Calculating a state occupancy distribution in multistate settings

EAPS HMMWG meeting - 22 September, 2023

Tim Riffe — opik; Universidad del País Vasco / Euskal Herriko Unibertsitatea; ikerbasque Basque Foundation for Science; Max-Planck-Institut für demografische Forschung

Iñaki Permanyer — CED Centre d'Estudis Demogràfics

Rustam Tursun Zade — opik; Universidad del País Vasco / Euskal Herriko Unibertsitatea

Magdalena Muszynska-Spielauer — Wittgenstein Centre for Demography and Global Human Capital (A collaboration of IIASA, VID/ÖAW, WU)

A multistate health model gives a multistate death distribution

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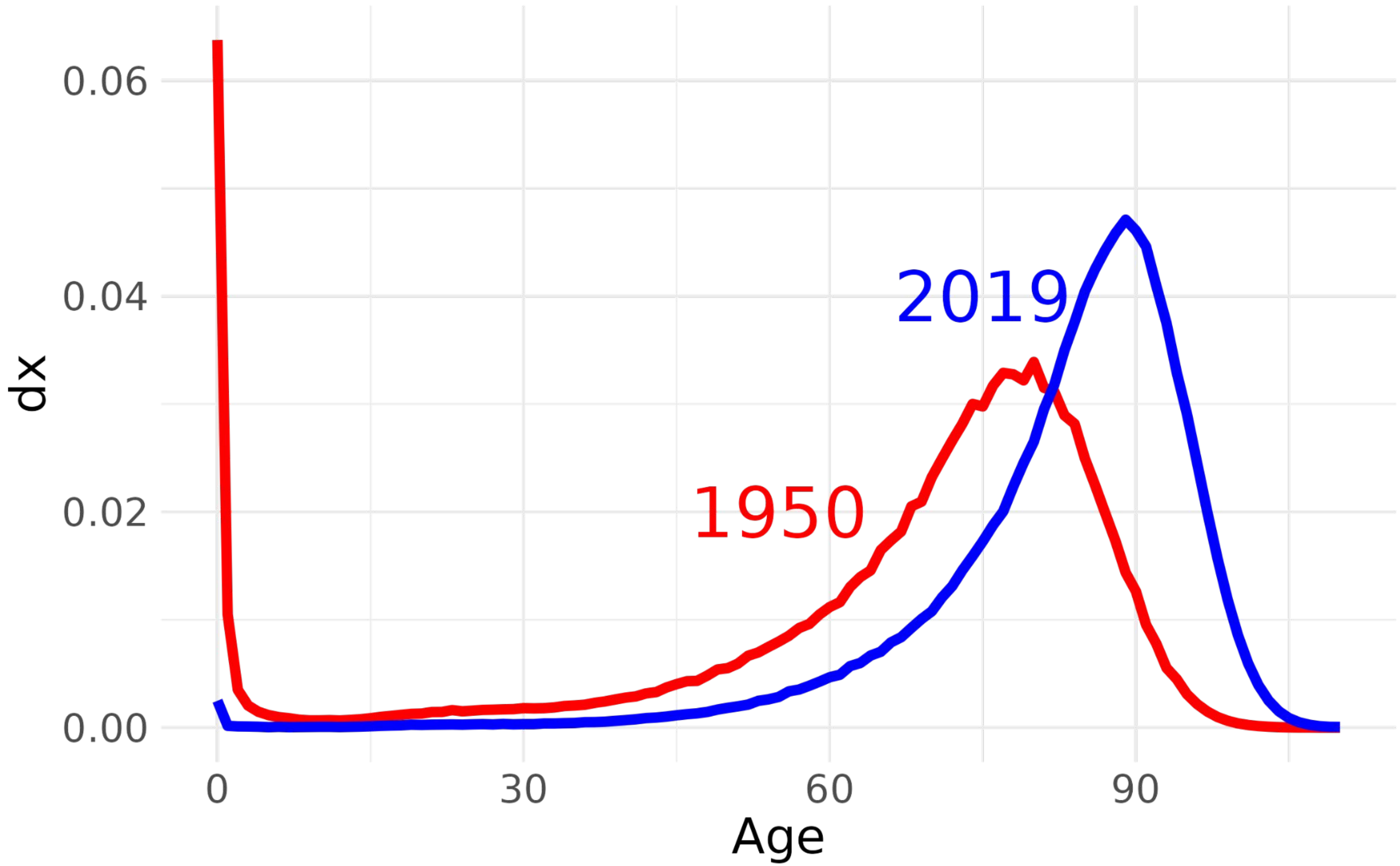
Magdalena Muszynska-Spielauer — Wittgenstein Centre FOR DEMOGRAPHY AND GLOBAL HUMAN CAPITAL, A COLLABORATION OF IIASA, VID/ÖAW, WU


