Homework Assignment #8

Note: You must provide sufficient detail in your derivations or proofs to earn full credit. No late homework will be graded.

1. GE claims that the life time distribution of its Everyday light bulbs is exponential with mean 1000 hours. If you test a random sample of 4 light bulbs and find that the average life time is 900 hours, do you have significant evidence against the GE's claim? Set up an appropriate hypothesis testing problem. Use the nominal level of 0.05 for your test. (Hint: You may view the exponential distribution as a gamma distribution. Also refer to Example 4.6.8 of the text.)

2. (Problem 1 continued) With data collected from a sample of 4 light bulbs, what is the power of your test if the actual mean life time is only 900 hours? (You may use any software for numerical calculations related to gamma distributions. If you use an online calculator, provide the URL.)

3. Suppose we have two independent random samples: $X_1, \cdots, X_n$ are exponential with mean μ_X and $Y_1, \cdots, Y_m$ are exponential with mean μ_Y.

(a) Find the LRT statistic of $H_0: \ \mu_X = \mu_Y$ versus $H_1: \ \mu_X \neq \mu_Y$.

(b) Show that the test can be based on the statistic $T = \frac{\sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} X_i + \sum_{i=1}^{m} Y_i}$.

(c) Find the distribution of T under the null hypothesis, and find a form of the level α test. (Refer to Problem 4.24 of the text if you wish.)

4. Let $X_1, \cdots, X_n$ be a random sample from Uniform $(0, \theta)$.

(a) Derive a likelihood ratio test for the hypothesis testing problem of $\theta \leq \theta_0$ versus $\theta > \theta_0$ with exact level α, where θ_0 is a given constant. Provide the critical region of the test as specific as possible.

(b) If the sample is $3, 5, 8, 12, 16, 22$, can you reject the null hypothesis with $\theta_0 = 25$ at level $\alpha = 0.05$?