

Calculating a state occupancy distribution in multistate settings

MPIDR Population Health Lab Talk - 26 September, 2023

Tim Riffe

Iñaki Permanyer

Rustam Tursun Zade

Magdalena Muszynska-Spielauer



Universidad
del País Vasco
CED
*Centre d'Estudis
Demogràfics*

erman ta zabal zazu



Euskal Herriko
Unibertsitatea



Universidad
del País Vasco

erman ta zabal zazu



Euskal Herriko
Unibertsitatea

ikerbasque
Basque Foundation for Science



MAX-PLANCK-INSTITUT
FÜR DEMOGRAFISCHE
FORSCHUNG



Wittgenstein Centre
FOR DEMOGRAPHY AND
GLOBAL HUMAN CAPITAL
A COLLABORATION OF IASA, VID/OHAK, WU

A multistate health model gives a multistate death distribution

MPIDR Population Health Lab Talk - 26 September, 2023

Tim Riffe

Iñaki Permanyer

Rustam Tursun Zade

Magdalena Muszynska-Spielauer



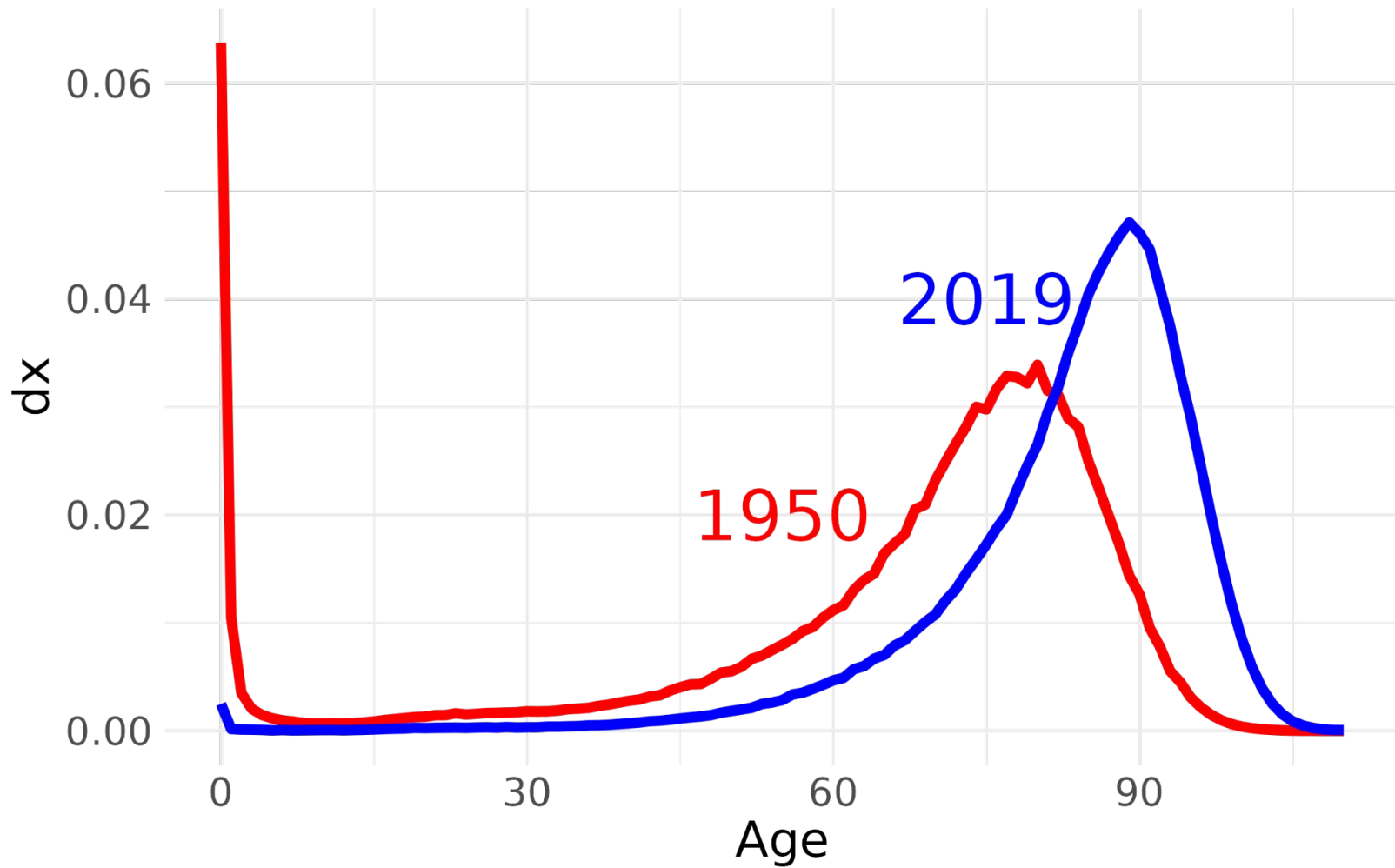
Wittgenstein Centre
FOR DEMOGRAPHY AND
GLOBAL HUMAN CAPITAL
A COLLABORATION OF IASA, VID/OAK, WU

