Homework Assignment #4

Note: You must provide sufficient detail in your derivations or proofs to earn full credit. No late homework will be graded.

1. Let X follow an exponential family of distributions with the density function

$$f_\theta(x) = a(\theta)h(x)exp(T(x)^T\theta)$$

where $\theta \in R^k$, and $a(\theta)$ and $h(x)$ are given functions. Calculate the mean of $T(x)$.

2. Let $X_1, \cdots, X_n$ be a random sample from the exponential distribution with $f_\theta(x) = exp\{-x+\theta\}$ on $x > \theta$. Let $X_{(1)}$ and $X_{(n)}$ be the smallest and the largest order statistics.

(a) Show that $X_{(1)}$ is the complete sufficient statistic for θ.
(b) Is $X_{(1)}$ independent of $X_{(n)} - X_{(1)}$?

3. Let $X_1, \cdots, X_n$ be a random sample from a Poisson family of distributions with mean λ.

(a) Is the sample mean $\bar{X}$ the complete sufficient statistic? Prove or disprove.
(b) Obviously, $\bar{X}$ is a method of moments estimator of λ. Another method of moments estimator of λ is the sample variance S^2. Compare the two estimators in terms of the mean squared error when $n = 3$.

4. Let $X_1, \cdots, X_n$ be a random sample from $Unif(-\theta, \theta)$.

(a) Find a method of moments estimator $T(x)$ of θ.
(b) Is $T(x)$ an unbiased estimator of θ?
(c) Find the maximum likelihood estimator of θ.