Chased by the Cops

The Blues Brothers' Car Chase

Timothy Lin & Joseph Lui

Atholton 3rd Period

Scene — Chased by the Cops

In the script, this scene is described as "The Great Car Chase."

In the film, the chase occurs from 2:00:48 to 2:03:42.

In the isolated clip, the moment of interest occurs from 49s to 53s.

Note: *Links to the youtube video of the chase scene, as well as particular significant moments, are located in Appendix A.*

In this scene, the blues brothers are being pursued by a fury of police cars in their '74 Dodge Monaco, the Bluesmobile. After they race around a bridge/tunnel system, they accelerate out of the tunnel on the entrance ramp and unrealistically fly into the air over a police car, just grazing it. Slightly tilted down on their left, they come back down and bounce significantly. The rest of the scene depicts their high speed chase through the streets of Chicago; at the end, after the blues brothers swerve to avoid a police car, a dozen or so police cars crash together into a huge pile and civic disorder ensues.

Differential Equation Model

The motion of the car following the impulse can be modeled by the IVP

$$my'' + \gamma y' + ky = F(t); y(0) = 0, y'(0) = 5.42 \text{ m/s} \tag{1}$$

where

$$m = 2218.49 \text{ kg}$$

$$\lambda = 26816.67 \text{ kg/s}$$

$$k = 87913.69 \text{ N/m}$$

$$F(t) = 48094.91 * \delta(t)$$

OR

$$F(t) = \begin{cases} 48094.91, & 0 \leq t \leq 0.25 \\ 0, & t > 0.25 \end{cases}$$

Mass m

The mass of the car is composed of the vehicle curb weight, weight of Dan Aykroyd and John Belushi (the actors playing the blues brothers), and the amount of gas in the car:

From the data for the Dodge Monaco 4-Door Sedan 400 V-8 2-bbl. TorqueFlite

$$\text{vehicle curb weight/mass} = 2000 \text{ kg}$$

Dan Aykroyd's current weight is reported to be 113 kg, but there are many reports that he is currently twice the man he was (in size), so he is literally perhaps 1.5 times the weight he used to be.

$$\text{mass of Dan Aykroyd} = 75.33 \text{ kg}$$

John Belushi's fansite gives his weight as 180-230 lbs. Assuming his weight is monotone and increasing, a good estimate for his weight 1980 is 190 lbs.

$$\text{mass of John Belushi} = 86.18 \text{ kg}$$

The total fuel capacity is 95 liters, and at one point in the chase, the fuel gauge reads approximately 10/12 units full. Given the density of gasoline at about 0.7197 kg/L,

$$\text{mass of gasoline in vehicle} = 56.98 \text{ kg}$$

In sum,

$$\text{total mass of vehicle and contents} = 2218.49\text{kg}$$

Spring Constant k

Based on the MVMA Specifications Form for the 1974 Dodge Monaco, the sum of the spring rates at each wheel is 502 lb/in. In metric/international units, the Monaco has a spring constant of 87913.69 N/m.

Damping Constant γ

The car's damping constant should be approximately set at a value where the car is critically damped with a 45 kg passenger and no gas in the car.
The springs are critically damped given $\gamma = 2\sqrt{km}$, where $k = 87913.69$ N/m and $m = 2045$ kg, then

$$\begin{aligned}\gamma &= 2\sqrt{87913.69 \text{ kg/s}^2 * 2045 \text{ kg}} \\ &= 26816.67 \text{ kg/s}\end{aligned}$$

Initial Conditions *y(0), y'(0)*

The car at the moment of the impulse is right on the ground, with no vertical displacement from $y = 0$ as defined by the ground.

The car reaches peak height right above the police car it jumps over, where it shears off the alert light of the police car. The police cars in that time period were all the Ford Custom 4-door Sedan Police Interceptor 428 V-8 Cruise-O-Matic, a common police interceptor model, which has a height of approximately 5 feet, or 1.5 meters.