0.06
0.04
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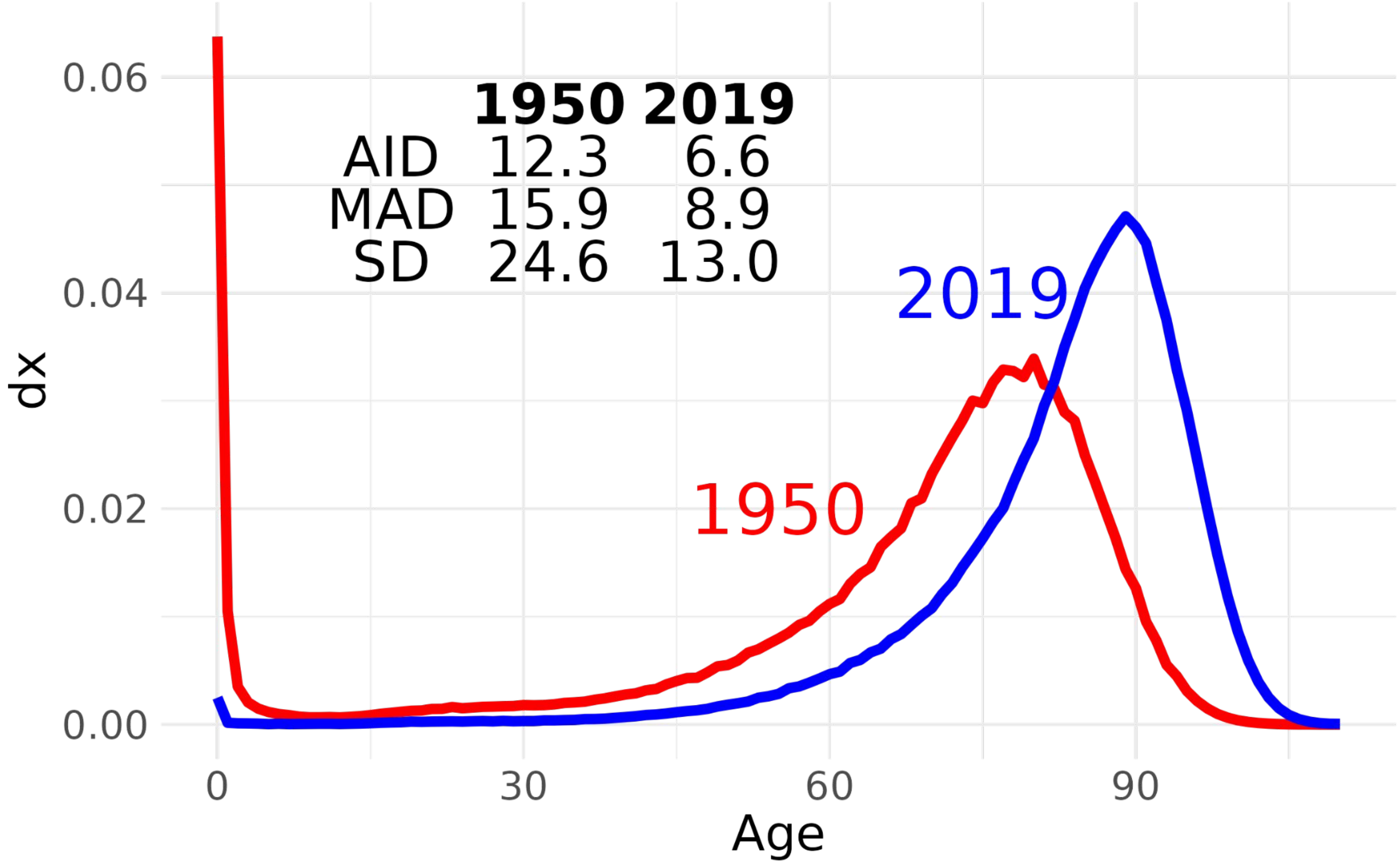
dx
2019
1950
0
30
60
90
Age



1950 2019
AID 12.3 6.6
MAD 15.9 8.9
SD 24.6 13.0
2019
1950
0.06
0.04
0.02
0.00
dx
0
30
60
90
Age

Prevalence-based approximations

Caswell and Zarulli *Population Health Metrics* (2018) 16:8
https://doi.org/10.1186/s12963-018-0165-5

Population Health Metrics

RESEARCH **Open Access**

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Matrix methods in health demography: a new approach to the stochastic analysis of healthy longevity and DALYs

Hal Caswell[1*] and Virginia Zarulli[2]

Population Health Metrics

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Check for updates

On the measurement of healthy lifespan inequality

Iñaki Permanyer[1,2*], Jeroen Spijker[1] and Amand Blanes[1]

Wittgenstein Centre FOR DEMOGRAPHY AND GLOBAL HUMAN CAPITAL

WIFO AUSTRIAN INSTITUTE OF ECONOMIC RESEARCH

ikerbasque

Healthy lifespan statistics derived from cross-sectional prevalence data using Sullivan's method are informative summary measures of population health

Magdalena Muszyńska-Spielauer*, Tim Riffe**, Martin Spielauer †

Incidence-based approximations

DEMOGRAPHIC RESEARCH
A peer-reviewed, open-access journal of population sciences

New Methods for Analyzing Active Life Expectancy

SARAH B. LADITKA, PhD
State University of New York Institute of Technology–Utica/Rome
DOUGLAS A. WOLF, PhD
Syracuse University

(1998) Journal of Aging and Health, 10(2)

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http://www.demographic-research.org/Volumes/Vol45/13/
DOI: 10.4054/DemRes.2021.45.13

Research Article

Healthy longevity from incidence-based models: More kinds of health than stars in the sky

Hal Caswell

Silke van Daalen

Incidence-based approximations

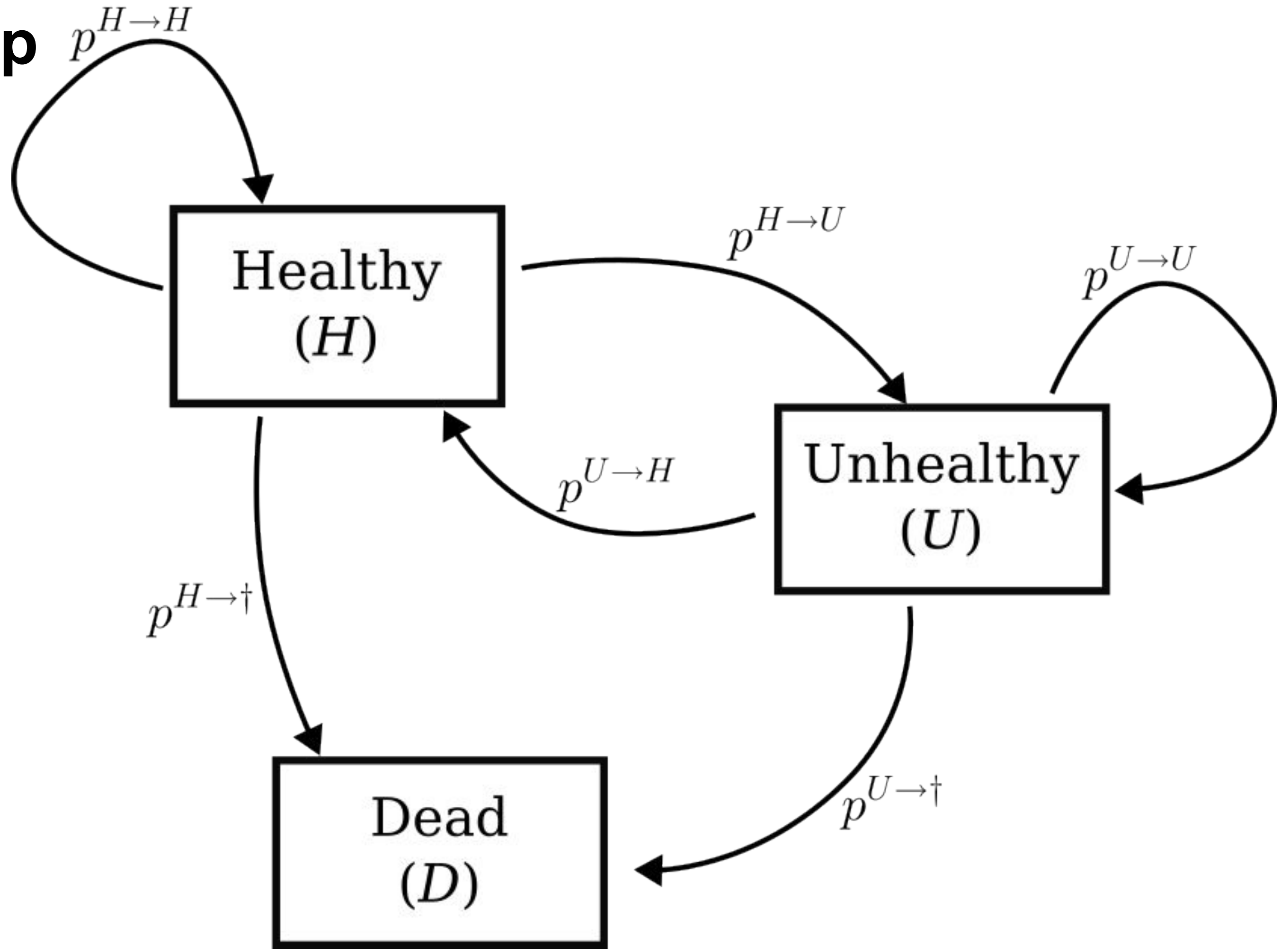
The first three moments suffice to calculate the mean, variance, and skewness of healthy longevity:

