Homework Assignment #9

1. Let $(X_1, Y_i), \cdots, (X_n, Y_n)$ be a random sample that follows the regression model

$$Y_i = \beta X_i + e_i, \quad i = 1, \cdots, n,$$

where the distribution of X_i is unspecified, but e_i are independent of X_i and are normally distributed with mean zero and known variance σ^2. Within the class of tests such that the conditional probability of the type I error given $(X_1, \cdots, X_n)$ is no greater than α, can you find the uniformly most powerful test for testing the null hypothesis $\beta = 0$ against the alternative hypothesis $\beta > 0$? If so, give an explicit critical region for the test.

2. Let $X_1, \cdots, X_n$ be a random sample from $n(\mu_1, \sigma^2)$, and $Y_1, \cdots, Y_n$ be a random sample from $n(\mu_2, 4\sigma^2)$, where all the parameters are unknown. The two samples are independent. We consider the null hypothesis $H_0 \ : \mu_1 = \mu_2$ versus the alternative hypothesis $H_1 \ : \mu_1 \neq \mu_2$.

(a) Find the maximum log-likelihood value l_0 under H_0.

(b) Find the maximum log-likelihood value l_1 without restrictions.

(c) Consider the likelihood ratio test statistic $T = 2(l_1 - l_0)$. Is the test equivalent to the test T^* which rejects H_0 when

$$\frac{(\bar{X} - \bar{Y})^2}{4S_x^2 + S_y^2} > k$$

for some constant k, where S_x^2 and S_y^2 are the two sample variances?

3. (Problem 2 continued.) Suppose that $n = 10$, and a level 0.05 test is carried out. (a) Find the exact value of k above for the test T^*; and (b) Find the approximate critical value for T using its asymptotic distribution, and then compare how close these two critical regions are.

4. A blood sample is analyzed by three labs to test the null hypothesis that the blood matches that of a suspect. The resulting p-values from the labs are 0.60, 0.25, and 0.01, respectively. (a) If the lab test results are independent, can you reject the null hypothesis at the significance level 0.05? (b) If the three test results are positively correlated, would you reach the same conclusion? Why?