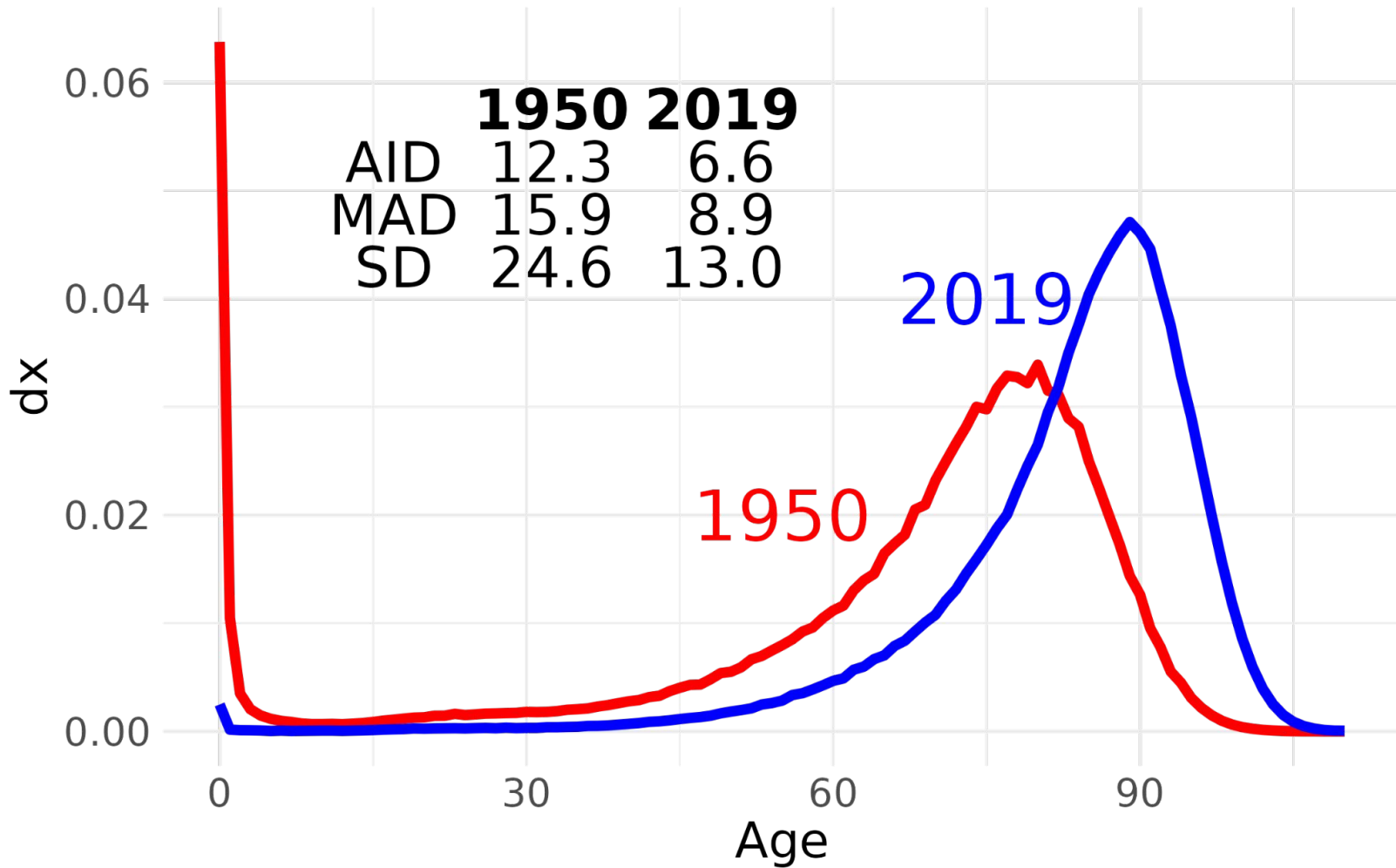


ikerbasque
Basque Foundation for Science



MAX-PLANCK-INSTITUT
FÜR DEMOGRAFISCHE
FORSCHUNG





Prevalence-based approximations

Caswell and Zarulli *Population Health Metrics* (2018) 16:8
<https://doi.org/10.1186/s12963-018-0165-5>

Population Health Metrics

RESEARCH

Open Access



Matrix methods in health demography: a new approach to the stochastic analysis of healthy longevity and DALYs

Population Health Metrics

Hal Caswell^{1*}  and Virginia Zarulli²

RESEARCH

Open Access



On the measurement of healthy lifespan inequality

Iñaki Permanyer^{1,2*} , Jeroen Spijker¹ and Amand Blanes¹



Wittgenstein Centre
FOR DEMOGRAPHY AND
GLOBAL HUMAN CAPITAL

WIFO

AUSTRIAN INSTITUTE OF
ECONOMIC RESEARCH

opik

iberbasque

Healthy lifespan statistics derived from cross-sectional prevalence data using Sullivan's method are informative summary measures of population health

Magdalena Muszyńska-Spielauer*, Tim Riffe**, Martin Spielauer †

Incidence-based approximations



DEMOGRAPHIC RESEARCH

A peer-reviewed, open-access journal of population sciences

New Methods for Analyzing Active Life Expectancy

SARAH B. LADITKA, PhD

State University of New York Institute of Technology–Utica/Rome

DOUGLAS A. WOLF, PhD

Syracuse University

(1998) *Journal of Aging and Health*, 10(2)

DEMOGRAPHIC RESEARCH

VOLUME 45, ARTICLE 13, PAGES 397–452

PUBLISHED 29 JULY 2021

<http://www.demographic-research.org/Volumes/Vol45/13/>

DOI: 10.4054/DemRes.2021.45.13

Research Article

Healthy longevity from incidence-based models: More kinds of health than stars in the sky

Hal Caswell

Silke van Daalen

Incidence-based approximations

The first three moments suffice to calculate the mean, variance, and skewness of healthy longevity:

$$E(\tilde{\rho}) = \tilde{\rho}_1 \quad (18)$$

$$V(\tilde{\rho}) = \tilde{\rho}_2 - (\tilde{\rho}_1 \circ \tilde{\rho}_1) \quad (19)$$

