Fundamentals of Database Systems
COMPSCI 351

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Relational Query Languages: Algebra

"Stand firm in your refusal to remain conscious during algebra.
In real life, I assure you, there is no such thing as algebra."

— Fran Lebowitz

Example: The Same Query in Different Languages

Consider our database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

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Query in English

What are the movies directed by 'Akira Kurosawa'?

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- Movie(title, year, country, run_time, genre), Director(id, title, year)
- Person(id, first_name, last_name, year_born), Actor(id, title, year, role)

Query in English

What are the movies directed by 'Akira Kurosawa'?

SQL

```
FROM    Movie m, Director d, Person p

                                    ;
```

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- Movie(title, year, country, run_time, genre), Director(id, title, year)
- Person(id, first_name, last_name, year_born), Actor(id, title, year, role)

Query in English

What are the movies directed by 'Akira Kurosawa'?

SQL

```
FROM    Movie m, Director d, Person p
WHERE   m.title=d.title AND m.year=d.year AND
        d.id=p.id AND p.first_name='Akira' AND p.last_name='Kurosawa';
```

Example: The Same Query in Different Languages

Consider our database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

Query in English

What are the movies directed by 'Akira Kurosawa'?

SQL

```
SELECT   m.title, m.year
FROM     MOVIE m, DIRECTOR d, PERSON p
WHERE    m.title=d.title AND m.year=d.year AND
         d.id=p.id AND p.first_name='Akira' AND p.last_name='Kurosawa';
```

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Relational algebra

MOVIE DIRECTOR PERSON

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Query in English

What are the movies directed by 'Akira Kurosawa'?

Relational algebra

$$\text{MOVIE} \bowtie \text{DIRECTOR} \bowtie \text{PERSON}$$

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Query in English

What are the movies directed by 'Akira Kurosawa'?

Relational algebra

$$\sigma_{\text{first\_name='Akira'}}(\text{MOVIE} \bowtie \text{DIRECTOR} \bowtie \text{PERSON})$$

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- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

Query in English

What are the movies directed by 'Akira Kurosawa'?

Relational algebra

$$\sigma_{\text{last\_name='Kurosawa'}}(\sigma_{\text{first\_name='Akira'}}(\text{MOVIE} \bowtie \text{DIRECTOR} \bowtie \text{PERSON}))$$

Example: The Same Query in Different Languages

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- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

Query in English

What are the movies directed by 'Akira Kurosawa'?

Relational algebra

$$\pi_{\text{title,year}}(\sigma_{\text{last\_name='Kurosawa'}}(\sigma_{\text{first\_name='Akira'}}(\text{MOVIE} \bowtie \text{DIRECTOR} \bowtie \text{PERSON})))$$

Algebras

An *algebra* is given by a set A and a set of operations on A

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In **Relational Algebra**

- A is the set of possible relations
- The partial operations on A
  - $\hookrightarrow$ take either 1 or 2 relations as input and
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In **Relational Algebra**

- A is the set of possible relations
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  - $\hookrightarrow$ take either 1 or 2 relations as input and
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For a relation r let $\#r$ denote its set of attributes over which r is defined

Operations of Relational Algebra

- attribute selection
- constant selection
- projection
- renaming
- union
- difference
- join

Attribute Selection

Attribute selection $\sigma_{A=B}$

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- takes a single relation r with $A, B \in \#r$, and produces

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- the relation $\sigma_{A=B}(r) = \{t \mid t \in r \wedge t(A) = t(B)\}$ over $\#r$

Example for Attribute Selection

Evaluate $\sigma_{\text{m.title=d.title}}$ on the following relation r

m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai
The Seven Samurai	Japan	Francis F. Coppola	The Godfather
The Godfather	USA	Akira Kurosawa	The Seven Samurai
The Godfather	USA	Francis F. Coppola	The Godfather

$\sigma_{\text{m.title=d.title}}(r)$

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$\sigma_{\text{m.title=d.title}}(r)$

m.title	m.country	d.name	d.title
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Constant Selection

Constant selection $\sigma_{A=c}$ with $c \in dom(A)$

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- takes a single relation r with $A \in \#r$, and produces

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- takes a single relation r with $A \in \#r$, and produces
- a relation $\sigma_{A=c}(r) = \{t \mid t \in r \wedge t(A) = c\}$ over $\#r$

Example for Constant Selection

Evaluate $\sigma_{\text{d.name='Akira Kurosawa'}}$ on the following relation r

m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai
The Godfather	USA	Francis F. Coppola	The Godfather

