ASEN 6008 - Final Project

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1 Introduction

This work investigates the VEEMJ trajectory to Saturn departing in 2026. The VEEMJS trajectory is fairly tricky to solve for as it contains a multi-revolution leg which could occur in three potential places (E-VGA, VGA-EGA1, EGA2-MGA). The trajectory also contains an Earth resonance orbit with multiple variations (2:1, 3:1, etc). The strategy goal for this mission is to find a trajectory which is low in both C3 and arrival V_∞ and minimizes the change in V_∞ at each flyby. While this will cause the time of flight to be relatively high, it allows the greatest mass budget for science experiments as necessary fuel will be minimized. Typically missions will not have a flight time this long, but it is an interesting excercise to find a trajectory to Saturn with a C3 below that needed for a Mars transfer.

Initial search was conducted using a brute force algorithm looking for various permutations of dates and location and type of the multi-rev leg as well as variations in the Earth resonance. A Type III or IV multi-rev trajectory between launch and VGA or VGA to EGA1 are preferred over the EGA2 to MGA leg due to the time required. This narrowed the search to a Type III multi-rev between VGA and EGA1 and a 3:1 Earth resonance launching around summer of 2026. Once found, numerical optimization using FMINCON and genetic algorithms were tried, but the FMINCON algorithm proved too succeptable to local optima and a genetic algorithm was used for the remainder of the trajectory search. Given an early August launch date and increases bounds for each of the flybys, the genetic algorithm was able to quickly solve for a trajectory minimizing the cost function below within just a few minutes.

$$J = 5\Delta V_{\infty,\text{total}} + 2C3 + V_{\infty,\text{Saturn}} \tag{1}$$

The cost function can be adjusted to return optimal trajectories emphasizing time of flight, $V_{\infty,\text{Saturn}}$ or other parameters as necessary.

1.1 Selected Trajectory

After optimization, the following trajectory was selected:

Launch	08-Aug-2026 02:00:24
C3	8.8785 (km^2/s^2)
$V_{\infty,\text{Saturn}}$	5.0923 km/s
Time of Flight	12.9833 years
$\Delta V_{\infty,\text{total}}$	17.293 m/s

Table 1: Selected Trajectory

While this is a long trajectory, current and past technology has allowed the completion of missions in excess of 13 years. The low C3 and low $\Delta V_{\infty,\text{total}}$ will allow for additional hardware to ensure the spacecraft's survival. One challenge will be powering the spacecraft due to the typical RTG decay of 4.8% per year.

2 Porkchop and Resonance Plots

The porkchop plots around the computed trajectory as well as the EGA resosonance plot are shown below. Porkchop plots are useful to show what combinations of V_∞, time of flight, or other parameters may exist but they do not guarantee that a particular trajectory does not impact the flyby planet.