Using the conservation of mechanical energy $PE_0 + KE_0 = PE_1 + KE_1$ with $PE = GPE = mgy$,

$$
\begin{aligned}
2218.49 \text{ kg} * 9.80 \text{ m}/s^2 * 1.5 \text{ m} &= \frac{1}{2} * 2218.49 \text{ kg} * v^2 \\
v^2 &= 2 * 1.5 \text{ m} * 9.80 \text{m}/s^2 \\
v &= -\sqrt{29.4 \text{ m}^2/\text{s}^2} \qquad\qquad = -5.42 m/s
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
y(0) &= 0 \text{ m} \\
y'(0) &= -5.42 \text{ m/s}
\end{aligned}
$$

External, Vertically Discontinuous Force *F(t)*

With a time approximated using a slowed version of the video, the maximum force of impact can be calculated by a change in momentum, where $m = 2218.49$ kg, $v = 5.42$ m/s, and $t \approx 0.25$ s:

$$
\begin{aligned}
\max_t F_{impact} &= \frac{\Delta p}{\Delta t} = \frac{mv}{t} \\
&= 48094.91 \text{ N}
\end{aligned}
$$

Setting $F(0) = F_{impact}$,

$$
F(t) = \begin{cases} 48094.91, & 0 \leq t \leq 0.25 \\ 0, & t > 0.25 \end{cases}
$$

Solution — Characteristic Polynomial & Method of Undetermined Coefficients

Homogeneous Solution by Characteristic Polynomial

Substituting in the values for m, γ, and k into the second order homogeneous equation $my'' + \gamma y' + ky = 0$ and assuming a homogeneous solution of $y = e^{rt}$,

$$
2218.49r^2 + 26816.67r + 87913.69 = 0
$$

$$
\begin{aligned}
r &= \frac{-26816.67 \pm \sqrt{26816.67^2 - 4 * 2218.49 * 87913.69}}{2 * 2218.49} \\
&= -6.0439 \pm 1.76039i
\end{aligned}
$$

$$
y = c_1 e^{-6.04t} \cos 1.76t + c_2 e^{-6.04t} \sin(1.76t)
$$

Nonhomogeneous Solution by Method of Undetermined Coefficients

$$2218.49y'' + 26816.67y' + 87913.69y = \begin{cases} 48094.91, & 0 \leq t \leq 0.25 \\ 0, & t > 0.25 \end{cases}$$

For the impulse region:

$$2218.49y_a'' + 26816.67y_a' + 87913.69y_a = 48094.91$$

Assuming a solution

$$\begin{aligned} y_a &= A \\ Y_a' &= 0 \\ 87913.69A &= 48094.91 \\ A &= 0.5471 \end{aligned}$$

$$y_a = 0.5471$$

$$\begin{cases} y_a = c_1e^{-6.04t}\cos 1.76t + c_2e^{-6.04t}\sin 1.76t + 0.5471 \\ y_a' = -6.04c_1e^{-6.04t}\cos 1.76t - 1.76c_1e^{-6.04t}\sin 1.76t \\ \qquad - 6.04c_2e^{-6.04t}\sin 1.76t + 1.76c_2e^{-6.04t}\cos 1.76t \end{cases}$$

$$\begin{cases} 0 = c_1 + 0.5471 \\ -5.42 = -6.04c_1 + 1.76c_2 \end{cases}$$

$$\begin{aligned} c_1 &= -0.55 \\ c_2 &= -4.96 \end{aligned}$$

$$y_a(t) = -0.55e^{-6.04t}\cos 1.76t - 4.96e^{-6.04t}\sin 1.76t + 0.55,\ \forall\ 0 \leq t \leq 0.25$$

For beyond the impulse region:

$$2218.49y_b'' + 26816.67y_b' + 87913.69y_b = 48094.91$$

The new initial conditions are

$$\begin{aligned} y_a(0.25) &= -0.027\text{ m} \\ y_a'(0.25) &= 1.83\text{ m/s} \end{aligned}$$