$\sigma_{\text{d.name='Akira Kurosawa'}}(r)$

m.title	m.country	d.name	d.title

Example for Constant Selection

Evaluate $\sigma_{\text{d.name='Akira Kurosawa'}}$ on the following relation r

m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai
The Godfather	USA	Francis F. Coppola	The Godfather

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m.title	m.country	d.name	d.title

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m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai
The Godfather	USA	Francis F. Coppola	The Godfather

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m.title	m.country	d.name	d.title

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Evaluate $\sigma_{\text{d.name='Akira Kurosawa'}}$ on the following relation r

m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai
The Godfather	USA	Francis F. Coppola	The Godfather

$\sigma_{\text{d.name='Akira Kurosawa'}}(r)$

m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai

Example for Constant Selection

Evaluate $\sigma_{\text{d.name='Akira Kurosawa'}}$ on the following relation r

m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai
The Godfather	USA	Francis F. Coppola	The Godfather

$\sigma_{\text{d.name='Akira Kurosawa'}}(r)$

m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai

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Evaluate $\sigma_{\text{d.name='Akira Kurosawa'}}$ on the following relation r

m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai
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m.title	m.country	d.name	d.title
The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai

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Evaluate $\sigma_{\text{d.name='Akira Kurosawa'}}$ on the following relation r

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The Seven Samurai	Japan	Akira Kurosawa	The Seven Samurai
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Projection

Projection $\pi_{A_1,\ldots,A_k}$

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- relation $\pi_{A_1,\ldots,A_k}(r) = \{t \mid \exists t' \in r \forall i \in \{1, \ldots, k\} t(A_i) = t'(A_i)\}$ over $\{A_1, \ldots, A_k\}$

Example for Projection

Evaluate $\pi_{\text{m.title,m.country}}$ on the following relation r

m.title	m.country	d.name	d.title
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$\pi_{\text{m.title,m.country}}(r)$

m.title	m.country
The Seven Samurai	Japan

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The Seven Samurai	Japan

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Evaluate $\pi_{\text{m.title,m.country}}$ on the following relation r

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m.title	m.country
The Seven Samurai	Japan

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$\pi_{\text{m.title,m.country}}(r)$

m.title	m.country
The Seven Samurai	Japan

Example for Projection

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m.title	m.country
The Seven Samurai	Japan

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m.title	m.country
The Seven Samurai	Japan
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$\pi_{\text{m.title,m.country}}(r)$

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Renaming

Simply changes some attribute names without changing the relation

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Renaming $\delta_{A_1 \mapsto B_1, \ldots, A_k \mapsto B_k}$ with $dom(B_i) = dom(A_i)$ for $i = 1, \ldots, k$

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Renaming $\delta_{A_1 \mapsto B_1, \ldots, A_k \mapsto B_k}$ with $dom(B_i) = dom(A_i)$ for $i = 1, \ldots, k$

- takes relation r with $A_1, \ldots, A_k \in \#r$ and $B_1, \ldots, B_k \notin \#r - \{A_1, \ldots, A_k\}$, and

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Simply changes some attribute names without changing the relation

Renaming $\delta_{A_1 \mapsto B_1, \ldots, A_k \mapsto B_k}$ with $dom(B_i) = dom(A_i)$ for $i = 1, \ldots, k$

- takes relation r with $A_1, \ldots, A_k \in \#r$ and $B_1, \ldots, B_k \notin \#r - \{A_1, \ldots, A_k\}$, and
- produces relation

$$\delta_{A_1 \mapsto B_1, \ldots, A_k \mapsto B_k}(r) = \{t \mid \exists t' \in r \forall i \in \{1, \ldots, k\} t(B_i) = t'(A_i) \wedge \forall C \in \#r - \{A_1, \ldots, A_k\} t(C) = t'(C)\}$$

over $(\#r - \{A_1, \ldots, A_k\}) \cup \{B_1, \ldots, B_k\}$

Example for Renaming

Evaluate $\delta_{\text{m.title} \mapsto \text{title}, \text{m.country} \mapsto \text{country}}$ on the following relation r

m.title	m.country
The Seven Samurai	Japan
The Godfather	USA

Example for Renaming

Evaluate $\delta_{\text{m.title}\mapsto\text{title},\text{m.country}\mapsto\text{country}}$ on the following relation r

m.title	m.country
The Seven Samurai	Japan
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$\delta_{\text{m.title}\mapsto\text{title},\text{m.country}\mapsto\text{country}}(r)$

title	country
The Seven Samurai	Japan
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The Godfather	USA

