Homework Assignment #5

Note: You must provide sufficient detail in your derivations or proofs to earn full credit. No late homework will be graded.

1. Let $X_1, \cdots, X_n$ be a random sample from $N(\theta, \theta^2)$. Find the maximum likelihood estimate of θ when $\bar{X} = 0$ is observed.

2. We have k independent observations $X_1, \cdots, X_k$ from a binomial distribution $b(n, p)$, where both n and p are unknown. In class, it was shown that the method-of-moments estimators are unspecified when $k = 1$.

(a) Find a method-of-moments estimator for (n, p) when $k > 1$.

(2) In the case of $k = 1$ ad $X = 8$, can you find the maximum likelihood estimator of (n, p)? If you cannot do it analytically, you may do it numerically for considering a reasonable range of n.

3. Let $(X_1, Y_1), \cdots, (X_n, Y_n)$ be a random sample from the bivariate normal distribution with mean (0,0), variances (σ^2, σ^2), and the correlation coefficient ρ.

(a) Find the maximum likelihood estimator of (σ^2, ρ).

(b) Find a method-of-moments estimator of (σ^2, ρ).

4. Let $X_1, \ldots, X_n$ be independent random variables with $X_i \sim \text{Poisson}(c_i\lambda)$ for $i = 1, \ldots, n$. Here $\lambda > 0$ is unknown while $c_1, \ldots, c_n$ are some known positive constants.

(a) Let $\tilde{\lambda} = n^{-1} \sum_{i=1}^{n} \{X_i/c_i\}$. Is $\tilde{\lambda}$ an unbiased estimator of λ? Calculate the mean squared error of $\tilde{\lambda}$.

(b) Find the maximum likelihood estimator $\hat{\lambda}$ of λ. Is $\hat{\lambda}$ unbiased? Is $\hat{\lambda}$ a better estimator compared to $\tilde{\lambda}$ under the mean squared error criterion?