$$
\begin{aligned}
E(\tilde{\boldsymbol{\rho}}) &= \tilde{\boldsymbol{\rho}}_1 & (18)\\
V(\tilde{\boldsymbol{\rho}}) &= \tilde{\boldsymbol{\rho}}_2 - (\tilde{\boldsymbol{\rho}}_1 \circ \tilde{\boldsymbol{\rho}}_1) & (19)\\
SD(\tilde{\boldsymbol{\rho}}) &= \sqrt{V(\tilde{\boldsymbol{\rho}})} & (20)\\
CV(\tilde{\boldsymbol{\rho}}) &= \mathcal{D}\left(\tilde{\boldsymbol{\rho}}_1\right)^{-1} SD(\tilde{\boldsymbol{\rho}}) & (21)\\
Sk\left(\tilde{\boldsymbol{\rho}}\right) &= \mathcal{D}\left[V(\tilde{\boldsymbol{\rho}})\right]^{-3/2} \left(\tilde{\boldsymbol{\rho}}_3 - 3\tilde{\boldsymbol{\rho}}_1 \circ \tilde{\boldsymbol{\rho}}_2 + 2\tilde{\boldsymbol{\rho}}_1 \circ \tilde{\boldsymbol{\rho}}_1 \circ \tilde{\boldsymbol{\rho}}_1\right). & (22)
\end{aligned}
$$

The vector $\tilde{\boldsymbol{\rho}}_m$ contains the mth moments of healthy longevity for all combinations of initial age and health stage. To obtain the moments of healthy longevity as a function

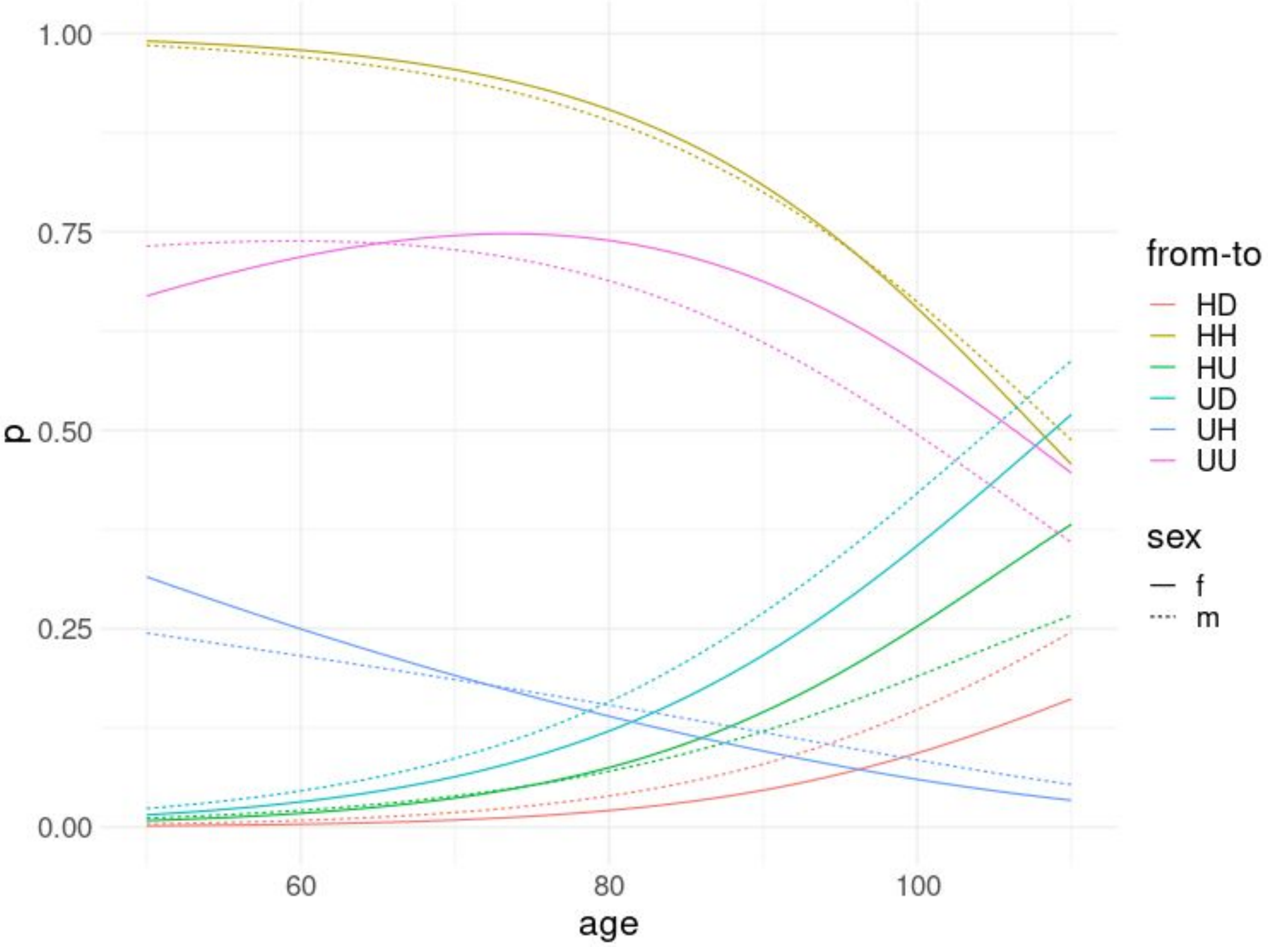
from Caswell & van Daalen (2021)