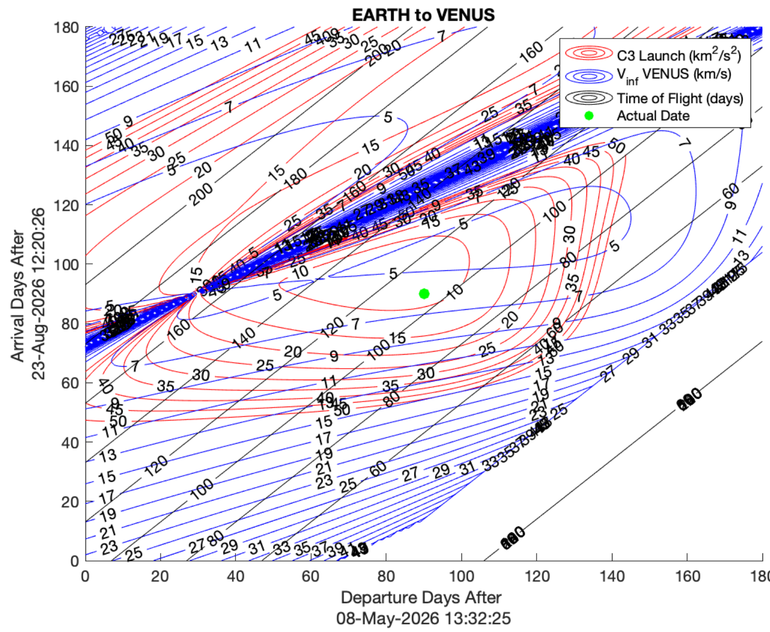

Figure 1: Launch to VGA

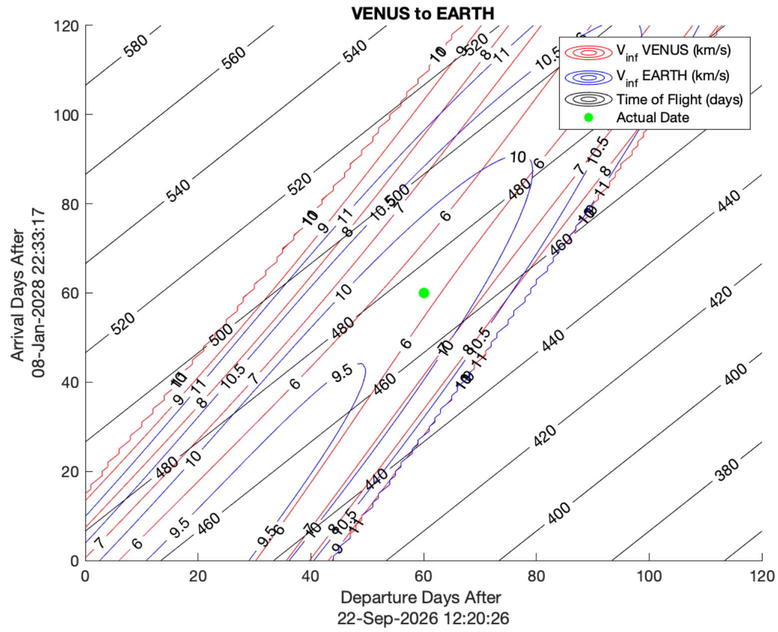

Figure 2: VGA to EGA1

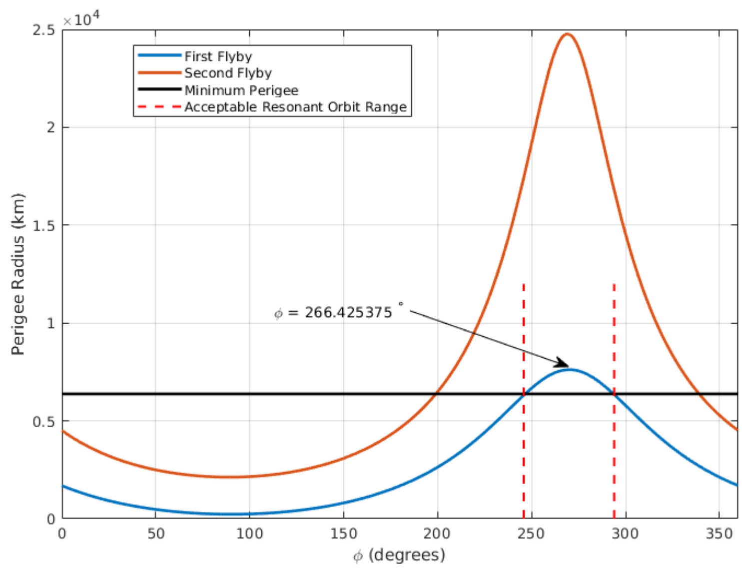

Figure 3: Earth Resonance

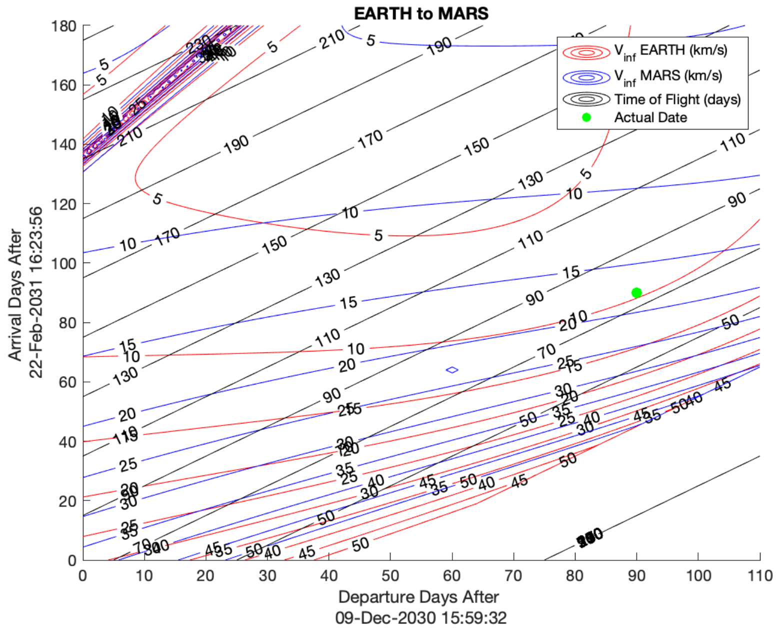

Figure 4: EGA2 to MGA

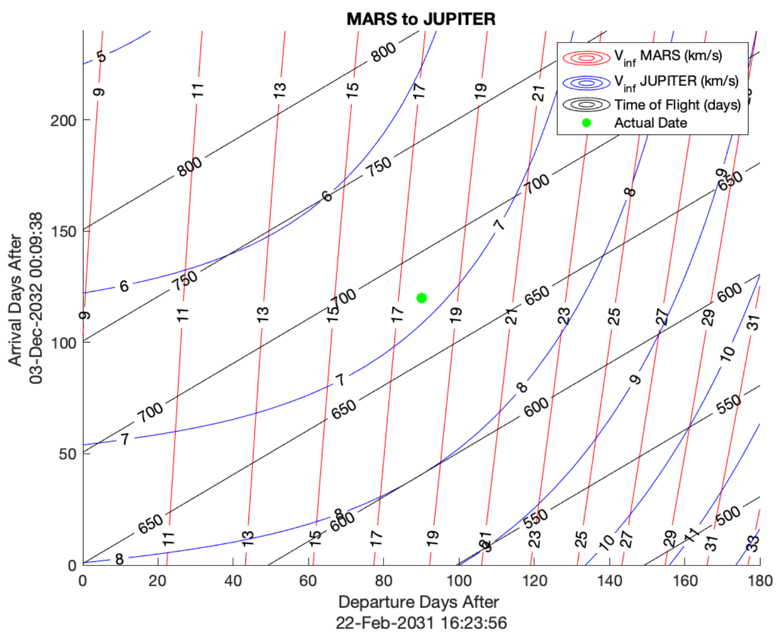

Figure 5: MGA to JGA

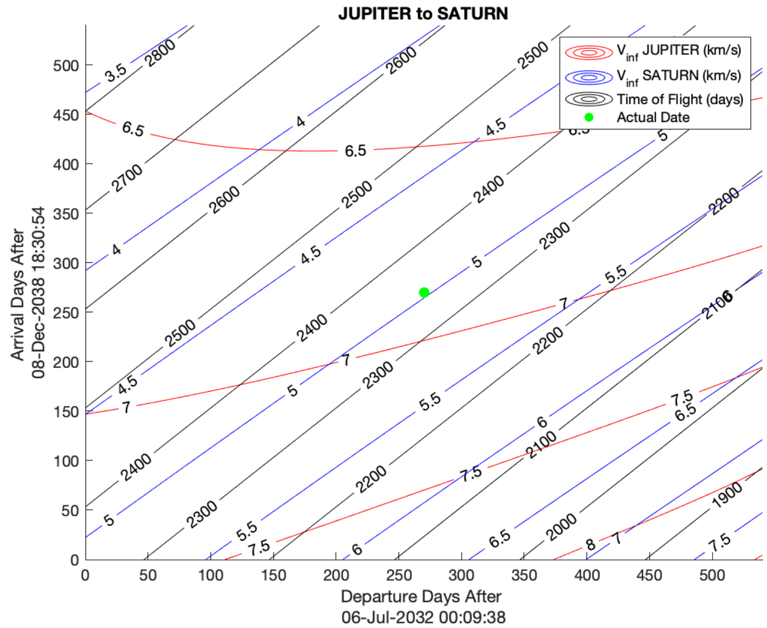

Figure 6: JGA to Saturn

3 Computed Values

The parameters of the MATLAB genetic algorithm solution are shown below and used as an initial guess for the high-fidelity GMAT computations.

3.1 Launch

Date = 08-Aug-2026 02:00:24
C3 = 8.8785(km^2/s^2)
RLA = -146.5967°
DLA = -0.2574°

3.2 VGA

Date = 21-Nov-2026 15:50:28
R_P = 7048.2082 km
h_P = 996.4082 km
B_R = -3627.430286 km
B_T = -13052.686747 km
$V_{\infty,-}$ = 5.84906497 km/s
$V_{\infty,+}$ = 5.84906497 km/s
$\Delta V_\infty = 6.90430824\times10^{-10}$ km/s

3.3 EGA1 - (Multi-Rev Type III from Venus)

Date = 08-Mar-2028 17:26:06
R_P = 7582.3396 km
h_P = 1204.1996 km

B_R = -804.903086 km
B_T = 11045.886486 km
$V_{\infty,-}$ = 9.63096103 km/s
$V_{\infty,+}$ = 9.63096100 km/s
ΔV_∞ = 2.80231482×10^{-08} km/s
ϕ = 266.425375 ° 3:1 Resonance

3.4 EGA2

Date = 09-Mar-2031 10:52:21
R_P = 24616.0192 km
h_P = 18237.8792 km
B_R = 3632.155139 km
B_T = 28373.999774 km
$V_{\infty,-}$ = 9.61366851 km/s
$V_{\infty,+}$ = 9.63096100 km/s
ΔV_∞ = 17.29249 m/s