Union

Union $\cup$

- takes two relations r and s with $\#r = \#s$, and produces
- the relation that is the set union $r \cup s$ over $\#r = \#s$

Example for Union

Evaluate $\sigma_{\text{Country='New Zealand'}}(r) \cup \sigma_{\text{Country='Japan'}}(r)$ on the following relation r

title	year	country	run_time	genre
13 Assassins	2010	Japan	126	Drama
La dolce vita	1960	Italy	174	Classic
Mana Waka	1937	New Zealand	85	History
Nosferatu	1922	Germany	80	Horror

$\sigma_{\text{Country='New Zealand'}}(r)$

title	year	country	run_time	genre

$\sigma_{\text{Country='Japan'}}(r)$

title	year	country	run_time	genre

$\sigma_{\text{Country='New Zealand'}}(r) \cup \sigma_{\text{Country='Japan'}}(r)$

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- takes two relations r and s as input, and

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$$r \bowtie s = \{t \mid \exists t_1 \in r, t_2 \in s(\forall A \in \#r(t(A) = t_1(A)) \wedge \forall B \in \#s(t(B) = t_2(B)))\}$$

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title	year	run_time	genre	id	role
13 Assassins	2010	126	Drama	2	Shinzaemon Shimada

Example for Natural Join

Evaluate $r \bowtie s$ over the following relations r and s

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Redundant Operators: Intersection and Cross-Product

Intersection $\cap$

- takes two relations r and s with $\#r = \#s$, and
- produces the set intersection $r \cap s = \{t \mid t \in r \wedge t \in s\}$ over $\#r \cup \#s$ as output

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Cross-product $\times$

- takes two relations r and s with schemata $\#r \cap \#s = \emptyset$, and
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$$r \times s = \{t \mid \exists t_1 \in r, t_2 \in s \forall A \in \#r(t(A) = t_1(A) \wedge \forall B \in \#s(t(B) = t_2(B)))\}$$

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$$r \div s = \{t \mid \begin{array}{l} \exists t_1 \in r \forall A \in \#r - \#s(t_1(A) = t(A) \wedge \\ \forall t_2 \in s \exists t_3 \in r \forall A \in \#r - \#s \\ \forall B \in \#s(t_3(A) = t(A) \wedge t_3(B) = t_2(B)))\} \end{array}$$

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GENIUS Question

Why is this operation redundant?

Relational Algebra as a Query Language

Use operations of relational algebra to define a query language

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Use operations of relational algebra to define a query language

For given database schema $\mathcal{S}$

- all our queries Q will be defined over $\mathcal{S}$
- $in(Q) = \mathcal{S}$ for all our queries Q
- $out(Q) = ans(Q)$, i.e., a single (answer) relation schema

Remarks

Notation

- use db to denote an $\mathcal{S}$-database
- for each $R \in \mathcal{S}$, use $db(R)$ to denote the R-relation in db

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- use db to denote an $\mathcal{S}$-database
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Note for equations in the definitions on the next pages

- left hand side refers to syntactic expression of the query language,
- right hand side contains Relational Algebra operations from before

The Query Language $\mathcal{L}_{\mathsf{ALG}}$ of Relational Algebra (1)

Each relation schema $R \in \mathcal{S}$ is a query in $\mathcal{L}_{\mathsf{ALG}}$

- with answer schema R, and the query mapping
- defined by $q(R)(db) = db(R)$

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Each pair $(A : c)$ with $c \in dom(A)$ is a query in $\mathcal{L}_{\mathsf{ALG}}$

- with answer schema $\{A\}$, and the query mapping
- defined by $q(A : c)(db) = \{(A : c)\}$

The Query Language $\mathcal{L}_{\text{ALG}}$ of Relational Algebra (2)

For each query $Q \in \mathcal{L}_{\text{ALG}}$ and attributes $A, B \in ans(Q)$ the expression $\sigma_{A=B}(Q)$ is also a query in $\mathcal{L}_{\text{ALG}}$

- with answer schema $ans(Q)$, and the query mapping
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For each query $Q \in \mathcal{L}_{\mathrm{ALG}}$, any attribute $A \in ans(Q)$ and any constant $c \in dom(A)$ the expression $\sigma_{A=c}(Q)$ is also a query in $\mathcal{L}_{\mathrm{ALG}}$

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The Query Language $\mathcal{L}_{\mathrm{ALG}}$ of Relational Algebra (3)

For each query $Q \in \mathcal{L}_{\mathrm{ALG}}$ and attributes $A_1, \ldots, A_k \in ans(Q)$ the expression $\pi_{A_1,\ldots,A_k}(Q)$ is also a query in $\mathcal{L}_{\mathrm{ALG}}$