Basic setup



$p^{H\to H}$
Healthy
(H)
$p^{H\to U}$
$p^{U\to U}$
$p^{U\to H}$
Unhealthy
(U)
$p^{H\to\dagger}$
$p^{U\to\dagger}$
Dead
(D)

Basic setup (example ADL)



Basic setup

$$\ell_{x+1}^{H} = \ell_x^{H} \cdot p_x^{H \to H} + \qquad \text{(remain healthy) +}$$

$$\ell_x^{U} \cdot p_x^{U \to H} \qquad \text{(return to health)}$$

Basic setup

$$\ell_{x+1}^{H} = \ell_x^{H} \cdot p_x^{H \to H} +$$

(remain healthy) +

$$\ell_x^{U} \cdot p_x^{U \to H}$$

(return to health)

(and similarly for unhealthy people)

$$\ell_{x+1}^{U} = \ell_x^{U} \cdot p_x^{U \to U} + \ell_x^{H} \cdot p_x^{H \to U}$$

Basic setup

$$HLE = \sum \ell_x^H$$

$$ULE = \sum \ell_x^U$$

Extending to age and duration: stocks

$$\ell^H(x+1, h+1) = \ell^H(x,h) \cdot p_x^{H \to H} + \quad \text{(remain healthy) +}$$

$$\ell^U(x,h) \cdot p_x^{U \to H} \quad \text{(return to health)}$$

Extending to age and duration: stocks

$$\ell^H(x+1, h+1) = \ell^H(x, h) \cdot p_x^{H \to H} +$$ (remain healthy) +

$$\ell^U(x, h) \cdot p_x^{U \to H}$$ (return to health)

$$\ell^U(x+1, h) = \ell^H(x, h) \cdot p_x^{H \to U} +$$ (health deterioration) +

$$\ell^U(x, h) \cdot p_x^{U \to U}$$ (remain unhealthy)

Extending to age and duration: stocks

$$\ell(x,h) = \ell^H(x,h) + \ell^U(x,h)$$

$$\ell(x) = \sum_h \ell(x,h)$$

$$C(x,h) = \frac{\ell(x,h)}{LE}$$

(stationary age-duration structure)

Extending to age and duration: deaths

$$d(x,h) = \ell^H(x,h) \cdot p_x^{H \to \dagger} +$$ (die while healthy) +

$$\ell^U(x,h) \cdot p_x^{U \to \dagger}$$ (die while unhealthy)

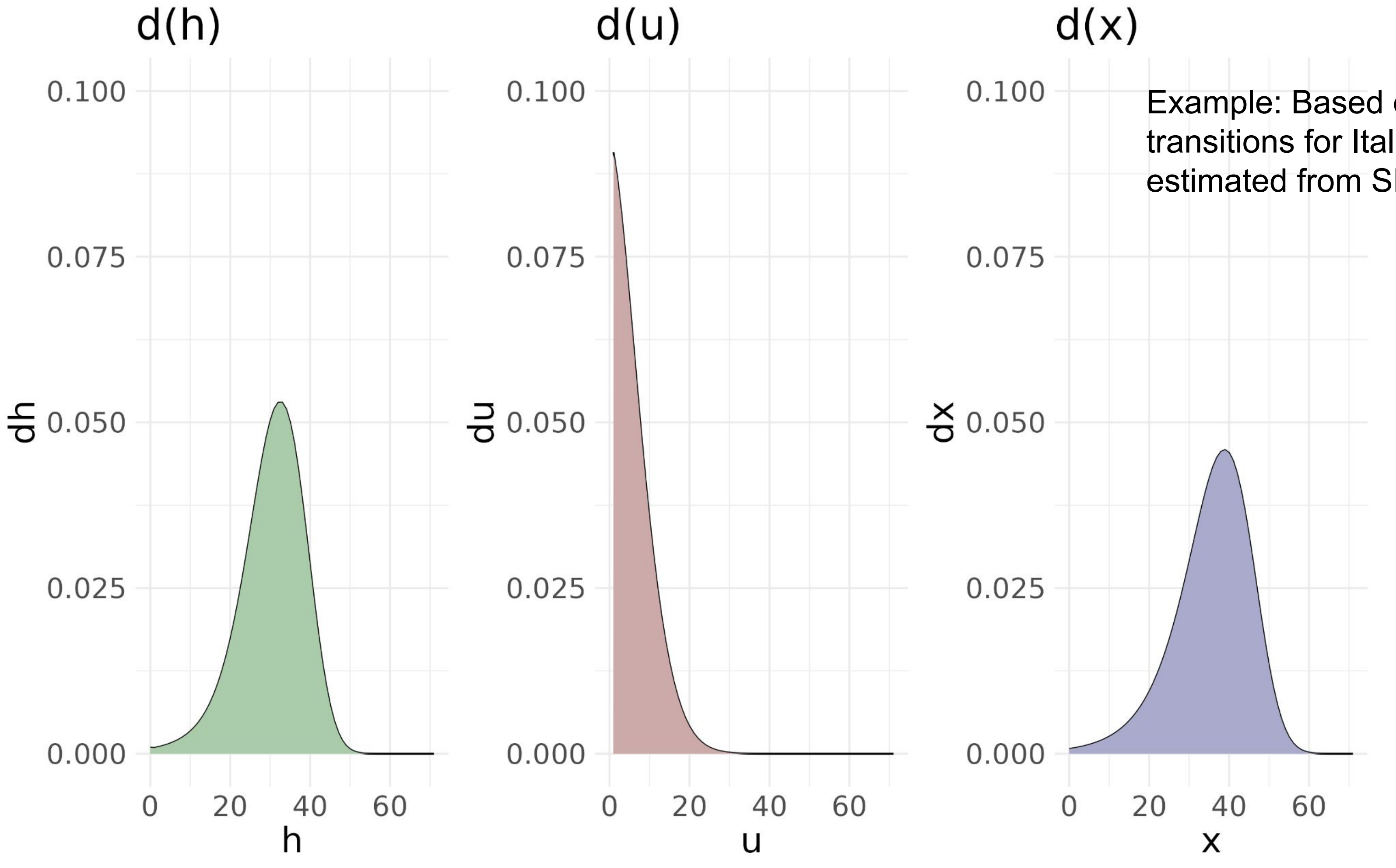
Extending to age and duration: deaths

$$d(x,h) = \ell^{H}(x,h) \cdot p_x^{H \to \dagger} +$$ (die while healthy) +

$$\ell^{U}(x,h) \cdot p_x^{U \to \dagger}$$ (die while unhealthy)