Solving for the second equation,

$$\begin{cases} y_b = c_1e^{-6.04t}\cos 1.76t + c_2e^{-6.04t}\sin 1.76t \\ y_b' = -6.04c_1e^{-6.04t}\cos 1.76 - 1.76c_1e^{-6.04t}sin 1.76t \\ \qquad - 6.04c_2e^{-6.04t}\sin 1.76t + 1.76c_2e^{-6.04t}\cos 1.76t \end{cases}$$

$$\begin{cases} -0.027 = c_1e^{-6.04/4}\cos 1.76/4 + c_2e^{-6.04/4}\sin 1.76/4 \\ 1.83 = -6.04c_1e^{-6.04/4}\cos 1.76/4 - 1.76c_1e^{-6.04/4}\sin 1.76/4 \\ \qquad - 6.04c_2e^{-6.04/4}\sin 1.76/4 + 1.76c_2e^{-6.04/4}\cos 1.76/4 \end{cases}$$

$$c_1 = -1.94$$
$$c_2 = 3.83$$
$$y_b(t) = -1.94e^{-6.04t}\cos 1.76t + 3.83e^{-6.04t}\sin 1.76t$$

Combining equations,

$$y(t) = \begin{cases} -0.55e^{-6.04t}\cos 1.76t - 4.96e^{-6.04t}\sin 1.76t + 0.55, & 0 \le t \le 0.25 \\ -1.94e^{-6.04t}\cos 1.76t + 3.83e^{-6.04t}\sin 1.76t, & t < 0.25 \end{cases} \tag{2}$$

Solution — Laplace Transforms

$2218.49y'' + 26816.67y' + 87913.69y = 48094.91 * \delta_0(t); y(0) = 0$ m, $y'(0) = -5.42$ m/s

$$2218.49[s^2Y + 5.42] + 26816.67s^1Y + 87913.69Y = 48094.91$$
$$Y[2218.49s^2 + 26816.67s + 87913.69] + 12024.22 = 48094.91$$

$$\begin{aligned} Y(s) &= \frac{36070.69}{2218.49s^2 + 26816.67s + 87913.69} \\ &= \frac{36070.69}{2218.49[s^2 + 12.09s + 39.63]} \\ &= \frac{36070.69}{2218.49} * \frac{1}{(s+6.04)^2 + 3.10} \\ &= \frac{16.26}{(s+6.04)^2 + 1.76^2} \\ &= \frac{16.26}{1.76} * \frac{1.76}{(s+6.04)^2 + 1.76^2} \\ &= 9.24 * \frac{1.76}{(s+6.04)^2 + 1.76^2} \\ y(t) &= 9.24e^{-6.04t}\sin 1.76t \end{aligned} \tag{3}$$

Solution — Systems of Differential Equations with Undetermined Coefficients

$$2218.49y'' + 26816.67y' + 87913.69y = \begin{cases} 48094.91, & 0 \le t \le 0.25 \\ 0, & t > 0.25 \end{cases}$$

Let $x_1 = y$, $x_1' = x_2 = y'$, and $x_2' = \frac{1}{m}[48094.91 - \gamma x_2 - kx_1]$,

$$\begin{cases} x_1' = x_2 \\ x_2' = -39.6277x_1 - 12.0878x_2 + 21.6791 \end{cases}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = x_1 \begin{pmatrix} 0 \\ -39.6277 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ -12.0878 \end{pmatrix} + \begin{pmatrix} 0 \\ 21.6791 \end{pmatrix}$$
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -39.6277 & -12.0878 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 21.6791 \end{pmatrix}, x(0) = \begin{pmatrix} 0 \\ -5.42 \end{pmatrix}$$