3.5 MGA

Date = 23-May-2031 12:01:52
R_P = 6798.0127 km
h_P = 3401.8227 km
B_R = -3227.061600 km
B_T = 6135.145630 km
$V_{\infty,-}$ = 17.78480568 km/s
$V_{\infty,+}$ = 17.78480568 km/s
ΔV_∞ = 2.67117883×10^{-10} km/s

3.6 JGA

Date = 25-Mar-2033 05:04:34
R_P = 326566.1711 km
h_P = 255074.1711 km
B_R = -36557.029282 km
B_T = -1348602.755828 km
$V_{\infty,-}$ = 6.94918488 km/s
$V_{\infty,+}$ = 6.94918488 km/s
ΔV_∞ = 8.77789397×10^{-10} km/s

3.7 Saturn

Date = 02-Aug-2039 05:54:12
V_∞ = 5.0923 km/s
ΔV SOI = Theoretical 649.478 m/s , GMAT 594.139 m/s (see SOI discussion below)
Inclination = 63.1226°
Period = 62.6966 (days)
R_P = 87,500 km
R_A = 6×10^6 km
a = 3.043828 ×10^6 km
e = 0.9712

4 Launch

Given a C3 of 8.8785(km^2/s^2) the theoretical magnitude of the velocity change necessary to transfer to Venus (TVI) from a 300km parking orbit can be calculated using patched conics by finding the difference in velocities of the spacecraft relative to the Earth in its parking orbit and its hyperbolic escape velocity relative to the Earth.

$$V_{\mathrm{LEO}} = \sqrt{\frac{\mu_{\mathrm{Earth}}}{R_{\mathrm{LEO}}}} = 7.72576 \text{ (km/s)} \tag{2}$$

$$V_{\mathrm{hyp}} = \sqrt{\frac{2\mu_{\mathrm{Sun}}}{R_{\mathrm{LEO}}} + \mathrm{C3}} = 11.3249 \text{ (km/s)} \tag{3}$$

$$\Delta V_{\mathrm{TVI}} = 3.59913 \text{ (km/s)} \tag{4}$$

This is within a few m/s of the GMAT calculated burn magnitude is 3.59657 km/s.

5 Delta-V_∞

Since a perfect flyby should have no change in V_∞, any difference in the Lambert approxiamate solution will likely be magnified in the high-fidelity GMAT simulation and real spacecraft trajectory. Thus the theoretical V_∞ should be as small as possible. The benefit of using a genetic algorithm for optimization is that it can easily drive the ΔV_∞ to zero if a high weight is used in the cost function. The exception of EGA2 which does not seem to drop below 17 m/s. All others were on the order of 10^{-8} km/s or lower.

6 Flyby Radii

The flyby radii for each gravity assist are well above the atmosphere for each planet with the exception of VGA. The VGA flyby altitude is 996 km relatively close to Venus's atmosphere which extends to an altitude of 250 km. This should not pose a problem provided the spacecraft's heliocentric orbit is well known.

Since this is a deep space mission it will likely be powered by an RTG and care must be taken to elminate any probability of collision with the Earth during EGA1 or EGA2. The lowest flyby is EGA1 with an altitude of 1204 km. This is above Galileo's flyby at 965 km, but the uncertainty in this distance will need to be further examined to guarantee a safe flyby.

7 Saturn Orbit Insertion

7.1 Orbit Selection

Similar to the Cassini mission which flew through a gap between Saturn's F and G rings, this mission will fly through a gap in Saturn's rings just 270 km wide known as the Maxwell gap centered at 87,500 km from the center of the planet [2]. The trajectory from Jupiter arrives at Saturn in a high inclination which makes the crossing easier to calcuate. The initial apoapsis target is 6,000,000 km which reduces the ΔV required (see next section) due to the highly eccentric orbit. From this initial orbit, Saturn's rings can be examined and the orbit can be further reduced to conduct passes near Saturn's moons, fuel permitting.

7.2 SOI ΔV

The theoretical ΔV required for SOI can be computed using the patched conics method. First compute the semi-major axis for an elliptical orbit with periapsis of radius of 87,500 km and apoapsis of 6,000,000 km. Then calcuate the velocity at periapsis for this elliptical orbit around Saturn.

$$a_{\text{elliptical}} = \frac{R_p + R_a}{2} \tag{5}$$

$$V_{\text{p, elliptical}} = \left(\frac{2\mu_{\text{Saturn}}}{R_{\text{p}}} - \frac{\mu_{\text{Saturn}}}{a_{\text{elliptical}}} \right)^{1/2} \tag{6}$$

Then compute the semi-major axis and velocity at periapsis for a hyperbolic orbit at the same point using the V_∞ found earlier using the trajectory search algorithm.

$$a_{\text{hyp}} = \frac{-\mu_{\text{Saturn}}}{V_\infty^2} \tag{7}$$

$$V_{\text{p, hyp}} = \left(\frac{2\mu_{\text{Saturn}}}{R_{\text{p}}} - \frac{\mu_{\text{Saturn}}}{a_{\text{hyp}}} \right)^{1/2} \tag{8}$$

$$\Delta V_\infty = V_{\text{p, elliptical}} - V_{\text{p, hyp}} = -649.478 \text{ m/s} \tag{9}$$

This is greater than the GMAT calculated burn of 594.139 m/s due to the GMAT trajectory arriving about one month later than predicted using the trajectory search algorithm.