- with answer schema $\{A_1, \ldots, A_k\}$, and the query mapping
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For each query $Q \in \mathcal{L}_{\mathrm{ALG}}$, any attributes $A_1, \ldots, A_k \in ans(Q)$ and attributes $B_1, \ldots, B_k \notin ans(Q) - \{A_1, \ldots, A_k\}$ with $dom(A_i) = dom(B_i)$ the expression $\delta_{A_1 \mapsto B_1,\ldots,A_k \mapsto B_k}(Q)$ is also a query in $\mathcal{L}_{\mathrm{ALG}}$

- with answer schema $(ans(Q) - \{A_1, \ldots, A_k\}) \cup \{B_1, \ldots, B_k\}$, and
- the query mapping defined by

$$q(\delta_{A_1 \mapsto B_1,\ldots,A_k \mapsto B_k}(Q))(db) = \delta_{A_1 \mapsto B_1,\ldots,A_k \mapsto B_k}(q(Q)(db))$$

The Query Language $\mathcal{L}_{\mathsf{ALG}}$ of Relational Algebra (4)

For any queries $Q_1, Q_2 \in \mathcal{L}_{\mathsf{ALG}}$ with $ans(Q_1) = ans(Q_2)$ the expression $Q_1 \cup Q_2$ is also a query in $\mathcal{L}_{\mathsf{ALG}}$

- with answer schema $ans(Q_1)$, and the query mapping
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- defined by $q(Q_1 - Q_2)(db) = q(Q_1)(db) - q(Q_2)(db)$

The Query Language $\mathcal{L}_{\mathrm{ALG}}$ of Relational Algebra (5)

For any queries $Q_1, Q_2 \in \mathcal{L}_{\mathrm{ALG}}$ the expression $Q_1 \bowtie Q_2$ is also a query in $\mathcal{L}_{\mathrm{ALG}}$

- with answer schema $ans(Q_1) \cup ans(Q_2)$, and the query mapping
- defined by $q(Q_1 \bowtie Q_2)(db) = q(Q_1)(db) \bowtie q(Q_2)(db)$

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
2	Jamie	Foxx	1967
3	Brad	Pitt	1963
4	Christoph	Waltz	1956
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Actor

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2	Django Unchained	2012	Django
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4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Actors appearing in all movies directed by Quentin Tarantino

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Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
2	Jamie	Foxx	1967
3	Brad	Pitt	1963
4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
2	Jamie	Foxx	1967
3	Brad	Pitt	1963
4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

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1	Quentin	Tarantino	1963
2	Jamie	Foxx	1967
3	Brad	Pitt	1963
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5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

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5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
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1	Inglorious Basterds	2009
5	Ex-machina	2015

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5	Alex	Garland	1970
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Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

id	first_name	last_name	year_born
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5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

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id	title	year
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Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id
4

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
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5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id
4

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
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5	Ex-machina	2015

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4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id
4

Evaluation of a Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

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3	Brad	Pitt	1963
4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year
2	Django Unchained	2012
3	Inglorious Basterds	2009
4	Inglorious Basterds	2009
4	Django Unchained	2012
6	Ex-machina	2015

Actors appearing in all movies directed by Quentin Tarantino

$$\pi_{\text{id,title,year}}(\text{Actor}) \div \pi_{\text{title,year}}(\text{Movie} \bowtie \text{Director} \bowtie \sigma_{\text{first\_name}='\text{Quentin}'}(\sigma_{\text{last\_name}='\text{Tarantino}'}(\text{Person})))$$

title	year
Inglorious Basterds	2009
Django Unchained	2012

id
4

Pop Quiz

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

In English, what are the results of the following queries?

- $\pi_{\text{title,year}}(\sigma_{\text{last\_name='Nicholson'}}(\sigma_{\text{first\_name='Jack'}}(\text{MOVIE} \bowtie \text{ACTOR} \bowtie \text{PERSON})))$

Pop Quiz

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

In English, what are the results of the following queries?

- $\pi_{\text{title,year}}(\sigma_{\text{last\_name='Nicholson'}}(\sigma_{\text{first\_name='Jack'}}(\text{MOVIE} \bowtie \text{ACTOR} \bowtie \text{PERSON})))$

  In which movies did 'Jack Nicholson' act?

Pop Quiz

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

In English, what are the results of the following queries?