A 2d death distribution!

$$1 = \sum_x \sum_h d(x,h)$$



Example: Based on ADL transitions for Italian women, estimated from SHARE

The first three moments suffice to calculate the mean, variance, and skewness of healthy longevity:

$$E(\tilde{\boldsymbol{\rho}}) = \tilde{\boldsymbol{\rho}}_1 \tag{18}$$

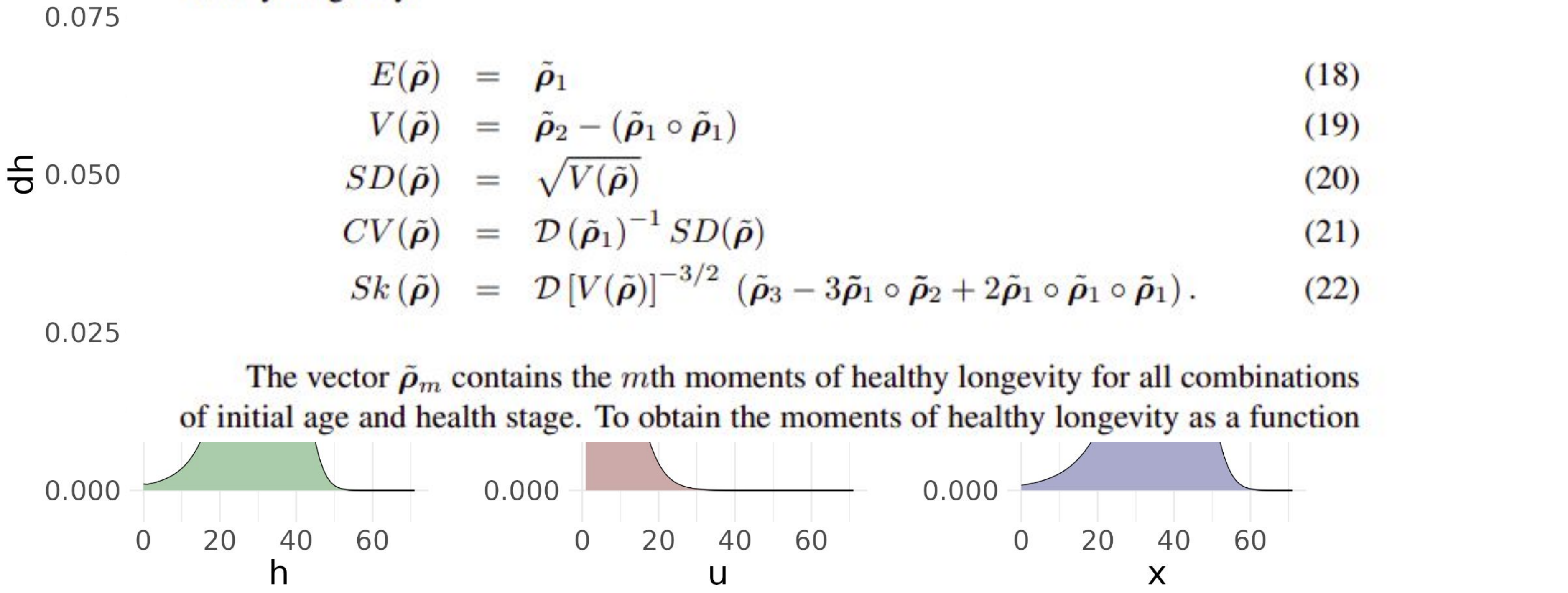
$$V(\tilde{\boldsymbol{\rho}}) = \tilde{\boldsymbol{\rho}}_2 - (\tilde{\boldsymbol{\rho}}_1 \circ \tilde{\boldsymbol{\rho}}_1) \tag{19}$$

$$SD(\tilde{\boldsymbol{\rho}}) = \sqrt{V(\tilde{\boldsymbol{\rho}})} \tag{20}$$

$$CV(\tilde{\boldsymbol{\rho}}) = \mathcal{D}(\tilde{\boldsymbol{\rho}}_1)^{-1} SD(\tilde{\boldsymbol{\rho}}) \tag{21}$$

$$Sk(\tilde{\boldsymbol{\rho}}) = \mathcal{D}[V(\tilde{\boldsymbol{\rho}})]^{-3/2} (\tilde{\boldsymbol{\rho}}_3 - 3\tilde{\boldsymbol{\rho}}_1 \circ \tilde{\boldsymbol{\rho}}_2 + 2\tilde{\boldsymbol{\rho}}_1 \circ \tilde{\boldsymbol{\rho}}_1 \circ \tilde{\boldsymbol{\rho}}_1). \tag{22}$$

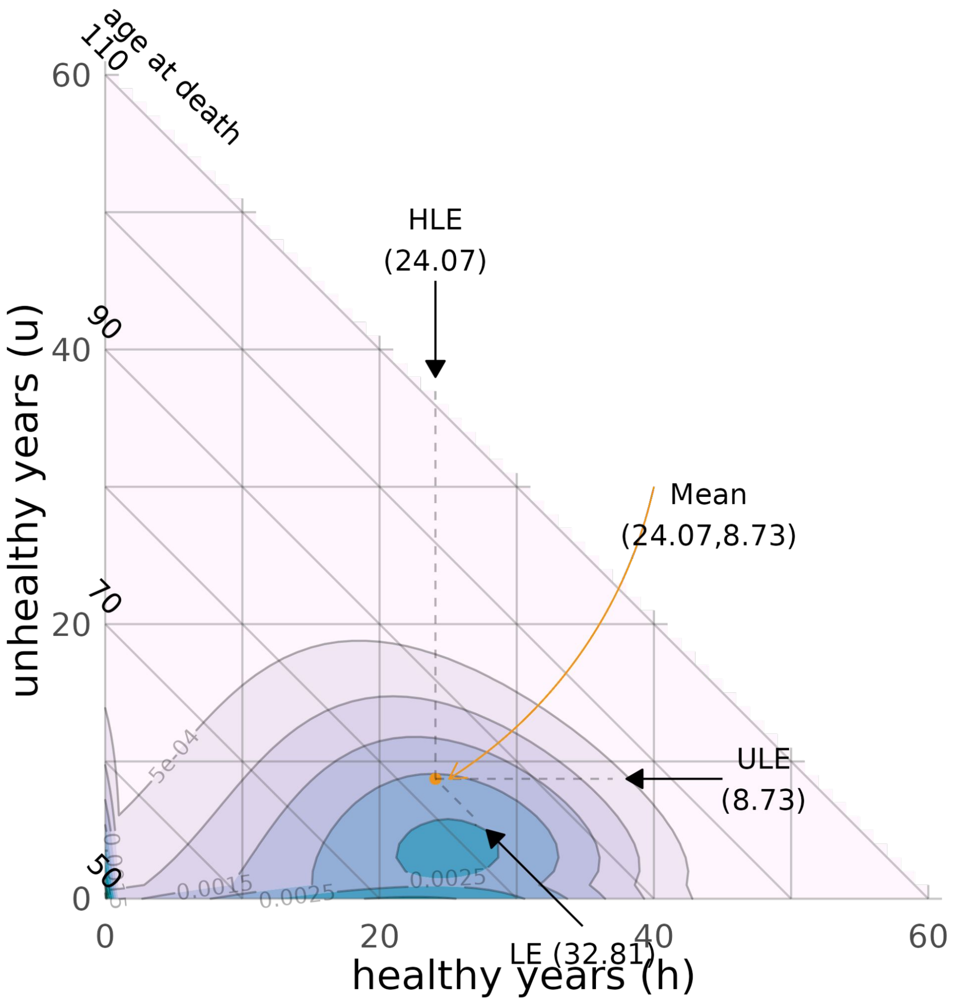
The vector $\tilde{\boldsymbol{\rho}}_m$ contains the mth moments of healthy longevity for all combinations of initial age and health stage. To obtain the moments of healthy longevity as a function



