

Mathematical Statistics

Lab 1

Exercise 1

Common midwife toad (*Alytes obstetricans*) lives in half of the ponds in some area, $P(\text{toad}) = 1/2$. If it is a sunny spring day (soil temperature is 14°C) then an observer can spot the toad with probability 0.6 – if the toad actually lives in a given pond. Look at the graphs below (Tanadini & Schmidt, 2011):

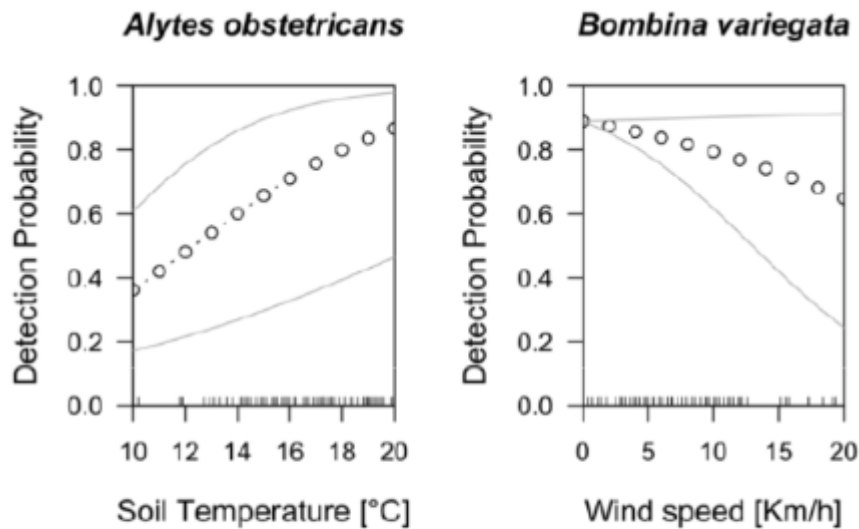


Figure 1. The relationship between meteorological variables and detection probabilities in two anurans. Thin gray lines are 95% confidence intervals. Small ticks inside the box indicate observed soil temperatures and wind speeds, respectively.
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If an observer visits a randomly selected pond in this area, what is the probability to observe a toad there?

Exercise 2

Observer visits ponds in a particular area and sees toads in 20% of the ponds in this particular area, $P(\text{sees a toad}) = 0.2$. The soil temperature was 14°C during the survey trip. We assume, that the observer does not see ghosts (if (s)he sees a toad, then a toad is actually there). What is the probability a toad actually lives in a randomly selected pond in this area?

$$P(\text{toad}) = ?$$

Exercise 3

Similar weather (soil temperature 14°C) lasts for several days in spring. Two observers visit separately (independently) ponds in a given area. Find the probability that at least one of them will spot a toad in a pond where a toad actually lives.

Exercise 4

Instead of experienced observers, schoolchildren went for a nature survey. Half of the ponds in this particular area are inhabited with the toads. Unfortunately schoolchildren can sometimes mix up two different species of toads – so sometimes a child may report seeing a common midwife toad in a pond where actually no toads live (with probability 0.1 a child thinks seeing a toad in a pond where actually no toads live). We assume that the soil temperature is still 14°C . What is the probability a child reporting a toad in a randomly selected pond in this particular area?

Exercise 5

Children report seeing toads in 15% of ponds. What is the proportion of ponds actually habited with common midwife toad?

Exercise 6

A supermarket sells light-bulbs from three different companies (A, B, C). Light-bulbs made by company A last at least a year with probability 0.9; company B light-bulbs work a year (or longer) with a probability 0.8 and company C light-bulbs go on working for at least a year with probability 0.4. On the shelf – space dedicated to light-bulbs company A products take up 50%, company B and C products both have 25% of the shelf-space.

What is the probability for me to get a light-bulb that will work at least a year from this supermarket if I just go there and pick a bulb randomly (I do not have any preliminary knowledge about the reliabilities of the light-bulbs).