaussian measurement noise levels, with all RFID tags in the readers' interior area

We assume the propagation path loss exponent $\alpha = 2$ and that the Gaussian measurement noise error is 1 dB. We run the simulation $N = 10000$ times and compare the average location errors of both localization algorithms.

It is observed from Fig. 1 that the performance of the proposed algorithm is consistently better than that of fingerprinting. With more than 4 reference tags, the location accuracy of the proposed algorithm is more than two meters than that of fingerprinting.

2) Propagation Path Loss Exponent: Using the fingerprinting localization approach, there is a tradeoff between the estimated location accuracy and the number of reference tags used. We distribute 4 reference tags as recommended by [3], and vary the propagation path loss exponent $\alpha = 2, 3, 4, 5, 6$. We assume the Gaussian measurement noise error is 1 dB. Similarly, we run the simulation $N = 10000$ times and compare the average location errors of both localization algorithms.

It is observed from Fig. 2 that the performance of both algorithms is almost consistent as the propagation path loss exponent α varies. The proposed algorithm outperforms fingerprinting by 1-meter localization accuracy in average.

3) Gaussian Measurement Noise: Here we also distribute 4 reference tags as recommended by [3], and vary the Gaussian measurement noise error $\sigma = 0.01, 0.05, 0.1, 0.2, 0.5, 0.8, 1, 1.2, 1.5$. The propagation path loss exponent $\alpha = 2$. Similarly, we run the simulation $N = 10000$ times and compare the average location errors of both localization algorithms.

It is observed from Fig. 3 that as the Gaussian measurement noise increases, the average localization accuracy deteriorate for both algorithms. However, the proposed algorithm outperforms fingerprinting by at least 1-meter localization accuracy.

4) Placement of Reference Points: Four reference tags are placed at (0,0), (15,0), (15,15) and (0,15). Two scenarios are compared here: one with the RFID tags uniformly distributed in the reference tags' interior area, and the other with the RFID tags uniformly distributed in the readers' interior area. It is noteworthy that in the second scenario, not all the RFID tags fall in the reference tags' interior area. In Fig. 4 we show the second scenario mentioned above with $N_1 = 10$ RFID tags. We run the simulation $N_2 = 1000$ times and take the average location errors of $N = N_1 \times N_2 = 10000$ occasions in both scenarios. We set the propagation path loss exponent $\alpha = 2$, and vary the Gaussian measurement noise error $\sigma = 0.01, 0.05, 0.1, 0.2, 0.5, 0.8, 1, 1.2, 1.5, 2$ dB.

By comparing the localization errors in Fig. 5 and Fig. 6, it is observed the fingerprinting localization algorithm works significantly better when the RFID tag is in the interior area of the reference tags. An intuitive explanation for this observation is that the estimated RFID tag location $\mathbf{x}_{est}$ is a linear combination of the reference tag locations $\{\mathbf{x}_j\}_{j=1}^{m}$. In the case of our network setup, given all $\{w_j\}_{j=1}^{m}$ positive, $\mathbf{x}_{est}$ falls in the interior area of $\{\mathbf{x}_j\}_{j=1}^{m}$, which leads to undesirable estimates in the scenario when not all the RFID tags satisfy this condition. However, for both scenarios we manage to use the proposed MDS algorithm to improve fingerprinting localization results, as shown in Fig. 5 and Fig. 6. For the second scenario, the proposed MDS algorithm improves on the localization accuracy against fingerprinting by more than 1 meter.

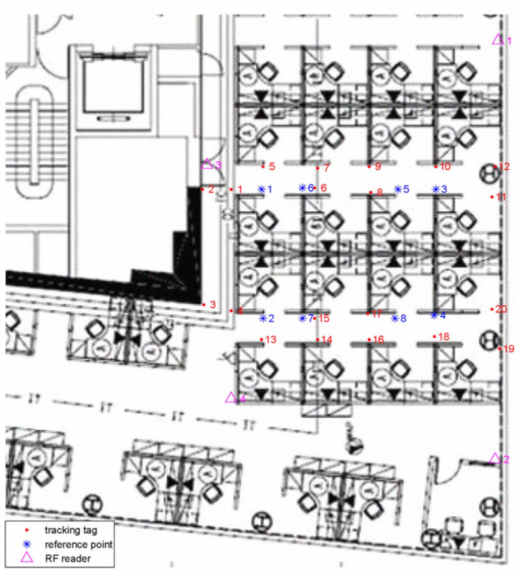

Fig. 7. Distribution of 8 reference points and RFID readers/tags in wireless network

B. Localization Experiments

In this section, we compare the performance of fingerprinting localization approach with that of our proposed algorithm using real data collected from our wireless mesh network. We conducted our localization experiments in an approach similar to fingerprinting. First we calibrated a group of reference points by moving an RFID tag in the wireless mesh network and measuring the RSS information to the RFID readers at each reference point. Next, the RSS information were transmitted via mesh routers to a database in the wireless network server. Finally, when the RFID tag entered the same wireless mesh network, we measured the RFID tag's RSS information and combined it with our pre-obtained RSS database in the server. We used this combined RSS information as inputs to localization algorithms. In location sensing at later stage, we would retrieve these reference points' RSS information from the database. In other words, the reference points function as reference tags.