Homogeneous Solution by Eigenanalysis

$$\begin{aligned} \left|A - \lambda I\right| &= 0 \\ \begin{vmatrix} 0-\lambda & 1 \\ -39.6277 & -12.0878-\lambda \end{vmatrix} &= 0 \\ \lambda^2 + 12.0878\lambda + 39.6277 &= 0 \\ \lambda &= -6.0439 \pm 1.7604i \end{aligned}$$

Solving for eigenvector when $\lambda_1 = -6.0439 + 1.7604i$,

$$\begin{pmatrix} 6.0439 - 1.7604i & 1 \\ -39.6277 & -6.0439 - 1.7604i \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\xi^{(1)} = \begin{pmatrix} 0.1506 + 0.0439i \\ -0.9876 \end{pmatrix}$$

$$\begin{aligned} x &= ce^{\lambda_1 t}\xi^{(1)} \\ &= ce^{(-6.0439+1.7604)t} \begin{pmatrix} 0.1506 + 0.0439i \\ -0.9876 \end{pmatrix} \\ &= ce^{-6.0439t}e^{1.7604it} \begin{pmatrix} 0.1506 + 0.0439i \\ -0.9876 \end{pmatrix} \\ &= ce^{-6.0439t}(\cos 1.7604t + i\sin 1.7604t) \begin{pmatrix} 0.1506 + 0.0439i \\ -0.9876 \end{pmatrix} \\ &= ce^{-6.0439t} \begin{pmatrix} 0.1506\cos 1.7604t + 0.1506i\sin 1.7604t + 0.0439i\cos 1.7604t + 0.0439i^2\sin 1.7604t \\ -0.9876\cos 1.7604t - 0.9876i\sin 1.7604t \end{pmatrix} \\ &= ce^{-6.0439t} \begin{pmatrix} 0.1506\cos 1.7604t - 0.0439\sin 1.7604t + i[0.1506\sin 1.7604t + 0.0439\cos 1.7604t] \\ -0.9876\cos 1.7604t - 0.9876i\sin 1.7604t \end{pmatrix} \\ &= c_1 e^{-6.0439t} \begin{pmatrix} 0.1506\cos 1.7604t - 0.0439\sin 1.7604t \\ -0.9876\cos 1.7604t \end{pmatrix} \\ &\quad + c_2 e^{-6.0439t} \begin{pmatrix} 0.1506\sin 1.7604t + 0.0439\cos 1.7604t \\ -0.9876\sin 1.7604t \end{pmatrix} \end{aligned}$$

$$\begin{aligned} x(t) =& c_1 e^{-6.0439t} \begin{pmatrix} 0.1506\cos 1.7604t - 0.0439\sin 1.7604t \\ -0.9876\cos 1.7604t \end{pmatrix} \\ &+ c_2 e^{-6.0439t} \begin{pmatrix} 0.1506\sin 1.7604t + 0.0439\cos 1.7604t \\ -0.9876\sin 1.7604t \end{pmatrix} + x_p(t) \end{aligned}$$

Nonhomogeneous Solution by Method of Undetermined Coefficients

Assuming a particular solution $x_p = \begin{pmatrix} A \\ B \end{pmatrix}$ with $x_p' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$,

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -39.6277 & -12.0878 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} + \begin{pmatrix} 0 \\ 21.6791 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -39.6277 & -12.0878 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ -21.6791 \end{pmatrix}$$

$$\text{rref} \begin{pmatrix} 0 & 1 & 0 \\ -39.6277 & -12.0878 & -21.6791 \end{pmatrix} \rightarrow \begin{matrix} A = 0.5471 \\ B = 0 \end{matrix}$$

$$\begin{aligned} x(t) =& c_1 e^{-6.0439t} \begin{pmatrix} 0.1506 \cos 1.7604t - 0.0439 \sin 1.7604t \\ -0.9876 \cos 1.7604t \end{pmatrix} \\ &+ c_2 e^{-6.0439t} \begin{pmatrix} 0.1506 \sin 1.7604t + 0.0439 \cos 1.7604t \\ -0.9876 \sin 1.7604t \end{pmatrix} + \begin{pmatrix} 0.5471 \\ 0 \end{pmatrix} \end{aligned}$$