Relationship between marginal death distributions

$$Var(x) = Var(h) + Var(u) + 2 \cdot Cov(h, u)$$

$$122 = 110 + 44 - 2 \cdot 16$$

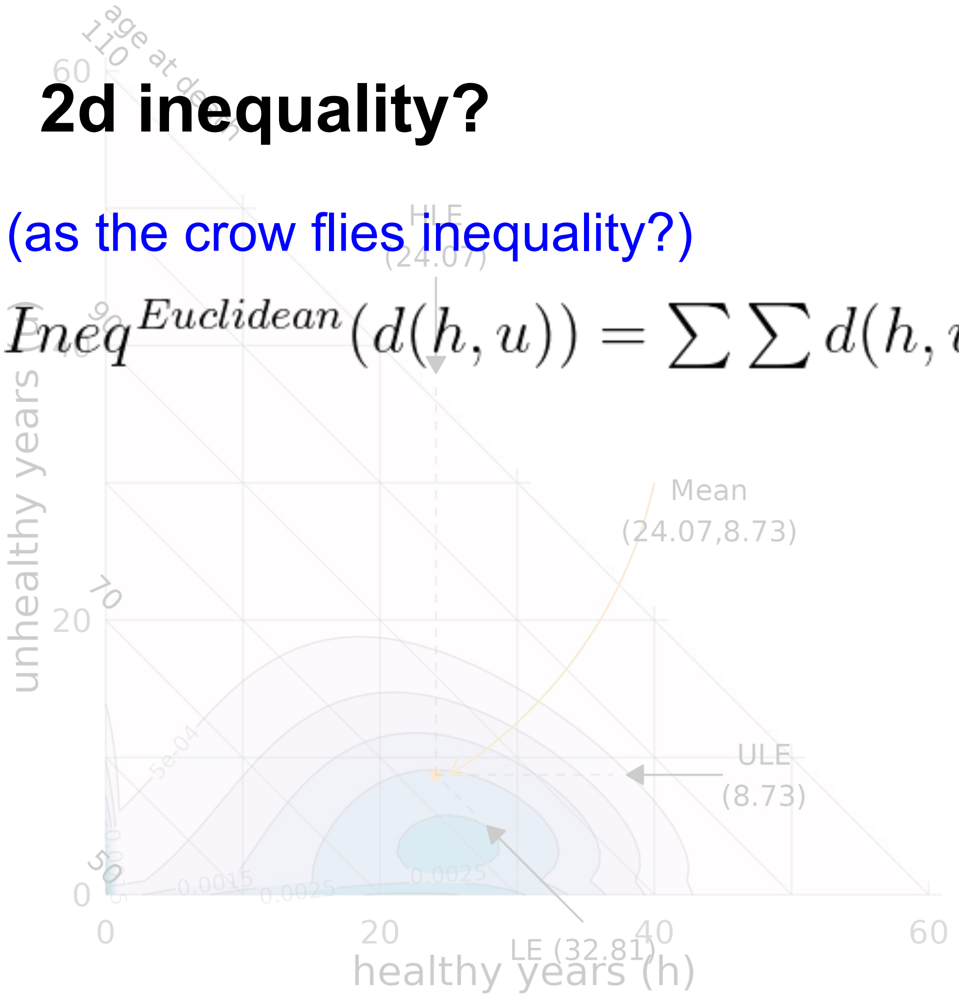


Example: Self-reported health (2 categories) Transitions recycled from Foltyn & Olsson (2021), based on US Health and Retirement Study data. Female “non-black” strata.

2d inequality?

(as the crow flies inequality?)

$$Ineq^{Euclidean}(d(h,u)) = \sum\sum d(h,u) \cdot \sqrt{(HLE-h)^2 + (ULE-u)^2}$$



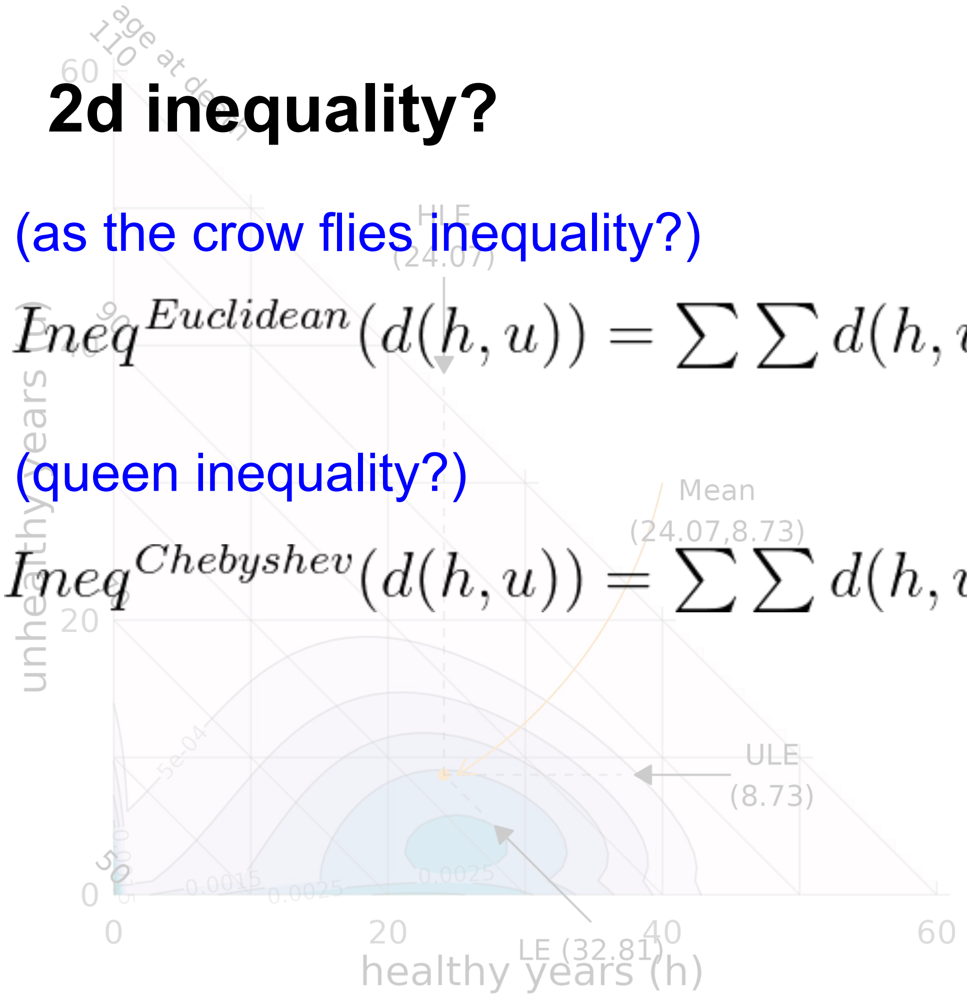
2d inequality?