- $\pi_{\text{title,year}}(\sigma_{\text{last\_name='Nicholson'}}(\sigma_{\text{first\_name='Jack'}}(\text{MOVIE} \bowtie \text{ACTOR} \bowtie \text{PERSON})))$
  In which movies did 'Jack Nicholson' act?
- $\pi_{\text{first\_name,last\_name}}(((\text{MOVIE} - \sigma_{\text{country='USA'}}(\text{MOVIE})) - \sigma_{\text{country='UK'}}(\text{MOVIE})) \bowtie \text{DIRECTOR} \bowtie \text{PERSON})$

Pop Quiz

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

In English, what are the results of the following queries?

- $\pi_{\text{title,year}}(\sigma_{\text{last\_name='Nicholson'}}(\sigma_{\text{first\_name='Jack'}}(\text{MOVIE} \bowtie \text{ACTOR} \bowtie \text{PERSON})))$

  In which movies did 'Jack Nicholson' act?

- $\pi_{\text{first\_name,last\_name}}(((\text{MOVIE} - \sigma_{\text{country='USA'}}(\text{MOVIE})) - \sigma_{\text{country='UK'}}(\text{MOVIE})) \bowtie \text{DIRECTOR} \bowtie \text{PERSON})$

  What are the names of people who directed movies not produced in the USA and not produced in the UK?

Pop Quiz

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

In English, what are the results of the following queries?

- $\pi_{\emptyset}(\sigma_{\text{role='James Bond'}}(\text{ACTOR}) \bowtie \text{MOVIE})$

Pop Quiz

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

In English, what are the results of the following queries?

- $\pi_{\emptyset}(\sigma_{\text{role='James Bond'}}(\text{ACTOR}) \bowtie \text{MOVIE})$

  Is there a movie in which the role 'James Bond' occurred?

Pop Quiz

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

In English, what are the results of the following queries?

- $\pi_{\emptyset}(\sigma_{\text{role='James Bond'}}(\text{ACTOR}) \bowtie \text{MOVIE})$

  Is there a movie in which the role 'James Bond' occurred?

- $\pi_{\text{first\_name,last\_name}}(((\pi_{\text{id}}(\text{DIRECTOR}) - \pi_{\text{id}}(\text{ACTOR})) \bowtie \text{DIRECTOR}) \bowtie \text{MOVIE} \bowtie \delta_{\text{id}\mapsto\text{aid}}(\text{ACTOR} \bowtie \text{PERSON}))$

Pop Quiz

Database schema from before

- $\textsc{Movie}$(title, year, country, run_time, genre), $\textsc{Director}$(id, title, year)
- $\textsc{Person}$(id, first_name, last_name, year_born), $\textsc{Actor}$(id, title, year, role)

In English, what are the results of the following queries?

- $\pi_{\emptyset}(\sigma_{\text{role='James Bond'}}(\textsc{Actor}) \bowtie \textsc{Movie})$

  Is there a movie in which the role 'James Bond' occurred?

- $\pi_{\text{first\_name,last\_name}}(((\pi_{\text{id}}(\textsc{Director}) - \pi_{\text{id}}(\textsc{Actor})) \bowtie \textsc{Director}) \bowtie \textsc{Movie} \bowtie \delta_{\text{id}\mapsto\text{aid}}(\textsc{Actor} \bowtie \textsc{Person}))$

  What are the names of actors appearing in movies directed by non-actors?

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in those movies that were the only ones directed by some director?

- Directors who directed at least two movies (same director, different movies):

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in those movies that were the only ones directed by some director?

- Directors who directed at least two movies (same director, different movies):
  - $Q_1 = \text{DIRECTOR} \bowtie \delta_{\text{title} \mapsto \text{title'}, \text{year} \mapsto \text{year'}}(\text{DIRECTOR})$

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in those movies that were the only ones directed by some director?

- Directors who directed at least two movies (same director, different movies):
  - $Q_1 = \text{DIRECTOR} \bowtie \delta_{\text{title} \mapsto \text{title'}, \text{year} \mapsto \text{year'}}(\text{DIRECTOR})$
  - $Q_2 = \sigma_{\text{title}=\text{title'}}(\sigma_{\text{year}=\text{year'}}(Q_1))$

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in those movies that were the only ones directed by some director?

- Directors who directed at least two movies (same director, different movies):
  - $Q_1 = \text{DIRECTOR} \bowtie \delta_{\text{title} \mapsto \text{title}', \text{year} \mapsto \text{year}'}(\text{DIRECTOR})$
  - $Q_2 = \sigma_{\text{title}=\text{title}'}(\sigma_{\text{year}=\text{year}'}(Q_1))$
  - $Q_3 = \pi_{\text{id,title,year}}(Q_1 - Q_2)$

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in those movies that were the only ones directed by some director?