Solving with the initial condition $x(0) = \begin{pmatrix} 0 \\ -5.42 \end{pmatrix}$,

$$\begin{pmatrix} 0 \\ -5.42 \end{pmatrix} = c_1 \begin{pmatrix} 0.1506 \\ -0.9876 \end{pmatrix} + c_2 \begin{pmatrix} 0.0439 \\ 0 \end{pmatrix} + \begin{pmatrix} 0.5471 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0.1506 & 0.0439 \\ -0.9876 & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -0.5471 \\ -5.42 \end{pmatrix}$$

$$\begin{aligned} c_1 &= 5.4881 \\ c_2 &= -31.2893 \end{aligned}$$

$$\begin{aligned} x(t) =& 5.4881 e^{-6.0439t} \begin{pmatrix} 0.1506 \cos 1.7604t - 0.0439 \sin 1.7604t \\ -0.9876 \cos 1.7604t \end{pmatrix} \\ &- 31.2893 e^{-6.0439t} \begin{pmatrix} 0.1506 \sin 1.7604t + 0.0439 \cos 1.7604t \\ -0.9876 \sin 1.7604t \end{pmatrix} \\ &+ \begin{pmatrix} 0.5471 \\ 0 \end{pmatrix} \end{aligned}$$

$$x(t) = e^{-6.0439t} \begin{pmatrix} -0.5471 \cos 1.7604t - 4.9531 \sin 1.7604t \\ -5.4200 \cos 1.7604t + 30.9013 \sin 1.7604t \end{pmatrix} + \begin{pmatrix} 0.5471 \\ 0 \end{pmatrix}$$

$$x(t) = e^{-6.04t} \begin{pmatrix} -0.55 \cos 1.76t - 4.95 \sin 1.76t \\ -5.42 \cos 1.76t + 30.90 \sin 1.76t \end{pmatrix} + \begin{pmatrix} 0.55 \\ 0 \end{pmatrix}$$

Because $x_1 = y$,

$$y_a(t) = -0.55 e^{-6.04t} \cos 1.76t - 4.95 e^{-6.04t} \sin 1.76t + 0.55, \forall\ 0 \leq t \leq 0.25$$

Full Solution Beyond Impulse

Given $y_a(t)$ found previously, the new set of initial conditions

$$y_b(0.25) = -0.0278$$
$$y_b'(0.25) = 1.8232$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -39.6277 & -12.0878 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The homogeneous solution to the differential systems of equations from above

$$\begin{aligned} x(t) =& c_1 e^{-6.0439t} \begin{pmatrix} 0.1506 \cos 1.7604t - 0.0439 \sin 1.7604t \\ -0.9876 \cos 1.7604t \end{pmatrix} \\ &+ c_2 e^{-6.0439t} \begin{pmatrix} 0.1506 \sin 1.7604t + 0.0439 \cos 1.7604t \\ -0.9876 \sin 1.7604t \end{pmatrix} \end{aligned}$$

and substituted with the new initial conditions gives a solution for after 0.25 s:

$$\begin{aligned} \begin{pmatrix} -0.0278 \\ 1.8232 \end{pmatrix} =& c_1 e^{-6.0439/4} \begin{pmatrix} 0.1506 \cos 1.7604/4 - 0.0439 \sin 1.7604/4 \\ -0.9876 \cos 1.7604/4 \end{pmatrix} \\ &+ c_2 e^{-6.0439/4} \begin{pmatrix} 0.1506 \sin 1.7604/4 + 0.0439 \cos 1.7604/4 \\ -0.9876 \sin 1.7604/4 \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 0.0259 & 0.0229 \\ -0.1972 & -0.0929 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -0.0278 \\ 1.8232 \end{pmatrix}$$

