Stats 511 Homework Assignment #2

Note: You must provide sufficient detail in your derivations or proofs to earn full credit. No late homework will be graded.

1. The Gamma distribution $\Gamma(a, b)$ has the probability density function

$$f(x) = \{b^a/\Gamma(a)\}x^{a-1}e^{-bx},\ x > 0.$$

Find the quantile function of $\Gamma(2, b)$ for $b > 0$ and compute its quartiles (Hint: well-defined constants or numerical approximations are acceptable in your answer).

2. Let $X_1, \cdots, X_n$ be a random sample from the Bernoulli distribution with success probability $p < 1/2$, and $\bar{X}_n$ be its sample mean.

(i) Find the approximate distribution of $T_n = \sqrt{\bar{X}_n} + \sqrt{1 - \bar{X}_n}$ for large n. (Hint: normal approximation here)

(ii) What if $p = 1/2$? (Explore as much as you can but we do not expect "the best answer".)

3. Let $X_1, \cdots, X_n$ be independent exponential random variables with mean 1.

(i) Find the probability density distribution of $X_{(n)} = \min\{X_1, \cdots, X_n\}$.

(ii) Find $P(\mathrm{X}_{(n)} = X_1)$.

4. Suppose that X_1, X_2, X_3 are independent standard normal random variables. If $T_n = a_1(X_1 - X_2)^2 + a_2(X_1 - X_3)^2 + a_3(X_2 - X_3)^2$ has a chi-square distribution (with integer degree of freedom), what are possible values of $a_1 + a_2 + a_3$?