8 GMAT Trajectory

The software GMAT provides high-fidelity trajectory computations precise enough to navigate a spacecraft. However, it requires a good initial guess to converge on a solution and can't be used alone for interplanetary missions. This section shows the computed GMAT solution and required TCMs.

8.1 Screenshots

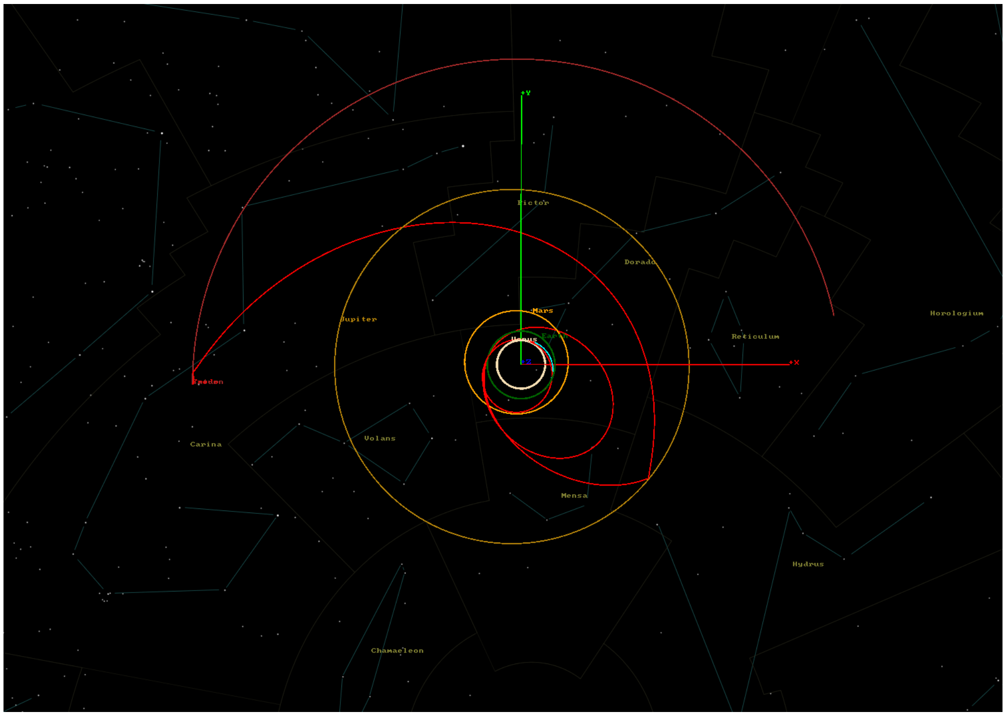

Figure 7: Overall Trajectory

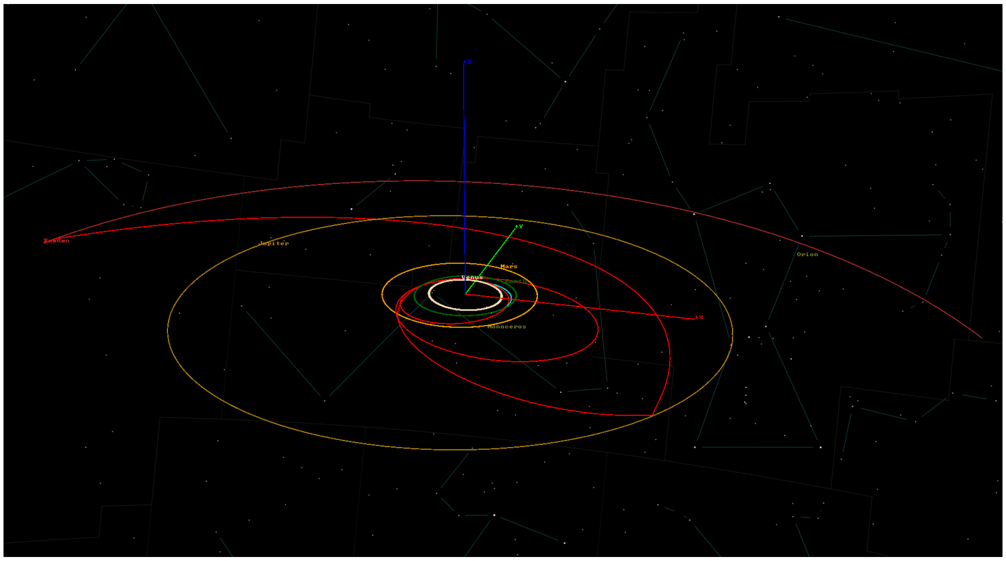

Figure 8: Overall Trajectory

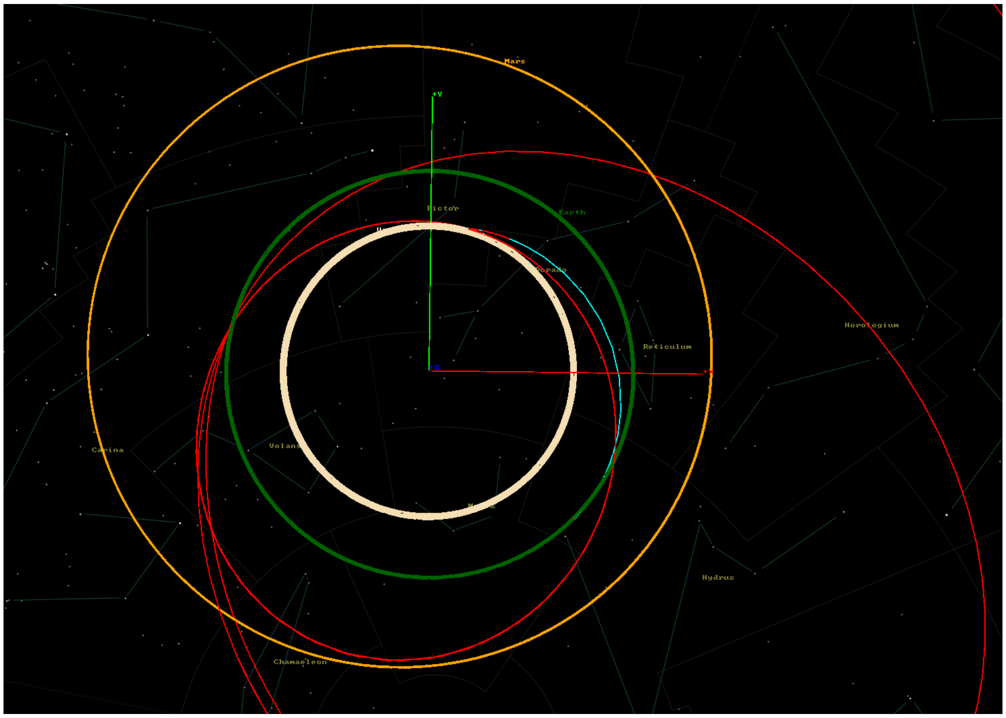

Figure 9: Inner Trajectory

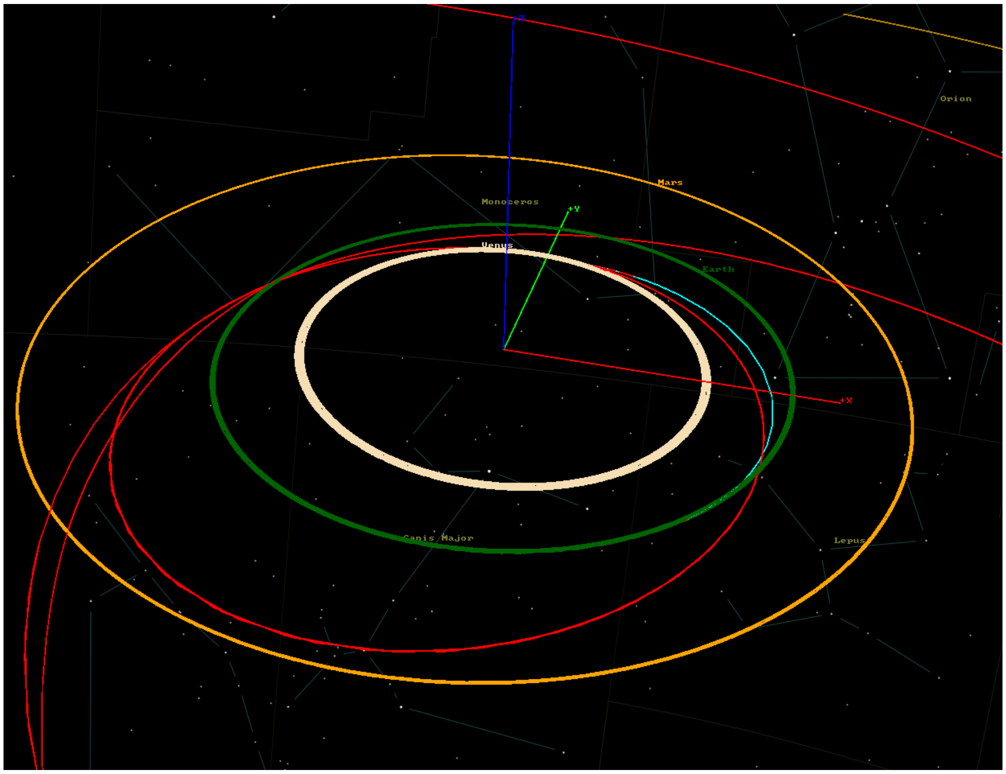

Figure 10: Inner Trajectory

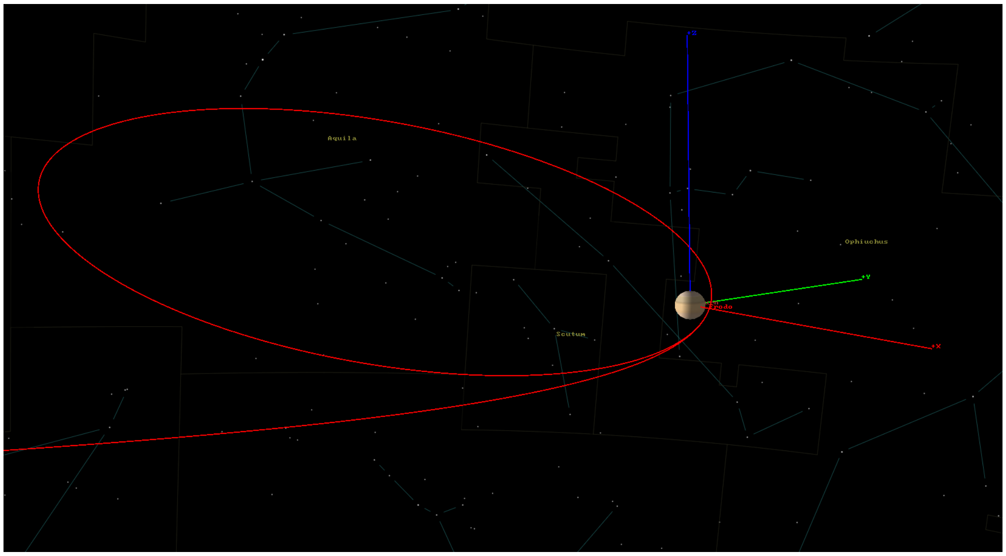

Figure 11: Saturn Initial Orbit

8.2 GMAT Maneuvers

This trajectory utilizes eight TCMs prior to SOI. Venus B-Plane parameters are targeted from launch resulting in no need for a TCM between Earth and Venus for the nominal trajectory and adjusted launch