(as the crow flies inequality?)

$$Ineq^{Euclidean}(d(h,u)) = \sum\sum d(h,u) \cdot \sqrt{(HLE-h)^2 + (ULE-u)^2}$$

(queen inequality?)

$$Ineq^{Chebyshev}(d(h,u)) = \sum\sum d(h,u) \cdot argmax(|h-HLE|, |u-ULE|)$$



2d inequality?

(as the crow flies inequality?)

$$Ineq^{Euclidean}(d(h,u)) = \sum\sum d(h,u) \cdot \sqrt{(HLE-h)^2 + (ULE-u)^2}$$

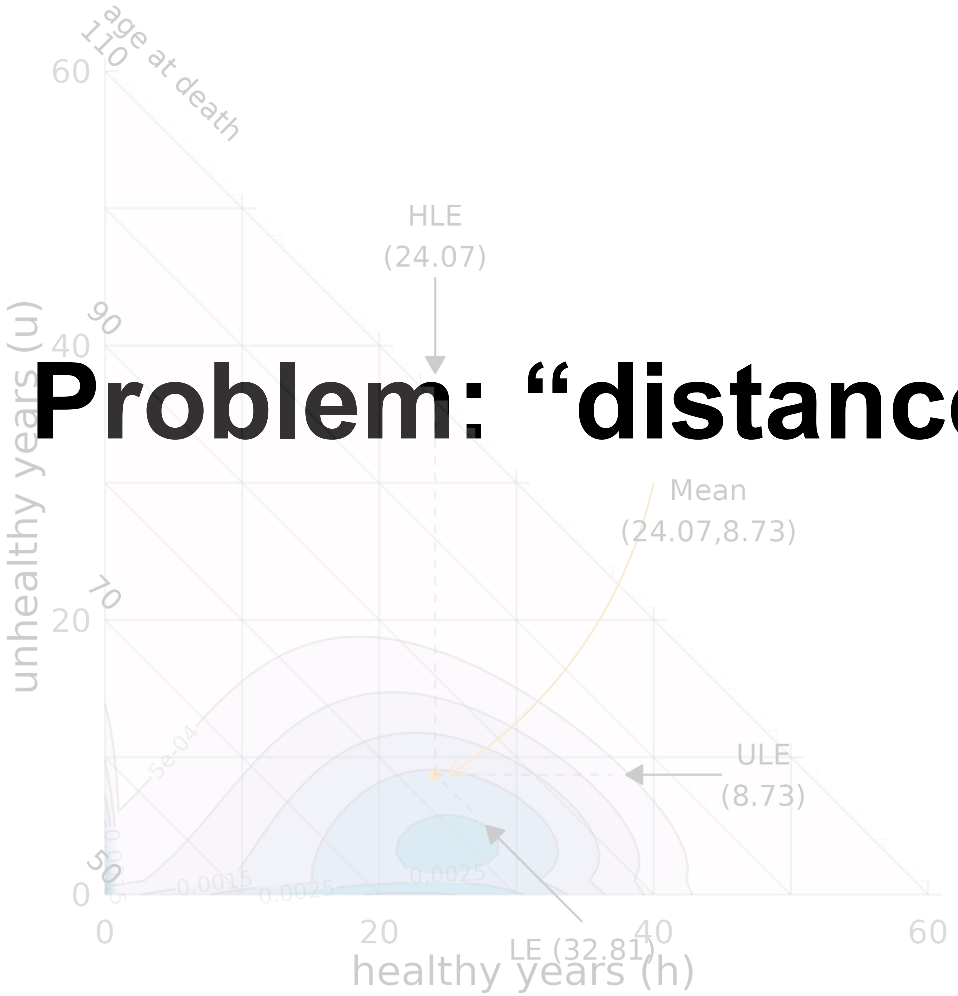
(queen inequality?)

