

Semantic Search for Quantity Expressions

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Overview

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- ▶ Motivation: Problem and State Of The Art

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- ▶ Our Approach: Structure Of The Search Engine

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- ▶ The Implementation
- ▶ Time for Questions

Motivation (1)

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- ▶ everything is quantified

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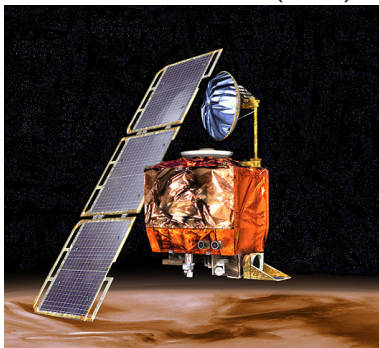
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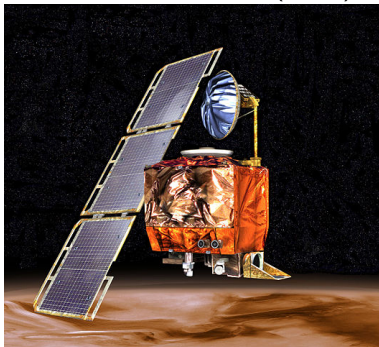
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- ▶ Different Units are a big problem

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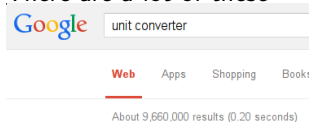
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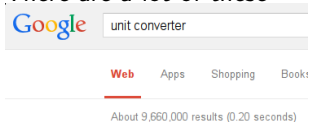
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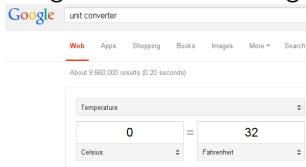


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- ▶ Google itself has one integrated



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- ▶ This is the kind of search engine we have built

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- ▶ For the first two points, we use a meta-mathematical approach

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- ▶ *MMT* is a software that allows us to make use of these concepts
- ▶ It is easy to write down your own theories and related them with views

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- ▶ We can use this to get a *Theory of Dimensions*:

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Dimension		
dim	:	type
none	:	dim
count	:	dim
length	:	dim
mass	:	dim
time	:	dim
current	:	dim
temperature	:	dim
luminous	:	dim
amount	:	dim
.	:	dim \rightarrow dim \rightarrow dim
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 5. The *sum* of two existing QEs (of the same dimension)
 6. The *quotient* of two existing QEs such as $1 \frac{\text{Meter}}{\text{Second}}$
- ▶ this results in the following *Theory of Quantity Expressions*:

Our Approach: The Unit System (5)

Quantity Expression	
import Dimension	
QE	: $\text{dim} \rightarrow \text{type}$
QENMul	: $\forall x : \text{dim}. \mathbb{R} \rightarrow \text{QE}(x) \rightarrow \text{QE}(x)$
QENDiv	: $\forall x : \text{dim}. \text{QE}(x) \rightarrow \mathbb{R} \rightarrow \text{QE}(x)$
QEAdd	: $\forall x : \text{dim}. \text{QE}(x) \rightarrow \text{QE}(x) \rightarrow \text{QE}(x)$
QEMul	: $\forall x, y : \text{dim}. \text{QE}(x) \rightarrow \text{QE}(y) \rightarrow \text{QE}(x \cdot y)$
QEDiv	: $\forall x, y : \text{dim}. \text{QE}(x) \rightarrow \text{QE}(y) \rightarrow \text{QE}\left(\frac{x}{y}\right)$

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Thou : QE (length)	
Foot	= QENMul (1000, Thou)
Yard	= QENMul (3, Foot)
Chain	= QENMul (22, Yard)
Furlong	= QENMul (10, Chain)
Mile	= QENMul (8, Furlong)

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- ▶ Here is a small part of it

Our Approach: The Unit System (6)