parameters, C3 = 8.8113 km^2/s^2, RLA = -146.2927°, DLA = -0.1972°. Not having a TCM prior to Venus is obviously an unrealistic expectation for a launch vehicle but works for the nominal GMAT scenario and an additional VGA B-plane TCM will just increase the ΔV used in the mission in the GMAT scenario.

A TCM occurs between 10 and 15 days prior to each flyby targeting the next gravity assist in the sequence. This greatly reduces the magnitude of each burn by utilizing the flyby planet's gravity. The inner planets have a second TCM after each flyby which refines the initial course correction occuring prior to the flyby. The total TCM ΔV required is 201.26 m/s and the overall ΔV required to arrive and complete orbit insertion is 795.40 m/s.

The GMAT solution arrives at Saturn a little over a month from the MATLAB solution which is not unusual given the approxiamate solution of the ephemerides used by the MATLAB algorithms. The TCMs and SOI burn parameters are shown in Table 2.

Maneuver	Target	Calendar Date	Julian Date	ΔV Components (m/s)	ΔV Magnitude (m/s)
1 V-EGA1	EGA1 B-Plane	11 Nov 2026 13:34:51	2461356.06586815	2.3183 -3.1389 -8.0682	8.9624
2 V-EGA1-2	EGA1 B-Plane	01 Dec 2026 12:00:00	2461376	2.1925e-04 7.2806e-04 4.7719e-05	7.2806e-04
3 E-EGA2	Earth RMAG	27 Feb 2028 12:00:00	2461829	0.4839 0.5079 -0.2573	0.7472
4 E-EGA2-2	EGA2 B-Plane	07 Jun 2028 12:00:00	2461930	0.1580 -4.8714 1.6507	5.1459
5 E-MGA	Mars RMAG	27 Feb 2031 03:00:19	2462924.62522555	-1.6030 30.8863 -10.6976	32.7257
6 E-MGA-2	MGA B-Plane	18 Mar 2031 12:00:00	2462944	-28.7862 2.4351 7.5703	29.8644
7 M-JGA	JGA B-Plane	08 May 2031 12:01:52	2462995.00129665	-58.5948 -5.7486 -66.4943	88.8138
8 J-SAT	Saturn Rp	08 Mar 2033 12:00:00	2463665	26.9662 -6.3862 21.3782	34.9998
9 SOI	Ra	12 Sep 2039 11:38:09	2466043.98482997	-594.1386	594.1386