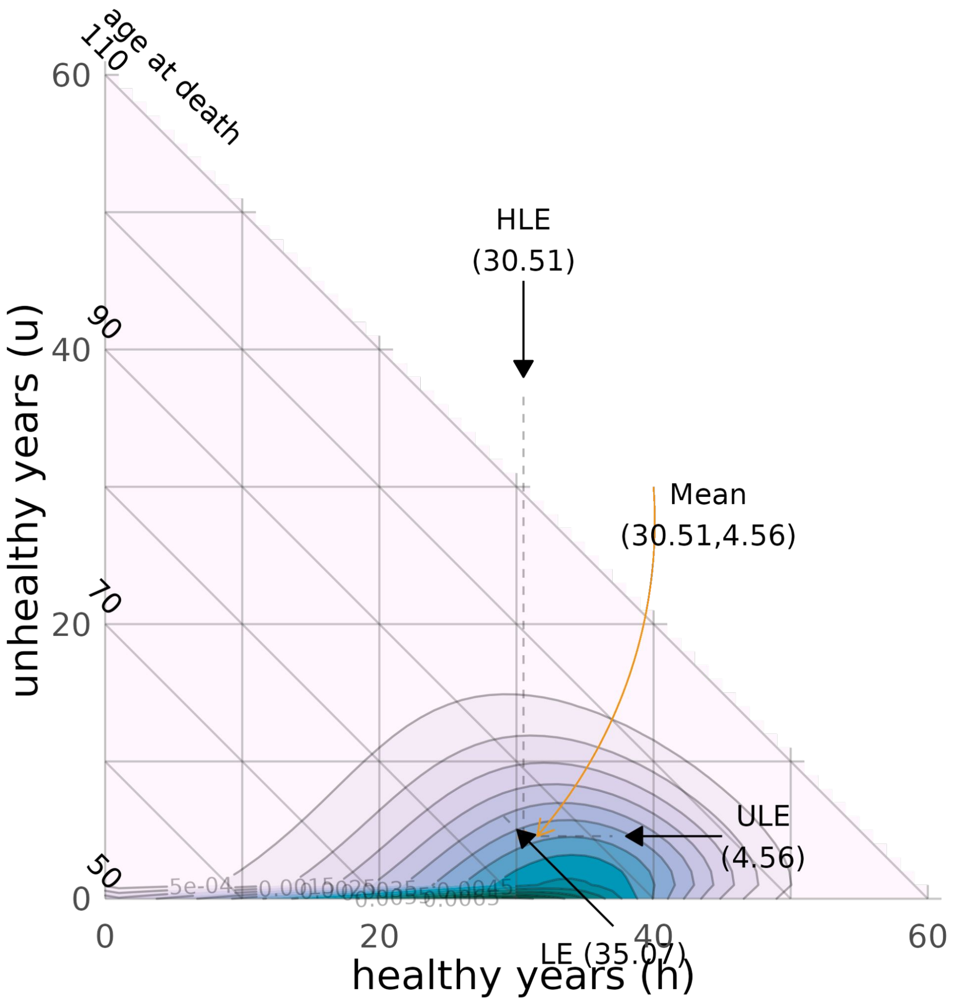
$$Ineq^{Chebyshev}(d(h,u)) = \sum\sum d(h,u) \cdot argmax(|h-HLE|, |u-ULE|)$$

(rook inequality? Minimum transition swaps)

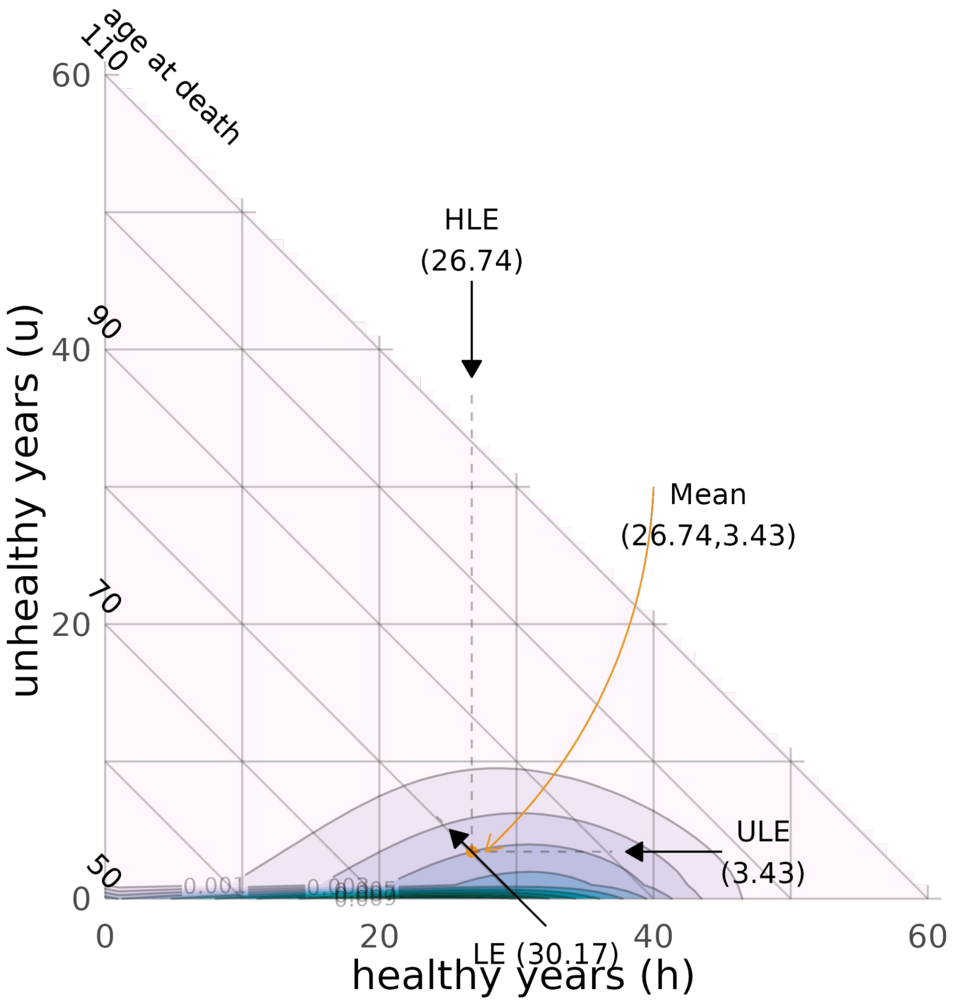
$$Ineq^{Manhattan}(d(h,u)) = \sum\sum d(h,u) \cdot (|HLE-h| + |ULE-u|)$$

Problem: “distance” unclear in 2d





Example: Activities of Daily Living (0 or 1+)
Transitions estimated from US Health and Retirement Study data. Female strata.



Example: Activities of Daily Living (0 or 1+)
Transitions estimated from US Health and
Retirement Study data. Male strata.

Questions to you

(i) Lifetable-style inequality from a 2d (or higher order) death distribution?

(ii) Stick with distribution statistics on the marginal distributions, but note the variance-covariance relationship.

(iii) An $e^{\dagger}$-style metric? These would be decomposable in interesting ways it seems.

(iv) Does the shape of d(x,h) have a useful message about morbidity compression? Iñaki has thoughts on this!

Thanks! tim.riffe@ehu.eus

(HLE lost due to death)

$$HLE^{\dagger} = \sum_x \ell_x^H \cdot p_x^{H \to \dagger} \cdot HLE_x^H + \ell_x^U \cdot p_x^{U \to \dagger} \cdot HLE_x^U$$

(HLE lost due to deterioration)

$$+ \sum_x \ell_x^H \cdot p_x^{H \to U} \cdot (HLE_x^H - HLE_x^U)$$

(HLE gained due to recovery)

$$- \sum_x \ell_x^U \cdot p_x^{U \to H} \cdot (HLE_x^H - HLE_x^U)$$