- Directors who directed at least two movies (same director, different movies):
  - $Q_1 = \text{DIRECTOR} \bowtie \delta_{\text{title} \mapsto \text{title}', \text{year} \mapsto \text{year}'}(\text{DIRECTOR})$
  - $Q_2 = \sigma_{\text{title}=\text{title}'}(\sigma_{\text{year}=\text{year}'}(Q_1))$
  - $Q_3 = \pi_{\text{id,title,year}}(Q_1 - Q_2)$
- Directors who directed only one movie:

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in those movies that were the only ones directed by some director?

- Directors who directed at least two movies (same director, different movies):
  - $Q_1 = \text{DIRECTOR} \bowtie \delta_{\text{title} \mapsto \text{title'}, \text{year} \mapsto \text{year'}}(\text{DIRECTOR})$
  - $Q_2 = \sigma_{\text{title}=\text{title'}}(\sigma_{\text{year}=\text{year'}}(Q_1))$
  - $Q_3 = \pi_{\text{id,title,year}}(Q_1 - Q_2)$
- Directors who directed only one movie:

  $Q_4 = \text{DIRECTOR} - Q_3$

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in those movies that were the only ones directed by some director?

- Directors who directed at least two movies (same director, different movies):
  - $Q_1 = \text{DIRECTOR} \bowtie \delta_{\text{title} \mapsto \text{title}', \text{year} \mapsto \text{year}'}(\text{DIRECTOR})$
  - $Q_2 = \sigma_{\text{title}=\text{title}'}(\sigma_{\text{year}=\text{year}'}(Q_1))$
  - $Q_3 = \pi_{\text{id,title,year}}(Q_1 - Q_2)$
- Directors who directed only one movie:

  $Q_4 = \text{DIRECTOR} - Q_3$
- Roles in movies whose director only directed that movie:

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in those movies that were the only ones directed by some director?

- Directors who directed at least two movies (same director, different movies):
  - $Q_1 = \text{DIRECTOR} \bowtie \delta_{\text{title} \mapsto \text{title}', \text{year} \mapsto \text{year}'}(\text{DIRECTOR})$
  - $Q_2 = \sigma_{\text{title}=\text{title}'}(\sigma_{\text{year}=\text{year}'}(Q_1))$
  - $Q_3 = \pi_{\text{id,title,year}}(Q_1 - Q_2)$
- Directors who directed only one movie:

  $Q_4 = \text{DIRECTOR} - Q_3$
- Roles in movies whose director only directed that movie:

  $Q_5 = \pi_{\text{role}}(\delta_{\text{id} \mapsto \text{aid}}(\text{ACTOR}) \bowtie Q_4)$

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
2	Jamie	Foxx	1967
4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Directors with at least two movies

$$Q_3 = \pi_{\text{id,title,year}}((\textsc{Director} \bowtie \delta_{\text{title}\mapsto\text{title}',\text{year}\mapsto\text{year}'}(\textsc{Director})) - \sigma_{\text{title}=\text{title}'}(\sigma_{\text{year}=\text{year}'}(\textsc{Director} \bowtie \delta_{\text{title}\mapsto\text{title}',\text{year}\mapsto\text{year}'}(\textsc{Director}))))$$

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
2	Jamie	Foxx	1967
4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Directors with at least two movies

$$Q_3 = \pi_{\text{id,title,year}}((\text{DIRECTOR} \bowtie \delta_{\text{title}\mapsto\text{title'},\text{year}\mapsto\text{year'}}(\text{DIRECTOR})) - \sigma_{\text{title=title'}}(\sigma_{\text{year=year'}}(\text{DIRECTOR} \bowtie \delta_{\text{title}\mapsto\text{title'},\text{year}\mapsto\text{year'}}(\text{DIRECTOR}))))$$

id	title	year	title'	year'
1	Django Unchained	2012	Django Unchained	2012
1	Inglorious Basterds	2009	Inglorious Basterds	2009
1	Django Unchained	2012	Inglorious Basterds	2009
1	Inglorious Basterds	2009	Django Unchained	2012
5	Ex-machina	2015	Ex-machina	2015

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
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5	Ex-machina	2015

Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
2	Jamie	Foxx	1967
4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Directors with at least two movies

$$Q_3 = \pi_{\text{id,title,year}}((\textsc{Director} \bowtie \delta_{\text{title}\mapsto\text{title}',\text{year}\mapsto\text{year}'}(\textsc{Director})) - \sigma_{\text{title}=\text{title}'}(\sigma_{\text{year}=\text{year}'}(\textsc{Director} \bowtie \delta_{\text{title}\mapsto\text{title}',\text{year}\mapsto\text{year}'}(\textsc{Director}))))$$

id	title	year	title'	year'
1	Django Unchained	2012	Django Unchained	2012
1	Inglorious Basterds	2009	Inglorious Basterds	2009
1	Django Unchained	2012	Inglorious Basterds	2009
1	Inglorious Basterds	2009	Django Unchained	2012
5	Ex-machina	2015	Ex-machina	2015

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
2	Jamie	Foxx	1967
4	Christoph	Waltz	1956
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Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Directors with at least two movies

$$Q_3 = \pi_{\text{id,title,year}}((\textsc{Director} \bowtie \delta_{\text{title}\mapsto\text{title'},\text{year}\mapsto\text{year'}}(\textsc{Director})) - \sigma_{\text{title}=\text{title'}}(\sigma_{\text{year}=\text{year'}}(\textsc{Director} \bowtie \delta_{\text{title}\mapsto\text{title'},\text{year}\mapsto\text{year'}}(\textsc{Director}))))$$

id	title	year	title'	year'
1	Django Unchained	2012	Inglorious Basterds	2009
1	Inglorious Basterds	2009	Django Unchained	2012

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
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Person

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Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Directors with at least two movies

$$Q_3 = \pi_{\text{id,title,year}}((\textsc{Director} \bowtie \delta_{\text{title}\mapsto\text{title}',\text{year}\mapsto\text{year}'}(\textsc{Director})) - \sigma_{\text{title}=\text{title}'}(\sigma_{\text{year}=\text{year}'}(\textsc{Director} \bowtie \delta_{\text{title}\mapsto\text{title}',\text{year}\mapsto\text{year}'}(\textsc{Director}))))$$

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

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id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
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id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
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Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Directors with only one movie

$$Q_4 = \text{Director} - Q_3$$

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
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Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
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4	Christoph	Waltz	1956
5	Alex	Garland	1970
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Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Directors with only one movie

$$Q_4 = \text{Director} - Q_3$$

Q_3=Directors with at least two movies

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
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4	Christoph	Waltz	1956
5	Alex	Garland	1970
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id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
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Directors with only one movie

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Q_3=Directors with at least two movies

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009

Directors with only one movie

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009
5	Ex-machina	2015

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
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id	title	year
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id	title	year	role
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4	Inglorious Basterds	2009	Col Hans Landa
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6	Ex-machina	2015	Ava

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Q_3=Directors with at least two movies

id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009

Directors with only one movie

id	title	year
5	Ex-machina	2015

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
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Django Unchained	2012	USA	165	Drama
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id	title	year
1	Django Unchained	2012
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id	title	year	role
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4	Inglorious Basterds	2009	Col Hans Landa
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id	title	year
1	Django Unchained	2012
1	Inglorious Basterds	2009

Directors with only one movie

id	title	year
5	Ex-machina	2015

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
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Person

id	first_name	last_name	year_born
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4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Roles in movies whose director only directed that movie

$$Q_5 = \pi_{\mathsf{role}}(\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\textsc{Actor}) \bowtie Q_4)$$

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
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4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Roles in movies whose director only directed that movie

$$Q_5 = \pi_{\mathsf{role}}(\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\textsc{Actor}) \bowtie Q_4)$$

Q_4=Directors with only one movie

id	title	year
5	Ex-machina	2015

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
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id	first_name	last_name	year_born
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4	Christoph	Waltz	1956
5	Alex	Garland	1970
6	Alicia	Vikander	1988

$\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\textsc{Actor})$

aid	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Roles in movies whose director only directed that movie

$$Q_5 = \pi_{\mathsf{role}}(\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\textsc{Actor}) \bowtie Q_4)$$

Q_4=Directors with only one movie

id	title	year
5	Ex-machina	2015

Q_5

aid	title	year	role	id
6	Ex-machina	2015	Ava	5

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
Ex-machina	2015	USA	108	Drama

Director

id	title	year
1	Django Unchained	2012
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Person

id	first_name	last_name	year_born
1	Quentin	Tarantino	1963
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4	Christoph	Waltz	1956
5	Alex	Garland	1970
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Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Roles in movies whose director only directed that movie

$$Q_5 = \pi_{\mathsf{role}}(\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\textsc{Actor}) \bowtie Q_4)$$