$$c_1 = -18.5654$$
$$c_2 = 19.7835$$

$$\begin{aligned} x(t) =& -18.5654 e^{-6.0439t} \begin{pmatrix} 0.1506 \cos 1.7604t - 0.0439 \sin 1.7604t \\ -0.9876 \cos 1.7604t \end{pmatrix} \\ &+ 19.7835 e^{-6.0439t} \begin{pmatrix} 0.1506 \sin 1.7604t + 0.0439 \cos 1.7604t \\ -0.9876 \sin 1.7604t \end{pmatrix} \\ y_b(t) =& -18.5654 e^{-6.0439t} (0.1506 \cos 1.7604t - 0.0439 \sin 1.7604t) \\ \\ &+ 19.7835 e^{-6.0439t} (0.1506 \sin 1.7604t + 0.0439 \cos 1.7604t \\ =& -1.93 e^{-6.04t} \cos 1.76t + 3.79 e^{-6.04t} \sin 1.76t \end{aligned}$$

Combining equations into a system,

$$y(t) = \begin{cases} -0.55 e^{-6.04t} \cos 1.76t - 4.95 e^{-6.04t} \sin 1.76t + 0.55, & 0 \le t \le 0.25 \\ -1.93 e^{-6.04t} \cos 1.76t + 3.79 e^{-6.04t} \sin 1.76t, & t > 0.25 \end{cases} \tag{4}$$

Solution — Numerical Approximation Using Euler's Method

Because a numerical approximation requires first order differential equations, the second order system $my'' + \gamma y' + ky = F(t)$ is decomposed into

$$\begin{cases} x_1' = x_2 \\ x_2' = \frac{1}{m}[F(t) - \gamma x_2 - kx_1] \end{cases} \quad \text{where} \quad \begin{aligned} m &= 2218.49 \text{ kg} \\ \gamma &= 87913.463543 \text{ N/m} \\ k &= 26816.639084 \\ F(t) &= \begin{cases} 48094.91, & 0 \le t \le 0.25 \\ 0, & t > 0.25 \end{cases} \\ y(0) &= 0.000000 \\ y'(0) &= 5.422177 \end{aligned}$$

Plugging these into a formula-enabled spreadsheet for Euler with a step size $h = 0.0001$,

t	y
0	0
0.01	-0.04388451
0.02	-0.07155155
0.03	-0.08828795
0.04	-0.09765895
0.05	-0.10206882
0.06	-0.10313893
0.07	-0.10196264
0.08	-0.09927729
0.09	-0.09558006
0.10	-0.09120625
0.11	-0.08638196
0.12	-0.08125961
0.13	-0.07594201
0.14	-0.07049846
0.15	-0.06497566

t	y
0.16	-0.05940512
0.17	-0.05380804
0.18	-0.04819871
0.19	-0.04258676
0.20	-0.03697865
0.21	-0.03137874
0.22	-0.02578992
0.23	-0.02021416
0.24	-0.01465274
0.25	-0.00910653
0.26	-0.00450481
0.27	-0.00140643
0.28	0.00067801
0.29	0.00207870
0.30	0.00301831

Comparing Solution Methods

Solution by Characteristic Polynomial:

$$y(t) = \begin{cases} -0.55e^{-6.04t}\cos 1.76t - 4.96e^{-6.04t}\sin 1.76t + 0.55, & 0 \le t \le 0.25 \\ -1.94e^{-6.04t}\cos 1.76t + 3.83e^{-6.04t}\sin 1.76t, & t < 0.25 \end{cases} \tag{5}$$

