Homework Assignment #10

Note: You must provide sufficient detail in your derivations or proofs to earn full credit. No late homework will be graded.

1. Let $X_1, \cdots, X_n$ be a random sample from $n(\mu, \sigma^2)$, where μ and σ are unknown.

(a) Find the likelihood ratio test for the null hypothesis $\sigma^2 = \sigma_0^2$ versus the alternative hypothesis $\sigma^2 \neq \sigma_0^2$ for a given value of σ_0^2.

(b) By inverting the above test, derive the level $1-\alpha$ confidence interval for σ^2.

2. Let $X_1, \cdots, X_n$ be a random sample from the Poisson distribution with mean λ, and let $T = \sum_{i=1}^n X_i$.

(a) Show that $(T - n\lambda)/\sqrt{2n\lambda}$ is asymptotically pivotal.

(b) Construct an approximate level $1-\alpha$ confidence set for λ based on the result in (a).

(c) For $n = 9$ and $T = 16$, find an approximate 95% confidence interval for λ.

3. In a series of 5 independent missile trials, only four launches were successful. Find a 90% lower confidence bound for the success probability.

4. Let $X_1, \cdots, X_n$ be a random sample from a distribution that takes three possible values as follows: $P(X = -1) = P(X = 1) = \theta$ and $P(X = 0) = 1 - 2\theta$.

(a) Show the likelihood ratio test for $H_0 : \ \theta = \theta_0$ versus $H_1 : \ \theta > \theta_0$ takes the form that $\sum_{i=1}^n I(X_i = 0) < k$ for some value of k.

(b) Assume that $k = k(\alpha)$ is known for any given level of α, find the level $1-\alpha$ lower confidence bound for θ.

(c) For $\theta_0 = 1/3$, $n = 10$ and $\alpha = 0.05$, find k numerically.