$$SD(\tilde{\rho}) = \sqrt{V(\tilde{\rho})} \quad (20)$$

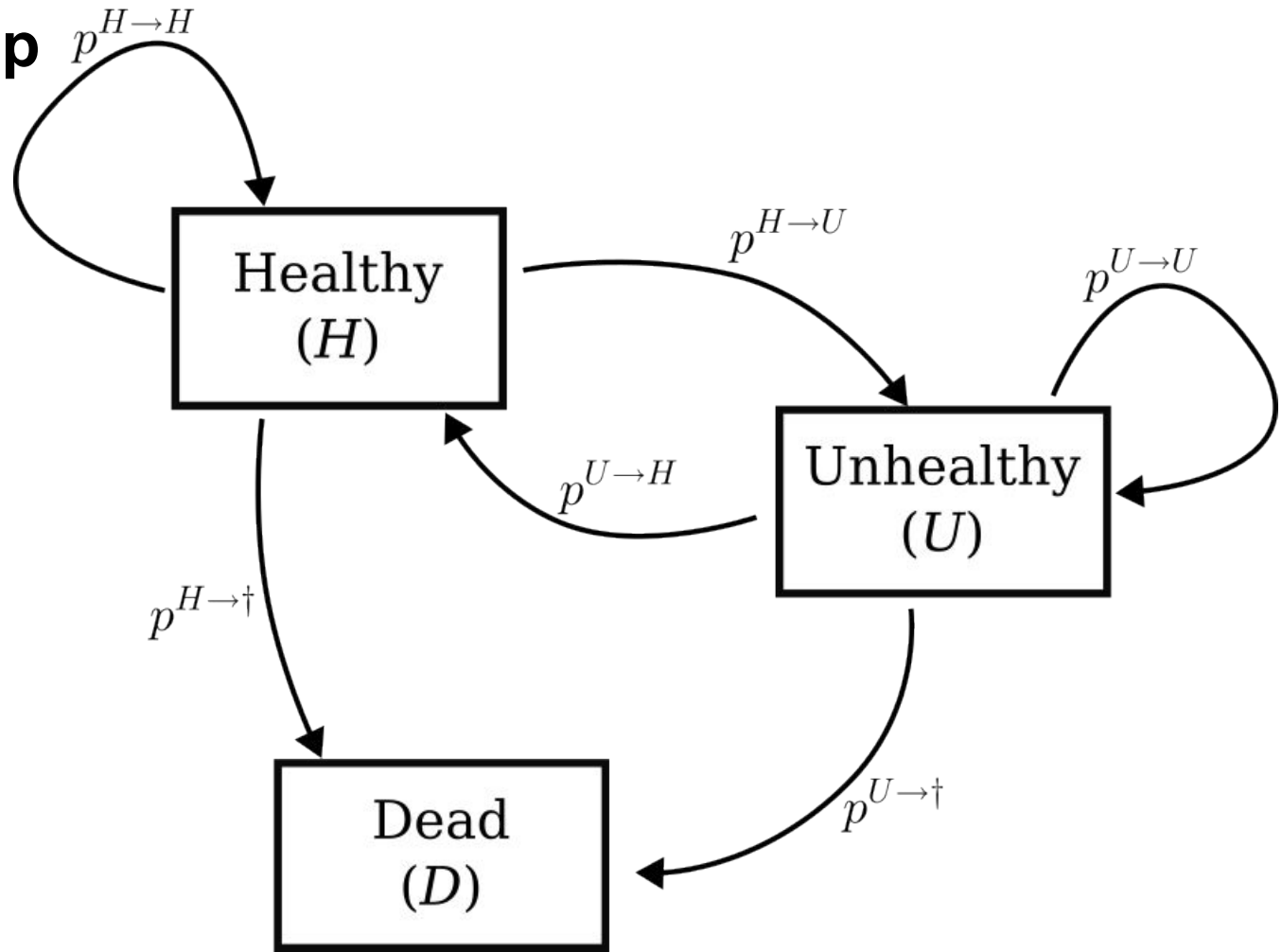
$$CV(\tilde{\rho}) = \mathcal{D}(\tilde{\rho}_1)^{-1} SD(\tilde{\rho}) \quad (21)$$

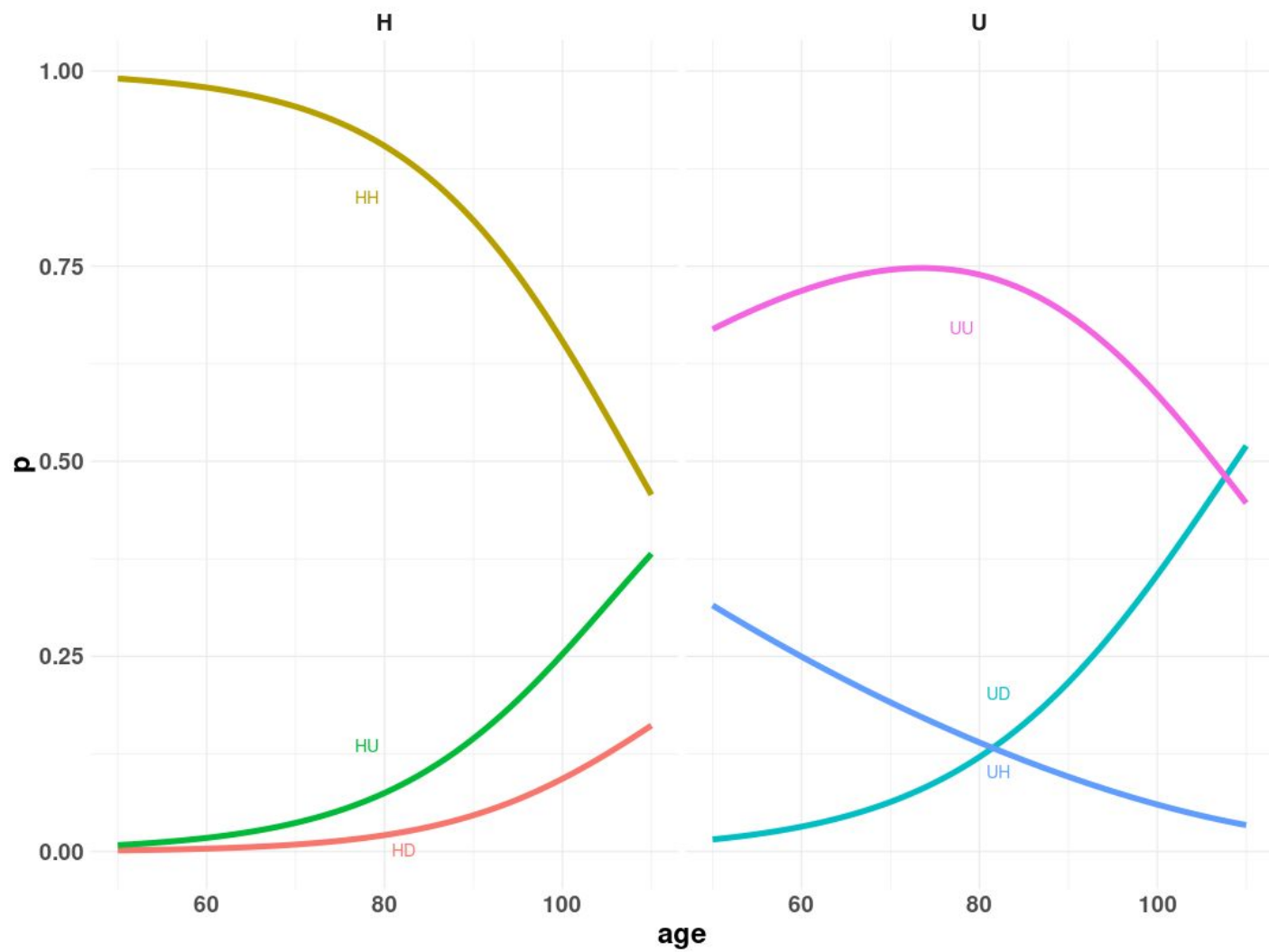
$$Sk(\tilde{\rho}) = \mathcal{D}[V(\tilde{\rho})]^{-3/2} (\tilde{\rho}_3 - 3\tilde{\rho}_1 \circ \tilde{\rho}_2 + 2\tilde{\rho}_1 \circ \tilde{\rho}_1 \circ \tilde{\rho}_1). \quad (22)$$