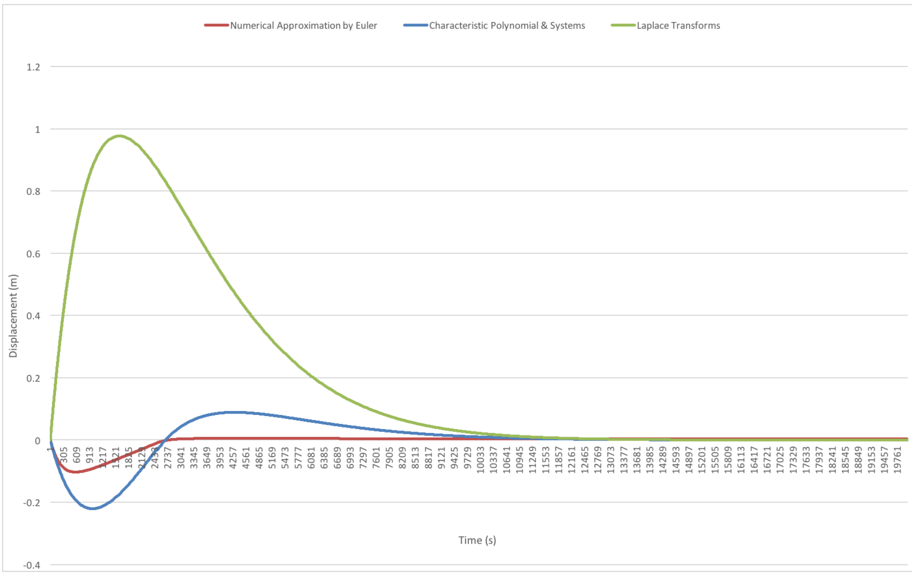
Solution by Laplace Transform:

$$y(t) = 9.24e^{-6.04t}\sin 1.76t$$

Solution by Systems of Differential Equations:

$$y(t) = \begin{cases} -0.55e^{-6.04t}\cos 1.76t - 4.95e^{-6.04t}\sin 1.76t + 0.55, & 0 \le t \le 0.25 \\ -1.93e^{-6.04t}\cos 1.76t + 3.79e^{-6.04t}\sin 1.76t, & t > 0.25 \end{cases}$$

Graphical Comparison



Between Characteristic Polynomial & Laplace Transform Methods. In the one derived using the Laplace transform, initially the solution jumps up to a height of nearly 1 meter into the air, before coming more gradually down to a steady-state equilibrium of 0. In contrast, the solution derived from the characteristic polynomial and the method of undetermined coefficients is compressed downwards first (while the car cannot go below the ground, y=0, if the wheels are compressed, the car can be measured to have been displaced in the negative direction), then the force of impact eventually brings the position y of the car up into the air about a tenth of a meter. The difference in behavior between the two solutions is due to the slightly different definitions of the force. Although both the Laplace transform and the characteristic polynomial eventually result in the same resulting equilibrium, they each have a different definition. The characteristic polynomial requires the initial conditions at time zero and then a calculation of the conditions at time equals a quarter of a second. However, the Laplace transform has a instantaneous, discontinuous definition of the force. Because of the differences in the definition of the force, both graphs have differences between them that eventually evens out to equilibrium.

Between Characteristic Polynomial and Numerical Approximation Methods. Although Euler with a fine enough step size can be as accurate as Runge-Kutta with a step size orders of magnitude larger, the constants we used were not accurate enough in and of themselves to counter the rounding error from doing so many more steps. The rounding error is the factor responsible for the slower response in the numerical approximation compared to the characteristic polynomial.

Appendix A — Scene Clips

Entire Car Chase: https://youtu.be/LMagP52BWG8
Moment of Vertically Discontinuous Force: https://youtu.be/LMagP52BWG8?t=52s
Dashboard Display with Fuel Gauge: https://youtu.be/LMagP52BWG8?t=1m59s

Appendix B — Data References

References

Specifications Form: Passenger Car: Dodge Monaco 1974. (1973, July). Detroit, MI: MVMA. Provides the spring constant for the Dodge Monaco 1974 model passenger car.

Movieclips. (2011, May 27). Chased by the cops - The Blues Brothers (7/9) movie clip (1980) hd [Video file]. Retrieved from https://youtu.be/LMagP52BWG8