Stats 511 Homework Assignment #1

Note: You must provide sufficient detail in your derivations or proofs to earn full credit. No late homework will be graded.

1. Give a hypothetical (but not unrealistic) example of two measurements where Pearson's correlation is very close to zero but there is a clear causal relationship. If you find the example from a public source, please cite properly.

2. Suppose that you can use a scale to measure the weight of a ball, or the total weight of two balls, or the difference in weight between two balls. Each measurement has no bias but is subject to a standard deviation of σ. Consider two schemes in estimating the weights of ball A and ball B as follows.

Scheme 1: take one measurement for each ball, obtaining the weights as w_A and w_B, respectively.

Scheme 2: take one measurement for the total weight of the two balls to get w_{A+B}, and then one measurement for the difference in weight between the two balls to get w_{A-B}.

Which scheme would you prefer in estimating the weights of the two individual balls? Explain.

3. Let $(X_1, Y_1), \cdots, (X_n, Y_n)$ be a random sample from the bivariate normal distribution with mean (0,0), variances (1,1), and correlation ρ. Let $T = \sum_{i=1}^{n} I(X_i > Y_i)$, where $I(.)$ is the indicator function. Find the distribution of T.

4. Let $X_1, \cdots, X_n$ be a random sample from the normal distribution with mean μ and variance σ^2. Let $T = \sum_{i=1}^{n}(X_i - \bar{X}_n)^2$, where $\bar{X}_n$ is the sample mean. Compare $n^{-1}T$ and $(n-1)^{-1}T$ as an estimator of σ^2 in terms of the mean and the variance. If you do not wish to do it analytically, write computer code in any platform to calculate the value for n = 10 numerically and explain how accurate you expect your answers to be.