The vector $\tilde{\rho}_m$ contains the m th moments of healthy longevity for all combinations of initial age and health stage. To obtain the moments of healthy longevity as a function

from Caswell & van Daalen (2021)

Basic setup





Basic setup

$$\ell_{x+1}^H = \ell_x^H \cdot p_x^{H \rightarrow H} +$$

(remain healthy) +

$$\ell_x^U \cdot p_x^{U \rightarrow H}$$

(return to health)

Basic setup

$$\ell_{x+1}^H = \ell_x^H \cdot p_x^{H \rightarrow H} + \ell_x^U \cdot p_x^{U \rightarrow H}$$

(remain healthy) +
(return to health)

(and similarly for unhealthy people)

$$\ell_{x+1}^U = \ell_x^U \cdot p_x^{U \rightarrow U} + \ell_x^H \cdot p_x^{H \rightarrow U}$$

Basic setup

$$HLE = \sum \ell_x^H$$

$$ULE = \sum \ell_x^U$$

Extending to age and duration: stocks

$$\ell^H(x+1, h+1) = \ell^H(x, h) \cdot p_x^{H \rightarrow H} + \ell^U(x, h) \cdot p_x^{U \rightarrow H}$$

(remain healthy) +
(return to health)

Extending to age and duration: stocks

$$\ell^H(x+1, h+1) = \ell^H(x, h) \cdot p_x^{H \rightarrow H} + \ell^U(x, h) \cdot p_x^{U \rightarrow H}$$

(remain healthy) +
(return to health)

$$\ell^U(x+1, h) = \ell^H(x, h) \cdot p_x^{H \rightarrow U} + \ell^U(x, h) \cdot p_x^{U \rightarrow U}$$

(health deterioration) +
(remain unhealthy)

Extending to age and duration: stocks

$$\ell(x, h) = \ell^H(x, h) + \ell^U(x, h)$$

$$\ell(x) = \sum_h \ell(x, h)$$

$$C(x, h) = \frac{\ell(x, h)}{LE}$$

(stationary age-duration structure)

Extending to age and duration: deaths

$$d(x, h) = \ell^H(x, h) \cdot p_x^{H \rightarrow \dagger} + \ell^U(x, h) \cdot p_x^{U \rightarrow \dagger}$$

(die while healthy) +
(die while unhealthy)

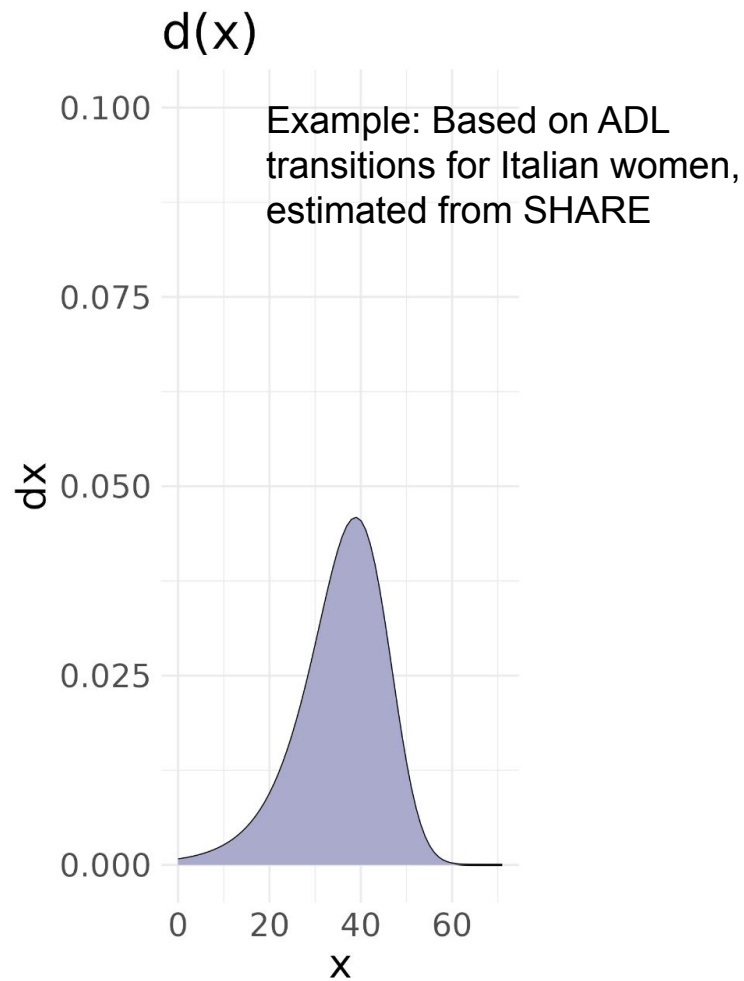
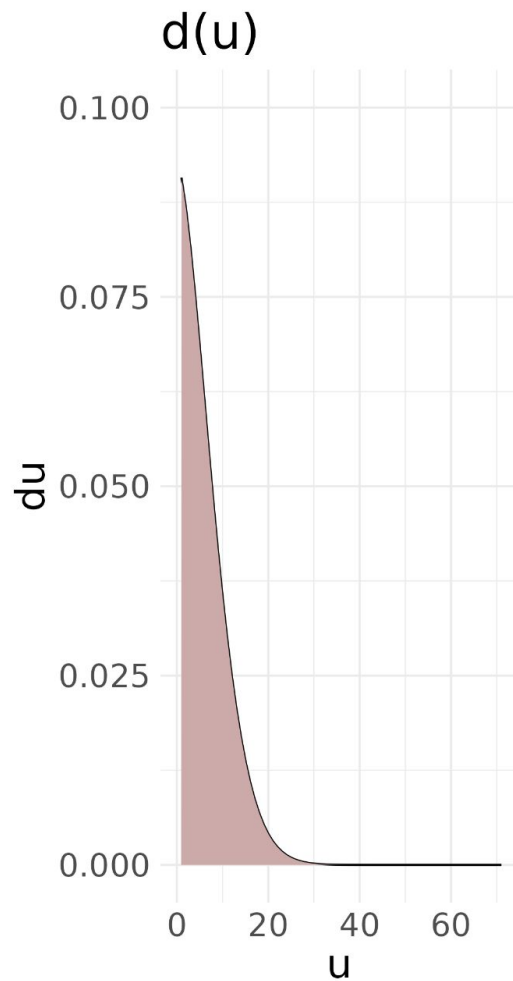
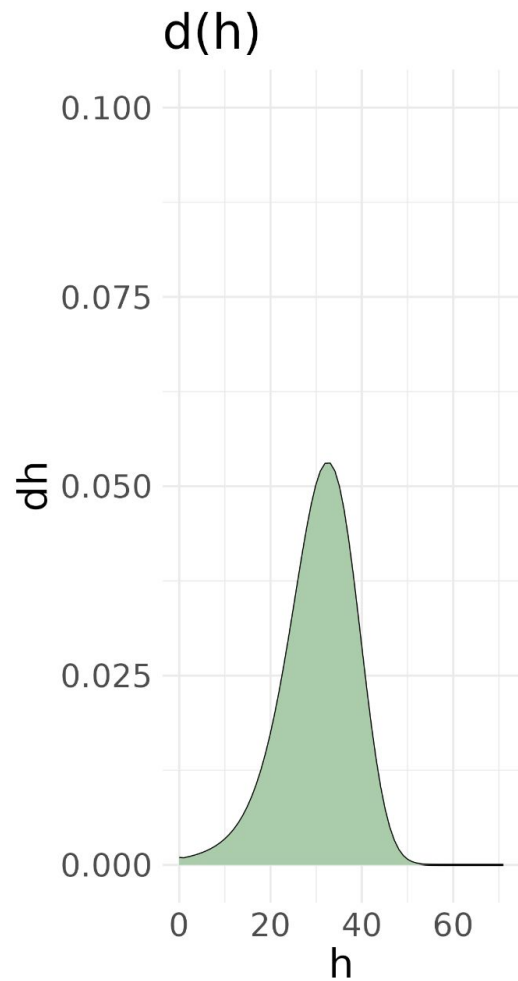
Extending to age and duration: deaths

