

Stats 511 Homework Assignment #3

Note: You must provide sufficient detail in your derivations or proofs to earn full credit. No late homework will be graded.

1. Let X_1, \dots, X_n be independent random variables taking values 0 or 1 with

$$P(X_i = 1) = e^{\theta - a_i} / (1 + e^{\theta - a_i}), \quad i = 1, \dots, n,$$

for some given constants a_i . Find a one-dimensional sufficient statistic for θ .

2. Let X_1, \dots, X_n be a random sample from a $\text{gamma}(\alpha, \beta)$ population.
(i) Find a two-dimensional sufficient statistic for (α, β) . (ii) If α is known, can you find a one-dimensional sufficient statistic for β ?

3. (a) Show that $T = (X_{(1)}, X_{(n)})$ for a random sample from the uniform distribution on $(\theta, 2\theta)$ with $\theta > 0$ is minimal sufficient for θ . (b) Show that the sample mean and the sample variance (\bar{X}, S^2) are not sufficient for θ .

4. Let $X = (x_1, x_2, x_3, x_4)$ follow the multinomial distribution with the cell probabilities given by $(\frac{1}{2} + \frac{\theta}{4}, \frac{1}{4}(1 - \theta), \frac{1}{4}(1 - \theta), \frac{\theta}{4})$.

(a) Show that this is an exponential family, and identify a minimal sufficient statistic.

(b) Is this a curved exponential family? Explain.