Table 2: TCMs and SOI Maneuver

9 Trajectory Comparison

9.1 Hohmann Transfer

Use of gravitational assists greatly reduced the ΔV required to reach Saturn. To compare this trajectory to an ideal Hohmann Transfer using patched conics, first compute the heliocentric arrival and departure velocities of the transfer ellipse.

$$a_{\text{transfer}} = \frac{R_{\text{Earth}} + R_{\text{Saturn}}}{2} \tag{10}$$

$$V_{\text{dep}} = \left(\frac{2\mu_{\text{Sun}}}{a_{\text{Earth}}} - \frac{\mu_{\text{Sun}}}{a_{\text{transfer}}}\right)^{1/2} \qquad V_{\text{arr}} = \left(\frac{2\mu_{\text{Sun}}}{a_{\text{Saturn}}} - \frac{\mu_{\text{Sun}}}{a_{\text{transfer}}}\right)^{1/2} \tag{11}$$

Next compute the departure and arrival velocity differences V_∞ from Earth and Saturn.

$$V_{\infty,\text{dep}} = V_{\text{dep}} - V_{\text{Earth}} \qquad V_{\infty,\text{arr}} = V_{\text{arr}} - V_{\text{Earth}} \tag{12}$$

With both V_∞s find the hyperbolic arrival and departure velocities of the spacecraft with respect to Earth and Saturn.

$$a_{\text{hyp, Earth}} = -\frac{-\mu_{\text{Earth}}}{V_{\infty,\text{dep}}^2} \qquad a_{\text{hyp, Saturn}} = -\frac{-\mu_{\text{Saturn}}}{V_{\infty,\text{arr}}^2} \tag{13}$$

$$V_{\text{dep, hyp}} = \left(\frac{2\mu_{\text{Earth}}}{R_{\text{LEO}}} - \frac{\mu_{\text{Earth}}}{a_{\text{hyp, Earth}}}\right)^{1/2} \qquad V_{\text{arr, hyp}} = \left(\frac{2\mu_{\text{Saturn}}}{R_{\text{p, Saturn}}} - \frac{\mu_{\text{Saturn}}}{a_{\text{hyp, Saturn}}}\right)^{1/2} \tag{14}$$

The magnitude of the ΔV required at departure and arrival is the difference of the spacecraft hyperbolic velocity from its initial Earth orbit and final Saturn orbit. The result is much greater than the initial trans-Venus injection and Saturn orbit insertion burns calculated earlier.

$$\Delta V_{\text{dep}} = V_{\text{dep, hyp}} - \left(\frac{\mu_{\text{Earth}}}{R_{\text{LEO}}}\right)^{1/2} \qquad \Delta V_{\text{arr}} = V_{\text{arr, hyp}} - \left(\frac{\mu_{\text{Saturn}}}{R_{\text{p, Saturn}}} - \frac{\mu_{\text{Saturn}}}{a_{\text{ellip, Saturn}}}\right)^{1/2} \tag{15}$$

$$\Delta V_{\text{Total}} = |\Delta V_{\text{dep}}| + |\Delta V_{\text{arr}}| = 7.99 \text{ (km/s)} \tag{16}$$

9.2 Other Missions

While this trajectory is quite long from launch to SOI, the required C3 is much lower than other missions like Cassini which had a C3 of 18.1. Cassini also utilized a deep space maneuver of 466 m/s which is not required for this mission [3]. The C3 for this mission is even lower than typical Mars mission such as Insight in 2016 which had a C3 of 11.9 - 17.7 during its 23 day launch window [1]. The low C3 and low ΔV enroute as well as SOI allows more mass for the spacecraft which could be used for additional science experiments and/or fuel for a prolonged mission around Saturn and its moons.

References

[1] Fernando Abilleira, Ray Frauenholz, Ken Fujii, Mark Wallace, and Tung-Han You. 2016 mars insight mission design and navigation. 2014.

[2] Michael Meltzer. *The Cassini-Huygens visit to Saturn: an historic mission to the ringed planet*. Springer, Cham, 2015 edition, 2015.

[3] F. Peralta and S. Flanagan. Cassini interplanetary trajectory design. *Control Engineering Practice*, 3(11):1603 – 1610, 1995.