$$d(x, h) = \ell^H(x, h) \cdot p_x^{H \rightarrow \dagger} + \ell^U(x, h) \cdot p_x^{U \rightarrow \dagger}$$

(die while healthy) +
(die while unhealthy)

A 2d death distribution!

$$1 = \sum_x \sum_h d(x, h)$$



d(h)

d(u)

d(x)

0.100

0.100

0.100

Example: Based on ADL transition
for Italian women, estimated from
SHARE

The first three moments suffice to calculate the mean, variance, and skewness of healthy longevity:

0.075

$$E(\tilde{\rho}) = \tilde{\rho}_1 \quad (18)$$

$$V(\tilde{\rho}) = \tilde{\rho}_2 - (\tilde{\rho}_1 \circ \tilde{\rho}_1) \quad (19)$$

$$SD(\tilde{\rho}) = \sqrt{V(\tilde{\rho})} \quad (20)$$

$$CV(\tilde{\rho}) = \mathcal{D}(\tilde{\rho}_1)^{-1} SD(\tilde{\rho}) \quad (21)$$

$$Sk(\tilde{\rho}) = \mathcal{D}[V(\tilde{\rho})]^{-3/2} (\tilde{\rho}_3 - 3\tilde{\rho}_1 \circ \tilde{\rho}_2 + 2\tilde{\rho}_1 \circ \tilde{\rho}_1 \circ \tilde{\rho}_1). \quad (22)$$

dh

0.050

0.025

The vector $\tilde{\rho}_m$ contains the m th moments of healthy longevity for all combinations of initial age and health stage. To obtain the moments of healthy longevity as a function

