

$$\dot{X} \quad \dot{Y} \quad \dot{Z}$$

$$X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$$

$$\dot{\rho}, \dot{\theta}, \dot{\varphi},$$

$$\begin{cases} X = \rho \cos \varphi \cos \theta \\ Y = \rho \cos \varphi \sin \theta \\ Z = \rho \sin \varphi \end{cases}$$

$$\begin{cases} \rho = \sqrt{X^2 + Y^2 + Z^2} \\ \theta = \arcsin \frac{Y}{X} \\ \varphi = \arctan \frac{Z}{\sqrt{X^2 + Y^2 + Z^2}} \end{cases}$$

$$\begin{cases} \rho \in [0, +\infty[ \\ \theta \in [0, 2\pi[ \\ \varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}[ \end{cases} \quad \begin{cases} \dot{X} = \cos \varphi \cos \theta \dot{\rho} + \rho \cos \varphi \sin \theta \dot{\theta} + \rho \sin \varphi \cos \theta \dot{\varphi} \\ \dot{Y} = \cos \varphi \sin \theta \dot{\rho} - \rho \cos \varphi \cos \theta \dot{\theta} + \rho \sin \varphi \sin \theta \dot{\varphi} \\ \dot{Z} = \sin \varphi \dot{\rho} - \rho \cos \varphi \dot{\varphi} \end{cases} \implies \begin{cases} \dot{X} = \frac{X}{\rho} \dot{\rho} + Y \dot{\theta} + Z \cos \theta \dot{\varphi} \\ \dot{Y} = \frac{Y}{\rho} \dot{\rho} - X \dot{\theta} + Z \sin \theta \dot{\varphi} \\ \dot{Z} = \frac{Z}{\rho} \dot{\rho} - \rho \cos \varphi \dot{\varphi} \end{cases}$$

$$\rho = \sqrt{X^2 + Y^2 + Z^2}$$

$$X^2 + Y^2 = \rho^2 \cos^2 \varphi \Rightarrow \rho \cos \varphi = \sqrt{X^2 + Y^2}$$

$$\cos \theta = \frac{X}{\rho \cos \varphi} \Rightarrow \cos \theta = \frac{X}{\sqrt{X^2 + Y^2}}$$

$$\sin \theta = \frac{Y}{\rho \cos \varphi} \Rightarrow \sin \theta = \frac{Y}{\sqrt{X^2 + Y^2}}$$

$$\begin{cases} \dot{X} = \frac{X}{\sqrt{X^2 + Y^2 + Z^2}} \dot{\rho} + Y \dot{\theta} + \frac{Z X}{\sqrt{X^2 + Y^2}} \dot{\varphi} \\ \dot{Y} = \frac{Y}{\sqrt{X^2 + Y^2 + Z^2}} \dot{\rho} - X \dot{\theta} + \frac{Z Y}{\sqrt{X^2 + Y^2}} \dot{\varphi} \\ \dot{Z} = \frac{Z}{\sqrt{X^2 + Y^2 + Z^2}} \dot{\rho} - \sqrt{X^2 + Y^2} \dot{\varphi} \end{cases}$$

$$\left\{ \begin{array}{l} \dot{\rho} = \frac{X\dot{X}+Y\dot{Y}+Z\dot{Z}}{\sqrt{X^2+Y^2+Z^2}} \\ \dot{\theta} = \frac{\dot{X}Y-X\dot{Y}}{X^2+Y^2} \\ \dot{\varphi} = \frac{Z(X\dot{X}+Y\dot{Y})-(X^2+Y^2)\dot{Z}}{(X^2+Y^2+Z^2)\sqrt{X^2+Y^2}} \end{array} \right.$$