Shown in Fig. 7 is a deployment of RFID readers, reference points and the RFID tags. Similar as in Section IV-A, we discuss the effects of reference points' number and the exponent β that defines Euclidean distance in the fingerprinting localization approach using real experimental data.

1) Reference Point Number: As shown in Fig. 7, we performed calibration at 8 reference points (as shown in Fig. 7) in our wireless mesh network test-bed and we deployed 20 RFID tags in the network. We selected $m = 3, 4, ..., 7, 8$ reference points for localization purpose. We then calculated the average location errors of all the 20 RFID tags and compared the results of both localization algorithms.

It is observed from Fig. 8 that the performance of the proposed algorithm is consistently better than that of fingerprinting. The location accuracy of the proposed algorithm is more than two meters than that of fingerprinting.

2) Euclidean Distance Exponent: It is noteworthy that the definition of Euclidean distance in signal strength in Equation 16 is not generalized, in the sense that the exponent in the equation can take values other than 2. We define the Euclidean distance in signal strength as follows.

$$E_j = \left[\sum_{i=1}^{n} (\theta_i - S_i)^{\beta} \right]^{1/\beta} \tag{16}$$

To explore whether this exponent is associated with the performance of our algorithm, we established a 4-reference-point database (Reference Points 1-4 in Fig. 7) and changed the exponent $\beta = 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6$. It is observed from Fig. 9 that the performance of both algorithms is almost consistent as the propagation path loss exponent β varies. Compared with fingerprinting, the proposed algorithm improves localization accuracy by about 2 meters in average.

In conclusion, the proposed MDS algorithm consistently performs better than the fingerprinting localization approach in both the simulation studies and the real localization experiments.

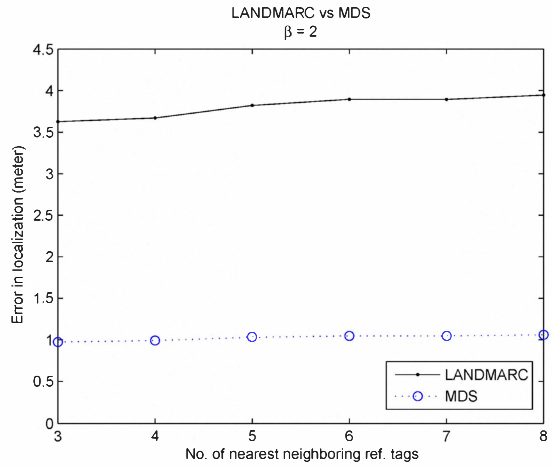

Fig. 8. Localization error distances with different number of reference points

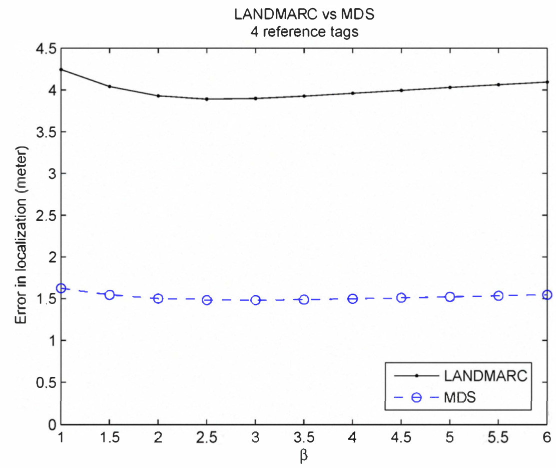

Fig. 9. Localization error distances with different Euclidean distance exponents

V. Conclusion

In this paper, we have presented a MDS-based indoor localization algorithm for wireless mesh networks. We used radio signal strength information to analyze the pairwise 'dissimilarities' between RFID tags and reference points, and by preprocessing this information we estimated the corresponding pairwise distances between the RFID tags and the reference points. We then applied an RF-based fingerprinting localization approach for an initial location estimation and subsequently applied classical MDS and Procrustes analysis to refine the fingerprinting localization result. The proposed location sensing approach improves the localization accuracy of the existing fingerprinting localization approach under different wireless network conditions.

Future work includes comparison of our proposed location sensing approach with other available wireless network localization methods, such as Maximum Likelihood Estimation [12], Modified Multidimensional Scaling [13] and Malguki Spring Model [14]. A future research agenda is to investigate whether these methods are appropriate for the scenario that all the RFID tags do not fall in the reference points' interior area. Currently we are implementing the fingerprint-MDS based algorithm in the Wi-Fi network test-bed in our lab, with Linksys Wireless-N Home Routers forming the wireless network infrastructure. Performance of the algorithms will be verified with real data collected during different periods of time in the presence of environment changes.

Acknowledgment

This work is done under the "Real-Time Secure RFID-based Track and Trace in Aerospace MRO Supply Chain" project, which is part of the Aerospace Programme funded by Science and Engineering Research Council (SERC), Agency for Science, Technology & Research (A*STAR), Singapore.

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