0.000

0

20

40

60

h

0.000

0

20

40

60

u

0.000

0

20

40

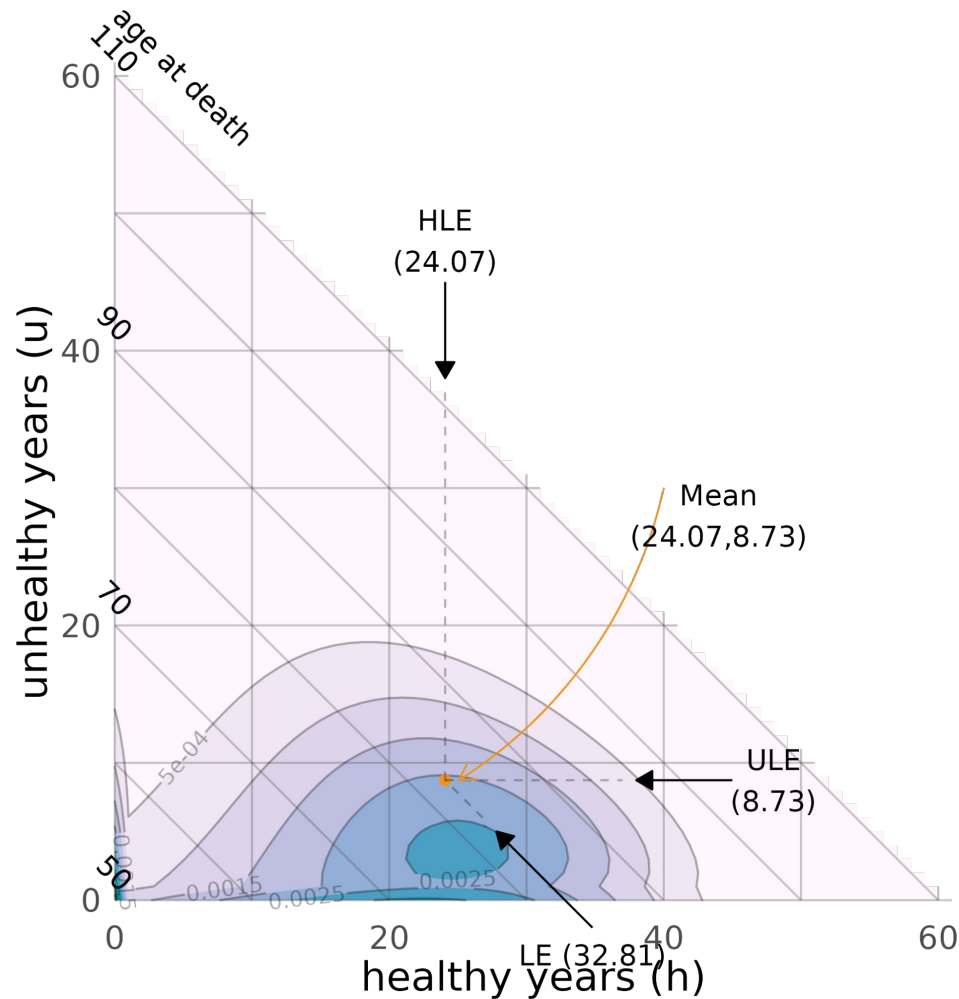
60

x

Relationship between marginal death distributions

$$Var(x) = Var(h) + Var(u) + 2 \cdot Cov(h, u)$$

$$122 = 110 + 44 - 2 \cdot 16$$

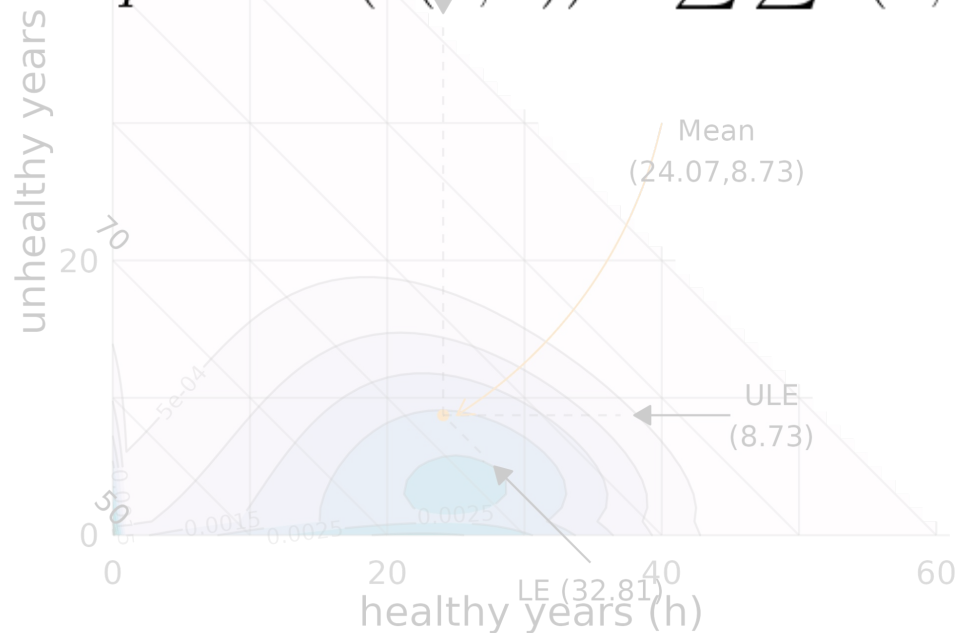


Example: Self-reported health (2 categories)
Transitions recycled from Foltyn & Olsson
(2021), based on US Health and Retirement
Study data. Female "non-black" strata.

2d inequality?

(as the crow flies inequality?)

$$Ineq^{Euclidean}(d(h, u)) = \sum \sum d(h, u) \cdot \sqrt{(HLE - h)^2 + (ULE - u)^2}$$



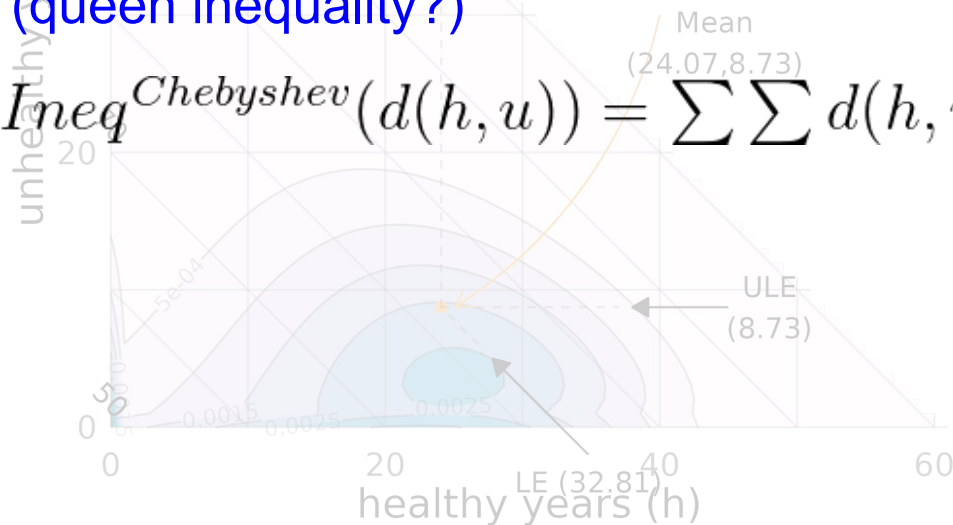
2d inequality?

(as the crow flies inequality?)

$$Ineq^{Euclidean}(d(h, u)) = \sum \sum d(h, u) \cdot \sqrt{(HLE - h)^2 + (ULE - u)^2}$$

(queen inequality?)

$$Ineq^{Chebyshev}(d(h, u)) = \sum \sum d(h, u) \cdot \operatorname{argmax}(|h - HLE|, |u - ULE|)$$



2d inequality?

(as the crow flies inequality?)