Q_4=Directors with only one movie

id	title	year
5	Ex-machina	2015

Q_5

role
Ava

Evaluation of another Query on a Database

Movie

title	year	country	run_time	genre
Inglorious Basterds	2009	USA	153	Drama
Django Unchained	2012	USA	165	Drama
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Director

id	title	year
1	Django Unchained	2012
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Person

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Actor

id	title	year	role
2	Django Unchained	2012	Django
4	Inglorious Basterds	2009	Col Hans Landa
4	Django Unchained	2012	Dr King Schultz
6	Ex-machina	2015	Ava

Roles in movies whose director only directed that movie

$$Q_5 = \pi_{\mathsf{role}}(\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\textsc{Actor}) \bowtie Q_4)$$

Q_4=Directors with only one movie

id	title	year
5	Ex-machina	2015

Q_5

role
Ava

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in *only* those movies that were the only ones directed by some director?

- Actors of movies directed by a director who has directed more than one movie:

Incremental Query Formulation

Database schema from before

- Movie(title, year, country, run_time, genre), Director(id, title, year)
- Person(id, first_name, last_name, year_born), Actor(id, title, year, role)

What are the roles played by actors appearing in *only* those movies that were the only ones directed by some director?

- Actors of movies directed by a director who has directed more than one movie:
  $Q_7 = \pi_{\mathsf{aid}}(\delta_{\mathsf{id} \mapsto \mathsf{aid}}(\textsc{Actor}) \bowtie Q_3)$

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in *only* those movies that were the only ones directed by some director?

- Actors of movies directed by a director who has directed more than one movie:
  $Q_7 = \pi_{\mathsf{aid}}(\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\text{ACTOR}) \bowtie Q_3)$
- Actors only appearing in movies which were the only ones directed by the director:

Incremental Query Formulation

Database schema from before

- MOVIE(title, year, country, run_time, genre), DIRECTOR(id, title, year)
- PERSON(id, first_name, last_name, year_born), ACTOR(id, title, year, role)

What are the roles played by actors appearing in *only* those movies that were the only ones directed by some director?

- Actors of movies directed by a director who has directed more than one movie:
  $Q_7 = \pi_{\mathsf{aid}}(\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\textsc{Actor}) \bowtie Q_3)$
- Actors only appearing in movies which were the only ones directed by the director:
  $Q_8 = \pi_{\mathsf{aid}}(\delta_{\mathsf{id}\mapsto\mathsf{aid}}(\textsc{Actor})) - Q_7$

Incremental Query Formulation

Database schema from before

- Movie(title, year, country, run_time, genre), Director(id, title, year)
- Person(id, first_name, last_name, year_born), Actor(id, title, year, role)

What are the roles played by actors appearing in *only* those movies that were the only ones directed by some director?

- Actors of movies directed by a director who has directed more than one movie:
  $Q_7 = \pi_{\text{aid}}(\delta_{\text{id}\mapsto\text{aid}}(\text{Actor}) \bowtie Q_3)$
- Actors only appearing in movies which were the only ones directed by the director:
  $Q_8 = \pi_{\text{aid}}(\delta_{\text{id}\mapsto\text{aid}}(\text{Actor})) - Q_7$
- Roles only appearing in movies which were the only ones directed by the director:

Incremental Query Formulation

Database schema from before

- Movie(title, year, country, run_time, genre), Director(id, title, year)
- Person(id, first_name, last_name, year_born), Actor(id, title, year, role)

What are the roles played by actors appearing in *only* those movies that were the only ones directed by some director?

- Actors of movies directed by a director who has directed more than one movie:
  $Q_7 = \pi_{\text{aid}}(\delta_{\text{id}\mapsto\text{aid}}(\text{Actor}) \bowtie Q_3)$
- Actors only appearing in movies which were the only ones directed by the director:
  $Q_8 = \pi_{\text{aid}}(\delta_{\text{id}\mapsto\text{aid}}(\text{Actor})) - Q_7$
- Roles only appearing in movies which were the only ones directed by the director:
  $Q = \pi_{\text{role}}(\delta_{\text{id}\mapsto\text{aid}}(\text{Actor}) \bowtie Q_8)$

Summary for Relational Algebra

- The relational model has brought forward simple and powerful query languages
- The operators in relational algebra include:
  - attribute and constant selection
  - projection and renaming
  - set union and difference
  - natural join
- Relational algebra expressions suggest an ordering on the operations
- It is possible to describe an algebra expression that produces the same result with the operators in a different order
  - This is used in query optimization
- Relational algebra operators are executed by the DBMS
  - see Figure for DBMS Architecture