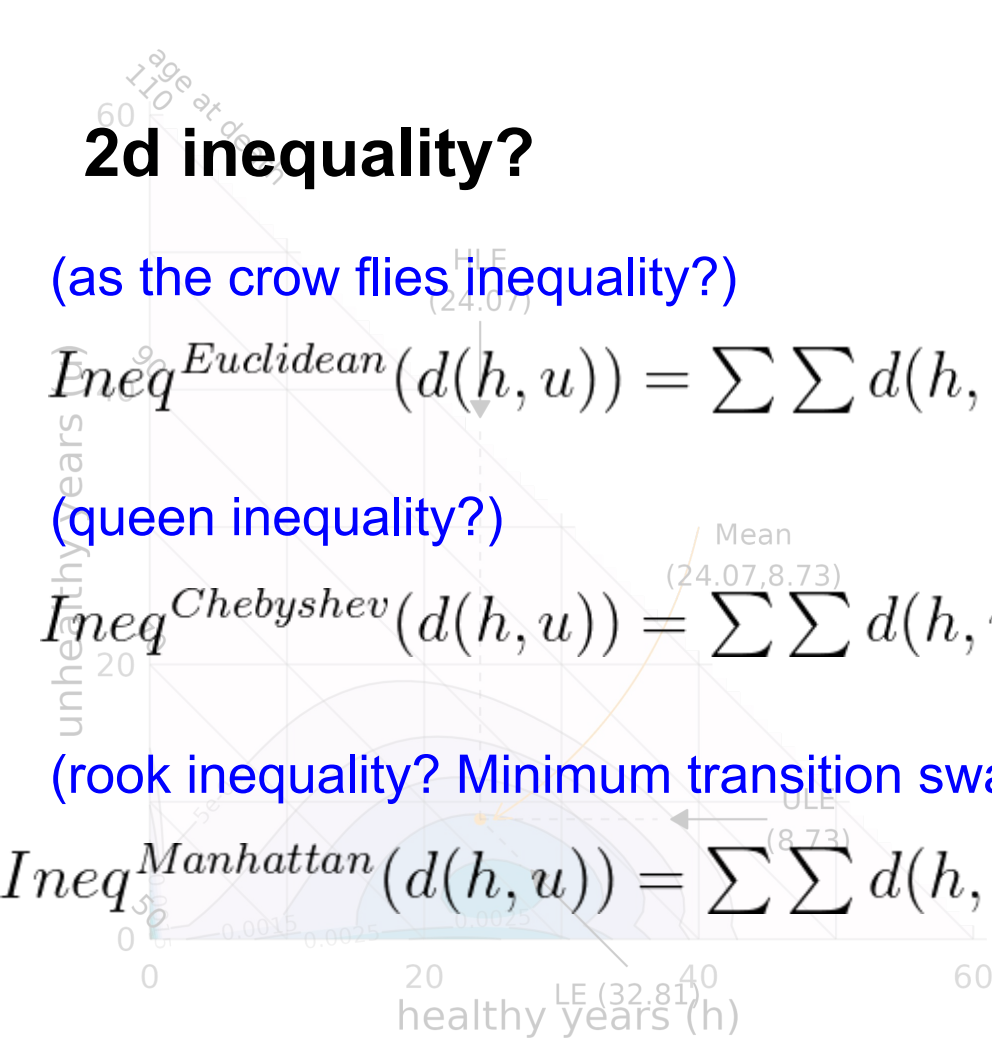
$$Ineq^{Euclidean}(d(h, u)) = \sum \sum d(h, u) \cdot \sqrt{(HLE - h)^2 + (ULE - u)^2}$$

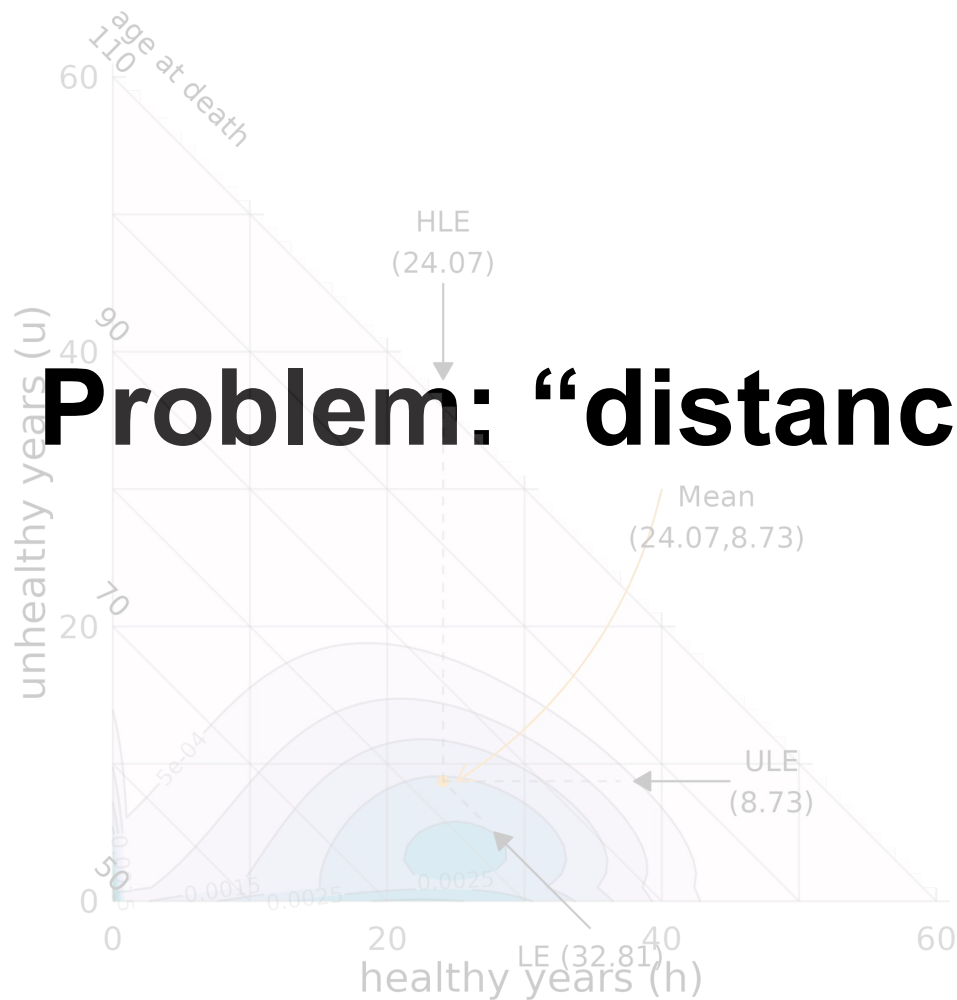
(queen inequality?)

$$Ineq^{Chebyshev}(d(h, u)) = \sum \sum d(h, u) \cdot \operatorname{argmax}(|h - HLE|, |u - ULE|)$$

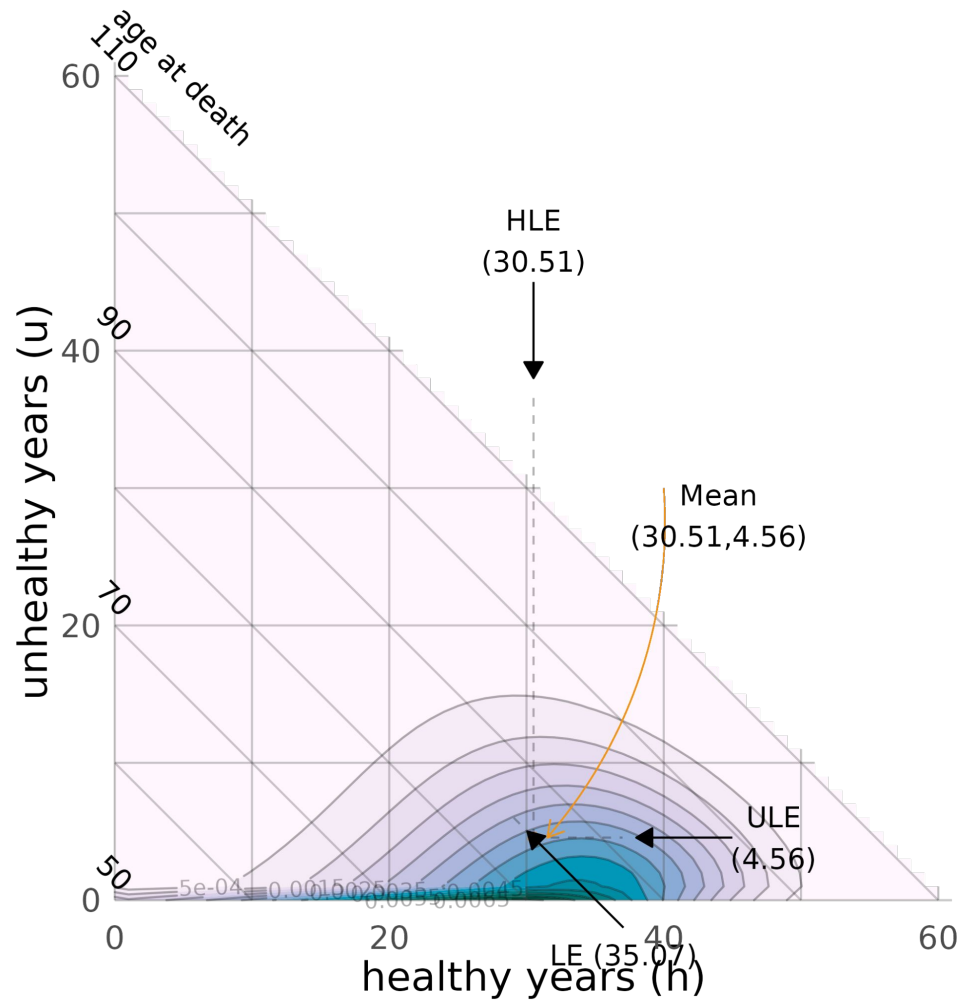
(rook inequality? Minimum transition swaps)

$$Ineq^{Manhattan}(d(h, u)) = \sum \sum d(h, u) \cdot (|HLE - h| + |ULE - u|)$$

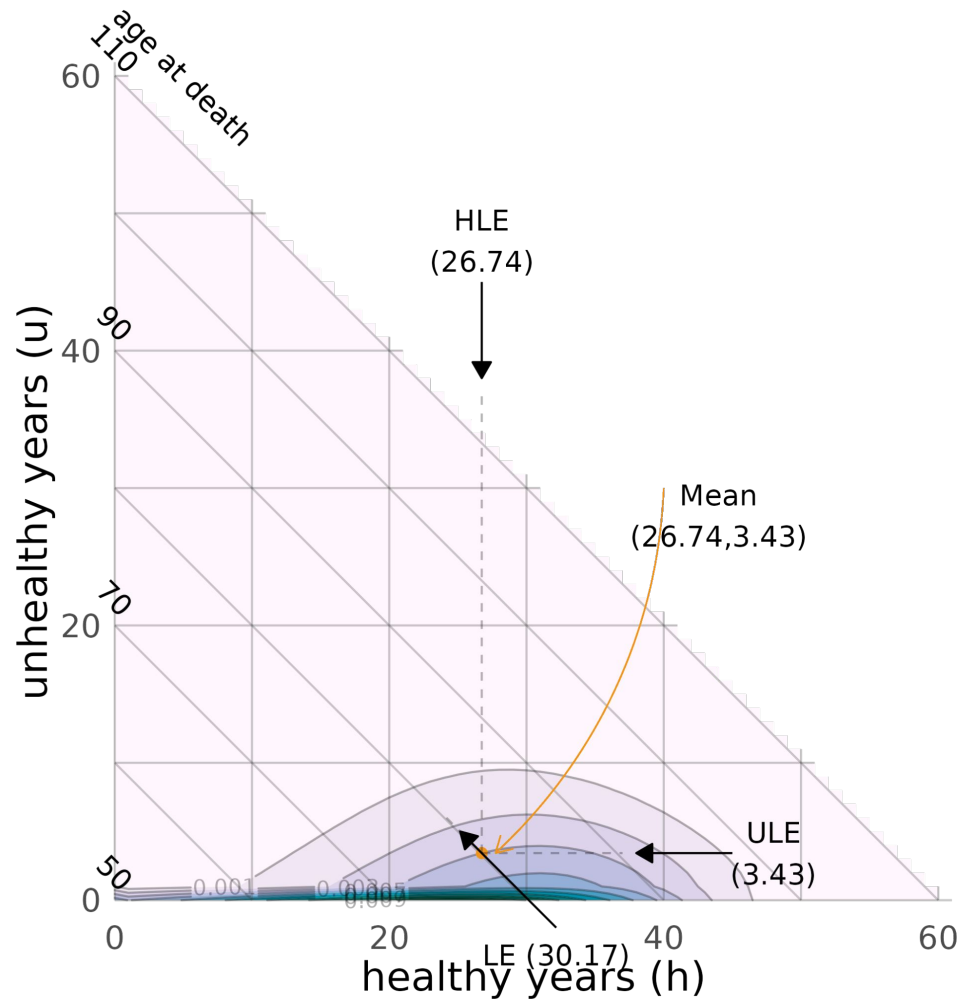




Problem: “distance” unclear in 2d



Example: Activities of Daily Living (0 or 1+)
Transitions estimated from US Health and Retirement Study data. Female strata.



Example: Activities of Daily Living (0 or 1+)
Transitions estimated from US Health and Retirement Study data. Male strata.

Questions to you

- (i) Lifetable-style inequality from a 2d (or higher order) death distribution?
- (ii) Stick with distribution statistics on the marginal distributions, but note the variance-covariance relationship.
- (iii) An e^+ -style metric? These would be decomposable in interesting ways it seems.
- (iv) Does the shape of $d(x,h)$ have a useful message about morbidity compression? Iñaki has thoughts on this!

Thanks!

tim.riffe@ehu.eus

(HLE lost due to death)

$$HLE^{\dagger} = \sum_x \ell_x^H \cdot p_x^{H \rightarrow \dagger} \cdot HLE_x^H + \ell_x^U \cdot p_x^{U \rightarrow \dagger} \cdot HLE_x^U$$

(HLE lost due to deterioration)

$$+ \sum_x \ell_x^H \cdot p_x^{H \rightarrow U} \cdot (HLE_x^H - HLE_x^U)$$

(HLE gained due to recovery)

$$- \sum_x \ell_x^U \cdot p_x^{U \rightarrow H} \cdot (HLE_x^H - HLE_x^U)$$



Save the Date!

25-27 Sept. 2024, Bilbao

Photo: Yves Alerie on unsplash